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Facilitation of Competing Bids
and the Price of a Takeover Target

Abstract

Initially uninformed bidders must incur costs to learn their (independent) valuations of a potential takeover target. The first bidder makes either a preemptive bid that will deter the second bidder from investigating, or a lower bid that will induce the second bidder to investigate and possibly compete. We show that the expected price of the target may be higher when the first bidder makes a deterring bid than when there is competitive bidding. Hence, by weakening the first bidder's incentive to choose a preemptive bid, regulatory and management policies to assist competing bidders may reduce both the expected takeover price and social welfare.

Two means by which to acquire a publicly-held firm are tender offer and merger. Until the passage of the (federal) Williams Act in 1968[1] and the state legislation that followed in its wake, cash tender offers were virtually unregulated in the United States. The Williams Act requires tender offers to remain open for a minimum period of twenty business days.[2] The Act further requires a bidder making a tender offer to disclose, *inter alia*, (i) the bidder's sources of funds, and (ii) the purpose of the tender offer, including planned re-organization, sale of assets, or change of management.[3] By providing competing bidders with more time and access to information, the Act facilitates competitive bidding.

The Williams Act sparked an intense debate among legal commentators on its delay and disclosure provisions, and on the appropriate conduct of the management of a target of a tender offer. Easterbrook and Fischel (1981 and 1982) argued that by reducing the gains from takeovers to first bidders, facilitation of competition in bidding would reduce first bidders' investigation of potential targets to a socially inefficient rate. They called for the Williams Act to be repealed, and for the management of targets to be bound by a 'rule of managerial passivity' that would prohibit management of targets from soliciting competing bids.[4]

Bebchuk (1982a, 1982b, and 1986) and Gilson (1981 and 1982) contended that competitive bidding for a takeover target benefits society by enabling acquisition of the target by a second bidder which can increase the value the target more than the first bidder. They advocated, instead, a 'rule of auctioneering', that "(1) provides, by regulating offerors, time for making competing bids; and (2) allows incumbent management to solicit such bids by providing information about the target to potential buyers."[5]

[1]15 United States Code, §§78l–78n (1976). Our discussion of the relevant securities law in this Introduction is based, in part, on Loss (1988), pp. 449–540.

[2]Securities and Exchange Commission Rule 14e-1.

[3]Securities and Exchange Commission Rule 14d-6 and Schedule 14D-1.

[4]See also Schwartz (1986).

[5]Bebchuk (1982a), p. 1030.

Although procedures for mergers are governed by corporation law of the individual states, bids to merge are indirectly subject to time and disclosure provisions of federal securities law in the following way. A number of states require that a merger be approved by majority vote of the outstanding shares of the target.[6] Under the (federal) Securities Exchange Act of 1934, however, most publicly-held corporations seeking proxies for a shareholder vote must prepare a proxy statement,[7] which must be filed with the Securities and Exchange Commission at least ten days before distribution.[8] [9] By setting time limits and requiring disclosure of information, federal securities law facilitates the entry of competitors to a merger.[10]

A number of preceding authors have formally analyzed the role of the management of a takeover target in maximizing the price of the target. Giammarino and Heinkel (1986) study the decision of management whether to accept a first bid when rejection allows a second bidder to enter. In Tiemann's (1988) model, one potential acquirer has better information about the possible improvement of the value of the target than the other. Tiemann shows that the management of the target can raise the expected price by disclosing information about itself only to the informationally-disadvantaged bidder. Shleifer and Vishny (1986) argue that the target management may pay greenmail to a low valuation acquirer in order to persuade another potential acquirer with higher valuation to investigate.

In contrast, we and others focus on the effects of government regulation on

[6]See, for instance, California General Corporations Law, §1201(a), Delaware General Corporation Law, §251, and New York Business Corporation Law, §903. There are exceptions, *e.g.*, under California law, a "short-form merger" is available if the acquirer owns 90% of the shares of the target and plans to merge the target into the parent corporation.

[7]§14A of 1934 Act.

[8]Securities and Exchange Commission Rule 14a-6.

[9]Even when a merger does not require a vote, the 1934 Act requires that a publicly-held corporation issue an information statement, providing substantially the same information as the proxy statement (§14C of 1934 Act).

[10]Management-led buyouts can be effected by either tender offer or merger hence are subject to federal and state regulation.

competition in bidding. In Fishman (1988), two potential acquirers can raise the value of the target by improvements of independent amount. Each acquirer must incur a cost to learn the size of his improvement. Having investigated, the first bidder must choose between a bid that leads a second bidder to investigate and a higher bid that deters investigation. If the second bidder should investigate and bid, he triggers an auction. In expectation, the price under an auction is higher than the price that would have been paid if the first bidder had deterred investigation. The lower the cost of investigation to the second bidder, the more inclined is the first bidder to choose a bid that leads to investigation. Hence, Fishman concluded that the expected price of the target will be maximized if the cost of investigation to the second bidder is minimized.

In an example due to Bhattacharya, generalized by Spatt (1989), each of several potential bidders must incur an entrance cost to participate in an auction of a target. This entrance cost would depend on time and disclosure provisions of the law. Bidders may have identical or independent valuations of the target. In the symmetric mixed-strategy equilibrium, each participates with a probability that makes his expected revenue equal to the expected entrance cost. Bhattacharya and Spatt found that the expected price of the target is decreasing in the number of potential bidders as long as there are at least two bidders. Hence exogenous elimination of potential bidders benefits the target. However, consistent with Fishman, they show that the expected price is decreasing in the entrance cost.[11]

All parties to the legal debate, as well as Tiemann (1988), Shleifer and Vishny (1986), Fishman (1988), and Bhattacharya and Spatt (1989) assumed that competitive bidding in takeovers is equivalent to an English auction in which each bidder may costlessly revise his bid as the price rises. In reality, the cost of making and revising takeover bids is far from trivial. It includes fees to counsel, investment bankers, and other outside advisors, the opportunity cost of executive time, and the cost of obtaining financing for the bid. Bidding costs are also affected by federal and state law governing takeovers. For instance, federal law specifies that share-

[11]For a detailed review of the literature on takeover bidding, see Spatt (1989).

holders who tender to one bidder must be allowed to withdraw their shares as long as the offer remains open.[12] This allows shareholders to tender to a competing bidder. The shorter the time available for shareholders to withdraw shares tendered to an earlier bidder, the higher the cost of making a counter-bid.

We present the economic setting of the model in the following section. Our framework is similar to Fishman's, differing only in one crucial respect—we assume that bidding is costly. In this setting, we show that the expected price of the target may be *lower* under competitive bidding than if the first bidder had bid to deter the potential competitor from investigating. In Section 2, each bidder may make at most one bid, so if the second bidder competes, he need only match the first bid to acquire the target. Section 3 provides an example in which each bidder may bid any number of times but must incur a cost each time he does so. If the second bidder should compete, the price of the target will *not* be bid up to the minimum of the two bidders' valuations net of bidding cost, because once a bidder believes that his value is exceeded by that of his competitor, he prefers to quit rather than incur any further cost of bidding.

Since the price of the target need not be higher with competitive bidding, lowering the cost of investigation may reduce the expected takeover price of the target. We show that by encouraging the first bidder to bid high to deter investigation, an increase in the second bidder's cost of investigation from its minimum level may increase both the expected price of the target and social welfare. Section 4 presents the implications of the analysis for government regulation of takeover bidding and the conduct of the management of a target. We conclude with empirical implications and suggestions for future work.

[12] Securities and Exchange Commission Rule 14d-7.

1 Economic Setting

There are two potential acquirers of some target. The value of the acquisition to the first bidder (FB) is v_1 while the value to the second bidder (SB) is v_2. These values depend on such idiosyncratic factors as the quality of the bidder's management, and the match between the bidder's existing operations and assets with those of the target. To represent an extreme form of bidder-specific valuation effects, we assume that v_1 and v_2 have independent and identical discrete marginal distribution λ_{v_1} with $\sum_{v_1} \lambda_{v_1} = 1$.[13] Let $\bar{v}$ be the largest possible value of v_1, and the expected value,

$$E(v_1) = E(v_2) = \sum_{v_1} v_1 \lambda_{v_1} < 0. \tag{1}$$

The objective of the shareholders and the management of the target is to maximize the expected price of the shares. Accordingly, the shareholders or management will sell the target if they receive a bid above the value of the cashflows of the target absent a takeover, which, for simplicity, we normalize to zero.[14] If there are competing bids, we assume that target will accept the higher bid. If the two bids are of equal amount, we break the tie by assuming that the target will be sold to the last bidder.

FB's decision to investigate occurs before the analysis proper begins. The events under consideration take the following sequence. Initially, at date 1, FB knows v_1, while in contrast, SB does not know v_2. FB makes a bid, $b_1 \in [0, v_1]$, and thereby alerts the potential competing bidder to the possibility of a profitable takeover. At date 2, SB must decide whether to investigate and learn v_2 at a cost of $c \geq 0$. Since,

[13] In reality, there is likely to be an element of common value to the acquisition, and the management of the target may have private information about this common-value element. A bidder may wish to make a bid that is contingent on the value of the acquisition, say by offering payment in its securities rather than cash, to elicit the target's information. See Hansen (1987) and Fishman (1989).

[14] We abstract from the free-rider problem in tendering described by Grossman and Hart (1980). However, as they note, this problem may be mitigated by the power of a successful bidder to dilute the minority holding.

by (1), $E(v_2) < 0$, SB will not bid unless he first investigates. Hence, if SB does not investigate, FB will acquire the target. If SB investigates, he must decide whether to bid.

SB's bid depends on the ensuing structure of the game: in Section 2, we assume that each bidder may bid at most once, while in Section 3, we allow both bidders to bid any number of times at a fixed cost per bid. Throughout the analysis, we adopt the convention that SB will investigate only if his expected return from investigation is strictly positive.[15]

In common with previous literature, we assume that bids cannot be withdrawn. Let the value of c and the prior distribution of v_1 and v_2 be common knowledge, and let the realized values of v_1 and v_2 be private information of the respective bidders if they should investigate. All parties are risk-neutral.

2 Facilitation of Competing Bids

In this section, we assume that each bidder may make at most one bid. We solve this game recursively: beginning with the second bidder's (SB's) bid, we consider SB's decision whether to investigate, and finally turn to the initial bid by the first bidder (FB). FB must consider that his bid will set a floor price for the target, and in this way, will affect SB's decision whether to investigate, and if SB should investigate, SB's decision whether to bid for the target.

