

# Face Recognition using Eigenvector and Principle Component Analysis

Dulal Chakraborty

Lecturer

Department of Information & Communication Technology,  
Comilla University, Comilla,  
Bangladesh

Sanjit Kumar Saha

Lecturer

Department of Computer Science & Engineering,  
Comilla University, Comilla,  
Bangladesh

Md. Al-Amin Bhuiyan

Professor

Department of Computer Science & Engineering, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh

## ABSTRACT

Face recognition is an important and challenging field in computer vision. This research present a system that is able to recognize a person's face by comparing facial structure to that of a known person which is achieved by using frontal view facing photographs of individuals to render a two-dimensional representation of a human head. Various symmetrization techniques are used for preprocessing the image in order to handle bad illumination and face alignment problem. We used Eigenface approach for face recognition. Eigenfaces are eigenvectors of covariance matrix, representing given image space. Any new face image can then be represented as a linear combination of these Eigenfaces. This makes it easier to match any two given images and thus face recognition process. The implemented eigenface-based technique classified the faces 95% correctly.

## Keywords

Principle component analysis, eigenvector, eigenvalue, eigenface, faces recognition.

## 1. INTRODUCTION

The face is primary focus of attention in social life which plays a major role in conveying identity and emotion. Although face recognition is challenging, the human ability to recognize faces is remarkable. We can recognize thousands of faces learned throughout our lifetime and identify familiar faces at a glance even after years of separation. This skill is quite robust, despite large changes in the visual stimulus due to viewing conditions, expression, aging, and distractions such as glasses, beards or changes in hair style.

Face recognition has fundamental importance in our social relationship, being extremely important for our simple everyday activities. It is a very high level task and has many applications. Developing a computational model of face recognition is quite difficult, because faces are complex, multi-dimensional visual stimuli. A formal method of classifying faces was first proposed by Francis Galton [1, 2] in 1888. During the 1980's work on face recognition remained largely dormant. Since the 1990's, the research interest in face recognition has grown significantly.

Investigations by numerous researchers [3, 4, 5] over the past several years have indicated that certain facial characteristics are used by human beings to identifying faces. There are three major research groups which propose three different approaches to the face recognition problem. The largest group [6, 7, 8] has dealt with facial characteristics which are used by human beings in recognizing individual faces. The second group [9, 10, 11, 12, 13] performs human face identification based on feature vectors extracted from profile silhouettes.

The third group [14, 15] uses feature vectors extracted from a frontal view of the face. Although there are three different approaches to the face recognition problem, there are two basic methods from which these three different approaches arise.

The first method is based on the information theory concepts, which is also known as the principal component analysis method. Based on the Karhunen-Loeve expansion in pattern recognition, M. Kirby and L. Sirovich [6, 7] have shown that any particular face could be economically represented in terms of a best coordinate system that they termed "eigenfaces". Later, M. Turk and A. Pentland [16] have proposed a face recognition method based on the eigenfaces approach.

The second method is based on extracting feature vectors from the basic parts of a face such as eyes, nose, mouth, and chin. In this method, with the help of deformable templates and extensive mathematics, key information from the basic parts of a face is gathered and then converted into a feature vector. L. Yullie and S. Cohen [17] played a great role in adapting deformable templates to contour extraction of face images.

After three decades of research effort, the Eigenface approach merged as the first real successful demonstration of automatic human face recognition. This is one of the methods which can be classified as appearance-based methods that use the whole face region as the raw input to a recognition system. The goal of an appearance-based face recognition algorithm is essentially to create low-dimensional representations of face images to perform recognition. In contrast, geometric feature-based methods attempt to distinguish between faces by comparing properties and relations between facial features, such as eyes, mouth, nose and chin. As a consequence, success of these methods depends heavily on the feature extraction and measurement process.

