FACIAL IMAGE ANALYSIS BASED ON TWO-DIMENSIONAL LINEAR DISCRIMINANT ANALYSIS EXPLOITING SYMMETRY

Konstantinos Papachristou, Anastasios Tefas and Ioannis Pitas

Department of Informatics, Aristotle University of Thessaloniki, Thessaloniki, Greece
{tefas,pitas}@aiia.csd.auth.gr

ABSTRACT

In this paper a novel subspace learning technique is introduced for facial image analysis. The proposed technique takes into account the symmetry nature of facial images. This information is exploited by properly incorporating a symmetry constraint into the objective function of the Two-Dimensional Linear Discriminant Analysis (2DLDA) to determine symmetric projection vectors. The performance of the proposed Symmetric Two-Dimensional Linear Discriminant Analysis was evaluated on real face recognition databases. Experimental results highlight the superiority of the proposed technique in comparison to standard approach.

***Index Terms*—** facial image analysis, subspace learning, symmetry constraint, two-dimensional linear discriminant analysis

1. INTRODUCTION

Subspace learning techniques have widely been used in many facial image analysis tasks [1], such as face detection, face recognition and facial expression recognition. Subspace learning techniques project the high-dimensional data on low-dimensional discriminant spaces and lead to methods which are faster, use less memory and have improved classification performance. Two of the most popular subspace learning techniques are Principal Component Analysis (PCA) [2] and Linear Discriminant Analysis (LDA) [3], respectively. PCA is an unsupervised subspace learning technique that projects the data onto a lower-dimensional space along the directions of maximum data variance. On the other hand, LDA is a supervised technique which determines a subspace where the projected data classes are optimally separated by maximizing the ratio of the between-class scatter matrix to the within-class scatter matrix.

The above subspace learning techniques are vector-based and, therefore, image data, such as facial images, should be vectorized before the application of PCA and LDA. When transforming images into vectors often leads to a high-dimensional vector space, where LDA usually faces the singularity of the within-class scatter matrix, while the determination of the corresponding projection vectors is very time-consuming. To overcome the problem of vectorizing, a number of LDA-based techniques have been proposed [4, 5, 6, 7, 8], which use two-dimensional data (images) instead of one-dimensional data (vectors) as input. This means that these techniques success to determine the corresponding projection vectors more efficiently than LDA in terms of accuracy and time, since the size of the between-class and within-class scatter matrices is quite smaller than the size of LDA scatter matrices.

Although the above techniques perform directly on image matrices, they ignore the a-priori knowledge that facial images are symmetric. Generally symmetry has been used in subspace learning. A method was proposed in [9] which combines the symmetry information of faces with PCA and LDA for human identification by selecting local facial regions which are stable to facial expression variations. In [10], it was shown that the performance of PCA and LDA can be improved by doubling the training set: for each sample, its symmetric version is also used. Symmetric extensions of PCA, LDA and Clustering based Discriminant Analysis techniques were proposed in [11], which determine symmetric projection vectors. In this paper, we exploit the symmetry nature of facial images by adding a symmetry constraint in the objective function of Two-Dimensional Linear Discriminant Analysis in order to learn subspaces equipped with more robustness and generalization ability.

The rest of this paper is organized as follows: In Section 2, the standard Two-Dimensional Linear Discriminant Analysis (2DLDA) technique is reviewed. Section 3 presents the proposed symmetric extension of 2DLDA. In Section 4, the experimental results of the proposed algorithm, compared with the standard one, are described. Finally, in Section 5 some concluding remarks are given.

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2. TWO-DIMENSIONAL LINEAR DISCRIMINANT ANALYSIS

Let $\mathcal{X} = \{\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_N\}$ denote the image set containing N sample images $\mathbf{X}_i \in \mathcal{R}^{m\times n}$. Two-Dimensional Linear Discriminant Analysis (2DLDA) [4] tries to find projection vectors $\mathbf{w}_i \in \mathcal{R}^{n\times 1}$ along which the classes of projected data $\mathbf{y}_i = \mathbf{X}_i\mathbf{w}_i$, where $\mathbf{y}_i \in \mathcal{R}^{m\times 1}$, are well separated. That is, the between-class scatter matrix:

$$\mathbf{S}_B = \sum_{i=1}^{c} n_i \left(\boldsymbol{M}_i - \boldsymbol{M}\right)^T \left(\boldsymbol{M}_i - \boldsymbol{M}\right) \quad (1)$$

and the within-class scatter matrix:

$$\mathbf{S}_W = \sum_{i=1}^{c}\sum_{k=1}^{n_i} \left(\boldsymbol{X}_k^i - \boldsymbol{M}_i\right)^T \left(\boldsymbol{X}_k^i - \boldsymbol{M}_i\right), \quad (2)$$

are defined. Here, $\mathbf{M}$ denotes the average image of all training sample images $\mathbf{X}_i$, c is the number of classes, $\boldsymbol{X}_k^i$ is the k-th sample image in the class i and $\boldsymbol{M}_i$, n_i are the average image and the number of samples in class i, respectively.