Since FB may bid only once, SB can acquire the target by matching b_1. If SB finds $v_2 \leq b_1$, he cannot gain from bidding. We assume that SB will match the first bid to acquire the target and realize a gain of $v_2 - b_1$ only if $v_2 > b_1$.[16] As v_1 and v_2 are independent, the first bid conveys no information to SB about v_2. Hence, given

[15]This ensures the existence of an optimal bid in the class of first bids by FB that deter investigation.

[16]Similar results may be proved with the alternative assumptions that if $v_2 = b_1$, SB (i) does bid, or (ii) randomizes between bidding and not.

a first bid of b_1, the expected return from investigation to SB,

$$\begin{aligned} R_2(b_1) &\stackrel{\text{def}}{=} -c + 0 \cdot \Pr(v_2 < b_1) + E_{v_2}(v_2 - b_1 | v_2 > b_1) \cdot \Pr(v_2 > b_1) \\ &= -c + E_{v_2}[\max\{v_2 - b_1, 0\}], \end{aligned} \tag{2}$$

where $E_{v_2}[\cdot]$ denotes the expectation with respect to the distribution of v_2. By our convention, SB will investigate only if

$$R_2(b_1) > 0.$$

To find FB's equilibrium bidding strategy, we distinguish two classes of bids—those that deter SB from investigating, and those that accommodate investigation. Suppose that there exists a bid $b_1 \in [0, v_1]$ such that SB does not investigate. With such a bid, FB will acquire the target with certainty and realize a gain of $v_1 - b_1$. Hence, the optimal *deterring bid* for FB is

$$\begin{aligned} &\arg\max \{v_1 - b_1 \mid R_2(b_1) \leq 0, \ b_1 \geq 0\} \\ &= \min \{b_1 \mid R_2(b_1) \leq 0, \ b_1 \geq 0\} \stackrel{\text{def}}{=} b_1^D(c), \end{aligned} \tag{3}$$

which is the minimum non-negative value of b_1 sufficient to deter investigation.[17] If such a bid exists, it does not vary with v_1, but is weakly decreasing in c.

Suppose that there exists some bid $b_1 \geq 0$ that leads SB to investigate. With this bid, FB's profit depends on whether SB finds $v_2 > b_1$. If $v_2 > b_1$, SB will acquire the target, and FB will gain nothing. If $v_2 \leq b_1$, FB will acquire the target for a profit of $v_1 - b_1$. Hence, the expected return to FB is $(v_1 - b_1) \ \Pr(v_2 \leq b_1)$, since v_1 and v_2 are independent. Thus, the optimal *accommodating bid* is

$$b_1^I(v_1, c) \stackrel{\text{def}}{=} \arg\max\{(v_1 - b_1) \ \Pr(v_2 \leq b_1) \mid R_2(b_1) > 0, b_1 \geq 0\}.^{18} \tag{4}$$

Consider those pairs (v_1, c) for which there exist both $b_1^D(c)$, the optimal deterring bid, and $b_1^I(v_1, c)$, the optimal accommodating bid. By (2), SB's expected

[17]This bid may be viewed as the lowest exercise price such that a risk-neutral call option premium on v_2 does not exceed c (aside from the constraint $b_1 \geq 0$).

[18]It will be shown in Lemma 1 that the optimal bid $b_1^I(v_1, c)$ exists for any v_1 in which FB chooses to accommodate investigation.

return from investigation, $R_2(b_1)$, is weakly decreasing in b_1. Hence, any deterring bid must be higher than any accommodating one, so in particular

$$b_1^D(c) > b_1^I(v_1, c). \tag{5}$$

To summarize, FB has two alternatives. First, he can bid high and acquire the target with certainty. Alternatively, he can bid low knowing that SB will investigate, but hoping that SB will draw a low v_2. If both of these alternatives are profitable, he will choose the bid that yields the larger expected profit. Assuming that FB will deter if these alternatives yield equal profit,[19] he will bid high if

$$v_1 - b_1^D(c) \;\geq\; [v_1 - b_1^I(v_1, c)] \;\; \Pr[v_2 \leq b_1^I(v_1, c)]. \tag{6}$$

If all profitable bids deter investigation, FB will choose the optimal such bid, and similarly, if all profitable bids lead SB to investigate, FB will choose the optimal accommodating bid.

Let $I(c)$ be the set of values v_1 such that FB makes accommodating bids, and $D(c)$ be the set of values such that FB makes a deterring bid. These sets are mutually exclusive. One of the two sets may be empty; for instance, if SB's cost of investigation $c \geq \overline{v}$, then SB will never investigate, hence $I(\overline{v})$ is empty.

To characterize FB's strategy, it is helpful to define

$$\hat{b}_1(v_1) \stackrel{\text{def}}{=} \arg\max\{(v_1 - b_1) \;\; \Pr(v_2 \leq b_1) \mid b_1 \geq 0\}. \tag{7}$$

By comparing (4) and (7), we see that $\hat{b}_1(v_1)$ solves the problem (4) without the constraint that the bid lead SB to investigate. We then have

Lemma 1. *For all $v_1 \in I(c)$,*

$$b_1^I(v_1, c) = \hat{b}_1(v_1),$$

[19]The same results may be proved with the alternative assumptions that when expected profits are equal, FB (i) accommodates, or (ii) randomizes between accommodating and deterring.

and hence the optimal accommodating bid by the first bidder is independent of c.[20]

Consider $v_1 \in I(c)$: by (6) and Lemma 1,

$$v_1 - b_1^D(c) \ < \ [v_1 - \hat{b}_1(v_1)] \ \Pr[v_2 \leq \hat{b}_1(v_1)].$$

For any $v_1' < v_1$, FB's profit from deterring will be lower than in the state v_1 by an amount $v_1 - v_1'$. FB's profit from accommodation, however, will be lower by less than $v_1 - v_1'$, because FB can do no worse than hold his accommodating bid unchanged at $\hat{b}_1(v_1)$. Thus, FB with valuation v_1' will also accommodate investigation. This proves that FB either deters in all states v_1, accommodates in all states v_1, or there exists a critical realization $v_1^\dagger(c)$, at or above which FB deters, and below which FB accommodates.

Consider FB with valuation $v_1^\dagger(c)$: by (3), if c were lowered, the deterring bid would weakly increase, thus weakly reducing the FB's profit from deterring. By Lemma 1, however, the reduction in c will not affect FB's profit from accommodating investigation. Therefore a reduction in c will weakly reduce the set of v_1 in which FB will deter investigation. We summarize the last two findings in

Lemma 2. *Either the first bidder deters investigation in all states $v_1 \geq 0$, accommodates in all states $v_1 \geq 0$, or there exists some cut-off, $v_1^\dagger(c)$, such that $I(c) = \{v_1 \mid 0 \leq v_1 < v_1^\dagger(c)\}$, and $D(c) = \{v_1 \mid v_1 \geq v_1^\dagger(c)\}$. Further, $D(c)$ is weakly increasing in c.*

Since each bidder may bid at most once, SB need only match the first bid to acquire the target. Hence the price at which the target is taken over will be FB's bid—regardless of which bidder acquires the target. Therefore, the expected takeover price of the target is

$$p(c) \stackrel{\text{def}}{=} \sum_{v_1 \in I(c)} b_1^I(v_1, c) \ \lambda_{v_1} + \sum_{v_1 \in D(c)} b_1^D(c) \ \lambda_{v_1}. \tag{8}$$

SB's cost of investigation, c, affects the expected takeover price in two ways. By (3), a lower cost will weakly increase the deterring bid $b_1^D(c)$. By Lemma 2, however, a lower cost will lead FB to deter investigation in fewer states v_1.

[20]Lemma 1 and the Proposition are proved in Appendix 1.

For sufficiently large c, $E_{v_2}[\max\{v_2, 0\}] \leq c$, so any non-negative bid will deter investigation, and hence $p(c) = 0$. If $c = 0$, it is impossible to deter investigation, hence $I(0) = \{v_1 | v_1 \geq 0\}$ and $D(0) = \emptyset$. By Lemma 2, the set $D(c)$ is weakly increasing in c. Consider the smallest cost of investigation, $\underline{c}$, low enough that only FB with the highest valuation $v_1 = \bar{v}$ makes a deterring bid. Then for all reductions in the cost of investigation, FB with all valuations will make accommodating bids, *i.e.*, $D(\underline{c} - \Delta c) = \emptyset$, for all $\Delta c > 0$.

In equilibrium, the price of the target is the first bid, whether the target is purchased by FB or SB. By Lemma 1, the change in the investigation cost does not affect the accomodating bids of FB with $v_1 < \bar{v}$. So the only change in the price of the target arises from the shift of FB with $v_1 = \bar{v}$ from deterring to accommodating investigation. By (5), FB's deterring bid strictly exceeds his accommodating bid. Thus, the reduction in the cost of investigation lowers the expected price of the target. This intuition is proved in the following result.

Proposition. *The expected takeover price is maximized at a positive value of the second bidder's cost of investigation.*

In this setting, social welfare may be measured as the expectation of the sum of the price received by the target, and the profits of FB and SB. Essentially, social welfare is the expectation of the valuation of the bidder who acquires the target less SB's cost of investigation (if incurred). In Appendix 1, we show that in the following example, an increase in c from zero raises social welfare.

Let $\lambda_{-2} = 0.39, \lambda_0 = 0.05, \lambda_1 = 0.38$, and $\lambda_2 = 0.18$. With $c = 0$, FB accommodates in all states $v_1 \geq 0$. In particular, when FB with valuation $\bar{v}$ bids $\hat{b}_1(\bar{v})$, SB investigates, and if $v_2 \in (\hat{b}_1(\bar{v}), \bar{v})$, acquires the target even though the target is socially more valuable in the hands of FB. A sufficient increase in c will induce FB with valuation $\bar{v}$ to switch to a deterring bid, and hence eliminate this source of inefficiency. The rise in c, however, means that SB will incur larger deadweight costs of investigation whenever FB has $v_1 \in I(c)$ and accommodates. On balance, in our example, an increase in c from 0 to 0.1584 raises expected social welfare.

