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# FACTOR PRICES AND INTERNATIONAL TRADE: <br> A UNIFYING PERSPECTIVE 

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#### Abstract

How do trade liberalizations affect relative factor prices and to what extent do they cause factors to reallocate across sectors? We first present a general framework that nests a wide range of models that have been used to study the link between globalization and factor prices. Under some restrictions, changes in the "factor content of trade" are sufficient statistics for the impact of trade on relative factor prices. We then study the determination of the factor content of trade in a specific version of our general framework featuring imperfect competition, increasing returns to scale, and heterogeneous producers. We show how heterogeneous firms' decisions shape the factor content of trade, and, therefore, the impact of trade liberalization on relative factor prices and between-sector factor allocation.


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## 1 Introduction

How do trade liberalizations affect the skill premium, or relative factor prices more generally, and to what extent do they cause factors to reallocate between sectors and across producers within sectors? This paper offers a unifying perspective on the fundamental forces that shape factor prices and factor allocation in a global economy.

In the first part of the paper we provide a set of sufficient statistics for the determination of factor prices across a wide range of international trade models. To do so, we consider a general framework that imposes only one key restriction: all efficiency units of a given factor employed in a country receive a common price. Given its generality, this framework nests the traditional Heckscher-Ohlin model, which emphasizes differences in factor intensities across sectors and factor endowments across countries. ${ }^{1}$ It also nests other models-emphasizing, e.g., differences in skill intensities between exporters and non-exporters within sectors, differences in the tradeability of skill-intensive and unskill-intensive goods, and complementarities between skilled labor and traded goods such as capital ${ }^{2}$ - that have been used to study the link between international trade and the skill premium. ${ }^{3}$

We show that within this framework, each factor price can be expressed as the product of two components. The first component is the inverse of the trade-adjusted factor supply, which is the domestic supply of that factor less the factor content of trade (FCT); the FCT is the quantity of that factor embodied in the country's net exports. A decrease in the trade-adjusted factor supply increases the factor's price, just like a decrease in its domestic supply. The second component is the factor payments for domestic absorption, which is the counterfactual payments to that factor if domestic sectoral absorption were produced domestically; this component depends on domestic sectoral expenditure shares and factor shares in sectoral revenues. An increase in the average revenue share of a factor increases the price of this factor. We use this decomposition to show how various mechanisms (which have been proposed to link trade to factor price) operate through these two components.

Under some additional restrictions, the ratio of factor payments for domestic absorption between any two factors is constant, so that changes in relative factor prices depend only on trade-adjusted factor supplies. In any model satisfying these restrictions, changes in the FCT

[^0]are sufficient statistics for the impact of trade on the relative price of any two factors if the domestic supplies of those two factors are fixed: changes in the economic environment - such as trade costs, foreign productivities, foreign factor supplies, foreign production functions, domestic productivities, or domestic supplies of other factors - affect domestic relative factor prices only through changes in the FCT. A similar result has been obtained previously by Deardorff and Staiger (1988) and Deardorff (2000) in perfectly competitive environments with constant returns to scale and common production technologies across producers within sectors. ${ }^{4}$ We show that this result applies more generally in models with imperfect competition, increasing returns to scale, and heterogeneous producers.

While our general framework makes a clear link between the FCT and factor prices, it takes the FCT as given. Hence, it does not provide insights into how changes in the economic environment, such as changes in trade costs, affect the FCT and relative factor prices. Moreover, while the FCT is a powerful sufficient statistic to assess the impact of trade on factor prices across a range of models, measuring the FCT in the data requires detailed information on factor employment and trade across highly disaggregated industries, which may be unavailable in practice; see e.g. Feenstra and Hanson (2000).

The second part of the paper studies the determination of the FCT. To do so, we specialize the general framework above to an environment with two-countries, two-factors (skilled and unskilled labor), and two-sectors, as in the Heckscher-Ohlin model; with sectoral productivity differences across countries, as in the Ricardian model; and with monopolistic competition and heterogeneous firms, as in Melitz (2003) and Bernard, Redding, and Schott (2007). In this model, the ratio of factor payments for domestic absorption between any two factors is constant, so that the FCT fully determines the relative price of skilled to unskilled labor (the skill premium), as in our general framework. Moreover, the FCT also fully determines the extent of between-sector factor reallocation and between-sector trade.

The FCT is shaped by comparative advantage, which is determined by cross-country differences in factor endowments and sectoral productivities - as in the standard HeckscherOhlin and Ricardian models, respectively. The strength of comparative advantage, however, is also shaped by firms' decisions to enter and to operate in each market, which are absent in these standard models. In particular, an increase in the mass of country 1 firms that sell in a given destination market in a given sector is equivalent - in terms of its impact on the FCT, the skill premium, between-sector factor reallocation, and between-sector trade - to an increase in country 1's exogenous Ricardian productivity in that sector. The mass of firms selling to a given destination increases either because of an increase in the mass of entering

[^1]firms or because of an increase in the fraction of entrants that operate in the destination. Moreover, the extent to which changes in the mass of firms selling in each destination affects the FCT depends on the degree of productivity heterogeneity. We use this logic to obtain the following results on the impact of trade liberalization on the FCT and, hence, on the skill premium and the extent of between-sector factor reallocation and between-sector trade.

We first show that a reduction in trade costs induces countries to expand production and exports in their comparative advantage sector and contract production elsewhere, as in the Heckscher-Ohlin and Ricardian models. This between-sector reallocation lowers the tradeadjusted supply of the factor used intensively in the comparative advantage sector (by raising its FCT) and hence raises its relative price. This effect is often referred to as the StolperSamuelson effect. We then study how the impact on the FCT - and, hence, the change in the skill premium and the extent of between-sector trade and factor reallocation-of moving from autarky to given aggregate trade shares is shaped by the extent of productivity heterogeneity and by heterogeneous firms' decisions to enter and operate in each market.

Greater within-sector productivity heterogeneity weakens ex-ante comparative advantage, reducing the magnitude of the change in the FCT - and, hence, the change in the skill premium and the extent of between-sector trade and factor reallocation-for a given change in trade shares. Given the extensive evidence of large productivity differences within narrowly-defined sectors, this comparative static exercise provides a rationale for empirical results suggesting that the FCT is not very large for many countries like the US, and that the extent of between-sector factor reallocation induced by trade and its impact on the skill premium are small in practice; see e.g. Goldberg and Pavcnik (2007). Endogenous entry and endogenous selection of firms into markets increases the magnitude of the change in the FCT and, hence, the change in the skill premium and the extent of between-sector trade and factor reallocation induced by a given change in trade shares. This result implies that measures of sectoral productivity and endowment differences across countries would underestimate the impact of trade liberalization on the skill premium and between-sector factor reallocation if firm entry decisions are not take into account. Note however that, given our earlier results, the extent of within-sector productivity heterogeneity, endogenous entry, and selection of firms into markets have no effect whatsoever on changes in factor prices, between-sector factor allocation, or between-sector trade, for given changes in the FCT.

Our results are related to recent papers in international trade identifying robust insights for welfare analysis across different models; see e.g., Arkolakis, Costinot, and RodriguezClare (Forthcoming) and Atkeson and Burstein (2010). Whereas these papers focus on the welfare implications of international trade, we focus on the distributional implications of international trade. We show that across a wide range of workhorse models, the effects of
international trade on the skill premium can be summarized by changes in the FCT. ${ }^{5}$
Our paper is most closely related to Bernard, Redding, and Schott (2007), henceforth BRS. Our contribution relative to BRS is as follows. First, we show that changes in the FCT are sufficient statistics for the impact of international trade on the skill premium and betweensector factor allocation. Second, we demonstrate analytically how the extent of within-sector productivity heterogeneity, endogenous entry, and selection of firms into markets each affects the impact of trade on the skill premium, between-sector trade and factor reallocation. Third, we revisit their finding that differences in factor endowments induce what BRS call "endogenous Ricardian productivity differences" at the industry level. ${ }^{6}$

## 2 Factor Prices: A Unifying Framework

In this section we present a general framework to examine the link between factor prices and trade. The key assumption in this framework is that in each country, all efficiency units of a given factor of production receive a common wage or price. We first derive a simple expression relating equilibrium factor prices to two components: trade-adjusted relative factor supplies and the factor payments for domestic absorption. We then show how changes in relative factor prices within a range of workhorse models of trade can be mapped into these two components. Finally, we describe a set of assumptions that are standard in the literature under which changes in the FCT are sufficient statistics for the impact of trade on relative factor prices.

### 2.1 General Framework

There are $N$ countries, indexed by $n=1, \ldots, N ; J$ sectors, indexed by $j=1, \ldots, J ;$ and $K$ factors of production, indexed by $k=1, \ldots, K$. Let $L_{k, i} \geq 0$ denote the stock of factor $k$ employed in country $i$. There is a common price, $w_{k, i} \geq 0$, for all units of factor $k$ employed in country $i .^{7}$ We denote by $E_{i}(j) \geq 0$ country $i$ 's total expenditure on sector $j$, and by

[^2]$\Lambda_{\text {in }}(j) \in[0,1]$ the share of country $n$ 's total expenditure in sector $j$ that is allocated to goods from country $i$, with $\sum_{i} \Lambda_{i n}(j)=1$.
Factor payments: Let $L_{k, i n}(j)$ denote the quantity of factor $k$ in country $i$, sector $j$ that is employed in supplying destination market $n$. At this point, $L_{k, i n}(j)$ is an accounting variable describing how factor usage is distributed across destination markets. In Section 2.2 we discuss how $L_{k, i n}(j)$ can be constructed in a range of specific models. The quantity of factor $k$ used in country $i$, sector $j$, across all destination markets is
$$
L_{k, i}(j)=\sum_{n} L_{k, i n}(j)
$$

The sum of $L_{k, i}(j)$ across industries must equal total employment of factor $k$ :

$$
\begin{equation*}
L_{k, i}=\sum_{j} L_{k, i}(j) \tag{1}
\end{equation*}
$$

Denote by $\lambda_{\text {in }}(j) \in[0,1]$ the share of country $i$ revenues from sales in country $n$ in sector $j$ that is paid to factors,

$$
\lambda_{i n}(j)=\frac{\sum_{k} w_{k, i} L_{k, i n}(j)}{\Lambda_{i n}(j) E_{n}(j)}
$$

and denote by $\alpha_{k, i n}(j) \in[0,1]$ the share of these factor payments that are paid to factor $k$,

$$
\alpha_{k, i n}(j)=\frac{w_{k, i} L_{k, i n}(j)}{\sum_{k^{\prime}} w_{k^{\prime}, i} L_{k^{\prime}, i n}(j)},
$$

where $\sum_{k} \alpha_{k, i n}(j)=1$ for all $n$ and $j$. Using these definitions, we can re-express equation (1) as

$$
\begin{equation*}
w_{k, i} L_{k, i}=\sum_{j} \sum_{n} \lambda_{i n}(j) \alpha_{k, i n}(j) \Lambda_{i n}(j) E_{n}(j) . \tag{2}
\end{equation*}
$$

Equation (2) is purely an accounting relationship, stating that the payments to a factor must equal the value of this factor used across all sectors in the production of goods bound for all destination markets.

Factor content of trade: Denote by $F C T_{k, i}$ the factor content of trade for factor $k$ in country $i$, i.e. the net exports of factor $k$ embodied in country $i$ 's trade:

$$
F C T_{k, i}=\sum_{j} \sum_{n}\left[L_{k, i n}(j)-L_{k, i i}(j) \frac{\Lambda_{n i}(j)}{\Lambda_{i i}(j)}\right] .
$$

Using our definitions of $\lambda_{i n}(j)$ and $\alpha_{k, i n}(j)$, we can express the payments for the FCT,
$w_{k, i} F C T_{k, i}$, as

$$
\begin{equation*}
w_{k, i} F C T_{k, i}=\sum_{j} \sum_{n \neq i}\left[\lambda_{i n}(j) \alpha_{k, i n}(j) \Lambda_{i n}(j) E_{n}(j)-\lambda_{i i}(j) \alpha_{k, i i}(j) \Lambda_{n i}(j) E_{i}(j)\right] . \tag{3}
\end{equation*}
$$

We can understand this expression for the payments for the FCT, $w_{k, i} F C T_{k, i}$, as follows. The first term in the summation in equation (3), $\lambda_{i n}(j) \alpha_{i n, k}(j) \Lambda_{i n}(j) E_{n}(j)$, simply represents the payments to factor $k$ embodied in country $i$ 's exports to destination market $n$. The second term in the summation, $\lambda_{i i}(j) \alpha_{k, i i}(j) \Lambda_{n i}(j) E_{i}(j)$, represents the counterfactual payments to factor $k$ in country $i$, had country $i$ produced for itself the value of goods that it imported from country $n$.

Note that constructing the FCT in the data requires input usage by destination country, which may be difficult to observe in practice. In Section 2.2, we discuss a range of models in which the construction of $F C T_{k, i}$ is simplified significantly.
Factor prices: To show how $F C T_{k, i}$ is related to $w_{k, i}$, we proceed as follows. By equation (2), equation (3), and the identity $\Lambda_{i i}(j)=1-\sum_{n \neq i} \Lambda_{n i}(j)$, we decompose payments to factor $k$ into two components:

$$
\begin{equation*}
w_{k, i} L_{k, i}=w_{k, i} F C T_{k, i}+\Phi_{k, i} . \tag{4}
\end{equation*}
$$

The first component is the payments for the FCT defined in equation (3). The second component is the factor payments for domestic absorption (FPD), $\Phi_{k, i}=\sum_{j} \lambda_{i i}(j) \alpha_{k, i i}(j) E_{i}(j)$, which is the counterfactual payments to the factor if domestic absorption were produced domestically. By equation (4), factor $k$ 's price is

$$
\begin{equation*}
w_{k, i}=\Phi_{k, i} / \mathcal{L}_{k, i}, \tag{5}
\end{equation*}
$$

where $\mathcal{L}_{k, i}=L_{k, i}-F C T_{k, i}$ denotes the trade-adjusted supply of factor $k$.
By comparing equations (2) and (5), it is apparent that for given values of $\lambda_{i i}(j), \alpha_{k, i i}(j)$, and $E_{i}(j)$, the price paid to factor $k$ in a trade equilibrium is equal to the price that would have been paid to factor $k$ in autarky had country $i$ 's stock of factor $k$ been $\mathcal{L}_{k, i}$ rather than $L_{k, i}$. If a country is a net exporter of factor $k$, then its factor price is determined as if it has a smaller stock of this factor. In this sense, we can think of $\mathcal{L}_{k, i}$ as the counterfactual stock of factor $k$ available in economy $i$ in the presence of international trade.

