Economics Bulletin

Factor market oligopsony and the location decision of free entry oligopoly

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Abstract

This paper examines the impact of oligopsony power on the location decision of undifferentiated oligopolistic firms with free entry. In the case where the distance of an oligopolistic firm from the output market is held constant, it shows that the optimum location moves away from the oligopsonistic input market if the demand function and the labor supply function are linear. In the case where the distance of an oligopolistic firm from the output market is a decision variable, it shows that the optimum location may not move toward the output market as demand increases if the demand function is convex. These results are significantly different from the conventional results based on the perfectly competitive factor market. It indicates that the presence of oligopsony power has important influence on the location decision of oligopolistic firms.

We are much indebted to Professor William Strange for his most helpful comments. Responsibility for any remaining errors is strictly ours.
Citation: Hwang, Chiung-I and Yeung-Nan Shieh, (2007) "Factor market oligopsony and the location decision of free entry oligopoly." *Economics Bulletin,* Vol. 18, No. 6 pp. 1-7
Submitted: October 6, 2006. **Accepted:** September 12, 2007.
URL: http://economicsbulletin.vanderbilt.edu/2007/volume18/EB-06R30002A.pdf

1. Introduction

Since Moses's 1958 pathfinding paper, *The Location and Theory of Production*, a large number of studies have attempted to integrate location theory with neoclassical production theory. Most of studies have focused on two polar cases: perfect competition and monopoly, see (Hurter and Martinich 1989). Little attention was devoted to the intermediate and more realistic cases: oligopoly and monopolistic competition. Recently, Mai and Hwang (1992) (henceforth MH) incorporated undifferentiated oligopoly into the Weber-Moses triangular model and attempted to fill this gap. Under the assumptions that (1) firms make Cournot conjectures about their rivals' production and location decisions; (2) the production function exhibits increasing returns to scale; (3) firms are price takers in the input market; (4) firms are free to enter and leave the industry, they obtained the following interesting propositions.[1]

MH1. *The optimum location is independent of a change in demand if the demand function is linear.*

MH2. *If the distance of an oligopolistic firm from the output market is held constant, the optimum location is independent of a change in demand.*

MH3. *In the case where the distance of an oligopolistic firm from the output market is a choice variable, the optimum location moves toward (away from) the output market as demand increases if the demand function is convex (concave).* (MH 1992, pp. 258-60).

These results are crucially dependent upon the assumption that oligopolistic firms are the price takers in the input markets. However, this assumption is too restrictive. As is well known in microeconomics, firms are likely to exert oligopsony power in buying resources because of real world market imperfections, see Bhaskar, Manning, To (2002). It would be important to examine the impact of market demand on the location decision of free-entry oligopoly when oligopsony power in the input markets prevails.[2]

The purpose of this paper is to introduce oligopsony market structure into MH model and examine the impact of output demand on the production and location decisions of oligopolistic firms. It will be shown that MH's propositions are no longer assured if the oligopolistic firm has oligopsony power in the input markets

2. An Oligopolistic Location Model

Following MH (1992), we assume that

[1] . MH (1992) also considered the impact of demand on the location decision when the production function exhibits constant or decreasing returns to scale. However, it can be shown that no solution exists if there are constant and decreasing returns to scale in production and free entry. In this note, we only consider the increasing returns to scale case.

[2]. Mai, Suwanskul and Yeh (1992), Yeh, Mai and Shieh (1996), Shieh and Mai (1997) and Shieh and Yeh (2004) have introduced monopsony into the location model. To our knowledge, no attempts have thus far been made to incorporate oligopsony power into the location model.

(a) N firms employ two transportable inputs (l and k) located at A and B to produce a homogenous product (q) which is sold in the output market C. The location triangle in Figure 1 illustrates the location problem of oligopolistic firms. In figure 1, the distance a and b and the angle $\pi/2 > \gamma > 0$ are known; h is the distance between the plant location (E) and C; s and z are the distances of plant location (E) from A and B, respectively; θ is the angle between CA and CE.