The objective of 2DLDA is to find the transformation matrix $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \ldots \mathbf{w}_d]$ that maximizes the ratio of the trace of the between-class scatter to the trace of the within-class scatter matrix:

$$J(\mathbf{W}) = \arg\max_{\mathbf{W}} \frac{tr[\mathbf{W}^T\mathbf{S}_B\mathbf{W}]}{tr[\mathbf{W}^T\mathbf{S}_W\mathbf{W}]}. \quad (3)$$

subject to the orthogonal constraints $\mathbf{w}_i^T\mathbf{w}_j,\ i \neq j,\ i,j = 1,\ldots,d$. That is to say, the objective is to maximize the between-class scatter matrix such that in the new low-dimensional space the class means are as far from each other as possible, and minimize the within-class scatter matrix such that samples from the same class are as close to their mean as possible.

The solution of (3) is approximated [12, 13] by the following generalized eigenvalue decomposition problem:

$$\mathbf{S}_B \cdot \mathbf{w} = \lambda \cdot \mathbf{S}_W \cdot \mathbf{w}, \quad (4)$$

by keeping the first d eigenvectors. For any image $\mathbf{X}_i$, d projected vectors $\mathbf{y}_i = \mathbf{X}_i\mathbf{w}_i,\ i = 1,\ldots,d$ are obtained forming an $m \times d$ matrix $\mathbf{Y} = [\mathbf{y}_1, \ldots, \mathbf{y}_d]$. The upper bound on d is $\min(c-1, n)$.

3. TWO-DIMENSIONAL LINEAR DISCRIMINANT ANALYSIS USING SYMMETRY

On facial image analysis, symmetry is a main characteristic since human faces are typical and common examples of symmetric objects. Therefore, it would be expected for the generated projection vectors $\mathbf{w}_i$, among other properties, to be symmetric in order to achieve a higher generalization capability and not to suffer from the over-training phenomenon. However, this does not usually happen either because the training set very often consists of a small number of samples, resulting in a poor pattern representation, or the sample images are not strictly symmetric. As can be shown in Figure 1, the sample images usually correspond to facial images under various lighting conditions, expressions (happiness, sadness, surprise, etc.), facial details (open or closed eyes) and unconstrained conditions. As a result the symmetry is not maintained in the 2DLDA output, resulting in bad pattern learning and generalization.

Fig. 1. Facial images under various illumination conditions, expressions and facial details or in unconstrained conditions ([14, 15, 16]).

In this section, we modify the 2DLDA technique by imposing a symmetry constraint [11] in its objective function for the determination of projection vectors that are symmetric, so that the samples are projected in symmetric discriminant subspaces. A way to measure the symmetry error of a vector $\mathbf{w} = [w_1, w_2, \ldots, w_{n-1}, w_n]^T$ is given by the following equation:

$$s_{\mathbf{w}} = \sum_{i=1}^{n/2} \left(w_i - w_{n+1-i}\right)^2 \quad (5)$$

An equivalent way of measuring the symmetry error of a vector $\mathbf{w}$ is to use the following $n \times n$ symmetry matrix:

$$\mathbf{A} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \ldots & 0 & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \ldots & -\frac{1}{\sqrt{2}} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & -\frac{1}{\sqrt{2}} & \ldots & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \ldots & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}. \quad (6)$$

It is straightforward to prove that:

$$s_{\mathbf{w}} = \mathbf{w}^T\mathbf{A}\mathbf{A}^T\mathbf{w} = \sum_{i=1}^{n/2} \left(w_i - w_{n+1-i}\right)^2. \quad (7)$$