Using equation (5), we express the price of factor $k_{1}$ relative to factor $k_{2}$ as

$$
\begin{equation*}
w_{k_{1}, i} / w_{k_{2}, i}=\left(\mathcal{L}_{k_{2}, i} / \mathcal{L}_{k_{1}, i}\right) \times\left(\Phi_{k_{1}, i} / \Phi_{k_{2}, i}\right) . \tag{6}
\end{equation*}
$$

Equation (6) decomposes the relative price of factor $k_{1}$ to factor $k_{2}$ into two terms: $(i)$ the trade-adjusted supply of $k_{2}$ relative to $k_{1}$ and (ii) the FPD of $k_{1}$ relative to $k_{2}$. An increase in $\mathcal{L}_{k_{2}, i} / \mathcal{L}_{k_{1}, i}$, either through a decrease in the relative supply of factor $k_{1}$ or an increase in the FCT of $k_{1}$, increases the relative price of $k_{1}$. Similarly, an increase in $\Phi_{k_{1}, i} / \Phi_{k_{2}, i}$, either through an increase in expenditure shares in sector intensive in factor $k_{1}$ or an increase in the average revenue share of factor $k_{1}$ across sectors, increases the relative price of $k_{1}$.

We summarize these results in the following proposition, which provides an equation for the change in the relative price of factor $k_{1}$ to factor $k_{2}$ between any two equilibria.

Proposition 1 If $w_{k_{1}, i}^{\prime} / w_{k_{2}, i}^{\prime}, \mathcal{L}_{k, i}^{\prime}$, and $\Phi_{k, i}^{\prime}$ denote the relative price of factor $k_{1}$ to factor $k_{2}$, the trade-adjusted supply of factor $k$, and the factor payments for domestic absorption of factor $k$ in a counterfactual equilibrium, then

$$
\begin{equation*}
\frac{w_{k_{1}, i}^{\prime}}{w_{k_{2}, i}^{\prime}} / \frac{w_{k_{1}, i}}{w_{k_{2}, i}}=\left[\frac{\mathcal{L}_{k_{2}, i}^{\prime}}{\mathcal{L}_{k_{1}, i}^{\prime}} / \frac{\mathcal{L}_{k_{2}, i}}{\mathcal{L}_{k_{1}, i}}\right] \times\left[\frac{\Phi_{k_{1}, i}^{\prime}}{\Phi_{k_{2}, i}^{\prime}} / \frac{\Phi_{k_{1}, i}}{\Phi_{k_{2}, i}}\right] \tag{7}
\end{equation*}
$$

Of course, both $\mathcal{L}_{k, i}$ and $\Phi_{k, i}$ are endogenous, and their equilibrium determination-and therefore, how they are affected by trade liberalization - is outside the scope of this accounting framework. In Section 3, we specialize our general framework to study the determination of these variables.

### 2.2 Mapping Specific Models into Framework

In this section we discuss how a variety of models of international trade, technological change, and the skill premium can be mapped into the general framework above. We also describe a range of model assumptions under which expression (7) and the calculation of the FCT can be simplified significantly.
Heckscher-Ohlin-like perfectly competitive models: Here we focus on perfectly competitive models with constant returns to scale in which all producers within a sector share a common factor intensity that does not depend on the destination in which output is sold. These assumptions are satisfied in the Heckscher-Ohlin model-see, e.g., Stolper and Samuelson (1941) -and its multi-sector and multi-factor extensions - see, e.g., Ethier (1984), Jones and Scheinkman (1977), and Costinot and Vogel (2010). In these models, $L_{k, i n}(j)$ can be constructed easily as the product of sector $j$ 's employment of factor $k, L_{k, i}(j)$, and the ratio of country $i$ sector $j$ revenues earned in market $n$ to total revenues earned in that sector, $\Lambda_{i n}(j) E_{n}(j) /\left(\sum_{n^{\prime}} \Lambda_{i n^{\prime}}(j) E_{n^{\prime}}(j)\right)$. Hence, the share of factor payments accruing to factor $k$ in sector $j$ production-i.e. the factor $k$ intensity of production in sector $j$-is the same
across destination markets, $\alpha_{k, i n}(j)=\alpha_{k, i}(j)$ for all $i, n, k$, and $j$, where

$$
\begin{equation*}
\alpha_{k, i}(j)=\frac{w_{k, i} L_{k, i}(j)}{\sum_{k^{\prime}} w_{k^{\prime}, i} L_{k^{\prime}, i}(j)} . \tag{8}
\end{equation*}
$$

Moreover, with constant returns to scale and perfect competition, firm profits are zero, so $\lambda_{\text {in }}(j)=1$ for all $i, n$, and $j$.

In any setting in which $\alpha_{k, i n}(j)$ and $\lambda_{\text {in }}(j)$ are common across destination markets, we can simplify significantly the construction of net exports of factor $k$. In particular, we have

$$
\begin{equation*}
F C T_{k, i}=\sum_{j} L_{k, i}(j) \omega_{i}(j) \tag{9}
\end{equation*}
$$

where

$$
\omega_{i}(j)=\frac{\sum_{n \neq i}\left[\Lambda_{i n}(j) E_{n}(j)-\Lambda_{n i}(j) E_{i}(j)\right]}{\sum_{n} \Lambda_{i n}(j) E_{n}(j)}
$$

denotes the ratio of country $i$ 's net exports in sector $j$ to country $i$ 's total revenue in sector $j$. The variables $L_{k, i}(j)$ and $\omega_{i}(j)$, and hence the factor $k$ content of trade, can be measured in principle using sectoral production and trade data. ${ }^{8}$

In this environment, the expression in Proposition 1 is simplified only because $\alpha_{k, i n}(j)=$ $\alpha_{k, i}(j)$ and $\lambda_{i n}(j)=1$. However, we can further simplify this expression under a few additional assumptions. If preferences and production functions are Cobb-Douglas and the Cobb-Douglas share parameters are unchanged across equilibria, then equation (7) simplifies to

$$
\begin{equation*}
\frac{w_{k_{1}, i}^{\prime}}{w_{k_{2}, i}^{\prime}} / \frac{w_{k_{1}, i}}{w_{k_{2}, i}}=\frac{\mathcal{L}_{k_{2}, i}^{\prime}}{\mathcal{L}_{k_{1}, i}^{\prime}} / \frac{\mathcal{L}_{k_{2}, i}}{\mathcal{L}_{k_{1}, i}} \tag{10}
\end{equation*}
$$

In this special case of our general framework, relative factor prices change only due to changes in trade-adjusted factor supplies. For fixed domestic supplies of factors $k_{1}$ and $k_{2}$, any change in the economic environment - such as trade costs, foreign productivities, foreign factor supplies, foreign production functions, domestic productivities, or domestic supplies of factors other than $k_{1}$ and $k_{2}$-affects domestic relative factor prices only through changes in the FCT.

Expression (10) was also obtained in Deardorf and Staiger (1988) and Deardorff (2000) in a perfectly competitive environment with constant returns to scale and common productivities across producers within each sector. Our result allows for heterogeneous productivities

[^3]within sectors, as in a multi-sector and multi-factor version of Eaton and Kortum (2002). ${ }^{9}$
Common factor intensities across sectors: A particular class of models nested by the perfectly competitive, constant returns to scale models above are those in which factor intensity is identical across producers, sectors, and destination markets, $\alpha_{k, i n}(j)=\alpha_{k, i}$. These assumptions are satisfied in, e.g., Katz and Murphy (1992), Krusell, Ohanian, Rios-Rul, and Violante (2000), and Burstein, Cravino, and Vogel (2010). Under these assumptions, equation (6) simplifies to
$$
w_{k_{1}, i} / w_{k_{2}, i}=\left(\mathcal{L}_{k_{2}, i} / \mathcal{L}_{k_{1}, i}\right) \times\left(\alpha_{k_{1}, i} / \alpha_{k_{2}, i}\right)
$$

Moreover, because $F C T_{k, i}$ equals $\alpha_{k, i} / w_{k, i}$ times country $i$ 's net aggregate exports ( $i$ 's trade balance), we have $\mathcal{L}_{k_{2}, i} / \mathcal{L}_{k_{1}, i}=L_{k_{2}, i} / L_{k_{1}, i}$. Hence, equation (7) becomes

$$
\frac{w_{k_{1}, i}^{\prime}}{w_{k_{2}, i}^{\prime}} / \frac{w_{k_{1}, i}}{w_{k_{2}, i}}=\left[\frac{L_{k_{2}, i}^{\prime}}{L_{k_{1}, i}^{\prime}} / \frac{L_{k_{2}, i}}{L_{k_{1}, i}}\right] \times\left[\frac{\alpha_{k_{1}, i}^{\prime}}{\alpha_{k_{2}, i}^{\prime}} / \frac{\alpha_{k_{1}, i}}{\alpha_{k_{2}, i}}\right] .
$$

In this class of models, changes in relative factor prices across two points in time are driven entirely by changes in relative factor supplies and by changes in relative factor intensities. Changes in relative factor intensities can be driven by technological change (see e.g. Katz and Murphy 1992), ${ }^{10}$ capital accumulation (see e.g. Krusell, Ohanian, Rios-Rul, and Violante 2000), and capital accumulation and international trade (see e.g. Burstein, Cravino, and Vogel 2010). ${ }^{11}$

Factor intensity varies by destination market: In Matsuyama (2007) and Burstein and Vogel (2010), markets are perfectly competitive, production is constant returns to scale, and average factor intensities vary depending on destination market. In Matsuyama (2007) producers are homogeneous within a sector, and trade costs are assumed to be skill intensive relative to production. In Burstein and Vogel (2010), for a given producer, skill intensity is independent of destination market, but the most productive producers tend to export and to be more skill intensive. Hence in these models, $\lambda_{i}(j)=1$ but $\alpha_{k, i n}(j)$ tends not to equal $\alpha_{k, i i}(j)$ for $n \neq i$.

[^4]With constant returns to scale it is straightforward to allocate aggregate sectoral factor employment, $L_{k, i}(j)$, to each destination market, $L_{k, i n}(j)$. Hence, these models fit into the general framework presented above. However, equation (7) simplifies only because $\lambda_{i i}(j)=1$. In general, changes in trade costs will affect relative factor prices through both trade-adjusted factor supplies and the factor payments for domestic absorption.

Heckscher-Ohlin-like imperfectly competitive models: In Section 3 we consider a range of models featuring imperfect competition, heterogeneous firms, and increasing returns to scale, as in, e.g., Romalis (2004) and BRS (2007). With imperfect competition, firms may earn profits, so $\lambda_{i n}(j)$ is not generally equal to one. Moreover, in some cases it is not straightforward to allocate sectoral employment, $L_{k, i}(j)$, across destination markets, $L_{k, i n}(j)$. This can be the case, for example, if a firm must incur entry costs that do not depend on the set of destination markets it supplies. However, we show that in the model of Section 3, Proposition 1 holds with the FCT being constructed using equation (9), and that equation (7) simplifies to equation (10).

It is straightforward to show that the same results hold in a two-factor version of Bernard, Eaton, Jensen, and Kortum (2003), which is an extension of Eaton and Kortum (2002) with Bertrand instead of perfect competition. Since there are constant returns to scale (and no fixed costs), allocating factors across destination markets is straightforward. With Frechet distributed productivities and CES demand, $\lambda_{i n}(j)$ is constant and equal across destination markets. ${ }^{12}$

## 3 The FCT in a Heterogeneous Firm Model

While the framework in Section 2 links the FCT to relative factor prices, it takes the FCT as given. Hence, it does not provide insights into how changes in the economic environment such as changes in trade costs affect the FCT, and therefore, relative factor prices. We now focus on understanding the determination of the FCT. To do so, we specialize the general framework above to an environment with two sectors, two factors (skilled and unskilled labor), two countries, and monopolistic competition, in which heterogeneous firms choose whether or not to enter and which markets to supply.

We use this model to obtain three sets of results. First, in Section 3, we show that in this environment, Proposition 1 holds, the FCT is given by equation (9), and the calculation of

[^5]the FCT in equation (7) simplifies to equation (10). Second, in Section 4, we demonstrate that the FCT and factor endowments fully determine not only the relative price of skilled to unskilled labor (the skill premium), but also the extent of between-sector factor reallocation and between-sector trade. Finally, in Section 5, we show how the extent of productivity heterogeneity between and within sectors, and heterogeneous firms' decisions to enter and operate in each market shape the impact of trade liberalization on the FCT, and, therefore, on factor allocation and the skill premium.

### 3.1 Model

Our model economy features two countries, $i=1,2$; two factors, which we refer to as skilled labor and unskilled labor; and two sectors, $j=x, y$, where $x$ is skill intensive. While factors are perfectly mobile across producers within a country, they are internationally immobile. The exogenous and fixed endowments of skilled and unskilled labor in country $i$ are denoted by $L_{s, i}$ and $L_{u, i}$, respectively. Each country produces a final non-tradeable good using output of both sectors. Output in each sector is produced using a continuum of differentiated intermediate goods, which are produced by firms using skilled and unskilled labor. International trade of intermediate goods is subject to variable and fixed costs. Factors are perfectly mobile across firms and sectors but are immobile across countries.

Preferences: The representative consumer's utility is defined over a non-tradeable final good, $Q_{i}$, that (for expositional purposes) places equal weight on the output of each sector

$$
Q_{i}=Q_{i}(x)^{1 / 2} Q_{i}(y)^{1 / 2},
$$

where $Q_{i}(j)$ denotes the output of sector $j$. The aggregate price index is $P_{i}=\frac{1}{2} P_{i}(x)^{1 / 2} P_{i}(y)^{1 / 2}$, where $P_{i}(j)$ is the price of sector $j$. Demand for the sector $j \operatorname{good}$ is $Q_{i}(j)=\frac{E_{i}}{2 P_{i}(j)}$, where $E_{i}=Q_{i} P_{i}$ denotes total expenditure in country $i$.
Sectoral aggregates: Sector $j$ 's output, $Q_{i}(j)$, is a CES aggregate of varieties

$$
Q_{i}(j)=\left(\int_{\omega \in \Omega_{j}} q_{i}(\omega, j)^{(\eta-1) / \eta} d \omega\right)^{\eta /(\eta-1)}
$$

Here, $q_{i}(\omega, j)$ denotes country $i$ consumption of variety $(\omega, j)$, and $\eta>1$ is the elasticity of substitution between varieties. The price index in sector $j$ is $P_{i}(j)=\left[\int_{\omega \in \Omega_{j}} p_{i}(\omega, j)^{1-\eta} d \omega\right]^{1 /(1-\eta)}$, where $p_{i}(\omega, j)$ denotes the price of good $(\omega, j)$ in country $i$. Demand for variety $(\omega, j)$ is $q_{i}(\omega, j)=\left(\frac{p_{i}(\omega, j)}{P_{i}(j)}\right)^{-\eta} Q_{i}(j)$.
Intermediate good technologies: There are a continuum of firms, each producing a
unique variety $(\omega, j)$. Firms face variable costs of production, fixed (market access) costs of selling in each country, and iceberg costs of international trade. Both fixed and variable costs use skilled and unskilled labor, where the factor intensity of production varies across sectors but is constant across firms within a sector and across fixed and variable costs within a firm.