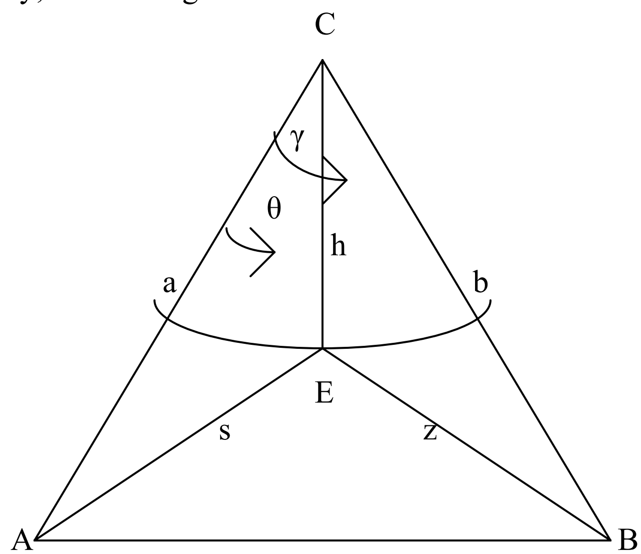

Figure 1. The Weber-Moses Triangle

(b) Firms make Cournot conjectures about their rivals' production and location decisions and enter the industry without any restrictions until there is no economic profit. Assume also that equilibra are symmetric. Thus, we can neglect the location dispersion of firms and focus on the impact of market demand on the location decision of a representative firm.

(c) The production function is homogeneous of degree n,

$$q = f(l, k) \tag{1}$$

with the following properties: $f_l l + f_k k = nq$, $f_{ll} l + f_{lk} k = (n-1)f_l$, $f_{kl} l + f_{kk} k = (n-1)f_k$, $f_{ll} l^2 + 2f_{lk} lk + f_{kk} k^2 = n(n-1)q$, where $f_l \equiv \partial q/\partial l > 0$, $f_k \equiv \partial q/\partial k > 0$, $f_{lk} \equiv f_{kl} \equiv \partial^2 q/\partial l \partial k > 0$, $f_{ll} \equiv \partial^2 q/\partial l^2 < 0$, $f_{kk} \equiv \partial^2 q/\partial k^2 < 0$. MH (1992) assumes that the production function is homothetic. To simplify our analysis and make calculation tractable, we assume that the production function is homogeneous of degree n.

(d) The industry inverse demand function for output is given by

$$P = P(Q, \alpha) \tag{2}$$

where $Q = \sum q^i$ is the market quantity demanded, α is a demand shift parameter. $P_Q \equiv \partial P/\partial Q < 0$, $P_\alpha \equiv \partial P/\partial \alpha > 0$ and $P_{Q\alpha} \equiv \partial^2 P/\partial Q \partial \alpha = 0$, cf. MH (1992, p. 256). Note that $\sum$ denotes $\sum_{i=1}^{N}$.

(e) The prices of inputs and output are evaluated at the plant location (E). The cost of purchasing inputs is the price of input at the source plus the freight cost, and the price of output is the market price minus the freight cost.
(f) Firms are the price taker in the K market, but have oligopsony power in the L market. Thus, they face an upward sloping labor supply curve, i.e.,

$$w = w(L),\ w_L \equiv dw/dL > 0. \tag{3}$$

where $L = \sum l^i$ is the quantity supplied of labor, cf. Chen and Lent (1992).
(g) Transportation rates are constant.
(i) The objective of each firm is to find the optimum location and production within the Weber-Moses triangle which maximizes the profit.

It is of interest to note that the inclusion of oligopsony power in the labor market constitutes the only point of departure from the MH model.

With these assumptions, the profit maximizing location problem of the representative firm is given by

$$\max \Pi = [P(Q, \alpha)\text{-}th]f(l, k) - [w(L)+t_1 s]l - (r+t_2 z)k \tag{4}$$

where $s = (a^2 + h^2 - 2ah\cos\theta)^{1/2}$, $z = [b^2 + h^2 - 2bh\cos(\gamma\text{-}\theta)]^{1/2}$; w(L) and r are the base prices of l and k at their sources A and B; t, t_1 and t_2 are constant transportation rates of q, l, k; s, z, and h are the distances from the plant location to the source location A, B and the market location C. It is worth mentioning that q, l, k, h and θ are choice variables and α, a, b, $\pi/2 > \gamma > 0$ are positive parameters.

