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This paper contains preliminary findings from research work still in progress and should not be quoted without prior approval of the author.

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FACTOR CONTENT FUNCTIONS

AND THE THEORY OF INTERNATIONAL TRADE*

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This paper introduces the concepts of direct and indirect factor trade utility functions and uses them to derive Marshallian and Hicksian factor content functions, which express the quantities of factors embodied in net imports as functions of the exogenous variables. The properties of these functions are discussed and they are used to derive a number of new results. In particular, it is shown that, in certain circumstances, the existence of gains from trade is necessary and sufficient for the Heckscher-Ohlin theorem to hold in its factor content form.

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FACTOR CONTENT FUNCTIONS AND THE THEORY OF INTERNATIONAL TRADE

I. Introduction

The principle that differences between countries in their relative factor endowments should have an important bearing on their trade patterns has been firmly entrenched in international trade theory since Ohlin's seminal work, Interregional and International Trade (1933). Not surprisingly, therefore, the formalisation of this principle embodied in the Heckscher-Ohlin theorem continues to dominate empirical studies of trade patterns.[1] Nevertheless, it is only relatively recently that significant progress has been made in extending the theory beyond the textbook two-good two-factor case. In particular, Deardorff (1982) has recently shown that the Heckscher-Ohlin theorem in its factor content version - implying that each country's exports should embody its relatively scarce factors - holds under very general conditions.[2]

The principal purpose of this paper is to carry further the process of generalising the Heckscher-Ohlin theorem and to show that considerable insight into this task can be obtained by relating the positive question of trade patterns to the normative one of the gains from trade. The link between the principle of comparative advantage and the gains from trade is well-known, but, to the best of our knowledge, our paper is the first to show that normative considerations are intimately bound up with generalisations of the Heckscher-Ohlin theorem. For example, whereas Deardorff's generalisations of the theorem yield sufficient conditions only, we are able to show that, in certain circumstances, the existence of gains from trade is necessary and sufficient for the Heckscher-Ohlin theorem to hold in one of its factor content versions.

In addition to these and other substantive contributions, a second objective of the paper is to introduce some powerful new techniques, which allow the trading pattern of a country to be summarised in terms of some convenient functions. Our technical contribution here amounts to an extension of the work of Woodland (1980) on direct and indirect trade utility functions.[3] In particular, by observing that prices must cover unit costs in competitive equilibrium, we derive an indirect factor trade utility function, the derivatives of which are factor content functions, which relate the factors embodied in

trade to the factor prices and factor endowments facing the economy. As well as permitting new theoretical results, these functions provide a basis for future empirical work on trade patterns.[4]

The plan of the paper is as follows. Section II introduces assumptions and notation, reviews the results of Woodland (1980) and extends them to derive Slutsky equations for net import demand functions. Section III introduces the indirect factor trade utility function, shows how it may be used to derive factor content functions and examines the properties of these functions. Section IV presents some results on the gains from trade and shows how these relate to generalisations of the Heckscher-Ohlin theorem. These results are probably the major substantive contribution of the paper. However, they rely on the restrictive assumption that specialisation in production does not take place; the consequences of relaxing this assumption are examined in Section V. Next, Section VI introduces the direct factor trade utility function and the factor trade expenditure function and uses these tools to derive a Slutsky-type decomposition of the derivatives of the factor content functions with respect to factor prices. This result may be viewed as a convenient summary of all the testable implications of the theory of comparative advantage. Finally, Section VII summarises the paper's conclusions, notes the relationship between factor content

functions and actual factor trade functions and mentions some directions for future research.

II. Preliminaries: Direct and Indirect Trade Utility Functions

We begin by describing the basic framework to be used in the paper, which is a general model of a competitive trading economy, similar to those used in many recent writings on international trade theory.

On the demand side, we assume that community preferences can be summarised by a utility function $u(x)$, which is a real-valued, continuous, weakly increasing and strictly quasi-concave function of the consumption vector $x = (x_1,\_\_\_,x_N)'$.[5] Maximisation of this function is carried out subject to the budget constraint, $p.x \leq I$, where I denotes the aggregate household's lump-sum income.[6] The solution to this maximisation problem is the vector of Marshallian demand functions, $x(p,I)$, which, on substitution into $u(x)$, yields the indirect utility function:

$$(2.1) \quad V(p,I) = u[x(p,I)]$$

$$(2.2) \quad = \max_{x} [u(x): \; p.x \leq I]$$

This function has a number of useful properties: it is finite and continuous for $p>0$ and $I\geq 0$; it is homogeneous of degree zero in p and I; it is weakly increasing and strictly quasi-convex in $p>0$ and non-decreasing in I. In addition, provided that the indirect utility function is differentiable and strictly increasing in I, the Marshallian demand functions may be expressed in terms of its derivatives via Roy's Identity:[7]

$$(2.3) \qquad V_p(p,I) = -V_I(p,I)\, x(p,I).$$

Before proceeding, we recall that demand behaviour may alternatively be characterised in terms of the dual problem of minimising the cost of attaining a given level of utility. This yields the expenditure function:

$$(2.4) \qquad e(p,u) = \underset{x}{\text{Min}} \, [p.x: u(x)\geq u]$$

This function is concave, weakly increasing and homogeneous of degree one in p, and non-decreasing in u. Moreover, by Shephard's Lemma, its partial derivatives with respect to p equal the Hicksian or utility-compensated demand functions:

$$(2.5) \qquad e_p(p,u) = x_p^c(p,u).$$

These may be related to the Marshallian demand functions

via the "Slutsky identity":