A sector $j$ firm from country $i$ with Hicks-neutral productivity $z \geq 1$ that hires $l_{s}$ units of skilled labor and $l_{u}$ units of unskilled labor in variable production activities produces $y=$ $z A_{i}(j) l_{s}^{\alpha_{s}(j)} l_{u}^{\alpha_{u}(j)}$ units of output, where $\alpha_{s}(j)+\alpha_{u}(j)=1$. Here, $\alpha_{k}(j)$ denotes the share of skilled $(k=s)$ and unskilled $(k=u)$ labor in production of all country $i$ firms in sector $j$, where we omit the dependence of $\alpha_{k}(j)$ on $i$ since factor intensities are equal in both countries. Because $x$ is skill intensive, we have $\alpha_{s}(x)>\alpha_{s}(y) . A_{i}(j)>0$ denotes country $i$ 's exogenous total factor productivity in sector $j$.

To facilitate exposition in our results below, we decompose $A_{i}(j)$ into two componentsnational TFP, $T_{i}$, and sectoral TFP, $T_{i}(j)$-so that $A_{i}(j)=T_{i} \times T_{i}(j)$. We normalize $T_{1}=1$. We define $a=A_{1}(x) A_{2}(y) / A_{1}(y) A_{2}(x)$ to be a measure of country 1's relative productivity advantage (if $a>1$ ) or disadvantage (if $a<1$ ) in sector $x$.

Firms from country $i$ must ship $\tau_{i n} q$ units of output in order for $q$ units to arrive in country $n$, with $\tau_{i i}=1$ and $\tau_{i n}=\tau_{n i}=\tau \geq 1$. We refer to $\tau$ as the iceberg transportation cost. Additionally, in order to supply a positive amount of goods to country $n$, a country $i$ firm incurs a fixed market access cost of $f_{\text {in }} \geq 0$ units of the sectoral composite input bundle in country $i$; we assume that these fixed costs are produced using the same input bundle as the production of intermediate goods in that sector. For simplicity, but without loss of generality for our results, we assume that variable and fixed trade costs are common across sectors. We denote by $f=f_{12} / f_{11}=f_{21} / f_{22}$ the relative fixed costs of international versus intra-national trade in all sectors and countries.

Under these assumptions on technology, a sector $j$ firm with productivity $z$ from country $i$ incurs a cost

$$
C_{i n}(q)=v_{i}(j)\left[\frac{q \tau_{i n}}{z}+f_{i n}\right]
$$

to supply $q>0$ units of goods to country $n$. We refer to $v_{i}(j)$ as the cost of the sector $j$ composite input bundle in country $i$, where

$$
\begin{equation*}
v_{i}(j)=\frac{1}{A_{i}(j)}\left[\frac{w_{s, i}}{\alpha_{s}(j)}\right]^{\alpha_{s}(j)}\left[\frac{w_{u, i}}{\alpha_{u}(j)}\right]^{\alpha_{u}(j)} . \tag{11}
\end{equation*}
$$

and where country $i$ 's wages for unskilled and skilled labor are $w_{s, i}$ and $w_{u, i}$, respectively. We denote by $c_{i n}(z, j)=v_{i}(j) \tau_{i n} / z$ the marginal cost of a firm with productivity $z$, sector
$j$, in country $i$ to supply a good to country $n$.
Conditional on a country $i$ firm paying the fixed cost to access market $n$, profit maximization implies that it charges a constant markup over its marginal cost, $p_{i n}(z, j)=\frac{\eta}{\eta-1} c_{i n}(z, j)$. In this case, a firm's market-specific revenue is proportional to its marginal cost,

$$
\begin{equation*}
r_{i n}(z, j)=\frac{E_{n}}{2 P_{n}(j)^{1-\eta}}\left[\frac{\eta}{\eta-1} c_{i n}(z, j)\right]^{1-\eta} \tag{12}
\end{equation*}
$$

and its market-specific variable profit is proportional to its revenue $\pi_{i n}(z, j)=r_{i n}(z, j) / \eta$.
Selection of firms into markets: A country $i$ firm chooses to supply market $n$ if the variable profit it earns there covers its fixed market access cost, $\pi_{i n}(z, j) \geq v_{i}(j) f_{\text {in }}(j)$. Denote by $z_{i n}^{*}(j)$ the productivity threshold at which the least productive sector $j$ firm from country $i$ sells in country $n$ :

$$
\begin{equation*}
z_{i n}^{*}(j)=\max \left\{\frac{\tau_{i n}}{P_{n}(j)}\left[\frac{2 \eta^{\eta} f_{i n}}{(\eta-1)^{1-\eta} E_{n}}\right]^{\frac{1}{\eta-1}} v_{i}(j)^{\frac{\eta}{\eta-1}}, 1\right\} . \tag{13}
\end{equation*}
$$

In order to understand the implications of endogenous selection for trade patterns and relative factor rewards, we consider specifications in which endogenous selection into markets is and is not active. In the specification in which endogenous selection is not active, we assume that $f_{i n}=0$ for all $i, n \in I$, so that every entrant sells to each market: $z_{i n}^{*}(j)=1$ for all $i, n \in I$ and $j \in J .{ }^{13}$ We refer to this as the case "without selection." This case corresponds to a multi-factor extension of Krugman (1980), as in Helpman and Krugman (1985) and in Romalis (2004).

In the specification in which endogenous selection is active, we assume that $f_{i n}$ is sufficiently large for all $i, n \in I$ and $j \in J$ such that there is selection into every market, i.e. $z_{i n}^{*}(j)>1$ for al $i, n \in I$ and $j \in J$. We refer to this as the case "with selection." This case corresponds to a multi-factor extension of Melitz (2003) - as in BRS-or of Chaney (2008). Note that the two cases we consider are not exhaustive. There are parameter values for which there exist country-pairs and sectors such that $z_{i n}^{*}(j)=1$ and $z_{k l}^{*}\left(j^{\prime}\right)>1$.

We define $t$ to be the relative size of international versus intra-national trade costs,

$$
t= \begin{cases}\tau^{\eta-1} & \text { without selection } \\ \tau^{\gamma} f^{\frac{\gamma+1-\eta}{\eta-1}} & \text { with selection }\end{cases}
$$

It is this relative cost $t$ that matters for our results throughout the paper, rather than $\tau$ and

[^6]$f$ separately. We assume that relative costs of international trade are strictly greater than those of intra-national trade, so that $t>1$. Under this assumption, any firm that exports also sells domestically.

Entry: In order to understand the implications of endogenous entry for trade patterns and relative factor prices, we consider two alternative specifications on the determination of the mass of entering firms in each sector, $M_{i}(j)$; we refer to these specifications as exogenous and endogenous entry. The difference between the two specifications is the timing regarding when entrepreneurs (potential entrants) realize their productivities.

In the specification with exogenous entry, we assume that entrepreneurs know their productivities ex-ante. In this case, the mass of entrepreneurs is fixed at $M_{i}(j)$, since if it were unbounded then only the most productive would enter. Firms in each sector/country draw their productivity $z$ from a Pareto distribution with shape parameter $\gamma$ and location parameter one: $G(z)=\operatorname{Pr}(Z \leq z)=1-z^{-\gamma}$. This case corresponds to, e.g., Chaney (2008), Arkolakis (Forthcoming), and Eaton et. al. (Forthcoming). For simplicity and without loss of generality, we assume in the exogenous entry case that $M_{i}(j)=M_{i} .{ }^{14}$

In the specification with endogenous entry, we assume that entrepreneurs are identical ex-ante. In this case, in each country/sector there is an unbounded mass of ex-ante identical potential entrants. To enter, an entrepreneur incurs a fixed entry cost of $f^{e}>0$ units of the sectoral composite input bundle (in the exogenous entry case, we assume that $f^{e}=0$ for all $j$ ). That is, sector $j$ startup costs in country $i$ are $f^{e} v_{i}(j)$. Upon entry, firms draw their productivity $z$ from the same distribution $G(z)$ defined above. This case corresponds to a version of Melitz (2003) and BRS (2007) with Pareto distributed productivities. The free entry condition, for all $j$, is given by

$$
\sum_{n=1}^{I} \int_{z_{i n}^{*}(j)}^{\infty}\left[\pi_{i n}(z, j)-v_{i}(j) f_{i n}\right] d G(z) \leq v_{i}(j) f^{e} \text { with equality if } M_{i}(j)>0
$$

Finally, in all that follows we focus exclusively on cases with incomplete specialization; i.e. in which $M_{i}(j)>0$ for all $i \in I$ and $j \in J$.
Trade balance: We assume trade balance in both countries. This implies that total expenditure equals total income (wages and profits) in each country,

$$
E_{i}=\sum_{k=s, u} w_{k, i} L_{k, i}+\sum_{j} \sum_{n} M_{i}(j)\left\{\int_{z_{i n}^{*}(j)}^{\infty}\left[\pi_{i n}(z, j)-v_{i}(j) f_{i n}\right] d G(z)-v_{i}(j) f^{e}\right\}
$$

[^7]
### 3.2 Equilibrium Characterization

In this section we derive the equations that we use to solve for equilibrium factor prices and trade patterns. We consider specifications (i) with endogenous or exogenous entry and (ii) with or without selection.

International trade: Denote by $\Lambda_{\text {in }}(j)$ the sector $j$ expenditure share in country $n$ on goods from country $i$. By definition, we have

$$
\Lambda_{i n}(j)=\frac{M_{i}(j) \int_{z_{i n}^{*}(j)}^{\infty} r_{i n}(z, j) d G(z)}{\sum_{k=1}^{I} M_{k}(j) \int_{z_{k n}^{*}(j)}^{\infty} r_{k n}(z, j) d G(z)}
$$

Substituting in for $G(z)$ and $r_{i n}(z, j)$ yields

$$
\begin{equation*}
\Lambda_{i n}(j)=\frac{M_{i}(j) v_{i}(j)^{1-\eta} z_{i n}^{*}(j)^{\eta-\gamma-1} \tau_{i n}^{1-\eta}}{\sum_{k=1}^{I} M_{k}(j) v_{k}(j)^{1-\eta} z_{k n}^{*}(j)^{\eta-\gamma-1} \tau_{k n}^{1-\eta}} \tag{14}
\end{equation*}
$$

In the specification without selection, in which $z_{i n}^{*}(j)=1$ for all $i, n \in I$ and $j \in J$, Equation (14) implies

$$
\begin{equation*}
\Lambda_{i n}(j)=\frac{M_{i}(j) v_{i}(j)^{1-\eta} \tau_{i n}^{1-\eta}}{\sum_{k=1}^{I} M_{k}(j) v_{k}(j)^{1-\eta} \tau_{k n}^{1-\eta}} \tag{15}
\end{equation*}
$$

In the specification with selection, in which $z_{i n}^{*}(j)>1$ for al $i, n \in I$ and $j \in J$, Equation (14) implies

$$
\begin{equation*}
\Lambda_{i n}(j)=\frac{M_{i}(j)\left[v_{i}(j)\right]^{\frac{\gamma \eta-\eta+1}{1-\eta}} f_{i n}^{\frac{\gamma-\eta+1}{1-\eta}} \tau_{i n}^{-\gamma}}{\sum_{k=1}^{I} M_{k}(j)\left[v_{k}(j)\right]^{\frac{\gamma \eta-\eta+1}{1-\eta}} f_{k n}^{\frac{\eta-\gamma-1}{\eta-1}} \tau_{k n}^{-\gamma}} . \tag{16}
\end{equation*}
$$

We denote by $\Delta_{i}=\frac{1}{2}\left[\Lambda_{n i}(x)+\Lambda_{n i}(y)\right]$, for $n \neq i$, country $i$ 's trade share. Note that $\Delta_{i}$ is the share of country $i$ 's expenditure allocated to imports from country $n \neq i$. We also denote by $\Theta_{i}=\Lambda_{n i}(y)-\Lambda_{n i}(x)$, for $n \neq i$, the share of country $i$ 's expenditure allocated to imports in sector $y$ minus the share of expenditures allocated to imports in sector $x$. The greater in absolute value is $\Theta_{i}$, the greater is the difference between net imports in the $x$ and $y$ sectors. Hence, for a given trade share $\Delta_{i}, \Theta_{i}$ indicates the importance of between sector trade relative to within sector trade.

Labor market clearing: In Appendix A we show that the labor market clearing conditionswhen entry is endogenous or exogenous and with or without selection-are given by

$$
\begin{equation*}
w_{k, i} L_{k, i}=\sum_{j} \sum_{n} \lambda \alpha_{k}(j) \Lambda_{i n}(j)\left(\frac{E_{n}}{2}\right) \tag{17}
\end{equation*}
$$

where $\lambda$ is the share of revenues paid to all factors in both sectors,

$$
\begin{equation*}
\lambda E_{i}=\sum_{k=s, u} w_{k, i} L_{k, i}, \tag{18}
\end{equation*}
$$

and is given by

$$
\lambda= \begin{cases}1 & \text { with endogenous entry }  \tag{19}\\ \frac{\gamma \eta-\eta+1}{\gamma \eta} & \text { with exogenous entry and with selection } \\ \frac{\eta-1}{\eta} & \text { with exogenous entry and without selection }\end{cases}
$$

in the different specifications of the model.
Equilibrium firm entry: In Appendix A we show that with endogenous entry, the mass of entering firms in each sector is given by

$$
\begin{equation*}
M_{i}(j) v_{i}(j) f^{e}=\widetilde{\lambda} \sum_{n} \Lambda_{i n}(j)\left(\frac{E_{n}}{2}\right) \tag{20}
\end{equation*}
$$

where $\widetilde{\lambda}=1 / \eta$ without selection and $\widetilde{\lambda}=(\eta-1) /(\gamma \eta)$ with selection.
Solving for an equilibrium: Equilibrium factor prices, total expenditures $E_{i}$, expenditure shares $\Lambda_{i n}(j)$, and entrants $M_{i}(j)$ can be solved for using factor market clearing as given by equation (17) (note that, by Walras' law, one equation is redundant), equation (18), expenditure shares $\Lambda_{i n}(j)$ as given by Equation (15) without selection and by Equation (16) with selection, and the free-entry conditions (with endogenous entry) as given by Equation (20). ${ }^{15}$

We compute production and consumption of the final non-tradeable good, $Q_{i}$, as follows. Given factor prices, nominal expenditures, and entry levels, the solution for sectoral price indices is provided in Appendix B. Using sectoral price indices and the definition of the aggregate price level, $P_{i}$, above we obtain $Q_{i}$. Our model and this solution procedure can be extended to any number of factors, sectors, and countries.

In some comparative static exercises, in Section 5, we simplify the model solution by assuming that countries and sectors are mirror symmetric: $A_{1}(x)=A_{2}(y), A_{1}(y)=A_{2}(x)$, $L_{s 1}=L_{u 2}, L_{u 1}=L_{s 2}$, and $\alpha_{x}=1-\alpha_{y}$. Mirror symmetry makes the model more tractable because $w_{s 1}=w_{u 2}, w_{u 1}=w_{s 2}$, and $E_{1}=E_{2}$.