To simplify our analysis, we first derive the input demand function of l and k in terms of q, h and θ based on the following constrained cost maximization problem at a given location,

$$\min L = [w(L) + t_1 s]l + (r + t_2 z)k + \lambda[q - f(l, k)] \tag{5}$$

where λ is the Lagrange multiplier. Setting the partial derivative of *L* with respect to l, k and λ equal to zero, we obtain the first-order conditions for a minimum as

$$\partial L/\partial l = ME + t_1 s - \lambda f_l = 0 \tag{6}$$
$$\partial L/\partial k = r + t_2 z - \lambda f_k = 0 \tag{7}$$
$$\partial L/\partial \lambda = q - f(l, k) = 0 \tag{8}$$

where ME is the marginal expense of l and $ME = [w(L) + w_L l] > 0$. The relationship between l (or k) and q can be derived by using the standard comparative static analysis,

$$\partial l/\partial q = (1/J)(A_1 f_{kk} - B_1 f_{lk}) \tag{9}$$
$$\partial k/\partial q = (1/J)(B_1 f_{ll} - A_1 f_{kl} - C_1 f_k) \tag{10}$$
$$J = (1/l)[nq(A_1 f_{kk} - B_1 f_{lk}) - C_1 l f_k^2]$$
$$= (1/k)[nq(B_1 f_{ll} - A_1 f_{kl}) - C_1 k f_k^2] \tag{11}$$

where A_1 = ME + t_1s, B_1 = r + t_2z, C_1 = $2w_L$ + $w_{LL}l$, and J < 0 by the second-order condition.

In MH (1992), oligopsony does not exist, i.e., ME = w and C_1 = 0, we can obtain

$$\partial l/\partial q = (l/nq) \text{ and } \partial k/\partial q = (k/nq). \tag{12}$$

The results in (9) – (11) and (12) are similar to those in the non-spatial model, cf. Silberberg (1978, pp. 201-202). Equation (12) indicates that if the production function is homogeneous, delivered prices are independent of input usage and the input proportion depends only upon the constant delivered price ratio. A change in output will not change input proportion. However, in the case where oligopsony exists, the delivered price ratio changes with input usage. A change in output and inputs will change the delivered price ratio and then input proportion. Thus, the expansion path is not an isocline and the result in (12) no longer applies.

It is worth mentioning that if the oligopolistic firm does not have oligopsony power, the delivered prices are constant, as MH (1992, p. 255) pointed out, the cost function can be written as the product of two functions: a function of output and another function of the delivered prices only, i.e., $C(q) = c(w + t_1s, r + t_2z)H(q)$. However, the delivered prices are a function of quantity used and the cost function would be $C(q) = [w(L) + t_1s]l(q,\theta,h) + (r+t_2z)k(q,\theta,h)$ if the oligopolistic firms have oligopsony power.

Substituting the input demand functions, $l = l(q,\theta,h)$ and $k = k(q,\theta,h)$ into (4), we obtain the profit as a function of q, θ and h. Via the envelope theorem, the first-order conditions for a maximum would be

$$\partial\Pi/\partial q = [(P + P_Qq) - th] - (ME + t_1s)(\partial l/\partial q) - (r + t_2z)(\partial k/\partial q) = 0 \tag{13}$$
$$\partial\Pi/\partial\theta = - t_1s_\theta l - t_2z_\theta k = 0 \tag{14}$$
$$\partial\Pi/\partial h = - tq - t_1s_h l - t_2z_h k = 0 \tag{15}$$

where $s_\theta \equiv \partial s/\partial\theta$, $z_\theta \equiv \partial z/\partial\theta$, $s_h \equiv \partial s/\partial h$, $z_h \equiv \partial z/\partial h$. Assume that the second-order conditions are satisfied and the possibility of the corner solution is excluded, cf. MH (1992). We can solve (13) - (15) for q, θ and h when entry is prohibited.

If free entry is allowed, each firm in the industry earns normal profit only. The following condition must be satisfied.

$$\Pi = [P(Nq, \alpha) - th]q - [w(Nl)+t_1s]l(q,\theta,h) - (r+t_2z)k(q,\theta,h) = 0 \tag{16}$$

Equations (13) – (16) can be solved for q, θ, h and N in terms of α and other parameters. This completes the model that comprises the basic analytical framework.

3. The Effect of Demand on Location Decision

We are now in a position to examine the effect of a change in demand for output on the optimum location. Following MH (1992), we consider two cases: (1) h is given and θ is the choice variable; (2) both h and θ are choice variables.