The low-dimensional representation of faces in the Eigenface approach is derived by applying Principle Component Analysis (PCA) to a representative dataset of images of faces. The system functions by projecting face images onto a feature space that spans the significant variations among known face images. These significant features are termed "Eigenfaces" because they are the principal components of the set of training face images. These features do not necessarily correspond to facial features such as eyes, nose and ears. They merely capture the image points that cause meaningful variations between the faces in the database that allow them to be differentiated. Face images are then classified within the low-dimensional model using a nearest-neighbor classifier. The Eigenface approach works well on test images unaffected by illumination changes. It is a well-known fact that intrapersonal differences (e.g. illumination effects, poses)

cause more variations between face images than interpersonal differences (identity). To handle this variability, methods usually take one of two approaches: measure some property in the image that is invariant or at least insensitive to illumination effects, or model the object in order to predict the variations caused by changes in illumination. Solutions that follow the former approach are so far still elusive, and may never exist at all. This suggests that appearance-based methods, which derive low-dimensional models of the face images used for training, are the only answer to this challenging problem.

## 2. EIGENVECTORS AND EIGENVALUES

“Eigen” is a German word meaning “proper” or “own”. An eigenvector of a matrix is a vector such that, if multiplied with the matrix, the result is always an integer multiple of that vector. This integer value is the corresponding eigenvalue of the eigenvector. This relationship can be described by the equation  $M \times u = c \times u$ , where  $u$  is an eigenvector of the matrix  $M$  and  $c$  is the corresponding eigenvalue.

### 2.1 Eigenfaces

Eigenfaces are the set of eigenvectors which used in computer vision problem for human face recognition. They can be simply defined as the eigenvectors which represent one of the dimension of face image space. All eigenvectors have an eigenvalue associated to it and the eigenvectors with the largest eigenvalues provide more information on the face variation than the ones with smaller eigenvalues.

### 2.2 Principle Component Analysis (PCA) Technique

PCA is one of the most successful techniques that have been used in face recognition. The objective of the Principal Component Analysis is to take the total variation on the training set of faces and to represent this variation with just some little variables. When we are working with great amounts of images, reduction of space dimension is very important. PCA intends to reduce the dimension of a group or to space it better so that the new base describes the typical model of the group. The maximum number of principal components is the number of variables in the original space. Even so to reduce the dimension, some principal components should be omitted.

## 3. FACE DETECTION

To locate the face, an image pyramid is formed from the original image. An image pyramid is a set of copies of the original image at different scale, thus representing a set of different resolutions. A mask is moved pixel wise over each image in the pyramid and at each position, the image section under the mask is passed to a function that assesses the similarity of the image section to a face. If the similarity value is high enough, the presence of a face at that position and resolution is assumed. From the position of the face, a first estimate of the eye position can be derived. A search for the exact eye position is started. The positions yielding the highest similarity values are taken as final estimates of the eye positions.

### 3.1 Face Image Normalization

After the face area has been detected, it is normalized before passing to the face recognition module. We apply a sequence of image pre-processing techniques so that the image is light and noise invariant. We also need to apply some standard face

recognition pre-requisite such as gray image conversion and scaling into a suitable sized image.

#### 3.1.1 Conversion to Gray Image and Scaling

Detected face is converted to grayscale using equation (1) and scaled to  $60 \times 60$  pixel using equation (2) and saved as a gray jpg image. Linear interpolation technique was employed to determine the scaled output image.

$$Gr_i = \frac{R_i + G_i + B_i}{3}, i=1, \dots, M \times N \quad (1)$$

Where  $Gr_i$  is the gray level value of  $i^{th}$  pixel of the gray image.  $R_i, G_i, B_i$  corresponds to red, green, blue value of the  $i^{th}$  pixel in the color image.

$$Q(x^q, y^q) = P\left(\frac{x^p}{60} x^q, \frac{y^p}{60} y^q\right) \quad (2)$$

Where, we want to re-scale image

$P[(0,0) - (x^p, y^p)]$  to image  $Q[(0,0) - (132,132)]$









Type of image	Red	Green	Blue
 RGB			
 Grayscale			

Figure 1. Conversion of RGB image to Grayscale image

## 4 FACE RECOGNITION

A simple approach to extracting the information contained in an image of a face is to somehow capture the variation in a collection of face images, independent of any judgment of features, and use this information to encode and compare individual face images.