[^8]
### 3.3 Mapping to General Framework

Since there is a common wage, $w_{k, i}$, for factor $k$ in country $i$, the model clearly fits into the general framework presented in Section 2. In the specification with exogenous entry, constructing $L_{k, i n}(j)$ is straightforward. It is the sum of factor $k$ employment in variable production and market access costs for supplying destination market $n$. With CES sectoral aggregators, the share of variable costs in total sectoral revenue is constant. With CES sectoral aggregators and Pareto-distributed productivity, the share of market access costs in total sectoral revenue is also constant. Hence, with common $\eta$ and $\gamma$ across sectors and countries, $\lambda_{\text {in }}(j)=\lambda$ for all destination markets and in each sector, where $\lambda=(\gamma \eta-\eta+1) /(\gamma \eta)$ with selection and $\lambda=(\eta-1) / \eta$ without selection. Since factor intensity is common across fixed and variable costs as well as across source and destination markets, we have $\alpha_{k, i n}(j)=\alpha_{k}(j)$. Hence, equation (2) from the general framework of Section 2 is simplified to equation (17) in our specialized model.

In the specification with endogenous entry, constructing $L_{k, i n}(j)$ is more subtle because there are multiple ways of allocating entry costs, $f^{e}$, across destination markets. However, for any construction of $L_{k, i n}(j)$ consistent with equilibrium sectoral factor allocation (i.e., $\left.L_{k, i}(j)=\sum_{n} L_{k, i n}(j)\right)$, we can again simplify equations (2) from the general framework of Section 2 to equation (17) in our model. To obtain equation (17), we make use of two results: $(i)$ free entry implies that revenues are equal to total costs (including entry, market access, and variable costs) in each sector, and (ii) fixed and variable costs have a common factor intensity in each sector. Note that to obtain this result in the specification with endogenous entry, we do not make use of Pareto distributed productivity or CES aggregators.

Given that factor market clearing conditions are given by equation (17), it follows that we can express the FCT using equation (9) in all specifications of our model. Finally, with Cobb-Douglas preferences and production functions and unchanged share parameters $\left(\alpha_{k}(j)=\alpha_{k}^{\prime}(j)\right)$, equation (7) from the general framework simplifies to equation (10), so that the change in the skill premium across two equilibria is given by

$$
\frac{w_{s, i}^{\prime}}{w_{u, i}^{\prime}} / \frac{w_{s, i}}{w_{u, i}}=\frac{\mathcal{L}_{u, i}^{\prime}}{\mathcal{L}_{s, i}^{\prime}} / \frac{\mathcal{L}_{u, i}}{\mathcal{L}_{s, i}} .
$$

Hence, in all specifications of our model, changes in the skill premium are fully determined by changes in trade-adjusted factor supplies. Moreover, since we impose that factor supplies are fixed parameters $\left(L_{k, i}=L_{k, i}^{\prime}\right)$, changes in the FCT are sufficient statistics for the impact of trade on the skill premium: changes in trade costs or in productivities affect the skill premium only through changes in the FCT.

## 4 The Skill Premium, Factor Allocation, and Trade

We now investigate the impact of trade liberalizations on the skill premium, factor allocation, and trade patterns in our model. We first show that if country 1 has a comparative advantage in the skill intensive good, then the trade-adjusted relative supply of skill, $\mathcal{L}_{s, i} / \mathcal{L}_{u, i}$, falls in country 1 and rises in country 2 when countries open to trade. We then show that changes in $\mathcal{L}_{s, i} / \mathcal{L}_{u, i}$ fully determine the impact of trade liberalization not only on the skill premium, as shown in the previous section, but also on between-sector factor allocation and betweensector trade. Through these results, we obtain a generalized version of what is often referred to as the Stolper-Samuelson effect. The Stolper-Samuelson effect relates changes in factor prices to exogenous changes in goods prices, whereas we relate changes in factor prices, factor allocation, and trade patterns to changes in trade costs, via changes in trade shares.

We say that country 1 has a comparative advantage in sector $x$ if the cost of the composite input bundle in sector $x$ relative to sector $y$ is relatively lower in country 1 than in country 2 in autarky: $v_{1}(x) / v_{1}(y)<v_{2}(x) / v_{2}(y)$ in autarky. According to this definition, country 1 has a comparative advantage in the skill-intensive sector if and only if

$$
\begin{equation*}
a\left(\frac{H_{1} / L_{1}}{H_{2} / L_{2}}\right)^{\alpha_{x}-\alpha_{y}}>1 \tag{CA}
\end{equation*}
$$

Condition CA follows from the definition of $v_{i}(j)$ in equation (11), from the factor-market clearing condition in equation (17), and from the observation that $\Lambda_{12}(j)=\Lambda_{21}(j)=0$ in autarky. Without loss of generality, we impose Condition CA throughout the remainder of the paper.

To understand Condition CA, consider two special cases that are standard in the literature. First, if $a=1$ so that there is no Ricardian comparative advantage, then country 1 has a comparative advantage in sector $x$ if and only if $H_{1} / L_{1}>H_{2} / L_{2}$, exactly as in the Heckscher-Ohlin model. Second, if endowment ratios are the same across countries so that there is no Heckscher-Ohlin-based comparative advantage, then country 1 has a comparative advantage in sector $x$ if and only if $a>1$, exactly as in the Ricardian model.

The consequences of moving from autarky $\left(\Delta_{1}, \Delta_{2}=0\right)$ to positive trade shares ( $\Delta_{1}^{\prime}, \Delta_{2}^{\prime}>$ 0 ) on the trade-adjusted relative supply of skill in country 1 are stated in the following proposition.

Proposition 2 If $\Delta_{1}, \Delta_{2}=0$ and $\Delta_{1}^{\prime}, \Delta_{2}^{\prime}>0$, then $\mathcal{L}_{s, 1}^{\prime} / \mathcal{L}_{u, 1}^{\prime}<\mathcal{L}_{s, 1} / \mathcal{L}_{u, 1}=L_{s, 1} / L_{u, 1}$.
Country 1 is a net exporter in the sector in which it has a comparative advantage, sector $x: \Theta_{1}=\Lambda_{21}(y)-\Lambda_{21}(x)>0$ if $\Delta_{1}>0$. Because the $x$ sector is skill intensive, country 1's
net exports embody a positive amount of skilled labor, $F C T_{s, 1}>0$, and a negative amount of unskilled labor, $F C T_{u, 1}<0$, if $\Delta_{1}>0$. Hence, moving from autarky to any positive trade shares reduces the trade-adjusted relative supply of skill in country 1.

For given trade shares $\Delta_{1}$ and $\Delta_{2}$, the level of the trade-adjusted relative supply of skill in either country, $\mathcal{L}_{s, i} / \mathcal{L}_{u, i}$, determines important economic outcomes in both countries: the skill premium $w_{s, i} / w_{u, i}$; between-sector factor allocation $L_{k, i}(j)$; and between-sector trade (the absolute value of $\Theta_{i}$ ). The following proposition states specifically how these economic outcomes vary across two equilibria with equal trade shares but different trade-adjusted factor supplies.

Proposition 3 In any two trade equilibria with equal trade shares $\Delta_{1}^{\prime}=\Delta_{1}>0$ and $\Delta_{2}^{\prime}=$ $\Delta_{2}>0$, the following eight statements are equivalent:

$$
\begin{array}{llll}
\text { (i) } & \mathcal{L}_{s, 1}^{\prime} / \mathcal{L}_{u, 1}^{\prime}<\mathcal{L}_{s, 1} / \mathcal{L}_{u, 1} & \text { (ii) } & \mathcal{L}_{s, 2}^{\prime} / \mathcal{L}_{u, 2}^{\prime}>\mathcal{L}_{s, 2} / \mathcal{L}_{u, 2} \\
\text { (iii) } & w_{s, 1}^{\prime} / w_{u, 1}^{\prime}>w_{s, 1} / w_{u, 1} & \text { (iv) } & w_{s, 2}^{\prime} / w_{u, 2}^{\prime}<w_{s, 2} / w_{u, 2} \\
\text { (v) } & L_{k, 1}^{\prime}(x)>L_{k, 1}(x) \text { for } k=s, u & \text { (vi) } & L_{k, 2}^{\prime}(x)<L_{k, 2}(x) \text { for } k=s, u \\
\text { (vii) } & \Theta_{1}^{\prime}>\Theta_{1} & \text { (viii) } \Theta_{2}^{\prime}<\Theta_{2}
\end{array}
$$

The intuition behind Proposition 3 can be understood as follows. A lower trade-adjusted relative supply of skill in country 1 (statement $i$ ) increases the skill premium in country 1 (statement $i i i$ ), as stated in Proposition 1. For fixed factor supplies, a lower trade-adjusted relative supply of skill requires a higher absolute value of the FCT of skilled and unskilled labor in country 1 , which requires that the extent of between-sector trade be greater (statement vii, since $\Theta_{1}>0$ from Condition CA). More between-sector trade requires that a greater share of factors be allocated to country 1's comparative advantage sector (statement $v)$. With trade balance and fixed trade shares, more between-sector trade in country 1 (statement vii) requires more between-sector trade in country 2 (statement viii, since $\Theta_{2}<0$ ), a greater share of factors allocated to country 2's CA sector (statement $v i$ ), and a greater absolute value of the FCT of skilled and unskilled, which is associated with both a higher trade-adjusted relative supply of skill (statement $i i$ ) and a lower skill premium (statement $i v)$.

Combining Propositions 2 and 3, we establish the following corollary.
Corollary 1 Reducing trade costs so that countries move from autarky $\left(\Delta_{1}, \Delta_{2}=0\right)$ to any positive level of trade $\left(\Delta_{1}^{\prime}, \Delta_{2}^{\prime}>0\right)$ raises the skill premium in country 1 and reduces it in country 2, reallocates factors towards the $x$ sector in country 1 and towards the $y$ sector in country 2, and generates positive net exports in the $x$ sector in country 1 and in the $y$ sector in country 2.

Intuitively, starting in autarky, a reduction in trade costs increases each country's net exports in its comparative advantage sector. This requires factors to reallocate towards that sector, which increases the relative demand and, therefore, the relative price of the factor that is used intensively in the comparative advantage sector.

## 5 Technology, Selection, and Entry

Changes in the trade-adjusted relative supply of skill are determined by changes in the FCT, which are endogenous. Our next goal is to study how key margins in our model-the extent of productivity heterogeneity between and within sectors, and heterogeneous firms' decisions to enter and operate in each market - shape the impact of trade liberalization on the FCT, and, therefore, on factor allocation and the skill premium.

These margins matter for equilibrium outcomes only through their impacts on expenditure shares $\Lambda_{i n}(j)$. This follows from Proposition 3, which shows that for given factor supplies and trade shares, changes in trade-adjusted relative factor supplies in each country are fully determined by changes in the extent of between-sector trade $\Theta_{1}$, which is itself determined by changes in $\Lambda_{i n}(j)$.

Equation (14) illustrates the various exogenous and endogenous determinants of these expenditure shares. First, composite input costs, $v_{i}(j)$, have a direct effect on expenditure shares through the prices charged by active firms. All else equal, lowering $v_{i}(j)$ increases $\Lambda_{i n}(j)$ for all $n$. From equation (11), composite input costs can be decomposed into two components: $(i)$ factor prices and intensities, $w_{k, i}^{\alpha_{k}(j)}$, as in the Heckscher-Ohlin model, and (ii) exogenous sectoral technologies, $A_{i}(j)$, as in the Ricardian model.

Second, the mass of operating firms from each country shapes expenditure shares: an increase in the mass of country $i$ firms operating in country $n$ increases $\Lambda_{\text {in }}(j)$, holding all else fixed. This mass of firms can be decomposed into two components: ( $i$ ) the mass of entering firms in country $i$, given by $M_{i}(j)$, and (ii) the fraction of country $i$ entrants that operate in country $n$, which is negatively related to $z_{i n}^{*}(j)$. All else equal, an increase in the mass of operating firms, either through an increase in $M_{i}(j)$ or a decrease in $z_{i n}^{*}(j)$, is equivalent, in terms of expenditure shares, to an increase in sectoral productivity $A_{i}(j)$.

Third, the extent of productivity heterogeneity affects the elasticity of expenditure shares to a change in the productivity cutoff, $z_{i n}^{*}(j)$. In particular, a greater dispersion of productivity, a lower $\gamma$, decreases the concentration of firms around the cutoff. This implies a smaller decrease in the mass of operating firms for a given increase in the productivity cutoff.

In what follows, we study how each of these margins affects the impact of trade liberalization on trade-adjusted relative supplies of skill, and therefore on the skill premium and the
extent of both between-sector factor reallocation and trade. In order to isolate the effects of these margins in our comparative static exercises, we choose trade costs, $t$, and relative country productivities, $T_{1} / T_{2}$, so that trade shares, $\Delta_{1}$ and $\Delta_{2}$, remain fixed. ${ }^{16}$ When comparing across equilibria under different parameter values, we always impose that factor supplies, factor shares, and the elasticity of substitution between varieties within sectors remain fixed: $L_{k, i}=L_{k, i}^{\prime}, \alpha_{k}(j)=\alpha_{k}^{\prime}(j)$, and $\eta=\eta^{\prime}$.

### 5.1 Productivity heterogeneity

Proposition 4 summarizes our findings about how productivity heterogeneity affects the impact of trade liberalization on trade-adjusted relative supplies of skill, and therefore on the skill premium and the extent of both between-sector factor reallocation and trade. ${ }^{17}$

Proposition 4 In the specification of the model with selection, the decline in the tradeadjusted relative supply of skill in country $1, \mathcal{L}_{s, 1} / \mathcal{L}_{u, 1}$, caused by moving from autarky to trade shares $\Delta_{1}, \Delta_{2}>0$ is greater the higher is a or the lower is $\gamma$ if either ( $i$ ) entry is exogenous, or (ii) entry is endogenous and countries and sectors are mirror symmetric.

Consider first the intuition for Proposition 4 in the exogenous entry case. Increasing $a$ (i.e., increasing country 1's relative productivity advantage in sector $x$ ) reduces country 1's cost in the $x$ sector relative to its cost in the $y$ sector, relative to that in country 2 . These changes in relative costs reinforce country 1's comparative advantage in sector $x$, inducing country 1 to specialize further in sector $x$. Hence, $\mathcal{L}_{s, 1} / \mathcal{L}_{u, 1}$ falls because the $x$ sector is skill intensive. ${ }^{18}$

Consider second the role of $\gamma$ in the exogenous entry case. As discussed above, the elasticity of $\Lambda_{i n}(j)$ to a change in the cutoff productivity $z_{i n}^{*}(j)$ is increasing in $\gamma$. To understand how this elasticity matters for economic outcomes, consider a change in $v_{i}(j)$, the composite input cost in sector $j$. The direct effect of such a change on $z_{i n}^{*}(j)$ is independent of $\gamma$. However, a given change in $z_{i n}^{*}(j)$ has a larger effect on sectoral expenditures the less dispersed are productivities, i.e., the higher is $\gamma$. Hence, higher values of $\gamma$ increase the

[^9]responsiveness of expenditure shares to a change in the cost of the composite input bundle. This implies that factor endowment differences and sectoral productivity differences, which affect the relative cost of the composite input bundle across countries, play a larger role in shaping expenditure shares, and therefore trade-adjusted relative supplies of skill, when $\gamma$ is higher.