The goal of the proposed 2DLDA is to impose this symmetry constraint in the objective functions of of 2DLDA by minimizing the quantity $tr[\mathbf{W}^T\mathbf{A}\mathbf{A}^T\mathbf{W}]$. That is, the objective of the proposed 2DLDA is to determine projection vectors $\mathbf{w}$, which both maximize class discrimination and are symmetric. Specifically, using the between-class $\mathbf{S}_B$ and within-class $\mathbf{S}_W$ scatter matrices as defined in (1), (2) and the trace as a measure of variance and symmetry, we want to maximize the trace of the quantity $\mathbf{W}^T\mathbf{S}_B\mathbf{W}$, so that the dispersion of samples from different classes will be maximized after the projection, while, at the same time, we want to minimize the trace of the $\mathbf{W}^T\mathbf{S}_W\mathbf{W}$ so that samples from the same classes will come as close as possible to their mean vector after the projection and we also want to minimize the trace of the quantity $\mathbf{W}^T\mathbf{A}\mathbf{A}^T\mathbf{W}$, in order to minimize the symmetry error of the projection vectors, where $\mathbf{W}$ contains the projection vectors $\mathbf{w}_i$. Consequently, we try to find the projection matrix $\mathbf{W}$ that maximizes the matrix trace ratio of the between-class scatter matrix to the within-class and symmetry scatter. Thus, we obtain the following objective function:

$$J(\mathbf{W}) = \arg \max_{\mathbf{W}^T\mathbf{W}=\mathbf{I}} \frac{tr[\mathbf{W}^T\mathbf{S}_B\mathbf{W}]}{(1-s)\, tr[\mathbf{W}^T\mathbf{S}_W\mathbf{W}] + s\, tr[\mathbf{W}^T\mathbf{A}\mathbf{A}^T\mathbf{W}]}. \tag{8}$$

subject to the orthogonal constraints $\mathbf{w}_i^T\mathbf{w}_j,\ i \neq j,\ i,j = 1,\ldots,d$. Here $s \in [0,1]$ is the symmetry factor that controls the symmetry of $\mathbf{w}$. Obviously, for $s = 0$ the proposed technique corresponds to 2DLDA, while as s is increasing to 1, the level of symmetry of the projection vectors is maximized.

The solution of (8) is given by the solution of following generalized eigenvalue decomposition problem:

$$\mathbf{S}_B \cdot \mathbf{w} = \lambda \cdot \left((1-s)\mathbf{S}_W + s\mathbf{A}\mathbf{A}^T\right) \cdot \mathbf{w}, \tag{9}$$

by keeping the d eigenvectors that correspond to the d largest eigenvalues. The upper bound on d, as in the case of 2DLDA, is $\min(c-1, n)$.

4. EXPERIMENTS

In this section, we present experiments conducted in order to evaluate the performance of the proposed symmetric 2DLDA technique in face recognition. We have employed four publicly available face recognition databases, namely ORL, AR, Extended YALE-B and LFW databases. In the following subsections, we describe the databases and experimental results.

4.1. Databases description

4.1.1. The ORL database

The ORL database [17] contains 400 images of 40 distinct persons (10 images each). The images were captured at different times and with different variations including lighting conditions, facial expressions (smiling/not smiling) and facial details (open/closed eyes, with/without glasses). Also, the images were taken in frontal position with a tolerance for some tilting and rotation of the face of up to 20 degrees. Some example facial images from the ORL database are displayed in Figure 2.

Fig. 2. Sample images from the ORL database.

4.1.2. The AR database

The AR database [14] contains over 4000 color images corresponding to 70 men's and 56 women's faces. The images were taken in frontal position with different facial expressions (anger, smiling and screaming), illumination conditions (left and/or right light on), and occlusions (sun glasses and scarf). Each person participated in two recording sessions, separated by two weeks (14 days) time. In our experiments, we used a subset from AR database, which contains cropped images from 100 persons (50 men and 50 women) [18]. Some example facial images from the AR database are displayed in Figure 3.

Fig. 3. Sample images from the AR database.

4.1.3. The Extended YALE-B database

The Extended YALE-B database [19] contains images of 38 persons in 9 poses and under 64 illumination conditions. We used the frontal cropped images only [15], in this work. Some example facial images from the Extended YALE-B database are displayed in Figure 4.

Fig. 4. Sample images from the Extended YALE-B database.

4.1.4. The LFW database

LFW [20] is an image dataset for unconstrained face verification. It contains more than 13,000 facial images collected from the web with large variations in pose, age, expression, illumination, etc. In our experiments, a subset with cropped images [16] was used corresponding to persons with 50 or more sample images. Some example facial images from the LFW database are displayed in Figure 5.