$$(2.6) \qquad x_p^c(p,u) = x_p[p,e(p,u)],$$

which, on differentiation, yields the fundamental result of consumer demand theory, the Slutsky equation:

$$(2.7) \qquad x_p^c = x_p + x_I x'.$$

Turning to production, we assume that the aggregate production possibilities set is convex and may be summarised by the implicit constraint $F(y,v) \leq 0$, where v is the vector of factor endowments, $v = (v_1,\_\_\_,v_M)$. In the face of a vector of commodity prices p and in the absence of factor-market distortions, competition will lead to an allocation of factors between sectors which maximises the value of net national output. The resulting maximal value is given by the national product function:

$$(2.8) \qquad g(p,v) = \underset{y}{\text{Max}}\ [p.y: F(y,v) \leq 0]$$

This function is homogeneous of degree one in both p and v; it is convex in p and concave in v; and its derivatives with respect to p and v equal the economy's commodity supply functions and inverse factor demand functions

respectively:

$$(2.9) \qquad g_p(p,v) = y(p,v)$$

$$(2.10) \qquad g_v(p,v) = w(p,v).$$

Assume now that the economy is opened to international trade. The value of GNP at domestic prices (which are not necessarily equal to world prices) is given by (2.8). This is distributed in full to consumers along with an additional lump-sum income b (which may reflect a net transfer from the rest of the world or a redistribution of tariff revenue).[8] The maximum utility thus attainable by consumers may then be written as a function of the exogenous variables p, b and v, by substituting from (2.8) into the indirect utility function (2.1):

$$(2.11) \qquad H(p,b,v) = V[p,g(p,v)+b]$$

The properties of this indirect trade utility function have been considered in detail by Woodland (1980). In particular, it is quasi-convex in p, weakly increasing in b, non-decreasing and quasi-concave in v and homogeneous of degree zero in (p,b). In addition, its partial derivatives with respect to p and b give, in a similar manner to Roy's Identity, the general-equilibrium net import demand

functions:

$$(2.12) \quad H_p(p,b,v) = -H_b(p,b,v)\, m(p,b,v),$$

where:

$$(2.13) \quad m(p,b,v) = x[p,g(p,v)+b] - y(p,v).$$

Equation (2.12) shows that the price derivatives of H are proportional to the net import demand functions, where the constant of proportionality is the derivative of H with respect to lump-sum income, b. The latter derivative is easily seen to equal $V_I(p,I)$, the aggregate household's marginal utility of income.

Finally, Woodland proves that the indirect trade utility function is minimised at the autarky price vector, p^0:

$$(2.14) \quad H(p^0,0,v) \leq H(p,0,v).$$

Woodland concludes from this result that the competitive system minimises the <u>rent</u> accruing to the factors of production, and relates this to the well-known fact that in the dual of a programming problem the price mechanism minimises rents. This observation forms the starting-point of our analysis in the next section.

In addition to deriving these properties of the indirect trade utility function, Woodland also shows that the net import demand functions may be viewed as the outcome of maximising a <u>direct</u> trade utility function:

$$(2.15) \quad U(m,v) = \underset{x}{\text{Max}} \, [u(x): F(x-m,v) \leq 0].$$

We may go further and note that, dual to this maximisation problem is the problem of minimising the cost of a vector of net imports subject to the constraint that a target level of utility be attained. This yields a <u>trade expenditure function</u>:

$$(2.16) \quad E(p,u,v) = \underset{m}{\text{Min}} \, [p.m: U(m,v) \geq u]$$

The properties of this function are similar to those of the standard expenditure function (2.4). In particular, its partial derivatives with respect to commodity prices give the economy's Hicksian net import demand functions:

$$(2.17) \quad E_p(p,u,v) = m^c(p,u,v)$$

$$(2.18) \quad = x^c(p,u) - y(p,v).$$

The derivatives of (2.17) are easily shown to be related to those of the Marshallian net import demand functions (2.13)

via a generalisation of the Slutsky equation:

$$(2.19) \qquad m_p^c = m_p + m_b m' .$$

Note an implication of (2.19): if $\partial m_j/\partial b$ and m_j have the same sign (i.e., if good j is an import good which is normal in demand or an export good which is inferior), the Marshallian import demand function for good j is a well-behaved function of its own price. Intuitively, an increase in the price of an import good is a worsening of the economy's terms of trade and so (assuming normality) the income effect reinforces the substitution effects in consumption and production.

III. The Indirect Factor Trade Utility Function

We now wish to extend the tools of the previous section to focus not on actual trade in commodities, but on <u>embodied</u> trade in factors. In order to do this, we must be more specific about the underlying technology. For the remainder of the paper therefore, we assume that technology in each sector can be represented by a unit cost function, $c^j(w)$, which is increasing, concave, twice differentiable and homogeneous of degree one in factor prices, w. This specification is rather more restrictive than (2.8) in the previous section; in particular, it rules out joint

production. However, it is consistent with the presence of intermediate inputs, provided the $c^j(w)$ functions are interpreted as cost functions net of intermediate inputs. (This requires that the Hawkins-Simon condition be satisfied.)

In a general trading equilibrium, the mix of commodities which is actually produced by the economy is itself endogenous. For the present, it is convenient to concentrate on the case where all goods continue to be produced, and we postpone until Section V consideration of the possibility that specialisation in production may take place. Since price is equal to unit cost in all sectors, therefore, we may define the <u>indirect factor trade utility function</u> by substituting the vector of unit cost functions into (2.11):[9]

$$(3.1) \quad L(w,b,v) = H[c(w),b,v]$$

$$(3.2) \quad = V[c(w),\ g\{c(w),v\}+b].$$

The remainder of this section is concerned with the properties of this function. The first property, which makes it especially convenient for studying problems related to the factor content of trade, is given by the following proposition:

Proposition 1: The derivatives of the indirect factor trade utility function L with respect to factor prices are proportional to the factor content of net imports, the constant of proportionality being the marginal utility of income.