In sharp contrast to the Proposition, Fishman (1988) showed that if each bidder can revise his bid costlessly as in an auction, then the expected takeover price will be maximized when SB's cost of investigation is minimized. Clearly, both Fishman's assumption of costless counter-bidding and our assumption that each bidder may bid at most once are polar cases. Accordingly, in the next section, we turn to analyze the more realistic intermediate situation.

3 Costly Counter-Bidding with Voluntary Termination: An Example

In this section, we provide an example to confirm that the main results of the previous section are consistent with a setting in which (i) each bidder may bid any number of times at a fixed cost per bid, and (ii) bidding ends only when one bidder quits voluntarily. We modify the assumptions of the preceding section in the following respects. First, each bidder must incur a fixed cost of γ to place a bid or to revise his bid. If the second bidder (SB) bids, the first bidder (FB) may counter-bid, then SB may counter-bid, and bidding continues in turn until one bidder quits. Whereas each bidder must incur the cost γ for each bid or counter-bid, SB need bear the cost of investigation, c, only once to learn v_2. Secondly, we assume that the marginal distribution of valuations, $\lambda_{v_1} > 0$ only for $v_1 \in \{0, 1, 2\}$, with $\lambda_0 + \lambda_1 + \lambda_2 = 1$.[21]

In an English auction, the auctioneer calls out an increasing price and the price rises until it reaches the valuation of the bidder with the lower valuation. To bias our results towards those of analyses that model takeover bidding as English auctions, we adopt the convention that if, at any stage, a bidder can raise the price of the target *without sacrificing expected profit*, then he will "close the gap" by bidding to raise the price.

[21]We show below that when bidding is costly, our earlier assumption that $E(v_2) < 0$ is not necessary to ensure that SB bids only after investigating.

3.1 Equilibrium

In order to explore the effect of changes in c on the expected takeover price and social welfare, we specify ranges for the parameters such that, in equilibrium, FB both accommodates and deters with positive probability. We use the concept of perfect Bayesian equilibrium.[22] In this setting, however, there potentially exist multiple distinct classes of equilibria. We first apply the Intuitive Criterion of Cho and Kreps (1987). Cho and Kreps developed the Intuitive Criterion in the context of a game of asymmetric information in which each player had only one move. In the present setting, however, consequent on a defection by FB, each bidder may bid more than once. In Appendix 2, we extend the Intuitive Criterion in a simple way to allow examination of such defections, and hence narrow the set of equilibria.

The class of equilibria of interest is:

I. FB:

$v_1 = 0$: Never bid.

$v_1 = 1$: Bid 0 with probability $1 - \mu$ or bid $b_1^D > 0$ with probability μ.

After a bid of zero, if SB bids, infer $v_2 = 1$ or 2, and quit.

$v_1 = 2$: Bid b_1^D.

II. SB:

If first bid is at least b_1^D, infer v_1 =1 or 2; do not investigate.

If first bid is below b_1^D, infer $v_1 = 1$, and investigate. In any later round, if FB makes a bid below b_1^D, infer $v_1 = 1$, while if FB makes a bid at or above b_1^D, infer $v_1 = 2$.

$v_2 = 0$: Quit.

$v_2 = 1$ or 2: Match the first bid.

[22]This essentially is Nash equilibrium with the further requirements that (i) beliefs be revised according to Bayes' Rule wherever possible, and (ii) the strategies continue to be a Nash equilibrium at all later stages of the game. See, for instance, Rasmusen (1989).

In the following, we provide ranges for the parameters under which the stated strategies and beliefs are an equilibrium.[23] The reader not interested in the technical details may skip directly to the next subsection, where we show that the expected price of the target and social welfare may increase with the cost of investigation.

As bidding is costly, a bidder with valuation 0 will never make a bid. We first require that SB bid only after investigating. If SB bid without investigating, he could do no worse than if he acquired the target with a bid of zero. Then his expected profit would be his expected valuation less the cost of bidding. We therefore assume $\lambda_1 + 2\lambda_2 - \gamma < 0$, or

$$1 - \lambda_0 + \lambda_2 < \gamma \quad . \tag{9}$$

Secondly, we require that FB with $v_1 = 1$ be indifferent between bids of 0 and b_1^D. If he bids zero, he incurs a cost γ, and SB investigates. With probability λ_0, SB draws $v_2 = 0$ and quits, so that FB gets $1 - \gamma$. With probability $\lambda_1 + \lambda_2$, SB matches, from which FB infers that $v_2 = 1$ or 2. If FB were then to counter-bid, he would at best break even, but with positive probability would lose an additional γ, so he prefers instead to quit with a loss of γ. Hence the expected profit to FB with $v_1 = 1$ from a bid of zero is

$$\lambda_0(1 - \gamma) + (\lambda_1 + \lambda_2)(-\gamma) = \lambda_0 - \gamma \quad .$$

If FB with $v_1 = 1$ bids b_1^D, he incurs a cost of γ, and in equilibrium SB does not investigate. Hence, FB's profit from bidding b_1^D is $1 - b_1^D - \gamma$. In a mixed strategy equilibrium, FB must receive equal expected profit from bidding 0 and b_1^D, so,

$$b_1^D = 1 - \lambda_0 \quad , \tag{10}$$

and this profit must exceed that from not bidding, $1 - b_1^D - \gamma > 0$, or

$$\lambda_0 > \gamma. \tag{11}$$

[23]In Appendix 2 we show that the stated beliefs satisfy our extension of the Intuitive Criterion.

Thirdly, we require that FB with $v_1 = 2$ prefer the deterring bid $b_1^D = 1 - \lambda_0$ to bidding zero. For analytic simplicity, we impose the slightly stronger constraint that even if, contrary to the proposed equilibrium, SB infers from a first bid of zero some probability that $v_1 = 2$, FB would still prefer to deter. Suppose that FB with $v_1 = 2$ were to bid 0. Since SB with $v_2 = 1$ believes that he faces $v_1 = 1$ or 2, he cannot make a positive profit, and has a chance of losing money, hence he quits. SB with $v_2 = 2$ will bid, at which point FB will bid $2 - \gamma$ to close the gap and acquire the target. Thus the expected profit to FB with $v_1 = 2$ from bidding 0 is

$$(\lambda_0 + \lambda_1)(2 - \gamma) + \lambda_2[-2\gamma + 2 - (2 - \gamma)] = 2(\lambda_0 + \lambda_1) - \gamma \quad .$$

Alternatively, by deterring, FB can gain $2 - b_1^D - \gamma = 1 + \lambda_0 - \gamma > 0$, by (10) and (11). It follows that FB will bid to deter if

$$\lambda_1 < \lambda_2 \quad . \tag{12}$$

Fourthly, we require that if $b_1 < b_1^D$, then SB investigates. Since $b_1 < b_1^D$ leads SB to infer that $v_1 = 1$, if SB investigates, he will bid if $v_2 = 1$ or 2. If SB should bid, FB with $v_1 = 1$ will infer that $v_2 = 1$ or 2, and will quit, hence SB with $v_2 = 1$ or 2 can acquire the target simply by matching the first bid.[24] So SB's expected return from investigation,

$$\begin{aligned} R_2(b_1) &= -c + \lambda_1(-\gamma + 1 - b_1) + \lambda_2(-\gamma + 2 - b_1) \\ &= -c + (\lambda_1 + \lambda_2)(-\gamma + 1 - b_1) + \lambda_2. \end{aligned}$$

But $b_1 < b_1^D$, so by (10) and (11), $b_1 < 1 - \lambda_0 \leq 1 - \gamma$. Hence $R_2(b_1) > -c + \lambda_2$, and SB will investigate if

$$\lambda_2 > c \, . \tag{13}$$

[24]At this stage, if FB with $v_1 = 1$ counter-bids $b_1' < b_1^D$, SB will continue to believe that $v_1 = 1$, hence FB will have incurred γ without changing SB's beliefs. If FB were to counter-bid $b_1' \geq b_1^D$, SB will infer that $v_1 = 2$, and will quit if $v_2 = 1$ or close the gap to $2 - \gamma$ if $v_2 = 2$. Hence, FB's expected profit from counter-bidding would be

$$-\gamma + \frac{\lambda_1}{\lambda_1 + \lambda_2}(1 - b_1') \leq -\gamma + \frac{\lambda_1}{\lambda_1 + \lambda_2}\lambda_0 < 0,$$

by (14) to follow. Thus, FB with $v_1 = 1$ will not counter-bid.

Fifthly, a bid of b_1^D or higher must deter investigation. To simplify the analysis, we will require, in addition, that conditional on a first bid of b_1^D or higher, if SB (contrary to the proposed equilibrium) were to investigate, he would bid only if $v_2 = 2$. Suppose SB with $v_2 = 1$ bids $b_2 \geq b_1$. In the proposed equilibrium, FB will believe $v_2 = 2$, so if $v_1 = 1$, FB will quit, and SB will realize $1 - \gamma - b_2$, while if $v_1 = 2$, FB will bid $2 - \gamma$ to close the gap and acquire the target, and SB will lose γ. By Bayes' Rule, the likelihood of $v_1 = 1$ conditional on a first bid b_1^D is $\mu\lambda_1/[\mu\lambda_1 + \lambda_2]$. So SB with $v_2 = 1$ will quit if

$$-\gamma + \frac{\mu\lambda_1}{\mu\lambda_1 + \lambda_2}(1 - b_2) < 0 \ .$$

By (10), $b_2 \geq b_1 \geq b_1^D$ implies $b_2 \geq 1 - \lambda_0$, hence the above will hold for all $b_2 \geq b_1^D$ and $\mu \leq 1$ if

$$\lambda_1(\lambda_0 - \gamma) < \gamma\lambda_2 \, . \tag{14}$$

Consider SB with $v_2 = 2$. Since he faces FB with $v_1 = 1$ or 2, he will match b_1, and FB will infer that $v_2 = 2$. Hence if $v_1 = 1$, FB will quit and SB will gain $2 - b_1 - \gamma$. If $v_1 = 2$, FB will counter-bid $2 - \gamma$ to close the gap, and SB will quit with a loss of γ. By Bayes' Rule, SB's expected return from investigation,

$$R_2(b_1) = -c + \lambda_2 \left[-\gamma + \frac{\mu\lambda_1}{\mu\lambda_1 + \lambda_2}(2 - b_1) \right] \ , \tag{15}$$

which is increasing in μ and decreasing in b_1. To ensure that a first bid of b_1^D or higher deters investigation, we require that $R_2(b_1^D) = 0$ for some value $\hat{\mu} \leq 1$. Substituting from (10) into (15), this condition is

$$\hat{\mu}(c) \stackrel{\text{def}}{=} \frac{\lambda_2}{\lambda_1 \left(\frac{1+\lambda_0}{(c/\lambda_2)+\gamma} - 1 \right)} \leq 1 \ . \tag{16}$$

By (11) and (13), $\hat{\mu}(c) > 0$. Hence, a first bid of $b_1 \geq b_1^D$ deters investigation for all $\mu \leq \hat{\mu}$.[25]

[25]If $\hat{\mu} > 1$, then $R_2(b_1^D) < 0$, hence FB can deter investigation with a bid of b_1^D for any $\mu \in [0, 1]$. This equilibrium is weakly dominated for FB by an equilibrium with a slightly lower bid that still deters.