In mathematical terms, the principal components of the distribution of faces, or the eigenvectors of the covariance matrix of the set of face images, treating an image as point (or vector) in a very high dimensional space is sought. Each image location contributes more or less to each eigenvector, so that it is possible to display these eigenvectors as a sort of ghostly face image which is called an "eigenface".

Sample face images and the corresponding eigenfaces are shown in Figure 2 and in Figure 3 respectively. Eigenfaces can be viewed as a sort of map of the variations between faces.



Figure 2. Training set face images

Each individual face can be represented exactly in terms of a linear combination of the eigenfaces. Each face can also be

approximated using only the "best" eigenfaces, those that have the largest eigenvalues, and which therefore account for the most variance within the set of face images. The best M eigenfaces span an M-dimensional subspace which we call the "face space" of all possible images.

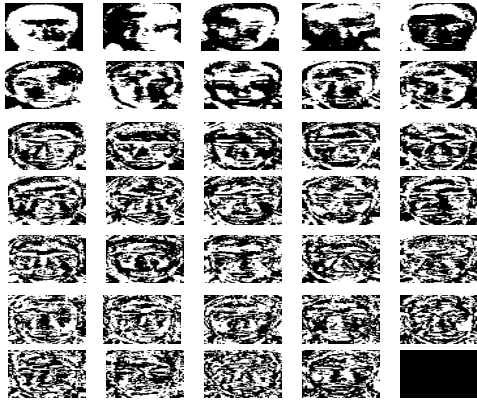


Figure 3. Eigenfaces with highest eigen values, that were calculated from the sample training set, given in Figure 2

Kirby and Sirovich [6, 7] developed a technique for efficiently representing pictures of faces using principal component analysis. Starting with an ensemble of original face images, they calculated a best coordinate system for image compression, where each coordinate is actually an image that they termed an "eigenpicture". In this research, we have followed the method which was proposed by M. Turk and A. Pentland [16] in order to develop a face recognition system based on the eigenfaces approach. They argued that, if a multitude of face images can be reconstructed by weighted sum of a small collection of characteristic features or eigenpictures, perhaps an efficient way to learn and recognize faces would be to build up the characteristic features by experience over time and recognize particular faces by comparing the feature weights needed to approximately reconstruct them with the weights associated with known individuals.

The basic steps involved in Face Recognition using Eigenfaces Approach are as follows:

1. Acquire initial set of face images known as Training Set ( $\Gamma_i$ ).
2. The average matrix  $\Psi$  has to be calculated. Then subtract this mean from the original faces ( $\Gamma_i$ ) to calculate the image vector ( $\phi_i$ ).

$$\psi = \frac{1}{M} \sum_{i=1}^M \Gamma_i$$

$$\phi_i = \Gamma_i - \psi$$

3. Find the covariance matrix C by

$$c = \frac{1}{M} \sum_{n=1}^M \phi_n \phi_n^T = AA^T$$

4. Compute the eigenvectors and eigenvalues of C.
5. The M significant eigenvectors are chosen as those with the largest corresponding eigenvalues
6. Project all the face images into these eigenvectors and form the feature vectors of each face image.

### 4.1 Training Set of Images

Let the training set consists of M images representing M image classes. Each of these images can be represented in

vector form. Let these images be  $\Gamma_1; \Gamma_2; \dots; \Gamma_M$ . The average face of the set is

$$\psi = \frac{1}{M} \sum_{i=1}^M \Gamma_i \quad (3)$$

Each face image differs from the average face of the distribution, and this distance is calculated by subtracting the average face from each face image. This gives us new image space .

$$\phi_i = \Gamma_i - \psi \quad (i = 1, 2, \dots, M) \quad (4)$$

### 4.2 Calculating Eigenfaces Definitions:

An  $N \times N$  matrix A is said to have an eigenvector X, and corresponding eigenvalue  $\lambda$  if

$$AX = \lambda X \quad (5)$$

Evidently, Eq. (7) can hold only if

$$\det|A - \lambda I| = 0 \quad (6)$$

which, if expanded out, is an Nth degree polynomial in  $\lambda$  whose root are the eigenvalues.