Proof: Differentiate (3.1) with respect to w:

$$(3.3) \qquad L_w(w,b,v) = c_w(w)\, H_p[c(w),b,v].$$

By Shephard's Lemma, the first term on the right-hand side is the M-by-N matrix of input-output coefficients A; and, by (2.12), the second term is proportional to the vector of net commodity imports:

$$(3.4) \qquad L_w(w,b,v) = -H_b[c(w),b,v]\, A(w)\, m[c(w),b,v].$$

Apart from the scalar $-H_b$, the right-hand side of (3.4) is an M-by-one vector, the i'th element of which gives the total quantity of factor i embodied in net imports (where all commodities are produced using the techniques of the home country). Written as a function of the exogenous variables, we shall refer to this henceforth as the vector of factor content functions, M(w,b,v), the i'th element of which is given by:

$$(3.5) \qquad M_i(w,b,v) = \sum_j a_{ij}(w)\, m_j[c(w),b,v].$$

Noting that $H_b[c(w),b,v] = L_b(w,b,v)$, the marginal utility of income, equation (3.4) may therefore be written in a form which makes clear that it is another generalisation of Roy's Identity:

$$(3.6) \qquad L_w(w,b,v) = -L_b(w,b,v)\, M(w,b,v).$$

Q.E.D.

Proposition 1 underlies the great usefulness of the indirect factor trade utility function in both theoretical and empirical work. Before considering these applications, we state some of the properties of L.

Proposition 2: The indirect factor trade utility function is: (i) quasi-convex in w; (ii) weakly increasing and quasi-concave in v; (iii) non-decreasing in b; and (iv) homogeneous of degree zero in the nominal variables (w,b).

The proof of Proposition 2 is given in the Appendix. In addition, we may note a further property of the indirect factor trade utility function (suppressing the arguments of the functions for simplicity):

$$(3.7) \qquad L_v = H_v = V_I g_v = V_I w.$$

In words, the derivatives of the indirect factor trade utility function with respect to factor endowments equal the competitive factor prices (measured in utility units). It should be stressed that this does not mean that the values of the w and v arguments of the function L cannot be chosen independently; the constraints embodied in the function will ensure that, in a competitive equilibrium, (3.7) will hold for any arbitrarily chosen values of w and v (provided these are consistent with the assumed pattern of specialisation).

Having characterised the formal properties of the indirect factor trade utility function, we would like to do the same for the factor content functions. Unfortunately, the dependence of these on income means that they are not well-behaved in general. (In Section VI, we shall discuss compensated factor content functions, which do not suffer from this drawback.) However, these income effects are eliminated if the factor content functions are evaluated at autarky factor prices, which gives us the following surprisingly strong result:

Proposition 3: Provided there are no free factors in autarky, the factor content functions evaluated at the autarky factor prices exhibit all the properties of a set of well-behaved neoclassical factor demand functions.

Proof: Differentiate equation (3.4) with respect to w:

$$(3.8) \qquad L_{ww} = -ML_{bw} - L_b M_w.$$

If there are no free factors in autarky, $M(w^0,0,v) = 0$. Hence, from (3.8), the properties of the Jacobian matrix of the factor content functions, M_w, depend solely on those of the matrix L_{ww}. We now wish to show that, at autarky factor prices, this matrix must be positive semi-definite.

Recalling the definition of the indirect factor trade utility function, (3.1), differentiate it twice with respect to w:

$$(3.9) \qquad L_w = c_w H_p$$

$$(3.10) \qquad L_{ww} = c_w H_{pp} c_w' + \sum_j H_j c^j_{ww}$$

where c^j_{ww} is the matrix of second partial derivatives of the unit cost function of sector j. In autarky, $H_j=0$ for all j, and so L_{ww} being positive semi definite is equivalent to H_{pp} being positive semi-definite.[10]

Next, to determine the sign of H_{pp}, differentiate (2.12) with respect to p:

(3.11) $$H_{pp} = -mH_{bp} - H_b m_p.$$

To simplify this, note that in autarky m is zero and that the vector m_p may be evaluated by differentiating (2.13):

(3.12) $$m_p = x_p + x_I g_p' - y_p,$$

which, from (2.9), (2.13) and the Slutsky equation (2.7) gives:

(3.13) $$m_p = x_p^c - y_p - x_I m'.$$

Collecting all these results, we find that, in autarky, the Jacobian of the factor content functions is:

(3.14) $$M_w = c_w(x_p^c - y_p)c_w'.$$

The right-hand side is a matrix quadratic form in a negative semi-definite matrix, which implies that the same is true of the left-hand side. Hence, in autarky, the factor content functions exhibit all the properties of a set of neoclassical factor demand functions.

Q.E.D.

IV. Gains from Trade

In this section we show how the indirect factor trade utility function may be used to express the gains from trade in a number of alternative ways and we then relate these to generalisations of the Heckscher-Ohlin theorem. We continue to assume that all goods are produced at all times.