Given the constraints (9), (11)–(14) and (16), a family of equilibria is described by the strategies and beliefs I and II above with $0 \leq \mu \leq \hat{\mu}$. The constraints are satisfied by the values $\lambda_0 = 0.62$, $\lambda_1 = 0.18$, $\lambda_2 = 0.2$, $\gamma = 0.6$, and $c = 0.03$ implying $\hat{\mu} = 0.958$.

3.2 Effect of Facilitation

Three features of the equilibrium are worthy of note. First, FB deters investigation both by (i) setting a floor price for the target, and (ii) credibly signalling to SB that v_1 is high.[26] Secondly, when FB with $v_1 = 1$ bids 0, SB investigates and bids 0 if $v_2 = 1$ or 2. SB's bid of zero suffices to convince FB that $v_2 \geq v_1$, hence FB quits rather than incur further bidding costs. So the "bidding war" ends after one round and the target is sold at a price of 0! Therefore, the deterring bid of $1 - \lambda_0$ *exceeds* the price of the target under competitive bidding. Finally, the expected price of the target,

$$[\lambda_1(1-\mu)\cdot 0 + (\lambda_1\mu + \lambda_2)(1-\lambda_0)] = (\lambda_1\mu + \lambda_2)(1-\lambda_0) , \tag{17}$$

increases with μ, *i.e.*, the more frequently FB bids to deter, the higher the expected price of the target.

Having derived the equilibrium, we now consider the effect of lowering the cost of investigation c on the expected price of the target. Since a reduction in c does not affect constraints (9), (11), (12), (14), and weakens constraints (13) and (16), the strategies and beliefs stated in I and II continue to be an equilibrium. Consider first the equilibrium with $\mu = \hat{\mu}$. When c is reduced, this equilibrium no longer exists because at a bid of b_1^D, investigation is now strictly profitable. Hence the surviving equilibria have $\mu < \hat{\mu}$. By (17), this implies that the expected price of the target will be lower.

[26]By contrast, in the model in which each bidder was restricted to at most one bid, FB bid to deter solely to set a floor price. We believe that, in general, both concerns will motivate deterring bids, as illustrated in the present equilibrium.

This reasoning extends to equilibria with $0 < \mu < \hat{\mu}$ in the following way. We assume that given some equilibrium characterized by μ, as c is reduced, the same equilibrium continues to obtain so long as it exists. By (15), $R_2(b_1^D)$ rises as c falls. Let μ^* be the value of μ such that $R_2(b_1^D) = R_2(1-\lambda_0) = 0$ when $c = 0$. Then for each $\mu \in (\mu^*, \hat{\mu})$, there exists some value of $c > 0$ such that $R_2(1-\lambda_0) = 0$, hence if c is reduced below that level, only equilibria with probabilities $\mu' < \mu$ remain. Thus reducing the cost of investigation successively removes equilibria in which the expected price of the target is higher, leaving equilibria with lower expected price.

For $\mu \in [0, \mu^*]$, SB will not investigate for all $c \geq 0$. Therefore the price of the target declines only weakly with c. If, however, we restrict attention to reductions in investigation cost that do promote investigation (by reducing μ), then the expected price increases *strictly*.[27]

Expected social welfare is the expected valuation of the acquiring bidder less the costs of investigation and bidding of both bidders, *i.e.*,

$$\lambda_1\mu + \lambda_1(1-\mu)[-c + \lambda_0 + \lambda_1(1-\gamma) + \lambda_2(2-\gamma)] + 2\lambda_2 \quad . \tag{18}$$

In the example with $\lambda_0 = 0.62$, $\lambda_1 = 0.18$, $\lambda_2 = 0.2$, and $\gamma = 0.6$, beginning with $c = 0.03$ and $\mu = \hat{\mu}(c) = 0.958$, a reduction in c will lower expected social welfare.

4 Implications for Regulation of Takeover Bids

In the legal debate on the time and disclosure provisions of the Williams Act and on the proper conduct of the management of a takeover target, it was assumed that competitive bidding for a takeover target would always lead to a higher price than a single bid that deterred competition.[28] Indeed, this premise was confirmed by Fishman (1988), who modelled competitive takeover bidding as an English auction.

[27]By our convention, SB will not investigate if $R_2(b_1) = 0$. Similar results may be proved with the alternative convention that, when $R_2(b_1) = 0$, SB investigates with positive probability less than one. See Appendix 2.

[28]Easterbrook and Fischel (1981, 1982), Bebchuk (1982a, 1982b, 1986), and Gilson (1981, 1982).

Our analysis shows that if, as is realistic, bidding is costly, competitive bidding may yield a price *lower* than that from a single deterring bid. This implies that the policy prescriptions based on the English-auction model must be qualified. For instance, Bebchuk (1982a, 1982b, 1986) argued that from the standpoint of shareholders of the target, the optimal degree of facilitation of competitive bidding balances the increase in the expected price of a target from competition against the reduction in the expected price resulting from the diminished incentive for first bidders to search for targets *ex-ante*. At a minimum, our suggests that the balance in Bebchuk's analysis must be tilted in the direction of less facilitation.[29]

From the standpoint of social welfare, we should consider the post-acquisition value of the target less the deadweight costs of investigation and bidding. An increase in the second bidder's cost of investigation will lead the first bidder to switch from accommodation to deterrence in some states of the world, thereby tending to raise social welfare in three ways: (i) there will be fewer instances in which a second bidder with valuation greater than the amount of the first bid but less than the first bidder's valuation will acquire the target, (ii) the cost of investigation will be incurred less frequently, and (iii) less investigation by the second bidder implies fewer bidding wars and a possible saving in costs of bidding.[30]

The increase in the cost of investigation, however, may reduce social welfare: (i) the second bidder will incur a higher cost whenever the first bidder accommodates, and (ii) there will more instances in which a first bidder with a lower valuation than the second bidder will acquire the target by a deterring bid. We have shown that, on balance, an increase in the cost of investigation may raise expected social welfare.

[29]We have not analyzed the *ex-ante* investigation decision of the first bidder. By increasing the first bidder's expected profit, a higher cost of investigation for competing bidders will encourage first bidders to investigate. Our arguments against facilitation to the maximum apply, *a fortiori*, when this factor is also taken into account.

[30]The first effect is strong in the basic model and weak in the example of costly bidding. We conjecture, however, that it will arise in a model of costly bidding in which at least two types of first bidder accommodate.

As discussed in the introduction, the cost of investigation to a competing bidder is affected by the delay and disclosure provisions of federal and state law and regulations. The more information that a first bidder discloses, the more will be freely available to a potential competitor. The shorter the time that a bid need remain open, the less time available for a potential competitor to investigate, and hence the greater his reliance on more costly methods of information collection and processing.[31]

The cost of investigation to competing bidders also may be influenced by the management of the target. At one extreme, the management of the target may institute a policy to provide free access to information about itself through regular briefings to analysts and published reports, while at the other, it may adopt a rule not to release any information other than that required by law and regulation.

There is, however, a problem of commitment for the management of a target in fixing the degree of assistance to competing bidders. Once a first bidder has announced his bid, the management of the target has an incentive to assist potential competitors, for that could only increase the price of the target. One solution is to set the cost of investigation by law and regulation, and then restrain the management of targets through a 'rule of managerial passivity'. The drawback of this approach is that the cost of investigation cannot be tailored to the situation of individual targets and their potential bidders.

If the first bidder is the management of the target seeking a buyout, the target will not have this *ex-post* incentive to assist a competing bidder. Surprisingly, this would be in the shareholders' interest insofar as it encourages the first bidder to make a high preemptive bid. The commitment problem thus leads to a bias in favour of management buyouts.

[31]Consistent with our analysis, Jarrell and Bradley (1980) observed that on average, premia in tender offers increased with the enactment of the Williams Act.

5 Concluding Remarks

There is a clear empirical distinction between our analysis and the English-auction models of takeover bidding. Under the auction model, the first bidder bids at a premium only if he seeks to deter investigation by a potential competitor. If the first bidder chooses to let the second bidder investigate, he will bid with zero premium.[32] But in reality, as Spatt (1989) has noted, bids at a substantial premium to market are frequently followed by the entry of competing bidders.

In contrast, our basic model is consistent with this empirical observation. Under the assumption that each bidder is allowed at most one bid, the first bidder will bid at a premium when accommodating investigation in order to increase the likelihood that the second bidder will draw a valuation below the first bid. In the example of costly bidding with voluntary termination, the first bidder with intermediate valuation does bid initially with zero premium when accommodating investigation. However, we conjecture that this is an artifact of the example; if at least two types of first bidder accommodate investigation, then the highest type among these may bid higher to distinguish himself and so avoid the cost of successive rounds of counter-bidding.

In an English auction, all the potential buyers assemble in one place and the auctioneer calls out an increasing price. The bidders might be charged an entrance fee, but they do not incur any additional cost to bid as the price rises. The auction models suggest that when competitive bidding occurs, the price of a takeover target will be the minimum of the bidders' valuations (perhaps less the cost of bidding). In our example with costly bidding, we show that a bidder's willingness to incur the bidding cost *per se* may signal a valuation high enough to persuade the competing

[32]To see this, suppose that the second bidder investigates. Then the price will be bid up to the minimum of the two bidders' valuations. So the probability that the first bidder acquires the target depends only on the relative magnitudes of the first and second bidders' *valuations*. To bid initially with positive premium would merely reduce the first bidder's gain from acquisition if he should acquire the target without providing any countervailing benefit.

bidder to quit. Thus the price at which the target is sold may be well below the minimum of the bidders' valuations less the bidding cost.