A matrix is called *symmetric* if it is equal to its transpose,

$$A = A^T \quad \text{or} \quad a_{ij} = a_{ji} \quad (7)$$

It is termed *orthogonal* if its transpose equals its inverse,

$$AA^T = A^T A = I \quad (8)$$

Finally, a real matrix is called *normal* if it commutes with its transpose,

$$AA^T = A^T A \quad (9)$$

**Theorem:** Eigenvalues of a real symmetric matrix are all real. Contrariwise, the eigenvalues of a real non symmetric matrix may include real values, but may also include pairs of complex conjugate values. The eigenvalues of a normal matrix with non degenerate eigenvalues are complete and orthogonal, spanning the N dimensional vector space. Let the training set of face images be  $\Gamma_1; \Gamma_2; \dots; \Gamma_M$  then the average of the set is defined by

$$\psi = \frac{1}{M} \sum_{i=1}^M \Gamma_i \quad (10)$$

Each face differs from the average by the vector

$$\phi_i = \Gamma_i - \psi \quad (11)$$

An example training set is shown in Figure 2. This set of very large vectors is then subject to principal component analysis, which seeks a set of M ortho-normal vectors,  $U_n$  which best describes the distribution of the data. The  $k^{th}$  vector,  $U_k$  is chosen such that

$$\lambda_k = \frac{1}{M} \sum_{n=1}^M (u_k^T \phi_n)^2 \quad (12)$$

is a maximum, subject to

$$u_l^T u_k = \delta_{lk} = \begin{cases} 1, & \text{if } l = k \\ 0, & \text{otherwise} \end{cases} \quad (13)$$

The vectors  $u_k$  and scalars  $\lambda_k$  are the eigenvectors and eigenvalues, respectively of the covariance matrix

$$c = \frac{1}{M} \sum_{n=1}^M \phi_n \phi_n^T = AA^T \quad (14)$$

where the matrix  $A = [\phi_1, \phi_2, \dots, \phi_M]$ . The covariance matrix C, however is  $N^2 \times N^2$  real symmetric matrix, and

determining the  $N^2$  eigenvectors and eigenvalues is an intractable task for typical image sizes.

If the number of data points in the image space is less than the dimension of the space ( $M < N^2$ , there will be only  $M-1$ , rather than  $N^2$  meaningful eigenvectors. The remaining eigenvectors will have associated eigenvalues of zero.

Consider the eigenvectors  $v_i$  of  $A^T A$  such that

$$A^T A V_l = \mu_l v_l \quad (15)$$

Pre multiplying both sides by A, we have

$$AA^T A V_l = \mu_l A v_l \quad (16)$$

from which we see that  $A v_l$  are the eigen vectors of

$$C = AA^T \quad (17)$$

Following these analysis, we construct the  $M \times M$  matrix

$$L = A^T A \quad (18)$$

where  $L_{mn} = \varphi_m^T \varphi_n$  and find the M eigenvectors,  $v_l$ , of L.

These vectors determine linear combinations of the M training set face images to form the eigenfaces  $u_l$ .

$$u_l = \sum_{k=1}^M v_{lk} \varphi_k, \quad l=1,2,\dots,M \quad (19)$$

With this analysis, the calculations are greatly reduced, from the order of the number of pixels in the images ( $N^2$ ) to the order of the number of images in the training set (M).

## 5. IMPLEMENTATION AND RESULT ANALYSIS

The experiment is implemented over a training set of 105 (3 each of 35 Images). Each image is in RGB level which is normalized to gray level and has dimension of  $60 \times 60$ . There are 7 subjects in the training set. Each subject has 5 images with frontal view with  $\pm 5^0$  different poses (like left, right, up and down). In our face database which images we have taken, they are all the students of department of Computer Science & Engineering, Jahangirnagar University of Bangladesh. Their average age is 22 years. All images are the same size. No image contains sunglasses, beard or mustaches in face area. Face images are taken in sunny bright light which we called day light condition and in dark light or low light condition. Then we use histogram equalized condition for face recognition. So we can say that in recognition procedure we use different illumination condition and all this condition it can recognize the known or unknown face.