Consider first Woodland's result, mentioned in Section II, that H is minimised at autarky commodity prices. Since p is directly related to w, given the assumption that specialisation in production does not take place, this immediately implies that L is minimised at autarky factor prices, w^0:[11]

$$(4.1) \qquad L(w^0,0,v) \leq L(w^1,0,v).$$

Here, w^1 is the factor price vector in any equilibrium other than that of autarky. From Proposition 2 (iii), L is non-decreasing in b. Hence:

$$(4.2) \qquad L(w^0,0,v) \leq L(w^1,b^1,v) \qquad \text{for any } b^1 \geq 0.$$

This provides a simple proof of an important result due to Ohyama (1972): Interpreting w^1 and b^1 as applying to a situation of restricted trade, a sufficient condition for this to be welfare-superior to autarky is that b^1, the

net revenue from tariffs and trade subsidies, be non-negative. (Deardorff (1982) suggests the term "natural trade" for this condition, while Smith (1982) calls it simply "restricted trade.")

This approach may be used to define a number of alternative measures of the gains from trade. Firstly, for any situation of restricted trade, we may ask what level of lump-sum transfer to the economy would yield the same utility as autarky. The value of b in question, b^*, is implicitly defined by (4.3), and, from (4.1) and Proposition 2 (iii), is non-positive:

$$(4.3) \qquad L(w^0,0,v) = L(w^1,b^*,v).$$

Now, consider the Taylor's series expansion of $L(w^1,b^*,v)$ around the restricted trade equilibrium (w^1,b^1,v):[12]

$$(4.4) \qquad L(w^1,b^*,v) = L(w^1,b^1,v) + (b^*-b^1)\, L_b(w^1,b^1,v).$$

This may be simplified by defining:

$$(4.5) \qquad G^1 = [\, L(w^1,b^1,v)-L(w^0,0,v)\,] \,/\, L_b(w^1,b^1,v).$$

G^1 is a compensating variation measure of the gains from trade; i.e., it equals the difference between the utility

levels in trade and in autarky, evaluated at the post-trade marginal utility of income. Using (4.3) and (4.5), (4.4) may therefore be written as:

$$(4.6) \qquad G^1 = b^1 - b^*.$$

Hence, b^1-b^* is a money measure of the gains from trade. Of course, if government intervention on average subsidises trade, b^1 may be sufficiently negative that restricted trade yields a lower utility level than autarky.

Naturally, a second measure of the gains from trade may be found by defining $\tilde{b}$ as the lump-sum transfer which, if provided in autarky, would yield the same utility as a given restricted trade situation:

$$(4.7) \qquad L(w^1,b^1,v) = L(w^0,\tilde{b},v).$$

Now, expand the right-hand side of (4.7) around the autarky equilibrium $(w^0,0,v)$:

$$(4.8) \qquad L(w^0,\tilde{b},v) = L(w^0,0,v) + \tilde{b}L_b(w^0,0,v).$$

This yields, in exactly the same manner as (4.6):

$$(4.9) \qquad G^0 = \tilde{b},$$

where G^0 is an equivalent variation measure of the gains from trade, equal to the difference between the restricted trade and autarky utility levels, evaluated at the marginal utility of income in autarky.

A different approach to measuring the gains from trade is to ask what proportion of the economy's factor endowment would be sufficient to yield the same level of utility as autarky in the face of the factor prices and lump-sum income of the restricted trade situation. This defines a non-negative scalar r^*:

$$(4.10) \quad L(w^0,0,v) = L(w^1,b^1,r^*v).$$

Provided there are gains from trade, r^* is less than unity, and is a physical measure, analogous to Debreu's (1951) coefficient of resource utilisation, of the extent to which welfare in autarky falls short of that in restricted trade. This may be seen explicitly by expanding the right-hand side of (4.10) around the restricted trade equilibrium (w^1,b^1,v):

$$(4.11) \quad L(w^1,b^1,r^*v) = L(w^1,b^1,v) + (r^*-1)\, v.L_v(w^1,b^1,v).$$

Making use of (3.7), this reduces to:

$$(4.12) \quad G^1 = (1-r^*)\, w^1.v.$$

Thus, $(1-r^*)$ is the ratio of the gains from trade to the value of national product (excluding any transfer b^1) in the restricted trade situation.[13]

We turn next to the relationship between the gains from trade and the factor content of trade. Once again, the method we adopt is to expand the indirect factor trade utility function by a Taylor's series expansion around either the autarky or the restricted trade point. Consider the latter expansion first:

$$(4.13) \quad L(w^0,0,v) = L(w^1,b^1,v) + (w^0-w^1).L_w(w^1,b^1,v) + (b^0-b^1)\, L_b(w^1,b^1,v).$$

This may be simplified by noting that from the homogeneity of L in nominal variables (Proposition 2(iv)):

$$(4.14) \quad w^1.L_w(w^1,b^1,v) + b^1\, L_b(w^1,b^1,v) = 0.$$

Substituting from (4.12) and (4.14), equation (4.13) may be written as follows:

$$(4.15) \quad G^1 = w^0.M^1,$$

where M^1 is the factor content of imports in the restricted trade situation. Hence, provided there are gains from trade, autarky factor prices are positively

correlated with the levels of embodied factor imports.[14] In other words, factors which are expensive in autarky tend on average to be imported when trade is opened up and vice versa. This result is clearly a multi-commodity generalisation of the Heckscher-Ohlin theorem and is closely related to one of Deardorff's (1982) results. However, Deardorff derived only a sufficient condition for the product $w^0.M^1$ to be non-negative, namely that the net revenue from restrictions on trade, b^1, be positive. By contrast, equation (4.15) provides (up to a first-order approximation) a necessary and sufficient condition.