We have shown that the English auction is a deficient model of takeover bidding. It may also be inadequate in other settings where buyers naturally arrive in sequence and incur costs to bid, such as the market for real property. Future work should be directed toward a general analysis of competitive bidding in which bidders may counter-bid indefinitely at a fixed cost per bid.

In the context of takeover bidding, such an analysis will provide a more general insight into the relation between the expected takeover price, the cost of investigation, and the cost of bidding. Presumably the cost of bidding is a motivation to bid higher and so avoid the cost of successive rounds of counter-bidding. On the other hand, by discouraging competing bidders, it may allow lower bids to succeed. Furthermore, the cost of bidding is dissipated, reducing the total improvement in value that the target may hope to appropriate. A general analysis should yield a clearer picture of the net outcome of these effects.

Appendix 1: Basic Model—Proofs and Example

Proof of Lemma 1.

- Suppose that, with the given c, the bid $\hat{b}_1(v_1)$ leads SB to investigate. Now $\hat{b}_1(v_1)$ solves the problem (4) without the constraint that the bid lead SB to investigate. So if $\hat{b}_1(v_1)$ leads to investigation, it must be the optimal accommodating bid.

- Suppose that the bid $\hat{b}_1(v_1)$ deters investigation. Then FB's profit from this bid will be $v_1 - \hat{b}_1(v_1)$. By (3), the definition of $b_1^D(c)$,

$$v_1 - b_1^D(c) \geq v_1 - \hat{b}_1(v_1) \geq [v_1 - \hat{b}_1(v_1)] \ \Pr[v_2 \leq \hat{b}_1(v_1)],$$

since $1 \geq \Pr[v_2 \leq \hat{b}_1(v_1)]$. But by (7), the definition of $\hat{b}_1(v_1)$,

$$[v_1 - \hat{b}_1(v_1)] \ \Pr[v_2 \leq \hat{b}_1(v_1)] \geq (v_1 - b_1) \ \Pr(v_2 \leq b_1),$$

for all $b_1 \geq 0$.

Therefore $v_1 - b_1^D(c) \geq (v_1 - b_1) \ \Pr(v_2 \leq b_1)$, for all $b_1 \geq 0$ that lead SB to investigate, hence by (6), FB prefers to bid $b_1^D(c)$, *i.e.*, $v_1 \notin I(c)$. ∎

Proof of Proposition.

- We first show that a reduction of SB's cost of investigation from $\underline{c}$ to $\underline{c} - \Delta c$ will reduce the expected price, p. By construction of $\underline{c}$, the set $D(\underline{c} - \Delta \underline{c}) = \emptyset$, hence by (8), the effect on p will be

$$\begin{aligned} \Delta p &= \textstyle\sum_{I(\underline{c}-\Delta c)} b_1^I(v_1, \underline{c} - \Delta c) \ \lambda_{v_1} \\ &\quad - \textstyle\sum_{I(\underline{c})} b_1^I(v_1, \underline{c}) \ \lambda_{v_1} - \sum_{D(\underline{c})} b_1^D(\underline{c}) \ \lambda_{v_1} \\ &= \textstyle\sum_{I(\underline{c})} [b_1^I(v_1, \underline{c} - \Delta c) - b_1^I(v_1, \underline{c})] \ \lambda_{v_1} \\ &\quad + \textstyle\sum_{D(\underline{c})} [b_1^I(v_1, \underline{c} - \Delta c) - b_1^D(\underline{c})] \ \lambda_{v_1}. \end{aligned} \tag{19}$$

By Lemma 2, if $v_1 \in I(\underline{c})$, then $v_1 \in I(\underline{c} - \Delta c)$ also, and by Lemma 1, for such v_1, $b_1^I(v_1, \underline{c} - \Delta c) = \hat{b}_1(v_1) = b_1^I(v_1, \underline{c})$. Substituting in (19),

$$\Delta p = \sum_{D(\underline{c})} [b_1^I(v_1, \underline{c}) - b_1^D(\underline{c})] \ \lambda_{v_1}.$$

But by (5), $b^I(v_1,\underline{c}) < b_1^D(\underline{c})$, thus, $\Delta p < 0$.

• We next show that if the cost of investigation, $c < \underline{c}$, then any reduction in the cost to $c - \Delta c$ will leave the expected price unchanged. By definition of $\underline{c}$, for all $\Delta c > 0$, $I(c - \Delta c) = I(c) = \{v_1 | v_1 \geq 0\}$ and $D(c - \Delta c) = D(c) = \emptyset$, hence, by (8), the effect on p will be

$$\Delta p = \sum_{I(\underline{c}-\Delta c)} b_1^I(v_1, \underline{c} - \Delta c)\ \lambda_{v_1} - \sum_{I(\underline{c})} b_1^I(v_1, \underline{c})\ \lambda_{v_1}.$$

By Lemma 1, for $v_1 \in I(c - \Delta c) = I(c)$, $b_1^I(v_1, c - \Delta c) = \hat{b}_1(v_1) = b_1^I(v_1, c)$, and thus $\Delta p = 0$.

• Finally, we show that $p(\underline{c}) > 0$. For $c < \underline{c}$, $I(c) = \{v_1 | v_1 \geq 0\}$. Hence by (8) and Lemma 1,

$$p(c) = \sum_{I(\underline{c})} b_1^I(v_1, \underline{c})\ \lambda_{v_1} = \sum_{I(\underline{c})} \hat{b}_1(v_1)\ \lambda_{v_1} \geq 0.$$

Therefore, by the first and second steps of this proof, $p(\underline{c}) > 0$. ∎

Example: Increase in c Raises Social Welfare

Let $\lambda_{v_1} > 0$, for $i = -2, 0, 1, 2$ only, with $\lambda_{-2} + \lambda_0 + \lambda_1 + \lambda_2 = 1$. By (1), we require that $E(v_2) < 0$, *i.e.*,

$$2\lambda_{-2} > \lambda_1 + 2\lambda_2 \quad . \tag{20}$$

If $c = 0$, FB accommodates in all states $v_1 \geq 0$, hence expected social welfare as a function of c is

$$W(0) = \sum_{v_1 \geq 0} \left\{ v_1 \Pr[v_2 \leq \hat{b}_1(v_1)] + \sum_{v_2 > \hat{b}_1(v_1)} v_2 \lambda_{v_2} \right\} \lambda_{v_1} \quad .$$

Consider an increase to $c > 0$ just sufficient to induce FB with $v_1 = 2$ to deter. For such c,

$$W(c) = \bar{v}\lambda_{\bar{v}} + \sum_{\bar{v} > v_1 \geq 0} \left\{ -c + v_1 \Pr[v_2 \leq \hat{b}_1(v_1)] + \sum_{v_2 > \hat{b}_1(v_1)} v_2 \lambda_{v_2} \right\} \lambda_{v_1} \quad .$$

The increase in c results in a change in expected welfare of

$$\Delta W \stackrel{\text{def}}{=} W(c) - W(0) = -c \ \Pr[\bar{v} > v_1 \geq 0] + \lambda_{\bar{v}} \left[\sum_{v_2 > \hat{b}_1(\bar{v})} (\bar{v} - v_2) \lambda_{v_2} \right] . \tag{21}$$

Assume that

$$\lambda_{-2} + \lambda_0 > \lambda_1 \ , \tag{22}$$

then the accommodating bid for FB with $v_1 = 2$ is

$$\hat{b}_1(2) = 0. \tag{23}$$

By (6), since FB with $v_1 = 2$ just prefers to deter, $2 - b_1^D = [2 - \hat{b}_1(2)] \ \Pr[v_2 \leq \hat{b}_1(2)]$, *i.e.*, $b_1^D = 2(\lambda_1 + \lambda_2)$. Let

$$\lambda_1 + \lambda_2 > \frac{1}{2} \ , \tag{24}$$

then $b_1^D > 1$.

By construction, c is just large enough to deter investigation, *i.e.*, by (2),

$$R_2(b_1^D) = -c + \sum_{v_2 > b_1^D} (v_2 - b_1^D) \lambda_{v_2} = -c + [2 - 2(\lambda_1 + \lambda_2)] \, \lambda_2 = 0 \, ,$$

so $c = 2\lambda_2(\lambda_{-2} + \lambda_0)$. Substituting for c and from (23) in (21),

$$\Delta W = \lambda_2 \left[-2(\lambda_{-2} + \lambda_0)(\lambda_0 + \lambda_1) + \lambda_1 \right].$$

Therefore expected social welfare will increase if

$$\lambda_1 > 2(\lambda_{-2} + \lambda_0)(\lambda_0 + \lambda_1) \ . \tag{25}$$

Conditions (20), (22), (24), and (25) are satisfied by $\lambda_{-2} = 0.39, \lambda_0 = 0.05, \lambda_1 = 0.38$, and $\lambda_2 = 0.18$.

Appendix 2: Equilibria in the Example of Costly Bidding with Voluntary Termination

In Section 3, we described a family of mixed strategy equilibria in which FB (first bidder) with $v_1 = 1$ randomizes between a low accommodating bid of 0 and a high deterring bid of $b_1^D = 1 - \lambda_0$. In this appendix, we define a refinement that directly extends the Intuitive Criterion of Cho and Kreps (1987) to our setting in which each bidder potentially can move any number of times. We next show that the equilibrium described in the main text survives this refinement. Finally, we describe three other families of equilibria that arise within the parameter ranges defined by conditions (9), (11)–(14) and (16), and show that (a) two families are eliminated by the Intuitive Criterion, and (b) the main results of the paper also hold in the third.

Essentially, the Intuitive Criterion, as applied to some equilibrium of a game of one move by each player, requires that if a defection from the equilibrium is not profitable (compared to the equilibrium payoff) for a first player of type t' under any inference by the next player, then the second player who observes such a defection should infer that the first player is not of type t'. If the defection is profitable for the first player of some type t under some inference by the second player which does not place any weight on t', then the equilibrium outcome must be ruled out.