Fig. 5. Sample images from the LFW database.

4.2. Experimental results

To estimate the recognition accuracy, we used the 5-fold cross validation procedure. More specifically, each database was divided into 5 non-overlapping subsets. Each experiment includes five training-test procedures (folds). In each fold, the standard and proposed 2DLDA were trained by using 4 subsets and testing was performed on the remaining subset. The proposed 2DLDA was used for $s = 0.0, 0.1, ..., 0.9999$. The projected samples were classified using the Nearest Centroid (NC) and k-Nearest Neighbor (kNN) classifiers. kNN was used for $k = 1, 3, 5$. For ease of representation, we will follow the notation kNN(n), where n is the number of nearest neighbors, in the case of kNN. Recognition accuracy was measured by using the mean classification rate over all five folds.

Table 1. Comparison of the best recognition accuracies (mean±std %), dimension and symmetry error of projection vectors of standard 2DLDA versus symmetric 2DLDA.

technique		Standard	Symmetric
ORL	kNN(1)	97.50 ± 2.34	**98.25** ± 1.68
	kNN(3)	95.75 ± 1.43	**96.50** ± 1.85
	kNN(5)	92.75 ± 1.63	**94.00** ± 1.63
	NC	94.00 ± 2.24	**94.25** ± 1.90
	symm. error	0.911151	0.066116
AR	kNN(1)	78.96 ± 3.40	**87.59** ± 2.08
	kNN(3)	65.04 ± 3.50	**77.30** ± 2.85
	kNN(5)	64.42 ± 2.49	**76.82** ± 3.41
	NC	68.69 ± 1.71	**71.01** ± 1.92
	symm. error	0.890714	0.072623
Extended YALE-B	kNN(1)	80.88 ± 3.62	**86.30** ± 2.76
	kNN(3)	77.10 ± 4.12	**84.14** ± 2.67
	kNN(5)	76.19 ± 5.42	**84.20** ± 1.90
	NC	24.47 ± 3.65	**56.41** ± 7.50
	symm. error	1.151940	0.08475
LFW	kNN(1)	51.46 ± 2.37	**52.13** ± 2.72
	kNN(3)	51.83 ± 3.54	**52.56** ± 2.71
	kNN(5)	51.28 ± 2.34	**52.13** ± 2.88
	NC	36.83 ± 1.59	36.83 ± 1.59
	symm. error	0.859193	0.020290

The results obtained are shown in Table 1. For each dataset, the first four rows illustrate the recognition accuracies obtained by applying the standard and proposed 2DLDA technique and the kNN(1), kNN(3), kNN(5) and NC classifiers respectively, while the average symmetry error of the projection vectors is given in the fifth one. The best results are shown in bold.

We observe that the proposed 2DLDA technique outperforms the standard one in all the databases. Indeed, an improvement in recognition accuracy is achieved when symmetry constraint is exploited. More specifically, in the ORL case the highest classification accuracies that are achieved for the standard and the proposed 2DLDA are 97.50% and 98.25%, respectively, while for the Extended YALE-B dataset the corresponding highest classification accuracies are 80.88% and 86.30%. In the AR case, the improvement is about 8.60%. Finally, in the LFW case, the highest classification accuracy corresponding to the proposed 2DLDA is 52.56%, while 51.83% is the respective one of the standard 2DLDA.

Therefore, we can conclude that for face databases containing facial images under conditions where the pose is not exactly frontal (ORL database case) or there is a variation in facial expression and in lighting conditions (AR and Extended YALE-B database) or in unconstrained conditions (LFW case), the proposed technique achieves better generalization by exploiting data symmetry and are not affected by the symmetry noise of the images. The symmetry error of the generated projection vectors decreased in the proposed 2DLDA technique, as expected. The value of s can be learned during training using a cross-validation procedure. In our experiments, the results corresponding to the proposed 2DLDA have been obtained for $s = 0.9$.

5. CONCLUSION

In this paper we proposed a subspace learning technique for facial image analysis. The proposed technique extends the Two-Dimensional Linear Discriminant Analysis (2DLDA) technique with the introduction of a symmetry constraint in its objective function taking into account the a-priori knowledge that symmetry appears in facial images. The performance of the proposed Symmetric Two-Dimensional Linear Discriminant Analysis has been evaluated in face recognition, where it has been found to outperform the standard 2DLDA technique.

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