Firstly we construct overall average image. This is the image which is formed by adding all images and dividing by number of images in training set. The eigenvectors of covariance matrix is formed by combining all deviation of training set's image from average image.

After finding overall average image, we have to find the eigenvectors of the covariance matrix. Since there are 35 images in the training set we need to find 35 eigenvectors that are used to represent our training set.

After finding eigenvectors, we use it to recognize the the training set of images. In our experiment we consider the training set of images for three conditions (day light, low light and histogram equalized) for both known and unknown faces where known faces are stored at face database and unknown

faces are not in the database. For known images, we found that the mean difference is almost zero. So the training set of images easily recognize with the corresponding image in the database. For unknown images, we found an exponential mean difference between database images and training set of images that mean the input images are not recognized with any of database images. The resultant analysis has given below with table and diagram.

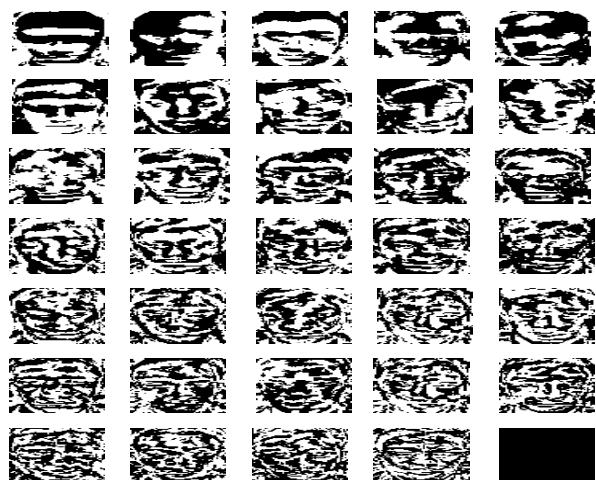
**Table 1. Training procedure of image**

Training Procedure	
No. of images taken for the training procedure	$35 \times 3$
Size	$60 \times 60$
Format	JPG
Output	Normalized images of the extracted face images
	Eigenface
	Average Eigenvector
	Mean Image

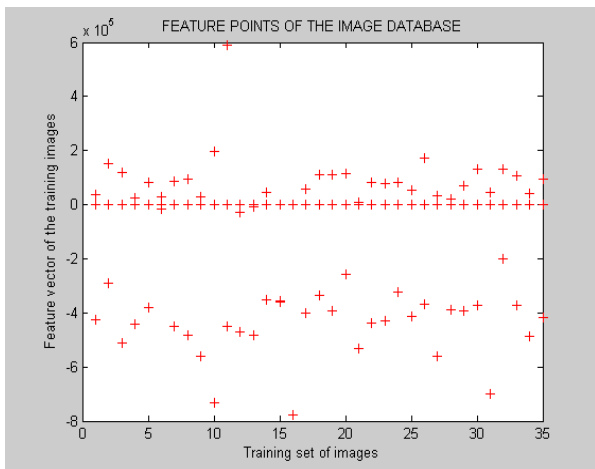
**Input image database in day light condition:**



**Generated eigen faces of image database in day light condition:**



Generated eigen vector for day light condition image database:



Result of Recognition Procedure in day light condition:

Recognition procedure in day light condition	
Total number of images taken for the test	10
Number of recognized images	10
Number of misrecognized images (no store in databases)	Nil
Number of misrecognized images (store in databases)	Nil
Accuracy	100%

Result of known image in day light condition:

Input Image	Image position in database	Mean difference	Resultant Position	Result
	1	0.0021	1	TRUE
	9	0.0048	9	TRUE
	20	0.0027	20	TRUE
	29	0.0039	29	TRUE
	34	0.0021	34	TRUE