In our setting, we begin by defining a defection to be the first move taken by a bidder that has zero probability of being made by that bidder (*regardless* of his type) at that stage of the game along any equilibrium path. Consider some defection by a given bidder (the defecting bidder), i: let the posterior beliefs of the next bidder, j, at that point be that the defecting bidder has $v_i = 0, 1, 2$ with likelihood $\beta = (\beta_0, \beta_1, \beta_2)$ with $\beta_0 + \beta_1 + \beta_2 = 1$. Given the defection and these beliefs, we consider subsequent strategies and beliefs of the two bidders that form a perfect Bayesian Equilibrium from the defection onward.

A perfect Bayesian equilibrium for the entire game satisfies the extended Intuitive Criterion if it is not ruled out by the following procedure. If for one or more types t' of the defecting bidder there does not exist any belief and subgame equilibrium such that the defection is preferable to the equilibrium payoff, then the next bidder must infer that the defecting bidder is not of type t'. If for some type t there exists a belief β that puts zero weight on type(s) t' and a subsequent subgame equilibrium such that the defection is preferable, then the equilibrium must be ruled out.[33] The procedure must be applied to every possible defection.[34]

We now show that the equilibrium in Section 3 of the main text satisfies our extension of the Intuitive Criterion. We first consider defections by FB in the first round of bidding. Since FB must incur a cost γ to bid, FB with $v_1 = 0$ will never profit by defecting from equilibrium. In equilibrium, both FB with $v_1 = 1$ and FB with $v_1 = 2$ can deter investigation and acquire the target for sure by bidding b_1^D. Hence, regardless of the subsequent beliefs of SB, a defection to a bid $b_1 > b_1^D$ is unprofitable for FB with both $v_1 = 1$ and $v_1 = 2$. Thus, if FB bids $b_1 > b_1^D$ initially, SB's inference that $v_1 = 2$ satisfies our refinement.

Consider a defection to a bid $b_1 \in (0, b_1^D)$ in the first round. Suppose that such a bid leads SB to infer that he faces FB with $v_1 = 2$. Then SB would not investigate and FB would acquire the target for sure. Given such an inference by SB, FB with $v_1 = 2$ prefers to defect than make the equilibrium bid of b_1^D. Furthermore, if FB with $v_1 = 1$ were to defect, he would enjoy profit of $-\gamma + 1 - b_1$, while his expected profit from his equilibrium strategy is

$$-\gamma + \lambda_0 < -\gamma + 1 - b_1,$$

since $b_1 < b_1^D = 1 - \lambda_0$. Since both types of FB would make this defection under some beliefs by SB, our extension of the Intuitive Criterion does not rule out the

[33]For purposes of this definition, a SB will be treated as a given type even if he has not yet learned his valuation. In consequence, the definition allows for defections by investigating and bidding when not called for in equilibrium.

[34]The original Cho and Kreps analysis allows iterated pruning of beliefs and message pairs. Since iteration is not essential for our purposes, we simplify the definition by allowing only a single stage of pruning.

equilibrium outcome or the beliefs that if FB bids $b_1 \in (0, b_1^D)$ initially, then SB infers $v_1 = 1$.

Next, consider a defection by FB in which he bids 0 initially, and then if SB investigates and matches (the equilibrium response for SB with $v_2 = 1$ or 2), FB counter-bids b_1'.[35] Obviously, this defection is unprofitable for FB with $v_1 = 0$. We will show that this defection cannot be profitable for FB with $v_1 = 2$. Let SB infer that he faces FB with $v_1 = 2$: this belief maximizes the profits to FB from the defection and hence leads SB to bid the most conservatively, to the defecting FB's advantage. In addition, we focus on $b_1 = 0$, as this defection maximizes FB's profits under the given inference by SB.

In the following subgame, FB bids 0 and incurs γ initially. Since this bid is an equilibrium move by FB with $v_1 = 1$, SB will follow the equilibrium, *i.e.*, investigate and if $v_2 = 0$, SB will quit and FB will acquire the target. If, however, $v_2 = 1$ or 2, SB will match with a bid of 0. Then, according to the defection, FB will counter-bid b_1', incurring γ again, and SB will infer that $v_1 = 2$, hence SB will quit if $v_2 = 1$, or close the gap if $v_2 = 2$. Thus, for FB with $v_1 = 2$, the expected profit from the defection is

$$-\gamma + 2\lambda_0 + (1 - \lambda_0)\left(-\gamma + \frac{2\lambda_1}{\lambda_1 + \lambda_2}\right) = -\gamma + 2\lambda_0 - (1 - \lambda_0)\gamma + 2\lambda_1.$$

His profit from his equilibrium strategy is $-\gamma + 1 + \lambda_0$, so his gain from defection is

$$\lambda_0 - (1 - \lambda_0)\gamma + 2\lambda_1 - 1 = \lambda_1 - \lambda_2 - (1 - \lambda_0)\gamma \ < \ 0,$$

by (12). Therefore, FB with $v_1 = 2$ will never defect. Consequently, our extension of the Intuitive Criterion will not rule our proposed beliefs by SB that FB making such a defection has $v_1 = 1$.

[35]If FB has $v_1 = 1$, such an initial bid is not a defection, but the counter-bid is. If FB has $v_1 = 2$, then such an initial bid is a defection, but it also is a move that can occur with positive probability on an equilibrium path (by FB with $v_1 = 1$), and hence is treated by SB as an equilibrium move; it is only when he counter-bids b_1' that SB recognizes a defection.

We next consider defections made by SB. We will show that SB will not defect, regardless of SB's type and of the inferences drawn by FB. The first possible defection by SB is if after FB bids b_1^D, SB investigates and then bids. Because of the cost of bidding, SB with $v_2 = 0$ cannot profit from bidding, so FB will infer either $v_2 = 1$ or 2. At that stage, let FB infer that he faces SB with $v_2 = 2$: this belief maximizes the profits to SB from the defection and hence leads FB to bid the most conservatively. Hence if $v_1 = 1$, FB will quit, while if $v_1 = 2$, FB will close the gap by bidding $2 - \gamma$.

This subgame is identical to that posed in the argument leading to conditions (14), (15), and (16). By that argument, SB with $v_2 = 1$ will not bid. Suppose that SB with $v_2 = 2$ bids some $b_2 \geq b_1^D$, then SB's expected return from investigation,

$$R_2(b_1) = -c + \lambda_2 \left[-\gamma + \frac{\mu\lambda_1}{\mu\lambda_1 + \lambda_2}(2 - b_2) \right] \leq -c + \lambda_2 \left[-\gamma + \frac{\mu\lambda_1}{\mu\lambda_1 + \lambda_2}(2 - b_1) \right] = 0,$$

by (15) and (16). Therefore, SB will not defect.

The second possible defection by SB is if FB with $v_1 = 1$ bids 0 initially, SB investigates, and then bids $b_2 > 0$ rather than match. Again, let FB have beliefs that maximize the profits to SB from the defection, *i.e.*, infer that he faces SB with $v_2 = 2$. Under these beliefs, FB with $v_1 = 1$ will quit. But, along the equilibrium, SB would bid $b_2 = 0$ and FB would quit as well. Thus, the defection clearly is inferior for SB with both $v_2 = 1$ and 2.

In summary, defection is unprofitable for SB at the point of investigation, and also after investigation, regardless of his type. So the beliefs proposed in the equilibrium of Section 3 cannot be ruled out, and therefore, the equilibrium satisfies our extension of the Intuitive Criterion as defined above.

We next describe three alternative families of equilibrium. We will eliminate the first two using the Intuitive Criterion; we then show that the main results of the paper concerning investigation cost, expected price of the target and social welfare apply for the third family of equilibria.

Family 1: Pure Strategy Separating Equilibria

There are two cases. In the first, FB with $v_1 = 1$ does not bid and FB with $v_1 = 2$ does bid. But if FB with $v_1 = 1$ bids zero, and even if SB makes the adverse inference that $v_1 = 1$, so that SB investigates, and quits only if $v_2 = 0$, FB will make a profit of $\lambda_0 - \gamma > 0$ by (11). Hence this case is not an equilibrium.

In the second case, FB with $v_1 = 1$ bids 0, and FB with $v_1 = 2$ bids some amount b_1^D which deters. Then if FB bids 0, SB will infer $v_1 = 1$, and accordingly, SB will quit if $v_2 = 0$, or match if $v_2 = 1$ or 2; hence FB gains $\lambda_0 - \gamma$. If $b_1^D < 1 - \lambda_0$, FB with $v_1 = 1$ would prefer to deter and gain $1 - b_1^D - \gamma > \lambda_0 - \gamma$, so this case is not an equilibrium. The case of $b_1^D = 1 - \lambda_0$ is a special case of the family of equilibria analyzed in the text with $\mu = 0$. Thus the remaining equilibria have $b_1^D > 1 - \lambda_0$ and are described as follows:

I. FB:

$v_1 = 0$: Never bid.

$v_1 = 1$: Bid 0; if SB bids, infer $v_2 = 1$ or 2, and quit.

$v_1 = 2$: Bid $b_1^D > 1 - \lambda_0$.

II. SB:

If $b_1 \geq b_1^D$, infer $v_1 = 2$, and do not investigate.

If $b_1 < b_1^D$, infer $v_1 = 1$ and investigate.

$v_2 = 0$: Quit.

$v_2 = 1$ or 2: Match the first bid.[36]

In this equilibrium, since $b_1^D > 1 - \lambda_0$, by (11), FB with $v_1 = 1$ strictly prefers to bid 0 over b_1^D. Consider a defection by FB to a bid slightly below b_1^D. FB with $v_1 = 1$ would still prefer to bid 0 even if the bid slightly below b_1^D always deterred investigation (as would occur if SB inferred that $v_1 = 2$). Accordingly, by the

[36]We do not specify SB's beliefs at subsequent stages of the game because we shall show that regardless of these, the equilibrium is ruled out.

Intuitive Criterion, SB must infer that FB has $v_1 = 2$, and the lower bid will still deter. Given these beliefs by SB, FB with $v_1 = 2$ would defect to the bid slightly lower than b_1^D, which rules out the equilibrium.