In the endogenous entry case, changes in $a$ and $\gamma$ have indirect effects on expenditure shares through $M_{i}(j)$, in addition to the direct effects we discuss above in the exogenous entry case. An increase in $a$ increases relative entry in the $x$ sector in country 1 relative to country 2 , which reinforces the direct effect. That is, endogenous entry magnifies exogenous comparative advantage. To understand the impact of an increase in $\gamma$ on entry, consider the following thought experiment: Starting in autarky, consider a move to trade first holding both $z_{i n}^{*}(j)$ and $M_{i}(j)$ fixed. International trade increases market-specific relative profits in a country's comparative advantage sector. Note that the impact on profits does not depend directly on $\gamma$ for a fixed $z_{i n}^{*}(j)$. Allowing now for changes in $z_{i n}^{*}(j)$ while still holding entry fixed, the previous discussion implies that changes in $z_{i n}^{*}(j)$ are also independent of $\gamma$. However, the less dispersed are productivities (i.e. the greater is $\gamma$ ), the greater is the change in a potential entrant's expected market-specific profit given equal-sized changes in $z_{i n}^{*}(j)$. Hence, given $M_{i}(j)$, opening up to trade induces larger changes in the expected value of firms at entry, the higher is $\gamma$. Hence, we should anticipate a rise in relative entry in the comparative advantage sector, and this rise should be greater the higher is $\gamma$. Thus, the indirect effect of a change in $\gamma$ on entry reinforces the direct effect of $\gamma$.

While Proposition 4 focuses on the specification of the model with selection, we obtain similar results in the specification without selection. In this specification, the parameter $a$ has the same effect as in Proposition 4. On the other hand, since the partial elasticity of expenditure shares with respect to composite input costs is $1-\eta$ instead of $(\gamma \eta+1-\eta) /(1-\eta)$, in this case Proposition 4 holds when $\gamma$ is replaced by $\eta$. Without selection, within-sector productivity heterogeneity does not matter for the trade-adjusted relative supply of skill.

### 5.2 Entry

Proposition 5 summarizes our findings about how the extent of endogenous entry affects the impact of trade liberalization on trade-adjusted relative supplies of skill, and therefore on the skill premium and the extent of both between-sector factor reallocation and trade.

Proposition 5 The decline in the trade-adjusted relative supply of skill in country $1, \mathcal{L}_{s, 1} / \mathcal{L}_{u, 1}$, caused by moving from autarky to trade shares $\Delta_{1}, \Delta_{2}>0$ is greater in the specification with endogenous entry than in the specification with exogenous entry.

Because factor prices are relatively lower in a country's comparative advantage sector, entry is relatively greater there. Recall that a larger mass of entrants in a given sector is equivalent - in terms of its implications for the skill premium and the extent of both betweensector factor reallocation and trade - to an increase in that sector's exogenous Ricardian productivity. Hence, endogenous entry increases $\mathcal{L}_{s, 1} / \mathcal{L}_{u, 1}$, just as an increase in exogenous Ricardian comparative advantage $a$.

### 5.3 Selection

Proposition 6 summarizes our findings about how the extent of selection affects the impact of trade liberalization on trade-adjusted relative supplies of skill, and therefore on the skill premium and the extent of both between-sector factor reallocation and trade.

Proposition 6 The decline in the trade-adjusted relative supply of skill in country $1, \mathcal{L}_{s, 1} / \mathcal{L}_{u, 1}$, caused by moving from autarky to trade shares $\Delta_{1}, \Delta_{2}>0$, is greater in the specification with selection than in the specification without selection if either (i) entry is exogenous, or (ii) entry is endogenous and countries and sectors are mirror symmetric.

This result follows directly from the following two observations; hence, we omit a formal proof of this proposition. First, trade patterns and factor prices obtained using the equations in the specification with selection limit to those obtained using the equations in the specification without selection, as $\gamma$ converges to $\eta-1$, when all parameters are the same across specifications (obviously with the exception of market access costs, which are assumed to be zero without selection). This is because, as $\gamma$ converges to $\eta-1$, almost all production occurs within an arbitrarily small mass of very productive firms. Hence, in this limiting case $\mathcal{L}_{s, 1} / \mathcal{L}_{u, 1}$ is equivalent in the specification with selection and the specification without selection. Second, in the specification with selection, the decline in $\mathcal{L}_{s, 1} / \mathcal{L}_{u, 1}$ (holding trade shares fixed) is greater the higher is $\gamma$, as shown in Proposition 4.

Intuitively, with endogenous selection the fraction of country 1 entrants, relative to country 2 entrants, that choose to sell in any given market is relatively larger in country 1's comparative advantage sector because country 1 has a relatively lower composite input cost in this sector: ${ }^{19}$

$$
\begin{equation*}
\frac{z_{1 n}^{*}(x)}{z_{2 n}^{*}(x)}<\frac{z_{1 n}^{*}(y)}{z_{2 n}^{*}(y)} \text { for all } n \tag{21}
\end{equation*}
$$

Recall that a larger fraction of firms that supply a given market is equivalent - in terms of its implications for the skill premium and the extent of both between-sector factor reallocation

[^10]and trade - to a larger exogenous sectoral productivity. Hence, endogenous selection reinforces ex-ante comparative advantage. Note that when $a=1$ this implies that the average productivity of country 1 firms supplying a given country is relatively lower in county 1 's comparative advantage sector, relative to country 2.

Relation to BRS: Proposition 6 and Condition (21) are reminiscent of a result in BRS that, with selection and endogenous entry, differences in endowments across countries lead to stronger selection for domestic production in a country's comparative advantage sector:

$$
\begin{equation*}
\frac{z_{11}^{*}(x)}{z_{11}^{*}(y)}>\frac{z_{22}^{*}(x)}{z_{22}^{*}(y)} \tag{22}
\end{equation*}
$$

That is, endogenous selection implies that the average productivity of firms that choose to produce for the domestic market is relatively greater in country 1's comparative advantage sector, compared to country 2 . This leads to their interpretation that differences in endowments across countries induce what they call "endogenous Ricardian productivity differences" at the industry level, which magnify Heckscher-Ohlin-based comparative advantage.

This interpretation may appear similar to our result in Proposition 6. However, BRS do not show that selection magnifies comparative advantage in the sense that it is equivalent - in terms of its implications for the skill premium or for the extent of either between-sector factor reallocation or trade - to an increase in exogenous Ricardian productivity in each country's comparative advantage sector.

Condition (21), which plays a central role in Proposition 6, differs from Condition (22) in three respects. First, Condition (21) depends on a comparison of cutoffs in a common destination market rather than in each country's domestic market. Second, Condition (21) emphasizes that a country is less selective in any given destination in its comparative advantage sector, relative to the other country. Third, while Condition (21) is satisfied with either endogenous or exogenous entry, Condition (22) is reversed in the specification with exogenous entry. In particular, the following lemma shows that whether the average productivity of domestic firms is higher or lower in a country's comparative advantage sector relative to another country depends on whether entry is endogenous or exogenous. ${ }^{20}$

Lemma 1 Consider the specification of our model with selection and suppose that $A_{i}(j)=1$ and, if entry exogenous, that $M_{i}(j)=1$. If entry is endogenous and trade shares are positive, then $z_{11}^{*}(x) / z_{11}^{*}(y)>z_{22}^{*}(x) / z_{22}^{*}(y)$. If entry is exogenous and there is no factor price equalization, then $z_{11}^{*}(x) / z_{11}^{*}(y)<z_{22}^{*}(x) / z_{22}^{*}(y)$.

[^11]Why does the relationship between the average productivity of domestic firms across sectors depend on whether entry is endogenous or exogenous? In our model, countries specialize in their comparative advantage sector. Recall from equation (14) that an expansion of the comparative advantage sector can occur along three margins: (i) firms of equal productivity can be larger, (ii) the productivity cutoff can be lower, and (iii) entry can be greater. With exogenous entry, only margins $(i)$ and (ii) are active. Equally productive firms are larger in the comparative advantage sector and, in order to have a larger mass of operating firms, the comparative advantage sector must be relatively less selective. With endogenous entry, all three margins are active. Moreover, margins (ii) and (iii) are not independent. When entry is endogenous, a relatively higher entry level in the comparative advantage sector makes survival relatively more difficult in the domestic market. Hence, this sector is larger while also being more selective. ${ }^{21}$

## 6 Conclusions

In this paper we have provided a unifying framework to study how factor prices and factor allocation respond to trade liberalizations. We derived a simple expression relating equilibrium factor prices to two components: trade-adjusted relative factor supplies and the relative factor payments for domestic absorption. We showed how changes in relative factor prices within a range of workhorse models of trade can be mapped into these two components and described a set of standard assumptions under which changes in the FCT are sufficient statistics for the impact of trade on relative factor prices. While these insights about the distributional implications of international trade hold across a range of models, measuring the sufficient statistics in practice can be difficult. Hence, delving into the specifics through which international trade impact relative factor prices remains an important research avenue.

We then specialized the general framework to an environment that combines the key elements of the Heckscher-Ohlin model, the Ricardian model, and the Melitz model. Changes in the FCT fully determine not only relative factor prices, but also the extent of betweensector factor reallocation and between-sector trade. We used this model to examine how the FCT is shaped by heterogenous firms' decisions to enter and to operate in each market. Endogenous entry and endogenous selection of firms into markets magnify the impact of trade on the FCT and hence the change in the skill premium and the extent of betweensector trade and factor reallocation, while greater within-sector productivity heterogeneity

[^12]weakens these effects. Given the extensive evidence of large productivity differences within narrowly-defined sectors, our prediction about the implications of within-sector productivity heterogeneity provides a rationale for empirical results suggesting that the FCT, the extent of between-sector factor reallocation induced by trade, and the impact of trade on the skill premium are small in practice.

## Appendix A: Additional Derivations

## Labor Market Clearing with Exogenous Entry

Variable input costs: With Cobb-Douglas production functions, payments to skilled and unskilled labor hired as a variable input in the production of a variety of sector $j$ in country $i$ that is bound for country $n$, denoted by $l_{s, i n}(z, j)$ and $l_{u i n}(z, j)$, are proportional to market-specific revenues

$$
\begin{equation*}
w_{k, i} l_{k, i n}(z, j)=\frac{\eta-1}{\eta} \alpha_{k}(j) r_{i n}(z, j) . \tag{23}
\end{equation*}
$$

Equation (23) implies that total payments to country $i$ labor employed in variable production in sector $j$ are $\frac{1}{2} \sum_{n} \frac{\eta-1}{\eta} \Lambda_{i n}(j) E_{n}$, of which a share $\alpha_{k}(j)$ is paid to factor $k$.

Market access input costs: Country $i$ 's total market access fixed costs associated with selling sector $j$ goods in country $n$ are given by

$$
F_{i n}(j)=M_{i}(j) f_{i n} v_{i}(j) z_{i n}^{*}(j)^{-\gamma} .
$$

Equation (12) implies that total sector $j$ revenue in country $i$ from goods shipped to country $n$ is

$$
\Lambda_{i n}(j) Q_{n}(j) P_{n}(j)=M_{i}(j) \frac{E_{n}}{2 P_{n}(j)^{1-\eta}}\left(\frac{\eta}{\eta-1}\right)^{1-\eta}\left[v_{i}(j) \tau_{i n}\right]^{1-\eta} \frac{\gamma}{\gamma+1-\eta} z_{i n}^{*}(j)^{\eta-\gamma-1}
$$

so that in general

$$
F_{i n}(j)=\frac{\eta^{\eta}}{(1-\eta)^{1-\eta}} \frac{\gamma+1-\eta}{\gamma \eta} P_{n}(j)^{1-\eta} v_{i}(j)^{\eta} f_{i n} \tau_{i n}^{\eta-1} \Lambda_{i n}(j) z_{i n}^{*}(j)^{1-\eta} .
$$

In the case with no selection $F_{i n}(j)=0$. In the case with selection

$$
\begin{equation*}
F_{i n}(j)=\frac{\gamma+1-\eta}{\gamma \eta} \frac{1}{2} \Lambda_{i n}(j) E_{n}, \tag{24}
\end{equation*}
$$

of which a share $\alpha_{k}(j)$ is paid to factor $k$.
Total factor payments without selection: With no selection into any market, variable labor costs represent a share $(\eta-1) / \eta$ of total revenues and market access payments are zero. Therefore, total labor payments equal $\frac{1}{2} \sum_{n}\left(\frac{\eta-1}{\eta}\right) \Lambda_{i n}(j) E_{n}$, of which a share $\alpha_{k}(j)$ is paid to factor $k$. Factor
market clearing implies

$$
\begin{equation*}
w_{k, i} L_{k, i}=\sum_{j} \sum_{n} \frac{\eta-1}{\eta} \alpha_{k}(j) \Lambda_{i n}(j) \frac{E_{n}}{2} \tag{25}
\end{equation*}
$$

In the exogenous entry case, Equation (25) and balanced trade imply $E_{n}=\frac{\eta}{\eta-1}\left(w_{s, n} L_{s, n}+w_{u, n} L_{u, n}\right)$. Hence, equation (25) is equivalent to equation (17), where $\lambda=(\eta-1) / \eta$.

Total factor payments with selection: Total payments to sector $j$ labor in country $i$ are the sum of variable input payments and market access fixed cost payments. With selection, these payments equal $\frac{1}{2} \sum_{n}\left(\frac{\gamma \eta-\eta+1}{\gamma \eta}\right) \Lambda_{i n}(j) E_{n}$, of which a share $\alpha_{k}(j)$ is paid to factor $k$. Hence, factor market clearing implies

$$
\begin{equation*}
w_{k, i} L_{k, i}=\sum_{j} \sum_{n}\left(\frac{\gamma \eta-\eta+1}{\gamma \eta}\right) \alpha_{k}(j) \Lambda_{i n}(j) \frac{E_{n}}{2} \tag{26}
\end{equation*}
$$

Equation (26) and balanced trade imply $E_{n}=\frac{\gamma \eta}{\gamma \eta-\eta+1}\left(w_{s, n} L_{s, n}+w_{u, n} L_{u, n}\right)$. Hence, equation (26) is equivalent to equation (17), where $\lambda=(\gamma \eta-\eta+1) / \gamma \eta$.