A different result is obtained if L is expanded around the autarky equilibrium point:

$$(4.16) \quad L(w^1,b^1,v) = L(w^0,b^0,v) + (w^1-w^0).L_w(w^0,0,v) + (b^1-b^0).L_b(w^0,0,v).$$

Manipulating this in a similar manner to (4.13) yields:

$$(4.17) \quad G^0 = -w^1.M^0 + b^1.$$

The main interest of (4.17) is that it highlights a source of gains from trade which is ignored in the literature: the possibility of gainfully disposing of factors which are free in autarky. For such factors the Kuhn-Tucker

conditions for minimisation of $L(w,b,v)$ subject to $w \geq 0$ imply that $L_i(w^0,0,v)$ may be positive and so that the corresponding $M_i(w^0,0,v)$ may be negative. Opening up the economy to trade may eliminate the surplus in these factors, so permitting gains from trade even if the net effect of restrictions on trade is to subsidise it (so that b^1 is negative).

V. The Consequences of Specialisation in Production

So far, we have assumed that all goods continue to be produced, but it is necessary to extend the analysis to allow for the possibility that some commodities cease to be produced at home after trade is opened up. For simplicity, we shall assume that all goods are essential in demand. This implies that all goods are consumed in the home country at all times and also that all goods are produced domestically in autarky.

The approach we have adopted to this issue is to work with a mixed indirect factor trade utility function, which is defined over all factor prices and also over the prices of those commodities which are not produced at home when trade is opened up.[15] Formally, we partition the commodity vector into two groups: a first group with prices q, which are produced at home; and a second group with prices r which are not profitable to produce

domestically in the trading equilibrium. Partitioning the vector of unit cost functions conformably, the relationship between price and unit cost for the two groups of commodities is therefore given by the following:

$$(5.1) \qquad c^q(w^1) = q^1,$$

$$(5.2) \qquad c^r(w^1) > r^1.$$

The mixed indirect factor trade utility function may now be defined as follows:

$$(5.3) \qquad \hat{L}(w,r,b,v) = H[c^q(w),r,b,v].$$

It is clear from the definition of $\hat{L}$ that its derivatives with respect to w are, like those of the function L, proportional to the factor content functions:

$$(5.4) \qquad \hat{L}_w = -\hat{L}_b M.$$

Similarly, its derivatives with respect to r are, like those of the function H, proportional to the actual import demand functions (i.e., to the demand functions) for non-competing imports:

$$(5.5) \qquad \hat{L}_r = -\hat{L}_b m^r = -\hat{L}_b x^r.$$

Hence, as far as the local properties of the function $\hat{L}$ are concerned, there is no need to add anything to what has been noted in earlier sections: in the neighbourhood of a given trading equilibrium, the properties of $\hat{L}$ as a function of w, b and v are identical to those of L, while its properties as a function of r are identical to those of H as a function of p. We can also extend the comparisons between autarky and restricted trade of the last section, provided we keep in mind that the autarky values of w and r cannot be chosen independently of one another but must satisfy:

$$(5.6) \qquad c^r(w^0) = r^0.$$

Expanding $\hat{L}$ around the trade equilibrium point, we therefore obtain an extension of equation (4.5):[16]

$$(5.7) \qquad \hat{L}^0 = \hat{L}^1 + (w^0-w^1).\hat{L}^1_w + (r^0-r^1).\hat{L}^1_r + (b^0-b^1)\hat{L}^1_b.$$

A series of simplifications similar to those which led to (4.15) now gives:

$$(5.8) \qquad G^1 = w^0.M^1 + \bar{p}^0.\bar{m}^1.$$

This shows that the gains from trade may be decomposed into two terms. The first term is the Heckscher-Ohlin term discussed in the last section. The second term may be

called the Ricardian specialisation term: it equals the value of non-competing imports in the restricted trade equilibrium, valued at the domestic cost of producing them in autarky. Clearly, if this term is sufficiently large, a country may gain from trade ($G^1>0$) and yet on average export in embodied form those factors which are relatively expensive in autarky ($w^0.M^1 < 0$).

VI. The Direct Factor Trade Utility Function and Hicksian Factor Content Functions

In this section, we introduce some new functions as alternatives to the indirect factor trade utility function and use them to derive a Slutsky-type decomposition of the derivatives with respect to factor prices of the factor content functions. For ease of reference, the relationships between the various functions are summarised in Table 1. Henceforward, we return to the case where all goods are produced domestically. Where necessary, this assumption can be relaxed along the lines indicated in the last section.

We have already seen in equation (2.15) that the direct trade utility function is defined as the maximum utility attainable when domestic production is augmented by importing a vector of final goods, m. An alternative

viewpoint is to ask what is the maximum utility attainable if all consumption must be produced using the techniques of the home country, but if its domestic factor endowment is augmented by "importing" a vector of factors, M. This leads to the direct factor trade utility function:

$$(6.1) \qquad \mathbb{U}(M,v) = \max_{x} [u(x): F(x,v+M) \leq 0].$$

The most important property of this function is stated in the following proposition:

Proposition 4: The direct factor trade utility function $\mathbb{U}$ is quasi-concave in the vector of net factor imports, M.

Proof: It is clear from the definition of $\mathbb{U}$ that it is increasing in M. Consider now two arbitrary net factor import vectors M_1 and M_2, and evaluate the function at a vector which is a linear combination of these two ($0 \leq a \leq 1$):

$$(6.2) \qquad \mathbb{U}[aM_1+(1-a)M_2,v] = \max [u(x): F\{x, v+aM_1+(1-a)M_2\} \leq 0],$$

$$(6.3) \qquad \geq \min [\max \{u(x): F(x,v+aM_1) \leq 0\}, \max \{u(x): F(x,v+(1-a)M_2) \leq 0\}],$$

(since F defines a convex production possibilities set),

$$(6.4) \qquad = \text{Min} \left[\bar{U}(aM_1, v), \bar{U}\{(1-a)M_2, v\} \right].$$

Hence, $\bar{U}$ is quasi-concave in M.