Family 2: Pure Strategy Pooled Equilibria

There are two cases. In the first, FB with $v_1 = 1$ or 2 pool at a deterring bid $b_1^D \neq 1 - \lambda_0$. (The case of $b_1^D = 1 - \lambda_0$ is treated in the main text as the special case of $\mu = 1$.) If $b_1^D > 1 - \lambda_0$, by the same reasoning as applied at the start of our analysis of the previous family, FB with $v_1 = 1$ would prefer to bid 0 even if SB makes the adverse inference that $v_1 = 1$ and investigates, than to bid b_1^D even if that deters with certainty. Hence this case is not an equilibrium.

Suppose that $b_1^D < 1 - \lambda_0$. Then, by (14), SB's expected return to investigation in the proposed equilibrium is precisely (15) with $\mu = 1$, *i.e.*,

$$
\begin{aligned}
R_2(b_1^D) &= -c + \lambda_2 \left[-\gamma + \frac{\lambda_1}{\lambda_1 + \lambda_2}(2 - b_1^D) \right] \\
&> -c + \lambda_2 \left[-\gamma + \frac{\hat{\mu}\lambda_1}{\hat{\mu}\lambda_1 + \lambda_2}(2 - (1 - \lambda_0)) \right] = 0,
\end{aligned}
$$

as $R_2(b_1^D)$ is increasing in μ and decreasing in b_1. Since $R_2(b_1^D) > 0$, SB will investigate, hence this case is not an equilibrium.

In the second case, FB with $v_1 = 1$ and 2 pool at an accommodating bid.

I. FB:

$v_1 = 0$: Never bid.

$v_1 = 1$: Bid 0; if SB bids, infer $v_2 = 2$, quit.

$v_1 = 2$: Bid 0; if SB bids, infer $v_2 = 2$, bid $2 - \gamma$ to close the gap.

II. SB:

If first bid is made, infer $v_1 = 1$ or 2, with sufficiently high probability weight on 1, and investigate.

$v_2 = 0, 1$: Quit.

$v_2 = 2$: Match the first bid.[37]

Consider a defection by FB with $v_1 = 2$ to a bid of $1 - \lambda_0 + \epsilon$, where $\epsilon > 0$. From the calculation leading to (10) and (11), even if such a bid always deterred investigation, FB with $v_1 = 1$ would still prefer a bid of 0. Hence, by the Intuitive Criterion, if FB were to bid $1 - \lambda_0 + \epsilon$, SB cannot believe that $v_1 = 1$, and so must infer that $v_1 = 2$. Under this belief, if FB with $v_1 = 2$ defected, SB would quit, hence, FB's profit would be $1 + \lambda_0 - \epsilon - \gamma > -\gamma + 2(\lambda_0 + \lambda_1)$, by (12), if ϵ is sufficiently small. Thus FB would defect, which rules out the pooled equilibrium.

Family 3: Equilibria With Randomized Investigation

In these equilibria, FB behaves as described in the main text. If FB bids 0, SB investigates, while if FB makes the higher pooled bid, which we now label b_1^p, SB investigates with probability $\theta > 0$. The case of $\theta = 0$ is treated in the main text.

I. FB:

$v_1 = 0$: Never bid.

$v_1 = 1$: Bid 0 with probability $1 - \mu$ or bid b_1^p with probability μ.

After any first bid, if SB bids, infer $v_2 = 1$ or 2, and quit.

$v_1 = 2$: Bid b_1^p.

After a bid of b_1^p, if SB bids, infer $v_2 = 2$, and close the gap by counter-bidding $2 - \gamma$.

II. SB:

If first bid is at least b_1^p, infer $v_1 = 1$ or 2, and investigate with probability $\theta > 0$.

$v_2 = 0$ or 1 : Quit.

$v_2 = 2$: Match the first bid.

If first bid is below b_1^p, infer $v_1 = 1$, and investigate. If in any later round, FB bids below b_1^p, infer $v_1 = 1$, while if FB bids at or above b_1^p, infer $v_1 = 2$.

[37]We do not specify SB's beliefs at subsequent stages of the game because we shall show that regardless of these, the equilibrium is ruled out.

$v_2 = 0$: Quit.

$v_2 = 1$ or 2: Match the first bid.

To characterize the equilibrium, first note that FB with $v_1 = 1$ must be indifferent between bidding 0 and bidding b_1^p, *i.e.*,

$$(1-0)\lambda_0 - \gamma = (1 - b_1^p - \gamma)(1-\theta) + (1 - b_1^p - \gamma)\theta(\lambda_0 + \lambda_1) - \gamma\theta\lambda_2,$$

or

$$\lambda_0 = (1 - b_1^p)[(1-\theta) + \theta(\lambda_0 + \lambda_1)] = (1 - b_1^p)(1 - \theta\lambda_2).$$

Hence

$$b_1^p = 1 - \frac{\lambda_0}{1 - \theta\lambda_2}, \tag{26}$$

and

$$\theta = \frac{1}{\lambda_2}\left(1 - \frac{\lambda_0}{1 - b_1^p}\right). \tag{27}$$

If $\theta = 1$, then $b_1^p = 1 - \frac{\lambda_0}{1-\lambda_2}$, while if $\theta = 0$, then $b_1^p = 1 - \lambda_0$. These are the minimum and maximum values of b_1^p.

SB must be indifferent between investigating and not,

$$0 = -c + (\lambda_0 + \lambda_1)(0) + \lambda_2\left[(2 - b_1^p)\frac{\mu\lambda_1}{\mu\lambda_1 + \lambda_2} + 0 - \gamma\right],$$

or

$$2 - b_1^p = (c + \gamma\lambda_2)\frac{\mu\lambda_1 + \lambda_2}{\mu\lambda_1\lambda_2},$$

hence

$$\mu(c, b_1^p) = \frac{\lambda_2}{\lambda_1\left[\frac{2 - b_1^p}{c/\lambda_2 + \gamma} - 1\right]}. \tag{28}$$

Given b_1^p, the equilibrium value $\mu(c, b_1^p)$ falls as c falls, *i.e.*, is increasing in c.

The equilibrium value $\mu(c, b_1^p)$ is increasing in b_1^p, so the lowest $b_1^p = 1 - \frac{\lambda_0}{1-\lambda_2}$ implies the lowest $\mu = \mu^*(c)$. Also, since $b_1^p \leq 1 - \lambda_0$,

$$\mu(c, b_1^p) \leq \mu(c, 1 - \lambda_0) \equiv \hat{\mu}(c, b_1^D),$$

or $\hat{\mu}(c)$ of (16) in the main text.

We now show that in this family, as in the equilibrium of the main text, a reduction in the cost of investigation may *lower* both the expected price of the target and expected social welfare. The expected price is

$$\begin{aligned} p &= \lambda_1\{(1-\mu)0+\mu b_1^p\}+\lambda_2\{[1-\theta+\theta(1-\lambda_2)]b_1^p+\theta\lambda_2(2-\gamma)\} \\ &= [\lambda_1\mu+\lambda_2(1-\theta\lambda_2)]b_1^p+\theta\lambda_2^2(2-\gamma). \end{aligned} \tag{29}$$

The mixed-strategy equilibrium of Family 3 is defined by two conditions, (26) and (27), and three variables, and hence there remains one degree of freedom. We follow the approach of the main text; beginning from any equilibrium with particular μ, b_1^p, and θ, as c is lowered, we keep to the given level of μ so long as this remains consistent with equilibrium. It follows by (28) that b_1^p rises, and so by (26) that θ falls. Eventually, for sufficiently low c, there exists no equilibrium with the given value of μ. We show that at this point, a further reduction in c causes the expected price to drop.

Consider $\mu(c, b_1^p)$ as a function of b_1^p parameterized by c. By (28), a reduction in c for given b_1^p reduces μ, while $\mu(c, b_1^p)$ is increasing for given c in b_1^p. It follows that c parameterizes a family of upward sloping schedules for μ in terms of b_1^p, with a reduction in c leading to a lower schedule. So as c is lowered, we reach a point where there just exists an equilibrium with the given μ; that equilibrium corresponds to the highest value of b_1^p. By (29), any further reduction in c, holding b_1^p and so θ constant, lowers the expected price as it reduces μ, the probability of a high first bid. It is also possible that with lower c, we shift to an equilibrium along the $\mu(c, b_1^p)$ schedule with *lower* b_1^p. But we shall show that along this schedule, *i.e.*, for given c, the expected price falls as b_1^p falls, thus the new equilibrium must have lower expected price.

We first show how the expected price behaves as c is reduced with μ held constant, and thus as b_1^p increases and θ falls. Substitute for b_1^p in terms of θ in (29),

$$\begin{aligned} p &= \lambda_1\mu b_1^p+\lambda_2(1-\theta\lambda_2)b_1^p+\theta\lambda_2^2(2-\gamma) \\ &= \lambda_1\mu\left(1-\frac{\lambda_0}{1-\theta\lambda_2}\right)+\lambda_2(1-\theta\lambda_2-\lambda_0)+\theta\lambda_2^2(2-\gamma). \end{aligned}$$

Differentiating,

$$\frac{\partial p}{\partial \theta} = \lambda_1\mu(-\lambda_0)[-(1-\theta\lambda_2)^{-2}(-\lambda_2)] - \lambda_2^2 + \lambda_2^2(2-\gamma)$$

or

$$\frac{1}{\lambda_2}\frac{\partial p}{\partial \theta} = -\frac{\lambda_0\lambda_1\mu}{(1-\theta\lambda_2)^2} + (1-\gamma)\lambda_2.$$

Thus $\frac{\partial p}{\partial \theta}$ may be positive or negative.