3.1. h is given

In this case, Equation (15) can be dropped from the first-order conditions. Totally differentiating (13), (14) and (16) and applying Cramer's rule, we obtain

$$(\partial\theta/\partial\alpha)_h = (-1/JD_3)P_\alpha t^2 z_\theta C_1 f_k[P_{QQ}q^3 + E - w_{LL}l^2(\partial l/\partial q)] \quad (17)$$
$$(\partial q/\partial\alpha)_h = (q^3/D_3)P_\alpha \Pi_{\theta\theta}[P_{QQ}q^3 + E - w_{LL}l^2(\partial l/\partial q)] \quad (18)$$
$$E = (w_L l/J)[(n-1)q(A_1 f_{kk} - B_1 f_{kl}) - C_1 l f_k^2] > 0 \quad (19)$$

where $z_\theta = -bhz^{-(1/2)}\sin(\gamma - \theta) < 0$, and $\Pi_{\theta\theta} < 0$, $(\partial l/\partial q) > 0$, and $J < 0$ and $D_3 \equiv (\Pi_{qq}\Pi_{\theta\theta} - \Pi_{\theta q}^2)P_Q q^2 - (\Pi_{qN}\Pi_{\theta\theta})P_Q(N - 1)q < 0$ by the second-order conditions. Note that $C_1 = 2w_L + w_{LL}l$ is normally positive, see Varian (1990, pp. 430-433).

Assume that the oligopolistic firm is a price taker in the L market, i.e., $C_1 = 0$. It follows from (17) that $(\partial\theta/\partial\alpha)_h = 0$. This result is consistent with MH2. However, when the oligopolistic firm has oligopsony power, the sign of $(\partial\theta/\partial\alpha)_h$ can not be determined. Thus, we can conclude that the effect of demand on the optimum location is ambiguous. Next, we consider the case where the demand function and the labor supply function are linear, i.e., $P_{QQ} = 0$ and $w_{LL} = 0$. It is easy to see from (17) that

$$(\partial\theta/\partial\alpha)_h = (-1/D_3 J)P_\alpha t^2 z_\theta C_1 f_k E > 0 \quad (20)$$

Thus, we can conclude that

Proposition 1. *If h is constant, the optimum location of an oligopolistic firm moves away from the oligopsonistic input market if the market demand function and the labor supply function are linear.*

It is clear that this result is significantly different from MH2. It shows that both the market demand function and the labor market supply condition play an important role in the determination of firm's location if oligopsony exists.

The economic interpretation of this proposition is as follows. If the demand function and the labor supply function are linear, a rise in the market demand will increase the output level of each oligopolistic firm when free entry allowed, i.e., $(\partial q/\partial\alpha)_h > 0$. Thus, if the production function exhibits increasing returns to scale, an increase in the output level leads to a fall in the input ratio (l/k) via factor substitution. Hence, the material pull of k is greater (smaller) than the material pull of l, i.e., $t_2 z_\theta k > t_1 s_\theta l$. As a result, the oligopolistic firm has an incentive to move its plant away the oligopsonistic labor market.

3.2. h and θ are choice variables

In this case, we apply the standard comparative static analysis to (13) – (16) and obtain

$$(\partial\theta/\partial\alpha) = (1/D_4)P_\alpha(\Pi_{hq}\Pi_{\theta h} - \Pi_{\theta q}\Pi_{hh})[P_{QQ}q^3 + E - w_{LL}l^2(\partial l/\partial q)] \quad (21)$$
$$(\partial h/\partial\alpha) = (1/D_4)P_\alpha(\Pi_{\theta q}\Pi_{\theta h} - \Pi_{\theta\theta}\Pi_{hq})[P_{QQ}q^3 + E - w_{LL}l^2(\partial l/\partial q)] \quad (22)$$

$$(\partial q/\partial \alpha) = (1/D_4)P_\alpha(\Pi_{\theta\theta}\Pi_{hh} - \Pi_{\theta h}^2)[P_{QQ}q^3 + E - w_{LL}l^2(\partial l/\partial q)] \quad (23)$$