Q.E.D.

We now proceed in a manner analogous to that in a standard presentation of consumer demand theory. A competitive equilibrium will be attained when (6.1) is maximised subject to the constraint that the value of net factor imports at given factor prices not exceed the lump-sum transfer, b:

$$(6.5) \qquad w.M \leq b.$$

This maximisation problem leads to factor content functions $\bar{M}(w,b,v)$ which, on substitution into (6,1), give an indirect factor trade utility function:

$$(6.6) \qquad \bar{L}(w,b,v) = \bar{U}[\, \bar{M}(w,b,v), v \,].$$

But this function is in fact the same as the function L which was introduced in Section III, as can be seen from the following. First, substitute from (6.1) into (6.6):

$$(6.7) \qquad \bar{L}(w,b,v) = \underset{M}{\text{Max}} \left[\underset{x}{\text{Max}} \{u(x): F(x, v+M) \leq 0\}: w.M \leq b \right].$$

Substituting $c_w(w)(x-y)$ for M, and noting that, since $-F_v$ is proportional to w in equilibrium,[17] the vector of net imports x-y can be produced using the same techniques as production for domestic consumption provided factor prices equal w:

$$(6.8) \quad \bar{L}(w,b,v) = \max_y [\max_x \{u(x): c(w).x \leq c(w).y+b\}: F(y,v)\leq 0],$$

$$(6.9) \quad = \max_x [u(x): c(w).x \leq \max_y\{ c(w).y: F(y,v)\leq 0\} + b].$$

$$(6.10) \quad = \max_x [u(x): c(w).x \leq g\{c(w),v\}+b],$$

$$(6.11) \quad = V [c(w), g\{c(w),v\}+b] \quad \text{(from (2.2))}$$

$$(6.12) \quad = L(w,b,v) \quad \text{(from (3.2))}.$$

Thus the direct factor trade utility function (6.1) and the indirect factor trade utility function (3.1) are indeed dual to one another, in the sense that knowledge of one is equivalent to knowledge of the other. Moreover, the factor content functions $\bar{M}(w,b,v)$ derived from maximising (6.1) are identical to the functions M(w,b,v) derived by differentiating (3.1), as given by equation (3.6).

The next step is to observe that, dual to the problem of maximising (6.1) subject to (6.5), is the problem of choosing a factor trade vector to minimise the cost of attaining a given utility level. This yields the <u>factor trade expenditure function</u>:

$$(6.13) \quad E(w,u,v) = \underset{M}{\text{Min}} \, [w.M: \bar{U}(M,v) \geq u].$$

The derivatives of this function are Hicksian or compensated factor content functions:

$$(6.14) \quad E_w(w,u,v) = M^c(w,u,v)$$

$$(6.15) \quad = c_w(w) \, m^c[c(w),u,v],$$

where the Hicksian net import demand functions m^c are defined by (2.18). Since $\bar{U}$ is quasi-concave in M from Proposition 4, it follows immediately that E is concave in w. Hence, by exact analogy with standard consumer theory, we may immediately state the following key proposition:

<u>Proposition 5</u>: The Hicksian factor content functions (6.14) exhibit, as functions of w, all the properties of a set of well-behaved neoclassical factor demand functions.

This implies, in particular, that the Jacobian of the vector M^C is negative semi-definite. Differentiation of (6.15) shows that this Jacobian equals:

$$(6.16) \quad M^C_w = E_{ww} = c_w m^C_p c'_w + \sum_j m_j c^j_{ww},$$

The first term on the right-hand side of (6.16) is negative semi-definite, since the matrix m^C_p (which equals $x^C_p - y_p$; i.e., equation (3.13) without the final income term), is the Jacobian of the Hicksian net import demand functions and is negative semi-definite. Proposition 5 ensures that this term cannot be outweighed by the second term in (6.16), which is a weighted average of negative semi-definite matrices c^j_{ww} (where the weights are both positive and negative and satisfy the constraint $m_j c^j = b$). The close relationship between Propositions 3 and 5 is obvious: income effects can be eliminated either by evaluating the Marshallian factor content functions at autarky or by holding the level of utility constant, and in either case the resulting functions are, in a general sense, decreasing functions of factor prices, w.

The final result is to note that we may relate the Hicksian and Marshallian factor content functions through a "Slutsky identity" (see Table 1), which, on differentiation with respect to w, yields a generalised Slutsky equation

for factor content functions:

$$(6.17) \qquad M_w^C = M_w + M_b M' .$$

Equation (6.17) may be said to summarise in a convenient form all the testable implications of the theory of comparative advantage. We may note one implication in particular. If $\partial M_i / \partial b$ and M_i have the same sign, the factor content function for factor i is a decreasing function of its own price. Assuming that factor i is imported in embodied form (so that M_i is positive), a sufficient (though over-strong) condition for this is that all final goods be normal in demand, since:

$$(6.18) \qquad M_b = Am_b = Ax_I .$$

By analogy with the earlier result on net import demand functions noted at the end of Section II, an increase in w_i when M_i is positive is a worsening of the terms of implicit factor trade, and so, if demand for the factor is normal, the income effect reinforces the substitution effects in consumption and production.