We next consider how the expected price behaves as μ or b_1^p is increased with c held constant (a movement to the right along the $\mu(c, b_1^p)$ schedule). From (29),

$$\begin{aligned}
\frac{p}{\lambda_2} &= \left[\frac{1}{\frac{2-b_1^p}{c/\lambda_2+\gamma}-1} + (1-\theta\lambda_2)\right] b_1^p + \theta\lambda_2(2-\gamma) \\
&= \left[\frac{c/\lambda_2+\gamma}{2-b_1^p-c/\lambda_2-\gamma} + 1 - \theta\lambda_2\right] b_1^p + \theta\lambda_2(2-\gamma) \\
&= \frac{c/\lambda_2+\gamma}{1+\frac{\lambda_0}{1-\theta\lambda_2}-\frac{c}{\lambda_2}-\gamma}\left[1-\frac{\lambda_0}{1-\theta\lambda_2}\right] + (1-\lambda_0-\theta\lambda_2) + \theta\lambda_2(2-\gamma) \\
&= \frac{c+\gamma\lambda_2}{\lambda_0\lambda_2+(\lambda_2-c-\gamma\lambda_2)(1-\theta\lambda_2)}(1-\lambda_0-\theta\lambda_2) + (1-\lambda_0) + \theta\lambda_2(1-\gamma).
\end{aligned}$$

Let

$$\begin{aligned}
F(\theta) &\stackrel{\text{def}}{=} \lambda_0\lambda_2 + (\lambda_2-c-\gamma\lambda_2)(1-\theta\gamma_2), \\
G(\theta) &\stackrel{\text{def}}{=} (c+\gamma\lambda_2)(1-\lambda_0-\theta\lambda_2).
\end{aligned}$$

Then

$$\frac{1}{\lambda_2}\frac{dp}{d\theta} = \frac{F(\theta)G'(\theta)-F'(\theta)G(\theta)}{[F(\theta)]^2} + \lambda_2(1-\gamma).$$

Now

$$F(\theta)G'(\theta) = [\lambda_0\lambda_2 + (\lambda_2-c-\gamma\lambda_2)(1-\theta\lambda_2)][-\lambda_2(c+\gamma\lambda_2)],$$

and

$$\begin{aligned}
-F'(\theta)G(\theta) &= -[-\lambda_2(\lambda_2-c-\gamma\lambda_2)](c+\gamma\lambda_2)(1-\lambda_0-\theta\gamma_2) \\
&= (\lambda_2-c-\gamma\lambda_2)(1-\lambda_0-\theta\lambda_2)[\lambda_2(c+\gamma\lambda_2)].
\end{aligned}$$

Hence

$$\begin{aligned} A \equiv F(\theta)G'(\theta) - F'(\theta)G(\theta) &= -\lambda_0\lambda_2^2(c+\gamma\lambda_2) - \lambda_0(\lambda_2 - c - \gamma\lambda_2)\lambda_2(c+\gamma\lambda_2) \\ &= -\lambda_0\lambda_2(c+\gamma\lambda_2)[\lambda_2 + (\lambda_2 - c - \gamma\lambda_2)] \\ &= -\lambda_0\lambda_2(c+\gamma\lambda_2)(2\lambda_2 - c - \gamma\lambda_2) \ < 0, \end{aligned}$$

since $\lambda_2 > c$, where A is independent of θ. Furthermore,

$$\lambda_2(1-\gamma)\ [F(\theta)]^2 = \lambda_2(1-\gamma)\ [\lambda_0\lambda_2 + (\lambda_2 - c - \gamma\lambda_2)(1-\theta\lambda_2)]^2.$$

Let

$$H(\theta) \stackrel{\text{def}}{=} A + \lambda_2(1-\gamma)[F(\theta)]^2. \tag{30}$$

Then

$$\frac{1}{\lambda_2}\frac{dp}{d\theta} = \frac{H(\theta)}{[F(\theta)]^2},$$

and $\frac{dp}{d\theta} < 0$ if and only if $H(\theta) < 0$.

By definition of $\hat{\mu}$, $\hat{\mu} \leq 1$ implies

$$\lambda_2(c+\gamma\lambda_2) \leq \lambda_1[\lambda_2(1+\lambda_0) - (c+\gamma\lambda_2)]$$

or

$$(1-\lambda_0)(c+\gamma\lambda_2) \leq \lambda_1\lambda_2(1+\lambda_0),$$

hence

$$c \ \leq \ \frac{\lambda_1\lambda_2}{1-\lambda_0}(1+\lambda_0) - \gamma\lambda_2 = \lambda_2\left[\frac{\lambda_1}{1-\lambda_0}(1+\lambda_0) - \gamma\right].$$

Now by (12), $\lambda_1 < \lambda_2$, hence $\lambda_1\lambda_0 < \lambda_2$, which implies that $\lambda_1(1+\lambda_0) < 1 - \lambda_0 = \lambda_1 + \lambda_2$, hence

$$\frac{\lambda_1}{1-\lambda_0}(1+\lambda_0) < 1.$$

Thus,

$$\lambda_2\left[\frac{\lambda_1}{1-\lambda_0}(1+\lambda_0) - \gamma\right] < \lambda_2(1-\gamma),$$

and

$$c < \lambda_2(1-\gamma). \tag{31}$$

Considering $F(\theta)$, by (31), $F(\theta)$ will be decreasing in θ, *i.e.*, $\theta = 0$ implies the largest $F(\theta)$, and hence the largest $H(\theta)$. Now if $\max_\theta H(\theta) < 0$, then for all other θ, $H(\theta) < 0$, and hence $\frac{dp}{d\theta} < 0$. At $\theta = 0$, by (30),

$$\frac{H(0)}{\lambda_2} = -\lambda_0(c + \gamma\lambda_2)(2\lambda_2 - c - \gamma\lambda_2) + (1-\gamma)\ [(1 + \lambda_0 - \gamma)\lambda_2 - c]^2. \qquad (32)$$

Taking first and second derivatives with respect to c, we find that the right-hand side is decreasing in c provided

$$c < \frac{1 + 2\lambda_0 - \gamma}{1 + \lambda_0 - \gamma}\ \lambda_2(1-\gamma),$$

which condition is implied by (31). Hence, $H(0)$ is decreasing with c in the relevant range. To ensure that $H(0) < 0$, we impose the constraint that the right-hand side of (32) be negative at $c = 0$, *i.e.*,

$$\gamma\lambda_0(2-\gamma) > (1-\gamma)(1 + \lambda_0 - \gamma)^2 \qquad (33)$$

Then $H(0) < 0$ for all c in the relevant range, and hence, the expected price of the target increases with b_1^p along the schedule $\mu(c, b_1^p)$. Condition (33) is satisfied by the numerical example given in Section 3 of the main text, $\lambda_0 = 0.62$, $\lambda_1 = 0.18$, $\lambda_2 = 0.2$, and $\gamma = 0.6$.

To summarize, starting with a given μ, as c is lowered, b_1^p will rise and the expected price may rise or fall. Eventually, c will fall to a point such that $b_1^p = 1 - \lambda_0$. Then a further reduction in c implies that only equilibria with lower μ will exist. All of these will have a lower expected price than the equilibrium with the original μ and $b_1^p = 1 - \lambda_0$.

We next show that a reduction in c may lower social welfare as well. The expected value of the target to the acquirer less total deadweight costs is

$$\begin{aligned} W &= \lambda_1\{\ -\gamma + (1-\mu)[-c + \lambda_0 \cdot 1 + \lambda_1(1-\gamma) + \lambda_2(2-\gamma)] \\ &\quad +\mu\ [(1-\theta)1 + \theta(-c + (\lambda_0 + \lambda_1)1 + \lambda_2(2-\gamma)\ )\ ]\ \} \\ &\quad +\lambda_2\{\ -\gamma + (1-\theta)2 + \theta\ [-c + (\lambda_0 + \lambda_1)2 + \lambda_2(2 - 2\gamma)\ ]\ \} \\ &= -(\lambda_1 + \lambda_2)\gamma + \lambda_1[1 + \lambda_2 - (\lambda_1 + \lambda_2)\gamma - c] + 2\lambda_2 \\ &\quad +\lambda_1\mu\ [\theta(\lambda_2 - \gamma\lambda_2 - c) - \lambda_2 + (\lambda_1 + \lambda_2)\gamma + c] - \theta\lambda_2(c + 2\gamma\lambda_2). \end{aligned}$$

Thus, increasing b_1^p for given c along the $\mu(c, b_1^p)$ schedule, the effect on W is

$$\frac{\partial W}{\partial \theta} = \lambda_1 \frac{\partial \mu}{\partial \theta}[\theta(\lambda_2 - \gamma\lambda_2 - c) - \lambda_2 + (\lambda_1 + \lambda_2)\gamma + c] + \lambda_1\mu(\lambda_2 - \gamma\lambda_2 - c) - \lambda_2(c + 2\gamma\lambda_2).$$

By (31), the coefficient of $\partial\mu/\partial\theta$ is increasing in θ. At $\theta = 0$, the coefficient is

$$\lambda_1[\lambda_1\gamma - (1-\gamma)\lambda_2 + c].$$

Suppose that

$$(1 - \lambda_0)\gamma + c > \lambda_2. \tag{34}$$

then $\lambda_1\gamma + c > (1-\gamma)\lambda_2$, and the coefficient of $\partial\mu/\partial\theta$ will be positive for all θ. Since $\partial\mu/\partial\theta < 0$, the first term of $\partial W/\partial\theta$ is negative.

The third term of $\partial W/\partial\theta$ is independent of μ. Hence, by (31), the sum of the second and third terms of $\partial W/\partial\theta$ is increasing in μ. At $\mu = 1$, this sum is $\lambda_1(\lambda_2 - \gamma\lambda_2 - c) - \lambda_2(c + 2\gamma\lambda_2)$. Hence if

$$\lambda_1(\lambda_2 - \gamma\lambda_2 - c) < \lambda_2(c + 2\gamma\lambda_2),$$

or

$$(1 - \lambda_0)c + 2\gamma\lambda_2^2 > \lambda_1\lambda_2(1-\gamma), \tag{35}$$

then the sum will be negative for all μ.

Given conditions (34) and (35),

$$\frac{\partial W}{\partial \theta} < 0.$$

Hence, for given c, as b_1^p is raised and concomitantly, μ raised and θ cut, social welfare will rise. It remains to determine whether for fixed b_1^p and θ, social welfare falls when c, and concomitantly μ, is lowered. At the highest value of $b_1^p = 1 - \lambda_0$, $\theta = 0$, hence

$$W = -(\lambda_1 + \lambda_2)\gamma + \lambda_1[1 + \lambda_2 - (\lambda_1 + \lambda_2)\gamma] + 2\lambda_2 + \lambda_1\mu\,[-\lambda_2 + (\lambda_1 + \lambda_2)\gamma + c].$$

By (34), $\partial W/\partial\mu > 0$, which is the desired result.

The numerical example given in Section 3 of the main text, $\lambda_0 = 0.62$, $\lambda_1 = 0.18$, $\lambda_2 = 0.2$, $\gamma = 0.6$, $c = 0.03$, satisfies conditions (33), (34), and (35).

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