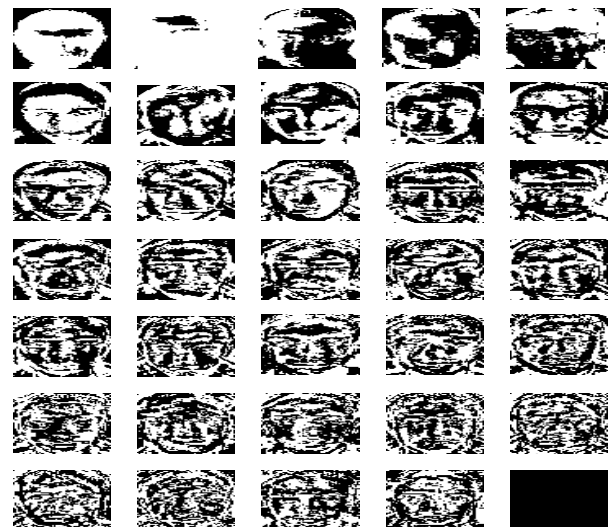
Input image database in low light condition:



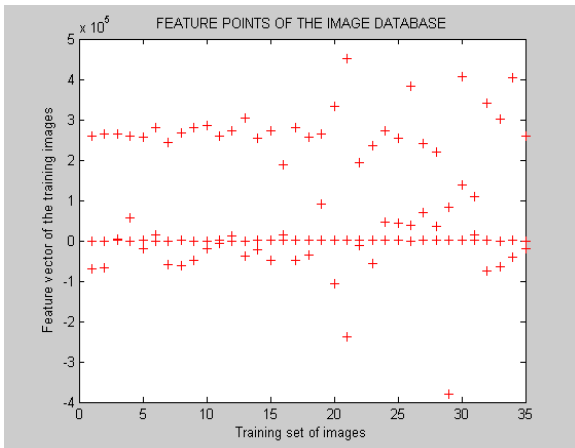
Result of unknown image in day light condition:

Input Image	Image position in database	Mean difference	Resultant Position	Result
	NULL	1.0e+005 * 0.3705	3	TRUE
	NULL	1.0e+005 * 0.2870	30	TRUE
	NULL	1.0e+005 * 0.5358	20	TRUE
	NULL	1.0e+005 * 1.2142	12	TRUE
	NULL	1.0e+005 * 0.1076	4	TRUE

Generated eigen faces for image database in low light condition:



**Generated eigen vector for low light condition:**



**Result of recognition procedure in low light condition:**

Recognition procedure in low light condition	
Total number of images taken for the test	10
Number of recognized images	10
Number of misrecognized images (no store in databases)	Nil
Number of misrecognized images (store in databases)	Nil
Accuracy	100 %

**Result of known image in low light condition:**

Input Image	Image position in database	Mean difference	Resultant Position	Result
	22	0.0018	22	TRUE
	26	0.0017	26	TRUE
	1	0.0038	1	TRUE
	9	0.0019	9	TRUE
	15	0.0020	15	TRUE

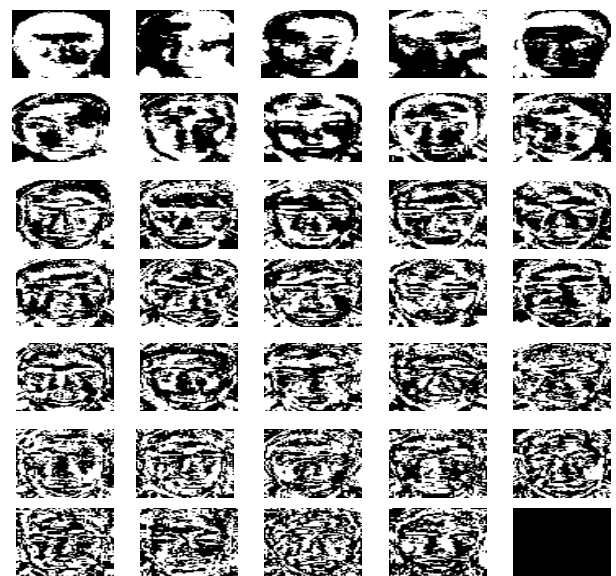
**Input image database in histogram equalized condition:**