VII. Conclusion

In this paper we have introduced the concepts of direct and indirect factor trade utility functions and have examined their properties. In addition, we have shown their relationship to factor content functions, which summarise in convenient form the dependence of implicit trade in factors (as embodied in commodities actually exported and imported) on the different variables which characterise a general equilibrium. This approach has thrown light on a number of issues in trade theory, most notably, we believe, the recent work on generalising the Heckscher-Ohlin theorem. In particular, we have shown an important and hitherto unnoticed link between two results in different branches of the literature. On the one hand, Deardorff (1982) has shown that "natural trade" (meaning a positive net revenue from intervention in trade) is sufficient for a number of generalisations of the Heckscher-Ohlin theorem to hold. On the other hand, Ohyama (1972) had earlier shown that the same condition is sufficient for gains from trade. We have related these results by demonstrating that the requirement of natural trade is not in itself crucial. Provided no specialisation in production takes place, the existence of gains from trade is itself necessary and sufficient (up to a first-order approximation) for the Heckscher-Ohlin theorem to hold in one of its multi-commodity forms. If not all

goods are produced when trade is opened up, then an additional Ricardian specialisation term must also be considered, and we have shown how this weakens the link between the gains from trade and the Heckscher-Ohlin theorem.

Another contribution of the paper is to present the properties of both Marshallian and Hicksian factor content functions. In general, the Marshallian functions were found to be well-behaved only in the neighbourhood of autarky. This may be contrasted with actual factor trade functions, which, as shown by Schweinberger (1978) and Neary (1980), exhibit all the standard neoclassical properties as functions of the factor rewards available on world markets in trading as well as in autarky equilibria. The difference between these two results follows simply from the assumption that only commodities and not factors enter the direct utility function. Relaxing this assumption by allowing some or all factors to be supplied on a choice-theoretic basis would imply that (because of income effects) actual factor trade functions would not be well-behaved either (except, of course, in the neighbourhood of autarky).

In conclusion, although we have touched on a great many topics, we do not believe that we have exhausted the potential of the new techniques introduced in this paper. At the theoretical level, it would be desirable to apply

factor content functions to many other issues, including the behaviour of economies with many consumers in which lump-sum redistribution cannot be carried out, and the properties of a general equilibrium of the world economy (with the tastes and technology of each country summarised by its own factor trade utility function). At the empirical level, it would be desirable to specify and estimate specific functional forms for factor content functions, with a view to testing the postulates of the neoclassical approach to trade theory. These and other extensions of our analysis seem important directions for future research.

APPENDIX: PROOF OF PROPOSITION 2

Parts (ii), (iii) and (iv) of this proposition follow directly from the properties of the functions H and c. To prove part (i), the quasi-convexity of the indirect factor trade utility function in w, we first characterise quasi-convexity as follows:

$$(A.1) \qquad L(w^3,b,v) \leq \text{Max}\{ L(w^1,b,v), L(w^2,b,v) \},$$

where:

$$(A.2) \qquad w^3 = aw^1 + (1-a)w^2.$$

Following Woodland (1980), we define the following three sets in consumption space:

$$(A.3) \qquad X^i = \{x: c(w^i).x \leq g[c(w^i),v]-b;\ x\geq 0\}, \quad i=1,2,3.$$

We have proved (A.1) if and only if it can be shown that:

$$(A.4) \qquad X^3 \subset X^1 \cup X^2.$$

Suppose there exists a vector x^*, such that $x^* \in X^3$ but $x^* \notin X^1 \cup X^2$. We now show that this implies a contradiction. Hence (A.4) must hold and therefore (A.1) is established.

It follows from the definition of x^* that:

(A.5) $\quad c(w^3).x^* \leq g[c(w^3),v] - b;$

But:

(A.6) $\quad c(w^1).x^* > g[c(w^1),v] - b,$

and:

(A.7) $\quad c(w^2).x^* > g[c(w^2),v] - b.$

Next, from the concavity of cost functions, it follows that:

(A.8) $\quad c(w^3) \geq ac(w^1) + (1-a)c(w^2),$

and so:

(A.9) $\quad c(w^3).x^* \geq [\ ac(w^1) + (1-a)c(w^2)\ ].x^*.$

Multiplying (A.6) and (A.7) by a and (1-a) respectively and adding, we can deduce that:

(A.10) $\quad c(w^3).x^* > a\{g[c(w^1),v]-b\} + (1-a)\ \{g[c(w^2),v]-b\}.$

Substituting

(A.11) $w^i.v = g[c(w^i),v], \qquad i=1,2,$

into (A.10), we can show that:

(A.12) $c(w^3).x^* > w^3.v - b,$

where w^3 is defined in (A.2). Hence, from (A.11) with i=3, we have shown that:

(A.13) $c(w^3).x^* > g[c(w^3),v] - b,$

which contradicts (A.5). Hence (A.4) holds.

Q.E.D.

FOOTNOTES

* This paper arose from an earlier one on the same subject by Schweinberger (1982). For helpful discussions and comments we are very grateful to A.V. Deardorff, A.K. Dixit, L.E.O. Svensson, A.D. Woodland and participants in a conference on Recent Developments in International Trade Theory at the University of Western Ontario, March 1983.

1. For recent reviews and extensions, see Deardorff (1983) and Leamer (1983).

2. The factor content version of the Heckscher-Ohlin theorem was first demonstrated by Vanek (1968) for the special case where factor prices are equalised between countries. For related work, see Dixit and Norman (1980), Ethier (1982) and Helpman (1983). Mention should also be made of Dixit and Woodland (1982), who adopt what might be called a "marginal" approach to generalising the Heckscher-Ohlin theorem, examining the determinants of trade between countries whose relative factor endowments differ to a "small" extent.