## Labor Market Clearing and Entry with Endogenous Entry

Labor market clearing: With free entry, total revenue equals total factor payments, sector by sector,

$$
\sum_{k} w_{k, i} L_{k, i}(j)=\sum_{n} \Lambda_{i n}(j)\left(\frac{E_{n}}{2}\right) .
$$

Moreover, the share of factor payments that accrue to factor $k$ is $\alpha_{k}(j)$ in sector $j$,

$$
\begin{equation*}
w_{k, i} L_{k, i}(j)=\alpha_{k}(j) \sum_{n} \Lambda_{i n}(j)\left(\frac{E_{n}}{2}\right) . \tag{27}
\end{equation*}
$$

By summing equation (27) across sectors, we obtain equation (17), where $\lambda=1$.
Entry: Total entry costs in sector $j$ are $M_{i}(j) v_{i}(j) f^{e}$. From the free entry condition, total entry costs, $M_{i}(j) v_{i}(j) f^{e}$, are equal to total revenues, $\sum_{n} \Lambda_{i n}(j)\left(\frac{E_{n}}{2}\right)$, minus variable production costs, $\sum_{n} \frac{\eta-1}{\eta} \Lambda_{i n}(j)\left(\frac{E_{n}}{2}\right)$, and market access costs, $F_{i n}(j)$. Together with $F_{i n}(j)=0$, without selection, and with equation (24), with selection, we obtain equation (20) both with and without selection.

## Price Indices

The sector $j$ price level in country $n$ equals

$$
\begin{equation*}
P_{n}(j)=\left(\frac{\gamma}{\gamma+1-\eta}\right)^{\frac{1}{1-\eta}} \frac{\eta}{\eta-1}\left[\sum_{i} M_{i}(j)\left[\tau_{i n} v_{i}(j)\right]^{1-\eta} z_{i n}^{*}(j)^{\eta-\gamma-1}\right]^{1 /(1-\eta)} . \tag{28}
\end{equation*}
$$

Without selection Equation (28) is equivalent to

$$
\begin{equation*}
P_{n}(j)^{\eta-1}=\frac{\lambda_{2}}{\sum_{i} M_{i}(j) \tau_{i n}^{1-\eta} v_{i}(j)^{1-\eta}} \tag{29}
\end{equation*}
$$

where $\lambda_{2}=\frac{\gamma+1-\eta}{\gamma}\left(\frac{\eta}{\eta-1}\right)^{\eta-1}$. With selection, Equation (28) is equivalent to

$$
\begin{equation*}
P_{n}(j)^{\gamma}=\frac{\lambda_{1}\left(Q_{n} P_{n}\right)^{\frac{\eta-\gamma-1}{\eta-1}}}{\sum_{i} M_{i}(j) v_{i}(j)^{\frac{\eta \gamma+1-\eta}{1-\eta}} f_{i n}^{\frac{\gamma+1-\eta}{1-\eta}} \tau_{i n}^{-\gamma}} \tag{30}
\end{equation*}
$$

where $\lambda_{1}=\frac{\gamma+1-\eta}{\gamma}\left(\frac{\eta}{\eta-1}\right)^{\eta-1}\left(\frac{1}{J \eta^{\eta}(\eta-1)^{1-\eta}}\right)^{\frac{\eta-\gamma-1}{\eta-1}}$.

## Appendix B: Proofs

Proof of Proposition 2. We proceed by contradiction. Suppose $\Lambda_{21}(y) \leq \Lambda_{21}(x)$. By equation (15) or (16), $\Lambda_{21}(y)<(=) \Lambda_{21}(x)$ is equivalent to

$$
\left[\frac{v_{2}(x) v_{1}(y)}{v_{2}(y) v_{1}(x)}\right]^{\zeta}<(=) \frac{M_{1}(y)}{M_{1}(x)} \frac{M_{2}(x)}{M_{2}(y)},
$$

where $\zeta=\eta-1>0$ without selection and $\zeta=(\gamma \eta-\eta+1) /(\eta-1)>0$ with selection, and where

$$
\frac{v_{2}(x) v_{1}(y)}{v_{2}(y) v_{1}(x)}=a \frac{w_{s, 2} / w_{u, 2}}{w_{s, 1} / w_{u, 1}} .
$$

With exogenous entry, $\frac{M_{1}(y)}{M_{1}(x)} \frac{M_{2}(x)}{M_{2}(y)}=1 .{ }^{22}$ With endogenous entry, equation (20) implies

$$
\frac{M_{1}(y)}{M_{1}(x)} \frac{M_{2}(x)}{M_{2}(y)}=\frac{v_{1}(x)}{v_{1}(y)} \frac{v_{2}(y)}{v_{2}(x)} \Gamma,
$$

where

$$
\Gamma=\frac{\sum_{n} \Lambda_{1 n}(y) E_{n}}{\sum_{n} \Lambda_{1 n}(x) E_{n}} \frac{\sum_{n} \Lambda_{2 n}(x) E_{n}}{\sum_{n} \Lambda_{2 n}(y) E_{n}} .
$$

Therefore, $\Lambda_{21}(y) \leq \Lambda_{21}(x)$ is equivalent to

$$
\begin{equation*}
\left[\frac{v_{2}(x) v_{1}(y)}{v_{2}(y) v_{1}(x)}\right]^{\zeta+\Xi} \leq \Gamma^{\Xi}, \tag{31}
\end{equation*}
$$

where $\Xi=0$ with exogenous entry and $\Xi=1$ with endogenous entry. In autarky, Inequality (31) is violated, since the left-hand-side is strictly greater than one under Condition CA and since $\Gamma=1$ simply because $\Lambda_{i i}(j)=1$ and $\Lambda_{i n}(j)=0$ for all $i \neq n$. Note that for arbitrarily small trade shares, Inequality (31) remains violated because $\Gamma$ and $\frac{v_{2}(x) v_{1}(y)}{v_{2}(y) v_{1}(x)}$ depend on trade costs only through the $\Lambda_{i n}(j)$ 's, are continuous in the $\Lambda_{i n}(j)$ 's, and for arbitrarily small trade shares $\Lambda_{i i}(j)$ and $\Lambda_{i n}(j)$ are arbitrarily close to their autarky values.

Since both the left- and right-hand sides of Inequality (31) are continuous in the $\Lambda_{\text {in }}(j)$ 's,

[^13]a necessary condition for Inequality (31) to be satisfied is that there exist trade costs such that $\Delta_{1}, \Delta_{2}>0$ and Inequality (31) is satisfied with equality; i.e. $\Lambda_{21}(y)=\Lambda_{21}(x)$. Equation (15) or (16), and $\Lambda_{21}(y)=\Lambda_{21}(x)$ imply $\Lambda_{i n}(x)=\Lambda_{i n}(y)$ for all $i, n$. Hence, $\Lambda_{21}(y)=\Lambda_{21}(x)$ implies $\Gamma=1$. Equation (17) and $\Lambda_{21}(y)=\Lambda_{21}(x)$ also imply that $\frac{v_{2}(x) v_{1}(y)}{v_{2}(y) v_{1}(x)}$ equals its autarky value, which by Condition CA is strictly greater than one. Hence, Inequality (31) can never be satisfied with equality. By continuity, Inequality (31) can never be satisfied. QED.

Proof of Proposition 3. We decompose the proof of Proposition 3 into four parts. First, we prove the equivalence of $(i)$ and (iii). Second, we prove the equivalence of (iii) and (vii). Third, we prove the equivalence of $(i i i)$ and $(v)$. The proofs for the equivalence of $(i i),(i v),(v i)$, and (vii) are identical, and therefore omitted. Fourth, we prove the equivalence of (vii) and (viii). Throughout the proof, we impose $L_{k, i}^{\prime}=L_{k, i}, \alpha_{k}^{\prime}(j)=\alpha_{k}(j), \Theta_{1}^{\prime}, \Theta_{1} \geq 0, \Delta_{1}^{\prime}=\Delta_{1}$, and $\Delta_{2}^{\prime}=\Delta_{2}$.
Part I: (i) $\mathcal{L}_{s, 1}^{\prime} / \mathcal{L}_{u, 1}^{\prime}<\mathcal{L}_{s, 1} / \mathcal{L}_{u, 1}$ if and only if (iii) $w_{s, 1}^{\prime} / w_{u, 1}^{\prime}>w_{s, 1} / w_{u, 1}$.
Part I follows directly from equation (10).
Part II: (iii) $w_{s, 1}^{\prime} / w_{u, 1}^{\prime}>w_{s, 1} / w_{u, 1}$ if and only if (vii) $\Theta_{1}^{\prime}>\Theta_{1}$.
The proof of Part II proceeds in 3 steps.
Step 1: The following inequalities are equivalent $(a) \Lambda_{11}^{\prime}(x)>\Lambda_{11}(x)$, (b) $\Lambda_{11}^{\prime}(y)<\Lambda_{11}(y)$, (c) $\Lambda_{12}^{\prime}(y)<\Lambda_{12}(y)$, (d) $\Lambda_{12}^{\prime}(x)>\Lambda_{12}(x)$, and $(e) \Theta_{1}^{\prime}>\Theta_{1}$.
$\Delta_{1}=\Delta_{1}^{\prime}$ and $\Delta_{2}=\Delta_{2}^{\prime}$, together with the identities $\Lambda_{12}(j)=1-\Lambda_{22}(j)$ and $\Lambda_{21}(j)=$ $1-\Lambda_{11}(j)$, directly imply that Inequalities $(a)$ and $(b)$ are equivalent, as are Inequalities $(c)$ and (d). In what follows, we first show that Inequality $(a)$ is equivalent to Inequality $(e)$ and we conclude by showing that Inequality $(c)$ is equivalent to Inequality $(e)$.

We have $\Lambda_{21}(x)=\Delta_{1}-\frac{1}{2} \Theta_{1}$. Hence, $\Theta_{1}^{\prime}>\Theta_{1}$ if and only if $\Lambda_{21}^{\prime}(x)<\Lambda_{21}(x)$, since $\Delta_{1}^{\prime}=\Delta_{1}$. Moreover, $\Lambda_{21}^{\prime}(x)<\Lambda_{21}(x)$ is equivalent to $\Lambda_{11}^{\prime}(x)>\Lambda_{11}(x)$, since $\Lambda_{11}^{(\prime)}(x)=1-\Lambda_{21}^{(\prime)}(x)$. Thus, Inequality $(e)$ is equivalent to Inequality $(a)$.

We conclude by showing that Inequality (c) is equivalent to Inequality (e). To show that Inequality (e) implies Inequality (c), we proceed by contradiction. Suppose that $\Theta_{1}^{\prime}>\Theta_{1}$ and $\Lambda_{12}^{\prime}(y) \geq \Lambda_{12}(y) . \Lambda_{12}^{\prime}(y) \geq \Lambda_{12}(y)$ is equivalent to $\Lambda_{12}^{\prime}(x) \leq \Lambda_{12}(x)$ while $\Theta_{1}^{\prime}>\Theta_{1}$ is equivalent to both $\Lambda_{21}^{\prime}(y)>\Lambda_{21}(y)$ and $\Lambda_{21}^{\prime}(x)<\Lambda_{21}(x)$. In the specification without or with selection, equation (15) or (16), $\Lambda_{12}^{\prime}(y) \geq \Lambda_{12}(y)$ and $\Lambda_{21}^{\prime}(y)>\Lambda_{21}(y)$ imply $t<t^{\prime}$; while equation (15) or (16), $\Lambda_{12}^{\prime}(x) \leq \Lambda_{12}(x)$, and $\Lambda_{21}^{\prime}(x)<\Lambda_{21}(x)$ imply $t>t^{\prime}$, a contradiction. Hence, in the specifications with and without selection, $\Theta_{1}^{\prime}>\Theta_{1}$ implies Inequality (c). Finally, to show that Inequality $(c)$ implies Inequality $(e)$, we proceed by contradiction. Suppose that $\Lambda_{12}^{\prime}(y)<\Lambda_{12}(y)$ and $\Theta_{1}^{\prime} \leq \Theta_{1}$. $\Lambda_{12}^{\prime}(y)<\Lambda_{12}(y)$ implies $\Lambda_{12}^{\prime}(x)>\Lambda_{12}(x)$ while $\Theta_{1}^{\prime} \leq \Theta_{1}$ implies both $\Lambda_{21}^{\prime}(y) \leq$ $\Lambda_{21}(y)$ and $\Lambda_{21}^{\prime}(x) \geq \Lambda_{21}(x)$. In the specification without selection (or with selection), equation (15) (or equation (16)), $\Lambda_{12}^{\prime}(y)<\Lambda_{12}(y)$ and $\Lambda_{21}^{\prime}(y) \leq \Lambda_{21}(y)$ imply $t>t^{\prime}$; while equation (15) (or equation (16)), $\Lambda_{12}^{\prime}(x)>\Lambda_{12}(x)$, and $\Lambda_{21}^{\prime}(x) \geq \Lambda_{21}(x)$ imply $t<t^{\prime}$, a contradiction. Hence, Inequality $(c)$ is equivalent to Inequality (e).

Step 2: If $\Delta_{1}=\Delta_{1}^{\prime}>0$ and $\Delta_{2}=\Delta_{2}^{\prime}>0$, then $E_{1} / E_{2}=E_{1}^{\prime} / E_{2}^{\prime}$.
$\Delta_{i}=\Delta_{i}^{\prime}$ for $i=1,2$ is equivalent to

$$
\begin{equation*}
\Lambda_{n i}(x)+\Lambda_{n i}(y)=\Lambda_{n i}^{\prime}(x)+\Lambda_{n i}^{\prime}(y), \text { for } i=1,2 \text { and } n \neq i \tag{32}
\end{equation*}
$$

and trade balance implies

$$
\begin{equation*}
\left[\Lambda_{21}^{(\prime)}(x)+\Lambda_{21}^{(\prime)}(y)\right] E_{1}^{(\prime)}=\left[\Lambda_{12}^{(\prime)}(x)+\Lambda_{12}^{(\prime)}(y)\right] E_{2}^{(\prime)} \tag{33}
\end{equation*}
$$

in both the original equilibrium (without ${ }^{\prime}$ ) and the new equilibrium (with '). Equations (32) and (33) yields $E_{1} / E_{2}=E_{1}^{\prime} / E_{2}^{\prime}$.