where $\Pi_{hq} = (-t/Jl)[(n-1)q(A_1f_{kk}-B_1f_{kl})+(C_1f_k/J)(tf_k+t_2z_h)$, $\Pi_{\theta h} = -(t_1s_{\theta h}l-t_2z_{\theta h}k)-[t_1s_\theta(\partial l/\partial h)+t_2z_\theta(\partial k/\partial h)]$, $\Pi_{\theta q} = (1/J)t_2z_\theta C_1f_k$, $\Pi_{hh}= -(t_1s_{hh}l+t_2z_{hh}k)-t_1s_h(\partial l/\partial h)+t_2z_h(\partial k/\partial h)]$, $\Pi_{\theta\theta} = -(t_1s_{\theta\theta}l-t_2z_{\theta\theta}k) - [t_1s_\theta(\partial l/\partial\theta)+t_2z_\theta(\partial k/\partial\theta)]$, $D_4 = P_Qq^2D_3 - q^2P_Q(N-1)(P_Q+P_{QQ}q)D_2$, D_2 and D_3 are the second-order and the third-order principal minors of Hessian determinant D_4, and $(\Pi_{\theta\theta}\Pi_{hh} - \Pi_{\theta h}^2) > 0$, $D_4 > 0$, by the second order conditions.

Assume that the market demand function is linear, i.e., $P_{QQ} = 0$. From (21) and (22), it is easy to see the signs of $(\partial\theta/\partial\alpha)$ and $(\partial h/\partial\alpha)$ are ambiguous. Thus, we have

Proposition 2. *When oligopsony power exists and the production function exhibits increasing returns to scale, the location decision of an oligopolistic firm depends upon a change in the market demand if the demand function is linear.*

This result is quite different from MH1. It shows that MH1 can not be applied to the oligopsony case.

Next, we consider the case where $C_1 = 0$ and $P_{QQ} \neq 0$. Applying these conditions into (21) and (22), we obtain

$$(\partial\theta/\partial\alpha) = [- t(n-1)/nD_4]q^3P_\alpha P_{QQ}\Pi_{\theta h} \quad (24)$$
$$(\partial h/\partial\alpha) = [t(n-1)/nD_4]q^3P_\alpha P_{QQ}\Pi_{\theta\theta} \quad (25)$$

where $(n-1) > 0$, $P_\alpha > 0$, $\Pi_{\theta\theta} < 0$, $D_4 > 0$. Since the signs of P_{QQ} and $\Pi_{\theta h}$ can not *a priori* be determined, the signs of $(\partial\theta/\partial\alpha)$ and $(\partial h/\partial\alpha)$ are ambiguous. Furthermore, from (24), we can show

$$(\partial h/\partial\alpha) > (<)\ 0, \text{ as } P_{QQ} < (>)\ 0 \quad (26)$$

In other words, an increase in demand causes each firm's output level to rise ($\partial q/\partial\alpha > 0$) and the input-output ratio to fall if the demand function is convex ($P_{QQ} > 0$). Hence, the relative strength of the input pulls to the market pull will decrease and the optimum location of the firm will move toward the output market. Clearly, this is MH3.

In the case where $C_1 > 0$ and $P_{QQ} \neq 0$, from (22), we can see that the sign of $(\partial h/\partial\alpha)$ is ambiguous. Thus, we can conclude that

Proposition 3. *When oligopsony power exists and the production function exhibits increasing returns to scale, the optimum location may not move toward (away from) the output market as demand increases if the demand function is convex (concave).*

Obviously, this result is different from MH3. In the case where oligopsony power exits, an increase in demand may cause each firm's output level to rise ($\partial q/\partial\alpha > 0$) and the input-output ratio to fall if the demand function is convex ($P_{QQ} > 0$). However, the relative strength of the input pulls and the output pull may not decrease. As a result, the optimum location may not move toward the output market.

4. Concluding Remarks

We have examined the impact of oligopsony power on the relationship between a change in demand for output and the location decision of undifferentiated oligopolistic firms. MH's study focuses on the case where oligopolistic firms are price takers in the labor market. Our work has generalized the study of MH in the sense that their results are valid only under some special circumstances.

Assuming that oligopolistic firms have oligopsony power, we show that the optimum location of an oligopolistic firm isn't independent of a change in demand if the demand function is linear. This indicates that MH1 can not be applied to the case where oligopsony power exists. In the case where the distance of an oligopolistic firm from the output market is held constant, we show that the optimum location moves away from the oligopsonistic input market if the demand function and the labor supply function are linear. In the case where the distance of an oligopolistic firm from the output market is a choice variable, we further show that the optimum location may not move toward (away from) the output market as demand increases if the demand function is convex (concave). These results are significantly different from MH2 and MH3. It indicates that the presence of oligopsony power has significant influence on the location decisions of undifferentiated oligopolistic firms.

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