3. Woodland's work is largely a formalisation and extension of the geometric techniques introduced by Meade (1952). Related tools have also been developed by Chipman (1979) and Dixit and Norman (1980), and, under the title "induced preference functions", by Rader (1964, 1978) and

Milne (1981).

4. Similar functions have been presented by Wong (1983), although he confines attention to the special case of two goods and two factors and is concerned with very different substantive issues.

5. Throughout the paper, we assume the existence of a single aggregate household. Our results could be extended in a relatively straightforward fashion to allow for many consumers, provided a social welfare function is continually implemented via lump-sum transfers, as in Woodland (1980), Section 4.

6. The notation p.x denotes the inner product of the two vectors p and x. In all other cases, vectors are column vectors except when they are transposed (as in x').

7. Subscripts denote partial derivatives. Thus, $V_I(p,I) = \partial V(p,I)/\partial I$ and $V_p(p,I)$ denotes the vector whose typical element is $\partial V(p,I)/\partial p_j$.

8. Unlike Woodland, we define b as positive when the household sector receives a net transfer.

9. An alternative approach, pursued by Wong (1983), is to define the function (which he calls the "input utility function") as: $L(w,b,v) = \text{Max}\ [u(x): x.c(w)=w.v+b] = V[c(w),w.v+b]$.

10. In fact, even out of autarky, the quadratic forms obtained by pre- and post-multiplying the Hessians of L and H by the appropriate price vectors are equal and so, a fortiori, they have the same sign: $w'L_{ww}w = p'H_{pp}p$. This follows from (3.11), noting that $w'c^j_{ww}w = 0$ from cost minimisation.

11. To see this directly, note that sufficient conditions for minimisation of L by choice of w are: $L_w = 0$ and L_{ww} positive definite. These are clearly satisfied only at $w = w^0$.

12. Note that the alternative expansion, of $L(w^1,b^1,v)$ around (w^1,b^*,v), does not lead to an easily interpretable result since $L_b(w^1,b^*,v)$ does not equal $L_b(w^0,0,v)$.

13. In this case too, an equivalent rather than compensating variation measure may be constructed by defining $\tilde{r}$ as that scalar which equates $L(w^1,b^1,v)$ to $L(w^0,0,\tilde{r}v)$. A Taylor's series expansion yields $G^0 = (\tilde{r}-1)\, w^0.v$.

14. We use the term "correlation" here in the sense of Dixit and Norman (1980) rather than Deardorff (1982); i.e., we do not require that either of the vectors on the right-hand side of (4.15) have zero mean.

15. This necessitates defining a different $\hat{L}$ function for each pattern of specialisation. An alternative approach, which would avoid this drawback, would be to work with a function defined over <u>all</u> commodity and factor prices as follows:

$$\bar{L}(w,p,b,v) = [H(\bar{p},b,v): \bar{p}_j = \text{Min } \{p_j, c^j(w)\},\ j=1,M].$$

However, a problem with this approach is that for many values of w and p the derivatives of $\bar{L}$ are undefined.

16. To simplify notation, superscripts 0 and 1 are used to denote the values of the exogenous variables at which the functions are evaluated; viz., $\hat{L}^0 = \hat{L}(w^0,r^0,0,v)$, etc.

17. By the envelope theorem, $\partial_M = -\lambda F_v$, where λ is the Lagrange multiplier attached to the constraint in (6.1); and by the first-order conditions for maximisation of (6.1) subject to (6.5), $\partial_M = \mu w$, where μ is the Lagrange multiplier attached to (6.5). Hence, $-F_v$ and w are proportional to one another.

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Table 1: Schematic Comparison of the Different Functions

	Consumption x	Net Imports m	Factor Content of Net Imports M
Direct utility function	$u(x)$	$U(m,v) = \text{Max}\ [u(x):\ F(x-m,v) \leq 0]$	$\bar{U}(M,v) = \text{Max}\ [u(x):\ F(x,v+M) \leq 0]$
maximised subject to:	$p.x \leq I$	$p.m \leq b$	$w.M \leq b$
--> Marshallian demand functions	$x(p,I)$	$m(p,b,v)$	$M(w,b,v)$
--> Indirect utility function	$V(p,I) = u[x(p,I)]$	$H(p,b,v) = U[m(p,b,v),v]$ $= V[p,g(p,v)+b]$	$L(w,b,v) = \bar{U}[M(w,b,v),v]$ $= H[c(w),b,v]$
--> Roy's Identity	$V_p(p,I) = -V_I(p,I)\ x(p,I)$	$H_p(p,b,v) = -H_b(p,b,v)\ m(p,b,v)$	$L_w(w,b,v) = -L_b(w,b,v)\ M(w,b,v)$
Expenditure function	$e(p,u) = \underset{x}{\text{Min}}\ [p.x:\ u(x) \geq u]$	$E(p,u,v) = \underset{m}{\text{Min}}\ [p.m:\ U(m,v) \geq u]$	$\bar{E}(w,u,v) = \underset{M}{\text{Min}}\ [w.M:\ \bar{U}(M,v) \geq u]$
--> Hicksian demand functions	$e_p(p,u) = x^c(p,u)$	$E_p(p,u,v) = m^c(p,u,v)$	$\bar{E}_w(w,u,v) = M^c(w,u,v)$
"Slutsky Identity"	$x^c(p,u) = x[p,e(p,u)]$	$m^c(p,u,v) = m[p,E(p,u,v),v]$	$M^c(w,u,v) = M[w,\bar{E}(w,u,v),v]$
--> Slutsky equation	$x^c_p = x_p + x_I x'$	$m^c_p = m_p + m_b m'$	$M^c_w = M_w + M_b M'$

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