Step 3: (iii) $w_{s, 1}^{\prime} / w_{u, 1}^{\prime}>w_{s, 1} / w_{u, 1}$ if and only if (vii) $\Theta_{1}^{\prime}>\Theta_{1}$.
By equation (17), we have

$$
\begin{equation*}
\frac{w_{s, 1}}{w_{u, 1}}=\frac{L_{u, 1}}{L_{s, 1}} \frac{\sum_{j} \sum_{n} \alpha_{s}(j) \Lambda_{1 n}(j) E_{n}}{\sum_{j} \sum_{n} \alpha_{u}(j) \Lambda_{1 n}(j) E_{n}} . \tag{34}
\end{equation*}
$$

With $L_{k, i}=L_{k, i}^{\prime}$ and $\alpha_{k}(j)=\alpha_{k}^{\prime}(j)$, we have $w_{s, 1}^{\prime} / w_{u, 1}^{\prime}>w_{s, 1} / w_{u, 1}$ if and only if

$$
\frac{\Lambda_{11}(y) E_{1}+\Lambda_{12}(y) E_{2}}{\Lambda_{11}^{\prime}(y) E_{1}^{\prime}+\Lambda_{12}^{\prime}(y) E_{2}^{\prime}}>\frac{\Lambda_{11}(x) E_{1}+\Lambda_{12}(x) E_{2}}{\Lambda_{11}^{\prime}(x) E_{1}^{\prime}+\Lambda_{12}^{\prime}(x) E_{2}^{\prime}} .
$$

By Step 2, the previous inequality is equivalent to

$$
\begin{equation*}
\frac{\Lambda_{11}(y) E_{1}+\Lambda_{12}(y) E_{2}}{\Lambda_{11}^{\prime}(y) E_{1}+\Lambda_{12}^{\prime}(y) E_{2}}>\frac{\Lambda_{11}(x) E_{1}+\Lambda_{12}(x) E_{2}}{\Lambda_{11}^{\prime}(x) E_{1}+\Lambda_{12}^{\prime}(x) E_{2}} \tag{35}
\end{equation*}
$$

By Step $1, \Theta_{1}^{\prime}>\Theta_{1}$ is equivalent to Inequalities $(a)-(d)$. Inequalities $(a)-(d)$ imply equation (35). Therefore $\Theta_{1}^{\prime}>\Theta_{1}$ implies $w_{s, 1}^{\prime} / w_{u, 1}^{\prime}>w_{s, 1} / w_{u, 1}$.

Now suppose that $w_{s, 1}^{\prime} / w_{u, 1}^{\prime}>w_{s, 1} / w_{u, 1}$. This is equivalent to equation (35). To prove that equation (35) implies $\Theta_{1}^{\prime}>\Theta_{1}$, we proceed by contradiction. Suppose that $\Theta_{1}^{\prime} \leq \Theta_{1}$. By Step 1 , this implies $\Lambda_{11}^{\prime}(x) \leq \Delta_{11}(x), \Lambda_{11}^{\prime}(y) \geq \Lambda_{11}(y), \Lambda_{12}^{\prime}(y) \geq \Lambda_{12}(y)$, and $\Lambda_{12}^{\prime}(x) \leq \Lambda_{12}(x)$. These four inequalities contradict equation (35). This concludes the proof of Step 3, and Part II follows directly.

Part III: (iii) $w_{s, 1}^{\prime} / w_{u, 1}^{\prime}>w_{s, 1} / w_{u, 1}$ if and only if $(v) L_{k, 1}^{\prime}(x)>L_{k, 1}(x)$ for $k=s, u$.
In the proof of Part III, we normalize $E_{1}=E_{1}^{\prime}=1$. By Step 2 of the proof of Part II, we have $E_{2}^{\prime}=E_{2}$. Moreover, with $E_{1}=E_{1}^{\prime}$, we have $w_{s, 1}^{\prime} / w_{u, 1}^{\prime}>w_{s, 1} / w_{u, 1}$ if and only if

$$
\begin{equation*}
w_{u, 1}^{\prime}<w_{u, 1} \tag{36}
\end{equation*}
$$

The proof of Part III proceeds in two steps.

Step 1: $w_{s, 1}^{\prime} / w_{u, 1}^{\prime}>w_{s, 1} / w_{u, 1}$ implies $L_{k, 1}^{\prime}(x)>L_{k, 1}(x)$ for $k=s, u$.
From equation (17), we have

$$
\begin{equation*}
w_{k, i} L_{k, i}(x)=\lambda \alpha_{k}(x) \sum_{n}\left[\Lambda_{i n}(x)\left(\frac{E_{n}}{2}\right)\right] \tag{37}
\end{equation*}
$$

By Part II, $w_{s, 1}^{\prime} / w_{u, 1}^{\prime}>w_{s, 1} / w_{u, 1}$ implies $\Lambda_{1 n}^{\prime}(x)>\Lambda_{1 n}(x)$ for $n=1,2$. By equation (37), $E_{n}^{\prime}=E_{n}$, and $\Lambda_{1 n}^{\prime}(x)>\Lambda_{1 n}(x)$ for $n=1,2$, we have

$$
\begin{equation*}
w_{u, 1}^{\prime} L_{u, 1}^{\prime}(x)=\lambda \alpha_{u}(x) \sum_{n}\left[\Lambda_{1 n}^{\prime}(x)\left(\frac{E_{n}^{\prime}}{2}\right)\right]>\lambda \alpha_{u}(x) \sum_{n}\left[\Lambda_{1 n}(x)\left(\frac{E_{n}}{2}\right)\right]=w_{u, 1} L_{u, 1}(x) \tag{38}
\end{equation*}
$$

Equations (36) and (38) imply $L_{u, 1}^{\prime}(x)>L_{u, 1}(x)$. We similarly have

$$
w_{s, 1}^{\prime} L_{s, 1}^{\prime}(y)=\lambda \alpha_{s}(y) \sum_{n}\left[\Lambda_{1 n}^{\prime}(y)\left(\frac{E_{n}^{\prime}}{1}\right)\right]<\lambda \alpha_{s}(y) \sum_{n}\left[\Lambda_{1 n}(y)\left(\frac{E_{n}}{1}\right)\right]=w_{s, 1} L_{s, 1}(y)
$$

and

$$
w_{s, 1}^{\prime}>w_{s, 1}
$$

which imply $L_{s, 1}^{\prime}(y)<L_{s, 1}(y)$. Since $L_{s, 1}(y)+L_{s, 1}(x)=L_{s, 1}$, we therefore have $L_{s, 1}^{\prime}(x)>L_{s, 1}(x)$. Hence, $w_{s, 1}^{\prime} / w_{u, 1}^{\prime}>w_{s, 1} / w_{u, 1}$ implies $L_{k, 1}^{\prime}(x)>L_{k, 1}(x)$ for $k=s, u$.
Step 2: $L_{k, 1}^{\prime}(x)>L_{k, 1}(x)$ for $k=s$, $u$ implies $w_{s, 1}^{\prime} / w_{u, 1}^{\prime}>w_{s, 1} / w_{u, 1}$.
We proceed by contradiction. Suppose that $L_{k, 1}^{\prime}(x)>L_{k, 1}(x)$ for $k=s, u$ and $w_{s, 1}^{\prime} / w_{u, 1}^{\prime} \leq$ $w_{s, 1} / w_{u, 1}$. By Part II, $w_{s, 1}^{\prime} / w_{u, 1}^{\prime} \leq w_{s, 1} / w_{u, 1}$ is equivalent to $\Theta_{1}^{\prime} \leq \Theta_{1}$, which, by Step 1 in the proof of Part II, implies $\Lambda_{1 n}^{\prime}(x) \leq \Lambda_{1 n}(x)$ for $n=1,2$. Therefore, $w_{s, 1}^{\prime} / w_{u, 1}^{\prime} \leq w_{s, 1} / w_{u, 1}$ implies

$$
\begin{equation*}
\lambda \alpha_{u}(x) \sum_{n}\left[\Lambda_{1 n}^{\prime}(x)\left(\frac{E_{n}^{\prime}}{2}\right)\right] \leq \lambda \alpha_{u}(x) \sum_{n}\left[\Lambda_{1 n}(x)\left(\frac{E_{n}}{2}\right)\right] . \tag{39}
\end{equation*}
$$

By equations (37) and (39), we have $w_{u, 1}^{\prime} L_{u, 1}^{\prime}(x) \leq w_{u, 1} L_{u, 1}(x)$. By $w_{s, 1}^{\prime} / w_{u, 1}^{\prime} \leq w_{s, 1} / w_{u, 1}$ and $E_{1}^{\prime}=E_{1}$, we also have $w_{u, 1}^{\prime} \geq w_{u, 1}$, so that $L_{u, 1}^{\prime}(x) \leq L_{u, 1}(x)$, a contradiction. Hence, $L_{k, 1}^{\prime}(x)>$ $L_{k, 1}(x)$ for $k=s, u$ implies $w_{s, 1}^{\prime} / w_{u, 1}^{\prime}>w_{s, 1} / w_{u, 1}$.
Part IV: $($ vii $) \Lambda_{21}^{\prime}(y)-\Lambda_{21}^{\prime}(x)>\Lambda_{21}(y)-\Lambda_{21}(x)$ if and only if (viii) $\Lambda_{12}^{\prime}(x)-\Lambda_{12}^{\prime}(y)>$ $\Lambda_{12}(x)-\Lambda_{12}(y)$.

From Step 1 of Part II, we have $\Lambda_{21}^{\prime}(y)-\Lambda_{21}^{\prime}(x)>\Lambda_{21}(y)-\Lambda_{21}(x)$ if and only if $\Lambda_{12}^{\prime}(y)<$ $\Lambda_{12}(y)$ and $\Lambda_{12}^{\prime}(x)>\Lambda_{12}(x)$. These inequalities imply $\Lambda_{12}^{\prime}(x)-\Lambda_{12}^{\prime}(y)>\Lambda_{12}(x)-\Lambda_{12}(y)$. The proof that $\Lambda_{12}^{\prime}(x)-\Lambda_{12}^{\prime}(y)>\Lambda_{12}(x)-\Lambda_{12}(y)$ implies $\Lambda_{21}^{\prime}(y)-\Lambda_{21}^{\prime}(x)>\Lambda_{21}(y)-\Lambda_{21}(x)$ is identical and omitted.

The proof of Proposition 3 follows directly from Parts I-IV and the similar, but omitted, proofs that $(i i),(i v),(v i)$, and (viii) are equivalent. QED.

Proof of Proposition 4 Part 1. First, $\Theta_{1} \geq \Theta_{1}^{\prime}$ if and only if

$$
\begin{equation*}
\left[\frac{1}{a^{\prime}}\left(\frac{w_{s, 1}^{\prime} / w_{u, 1}^{\prime}}{w_{s, 2}^{\prime} / w_{u, 2}^{\prime}}\right)^{\left(\alpha_{x}-\alpha_{y}\right)}\right]^{\frac{\eta \gamma^{\prime}-\eta+1}{\eta-1}} \geq\left[\frac{1}{a}\left(\frac{w_{s, 1} / w_{u, 1}}{w_{s, 2} / w_{u, 2}}\right)^{\left(\alpha_{x}-\alpha_{y}\right)}\right]^{\frac{\eta \gamma-\eta+1}{\eta-1}} \tag{40}
\end{equation*}
$$

Second, $\Theta_{1}^{\prime}>0$ implies

$$
\begin{equation*}
1>\left[\frac{1}{a^{\prime}}\left(\frac{w_{s, 1}^{\prime} / w_{u, 1}^{\prime}}{w_{s, 2}^{\prime} / w_{u, 2}^{\prime}}\right)^{\left(\alpha_{x}-\alpha_{y}\right)}\right]^{\frac{\eta \gamma^{\prime}-\eta+1}{\eta-1}} \tag{41}
\end{equation*}
$$

Third, Proposition 3 and $\Theta_{1} \geq \Theta_{1}^{\prime}>0$ imply

$$
\begin{align*}
w_{s, 1} / w_{u, 1} & \geq w_{s, 1}^{\prime} / w_{u, 1}^{\prime}  \tag{42}\\
w_{s, 2} / w_{u, 2} & \leq w_{s, 2}^{\prime} / w_{u, 2}^{\prime} \tag{43}
\end{align*}
$$

We now prove the comparative static result for $\gamma$. We proceed by contradiction. Suppose that $\gamma^{\prime}>\gamma$ and that $\Theta_{1} \geq \Theta_{1}^{\prime}>0$. Then

$$
\begin{equation*}
\left[\frac{1}{a^{\prime}}\left(\frac{w_{s, 1}^{\prime} / w_{u, 1}^{\prime}}{w_{s, 2}^{\prime} / w_{u, 2}^{\prime}}\right)^{\left(\alpha_{x}-\alpha_{y}\right)}\right]^{\frac{\eta \gamma^{\prime}-\eta+1}{\eta \gamma-\eta+1}} \geq \frac{1}{a^{\prime}}\left(\frac{w_{s, 1} / w_{u, 1}}{w_{s, 2} / w_{u, 2}}\right)^{\left(\alpha_{x}-\alpha_{y}\right)} \geq \frac{1}{a^{\prime}}\left(\frac{w_{s, 1}^{\prime} / w_{u, 1}^{\prime}}{w_{s, 2}^{\prime} / w_{u, 2}^{\prime}}\right)^{\left(\alpha_{x}-\alpha_{y}\right)} \tag{44}
\end{equation*}
$$

where the first weak inequality follows from Condition (40), $a=a^{\prime}$, and $(\eta \gamma-\eta+1) /(\eta-1)>0$ while the second weak inequality follows from Conditions (42) and (43). Condition (44) and $\gamma^{\prime} / \gamma>$ 1 contradict Condition (41). Thus, if $t$ and $T_{1} / T_{2}$ are chosen to match fixed values of $\Delta_{1}, \Delta_{2}>0$, and if $\gamma^{\prime}>\gamma, \Theta_{1}, \Theta_{1}^{\prime}>0$, then $\Theta_{1}^{\prime}>\Theta_{1}$. Combined with Proposition 3, this yields the desired comparative static result for $\gamma$.

We now prove the comparative static result for $a$. We proceed by contradiction. Suppose that $a^{\prime}>a$ and that $\Theta_{1} \geq \Theta_{1}^{\prime}>0$. Then Condition (40) implies

$$
\frac{1}{a^{\prime}} \times\left(\frac{w_{s, 1}^{\prime} / w_{u, 1}^{\prime}}{w_{s, 2}^{\prime} / w_{u, 2}^{\prime}}\right)^{\left(\alpha_{x}-\alpha_{y}\right)} \geq \frac{1}{a} \times\left(\frac{w_{s, 1} / w_{u, 1}}{w_{s, 2} / w_{u, 2}}\right)^{\left(\alpha_{x}-\alpha_{y}\right)}
$$

which, contradicts Conditions (42) and (43). Thus, if $t$ and $T_{1} / T_{2}$ are chosen to match fixed values of $\Delta_{1}, \Delta_{2}>0$, and if $a^{\prime}>a$, then $\Theta_{1}^{\prime}>\Theta_{1}$. Combined with Proposition 3, this yields the desired comparative static result for $a$. QED.

Proof of Proposition 4 Part 2. The proof requires two preliminary steps and uses the following notation: $\zeta=\frac{M_{2}(x)}{M_{1}(x)}\left(\frac{v_{2}(x)}{v_{1}(x)}\right)^{1-\frac{\eta \gamma}{\eta-1}}$.
Step 1. $\frac{d}{d \gamma}\left(\frac{w_{s, 1}}{w_{u, 1}}\right) \leq 0 \Leftrightarrow \frac{d}{d \gamma} \zeta \geq 0$.