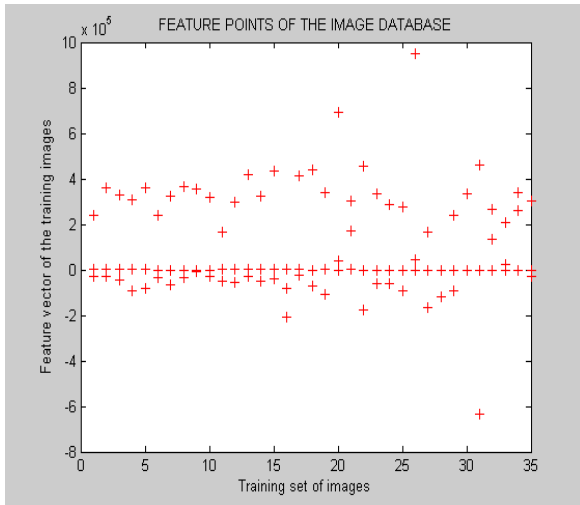
**Result of unknown image in low light condition:**

Input Image	Image position in database	Mean difference	Resultant Position	Result
	NULL	1.0e+005 * 0.4373	1	TRUE
	NULL	1.0e+005 * 0.1483	20	TRUE
	NULL	1.0e+005 * 0.0997	32	TRUE
	NULL	1.0e+005 * 1.3318	21	TRUE
	NULL	1.0e+005 * 0.4225	22	TRUE

**Generated eigen faces in histogram equalized condition:**



**Generated eigen vector for histogram equalized image database:**



**Result of known image in histogram equalized condition:**

Input Image	Image position in database	Mean difference	Resultant Position	Result
	28	0.0045	28	TRUE
	1	0.0004	1	TRUE
	18	0.0031	18	TRUE
	15	0.0028	15	TRUE
	22	0.0035	22	TRUE

**Result of unknown image in histogram equalized condition:**

Input Image	Image position in database	Mean difference	Resultant Position	Result
	NULL	1.0e+005 * 1.1427	31	TRUE
	NULL	1.0e+005 * 0.6798	29	TRUE
	NULL	1.0e+005 * 0.3922	21	TRUE
	NULL	1.0e+005 * 0.1790	23	TRUE
	NULL	1.0e+005 * 0.2576	1	TRUE

**Result of recognition procedure in histogram equalized condition:**

Recognition procedure in day light condition	
Total number of images taken for the test	10
Number of recognized images	10
Number of misrecognized images (no store in databases)	Nil
Number of misrecognized images (store in databases)	Nil
Accuracy	100%

**6. CONCLUSION**

In this paper eigenface based face recognition has been described. The eigenface approach for face recognition process is fast and simple which works well under constrained environment. It is one of the best practical solutions for the problem of face recognition. Eigenfaces method is a principal component analysis approach, where the eigenvectors of the covariance matrix of a small set of characteristic pictures are sought. These eigenvectors are called eigenfaces due to their resemblance of face images. Recognition is performed by obtaining feature vectors from the eigenvectors space.

Many applications which require face recognition do not require perfect identification but just low error rate. So instead of searching large database of faces, it is better to give small set of likely matches. By using Eigenface approach, this small set of likely matches for given images can be easily obtained. For given set of images, due to high dimensionality of images, the space spanned is very large. But in reality, all these images are closely related and actually span a lower dimensional space. By using eigenface approach, we try to reduce this dimensionality. The lower the dimensionality of this image space, the easier it would be for face recognition. Any new image can be expressed as linear combination of these eigenfaces. This makes it easier to match any two images and thus face recognition.

One of the limitation for eigenface approach is in the treatment of face images with varied facial expressions and with glasses. Also as images may have different illumination conditions. This can be removed by RMS (root mean square) contrast stretching and histogram equalization.

In the present work, we used 105 face images as face database. The procedure we used is quite satisfactory. It can recognize both the known and unknown images in the database in various conditions with accuracy 100%. We think when we use a huge database such as thousand number of images or more, then some misdetection (2%-5%) may happened in the recognition procedure. Our future work may include an improvement to the eigenfaces technology and recognition procedure for live video stream.

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