With endogenous entry $w_{u, n} L_{u, n}+w_{s, n} L_{s, n}=Q_{n} P_{n}$ and with mirror-symmetry $Q_{1} P_{1}=Q_{2} P_{2}$, $\alpha_{y}=1-\alpha_{x}, \Lambda_{12}(y)=1-\Lambda_{11}(x), \Lambda_{11}(y)=1-\Lambda_{12}(x)$, and $\Delta_{2}=\frac{1}{2}\left[\Lambda_{12}(x)+1-\Lambda_{11}(x)\right]$, in which case equation (17) is equivalent to

$$
w_{s, 1} L_{s, 1}=\frac{Q_{1} P_{1}}{2}\left\{\left(2 \alpha_{x}-1\right)\left[2 \Delta_{2}-1+2 \Lambda_{11}(x)\right]+2\left(1-\alpha_{x}\right)\right\}
$$

Choosing $Q_{1} P_{1}$ as the numeraire, implies $\frac{d}{d \gamma}\left(\frac{w_{s, 1}}{w_{u, 1}}\right) \leq 0$ if and only if $\frac{d}{d \gamma} \Lambda_{11}(x) \leq 0$. Differentiating $\Delta_{2}$ with respect to $\gamma$ and setting $\frac{d}{d \gamma} \Delta_{2}=0$ yields $\frac{d}{d \gamma} \Lambda_{12}(x)=\frac{d}{d \gamma} \Lambda_{11}(x)$. Equation (16) implies

$$
\frac{d}{d \gamma} \Lambda_{12}(x)=-\Lambda_{12}(x)^{2}\left\{\zeta \frac{d t}{d \gamma}+t \frac{d \zeta}{d \gamma}\right\}
$$

and

$$
\begin{equation*}
\frac{d}{d \gamma} \Lambda_{11}(x)=-\Lambda_{11}(x)^{2}\left\{-t^{-2} \zeta \frac{d t}{d \gamma}+t^{-1} \frac{d \zeta}{d \gamma}\right\} \tag{45}
\end{equation*}
$$

Hence, $\frac{d}{d \gamma} \Lambda_{12}(x)=\frac{d}{d \gamma} \Lambda_{11}(x)$ if and only if

$$
\begin{equation*}
\frac{d t}{d \gamma}=\frac{\Lambda_{11}(x)^{2}-t^{2} \Lambda_{12}(x)^{2}}{\Lambda_{11}(x)^{2}+t^{2} \Lambda_{12}(x)^{2}}\left(\frac{t}{\zeta}\right)\left(\frac{d \zeta}{d \gamma}\right) \tag{46}
\end{equation*}
$$

Equations (45) and (46) imply

$$
\begin{aligned}
\frac{d}{d \gamma} \Lambda_{11}(x) & =-t^{-1} \Lambda_{11}(x)^{2}\left\{-\frac{\Lambda_{11}(x)^{2}-t^{2} \Lambda_{12}(x)^{2}}{\Lambda_{11}(x)^{2}+t^{2} \Lambda_{12}(x)^{2}}+1\right\}\left(\frac{d \zeta}{d \gamma}\right) \\
& =-\frac{2 t \Lambda_{11}(x)^{2} \Lambda_{12}(x)^{2}}{\Lambda_{11}(x)^{2}+t^{2} \Lambda_{12}(x)^{2}}\left(\frac{d \zeta}{d \gamma}\right)
\end{aligned}
$$

Hence, $\frac{d}{d \gamma} \Lambda_{11}(x)$ has the opposite sign as $\frac{d \zeta}{d \gamma}$. Hence, $\frac{d}{d \gamma}\left(\frac{w_{s, 1}}{w_{u, 1}}\right) \leq 0 \Leftrightarrow \frac{d}{d \gamma} \Lambda_{11}(x) \leq 0 \Leftrightarrow \frac{d}{d \gamma} \zeta \geq 0$.
Step 2. If $\frac{d}{d \gamma} \zeta>0$ then $\frac{d}{d \gamma} t<0$.
Equation (46) and $\frac{d}{d \gamma} \zeta>0$ imply $\frac{d}{d \gamma} t<0$ if and only if $\Lambda_{11}(x)<t \Lambda_{12}(x)$. Equation (16) implies both $(i) \Lambda_{11}(x)<t \Lambda_{12}(x)$ is equivalent to $\zeta<1$ and $(i i) \Lambda_{12}(x)>\Lambda_{12}(y)$ is equivalent to $\zeta<1$. Hence, $\Theta_{1}>0$-which implies $\Lambda_{12}(x)>\Lambda_{12}(y)$-implies $\Lambda_{11}(x)<t \Lambda_{12}(x)$, which itself implies $\frac{d}{d \gamma} t<0$.

We now use Steps 1 and 2 to prove Proposition 4 Part 2. With mirror-symmetry

$$
\zeta=\frac{\left(t^{2}+1\right)\left[\frac{v_{1}(x)}{v_{1}(y)}\right]^{\frac{\eta \gamma}{\eta-1}}-2 t}{\left(t^{2}+1\right)-2 t\left[\frac{v_{1}(x)}{v_{1}(y)}\right]^{\frac{\eta \gamma}{\eta-1}}}
$$

To obtain a contradiction, suppose that $\frac{d}{d \gamma} \zeta>0 . \frac{d}{d \gamma} \zeta>0$ if and only if

$$
\begin{aligned}
&\left\{t^{2}+1-2 t\left[\frac{v_{1}(x)}{v_{1}(y)}\right]^{\frac{\eta \gamma}{\eta-1}}\right\} {\left[2\left(t\left[\frac{v_{1}(x)}{v_{1}(y)}\right]^{\frac{\eta \gamma}{\eta-1}}-1\right) \frac{d t}{d \gamma}+\left(t^{2}+1\right) \frac{d\left[\frac{v_{1}(x)}{v_{1}(y)}\right]^{\frac{\eta \gamma}{\eta-1}}}{d \gamma}\right]>} \\
& 2\left(\left(t^{2}+1\right)\left[\frac{v_{1}(x)}{v_{1}(y)}\right]^{\frac{\eta \gamma}{\eta-1}}-2 t\right)\left[\left(t-\left[\frac{v_{1}(x)}{v_{1}(y)}\right]^{\frac{\eta \gamma}{\eta-1}}\right) \frac{d t}{d \gamma}-t \frac{d\left[\frac{v_{1}(x)}{v_{1}(y)}\right]^{\frac{\eta \gamma}{\eta-1}}}{d \gamma}\right]
\end{aligned}
$$

Since $t>1, \frac{d}{d \gamma} \zeta>0$ if and only if

$$
\begin{equation*}
\left(1-t^{2}\right) \frac{d}{d \gamma}\left[\frac{v_{1}(x)}{v_{1}(y)}\right]^{\frac{\eta \gamma}{\eta-1}}+2\left(\left[\frac{v_{1}(x)}{v_{1}(y)}\right]^{\frac{2 \eta \gamma}{\eta-1}}-1\right) \frac{d t}{d \gamma}<0 \tag{47}
\end{equation*}
$$

We consider the first and second terms of Condition (47) separately. According to Step 1, $\frac{d}{d \gamma} \zeta>0$ implies $\frac{d}{d \gamma}\left(\frac{w_{s, 1}}{w_{u, 1}}\right)<0$. Hence, $\frac{d}{d \gamma} \zeta>0$ implies $\frac{d}{d \gamma}\left[\frac{v_{1}(x)}{v_{1}(y)}\right]^{\frac{\eta \gamma}{\eta-1}}<0$. The first term in Condition (47) is, therefore, positive. In the second term we know that $\left[\frac{v_{1}(x)}{v_{1}(y)}\right]^{\frac{2 \eta \gamma}{\eta-1}}<1$, which follows from the fact that $\Theta_{1}>0$ is equivalent to $\zeta<1$, and $\zeta<1$ implies $v_{1}(x)<v_{1}(y)$. In the second term we also have $\frac{d t}{d \gamma}<0$, which follows from Step 2. Hence, the second term in Condition (47) is positive. This contradicts $\frac{d}{d \gamma} \zeta>0$. QED.

Proof of Proposition 5. The proof of Proposition 5 requires additional notation and two preliminary steps. Let $\vartheta_{1}=\frac{\Lambda_{11}(x) \Delta_{2} / \Delta_{1}+\Lambda_{12}(x)}{\Lambda_{11}(y) \Delta_{2} / \Delta_{1}+\Lambda_{12}(y)}$ denote the ratio of country 1's revenue in the $x$ sector to country 1's revenue in the $y$ sector. Let $k_{i}\left(\vartheta_{i}\right)=\frac{M_{i}^{T}(x) / M_{i}^{T}(y)}{M_{i}^{A}(x) / M_{i}^{A}(y)}$, where $M_{i}^{T}(j)$ and $M_{i}^{A}(j)$ denote the mass of sector $j$ entrants in country $i$ in a trade equilibrium for a given $\vartheta_{i}$ and in the autarky equilibrium $\left(\vartheta_{i}^{A}=1\right)$, respectively.

Step 1. $k_{1}$ is increasing in $\vartheta_{1}$.
Equations (11) and (20) imply

$$
\frac{M_{i}(x)}{M_{i}(y)}=\frac{\alpha_{x}^{\alpha_{x}}\left(1-\alpha_{x}\right)^{1-\alpha_{x}}}{\alpha_{y}^{\alpha_{y}}\left(1-\alpha_{y}\right)^{1-\alpha_{y}}} \frac{A_{i}(x)}{A_{i}(y)}\left(\frac{w_{u, i}}{w_{s, i}}\right)^{\alpha_{x}-\alpha_{y}} \frac{\sum_{n} \Lambda_{i n}(x) Q_{n} P_{n}}{\sum_{n} \Lambda_{i n}(y) Q_{n} P_{n}}
$$

so that

$$
\begin{equation*}
k_{i}=\left(\frac{w_{u, i}^{T} / w_{s, i}^{T}}{w_{u, i}^{A} / w_{s, i}^{A}}\right)^{\alpha_{x}-\alpha_{y}} \frac{\sum_{n} \Lambda_{i n}^{T}(x) Q_{n} P_{n}}{\sum_{n} \Lambda_{i n}^{T}(y) Q_{n} P_{n}} \tag{48}
\end{equation*}
$$

Re-expressing Equation (48) as a function of the $\Lambda \mathrm{s}$ and $\Delta \mathrm{s}$, using equation (17), yields

$$
\begin{equation*}
k_{1}\left(\vartheta_{1}\right)=\left(\frac{1}{\alpha_{y}} \frac{2}{\frac{\alpha_{x}}{\alpha_{y}}+1}-1\right)^{\alpha_{y}-\alpha_{x}}\left(\frac{1}{\alpha_{y}} \frac{\vartheta_{1}+1}{\frac{\alpha_{x}}{\alpha_{y}} \vartheta_{1}+1}-1\right)^{\alpha_{x}-\alpha_{y}} \vartheta_{1} . \tag{49}
\end{equation*}
$$

Hence, $k_{1}\left(\vartheta_{1}^{\prime}\right)>k_{1}\left(\vartheta_{1}\right)$ if and only if

$$
\begin{equation*}
\left(\frac{\alpha_{y}+\alpha_{x} \vartheta_{1}}{\alpha_{y}+\alpha_{x} \vartheta_{1}^{\prime}} \frac{1-\alpha_{y}^{2}+\vartheta_{1}^{\prime}\left(1-\alpha_{x} \alpha_{y}\right)}{1-\alpha_{y}^{2}+\vartheta_{1}\left(1-\alpha_{x} \alpha_{y}\right)}\right)^{\alpha_{x}-\alpha_{y}} \frac{\vartheta_{1}^{\prime}}{\vartheta_{1}}>1 \tag{50}
\end{equation*}
$$

If $\vartheta_{1}^{\prime}>\vartheta_{1}$, then

$$
\begin{equation*}
\left(\frac{\vartheta_{1}}{\vartheta_{1}^{\prime}}\right)^{\alpha_{x}-\alpha_{y}}<\left(\frac{\alpha_{y}+\alpha_{x} \vartheta_{1}}{\alpha_{y}+\alpha_{x} \vartheta_{1}^{\prime}} \frac{1-\alpha_{y}^{2}+\vartheta_{1}^{\prime}\left(1-\alpha_{x} \alpha_{y}\right)}{1-\alpha_{y}^{2}+\vartheta_{1}\left(1-\alpha_{x} \alpha_{y}\right)}\right)^{\alpha_{x}-\alpha_{y}} \tag{51}
\end{equation*}
$$

Condition (51) and $0 \leq \alpha_{x}-\alpha_{y} \leq 1$ imply Condition (50), so that $k_{1}$ is increasing in $\vartheta_{1}$.
Step 2. If $\Delta_{1}^{\prime}=\Delta_{1}, \Delta_{2}^{\prime}=\Delta_{2}$, and $\Theta_{1}^{\prime} \geq \Theta_{1}$, then $\vartheta_{1}^{\prime}=\vartheta_{1}\left(\Theta_{1}^{\prime}\right) \geq \vartheta_{1}\left(\Theta_{1}\right)=\vartheta_{1}$.
Choose $Q_{1} P_{1}$ as the numeraire, which implies that $Q_{2} P_{2}$ is fixed given fixed trade shares and that $\Lambda_{1 n}(x) Q_{n} P_{n}+\Lambda_{1 n}(y) Q_{n} P_{n}$ is fixed. Hence, a sufficient condition under which $\vartheta_{1}^{\prime} \geq \vartheta_{1}$ is $\Lambda_{1 n}^{\prime}(x) \geq \Lambda_{1 n}(x)$ for $n=1,2$. We have $\Lambda_{11}^{\prime}(x)=1-\Delta_{1}^{\prime}+\frac{1}{2} \Theta_{1}^{\prime} \geq 1-\Delta_{1}+\frac{1}{2} \Theta_{1}=\Lambda_{11}(x)$, so that it only remains to show that $\Lambda_{12}^{\prime}(x) \geq \Lambda_{12}(x)$. We have

$$
\Lambda_{12}(x)=\left[1+\left(\frac{1}{\Delta_{1}-\frac{1}{2} \Theta_{1}}-1\right)^{-1} t\right]^{-1}
$$

so that $\Lambda_{12}^{\prime}(x) \geq \Lambda_{12}(x)$ if and only if

$$
\phi_{1}^{\prime} t^{\prime} \leq \phi_{1} t
$$

where

$$
\phi_{1}=\left(\frac{1}{\Delta_{1}-\frac{1}{2} \Theta_{1}}-1\right)^{-1}
$$

Moreover, $\Delta_{2}=\Delta_{2}^{\prime}$ implies

$$
\begin{equation*}
\left(1+\phi_{1} t\right)^{-1}+\left(1+\phi_{2} t\right)^{-1}=\left(1+\phi_{1}^{\prime} t^{\prime}\right)^{-1}+\left(1+\phi_{2}^{\prime} t^{\prime}\right)^{-1} \tag{52}
\end{equation*}
$$

where

$$
\phi_{2}=\left(\frac{1}{\Delta_{1}+\frac{1}{2} \Theta_{1}}-1\right)^{-1}
$$

To obtain a contradiction, suppose that $\phi_{1}^{\prime} t^{\prime}>\phi_{1} t$. Then Equation (52) implies $\phi_{2}^{\prime} t^{\prime}<\phi_{2} t$. Hence, $\phi_{2}^{\prime} / \phi_{2}<\phi_{1}^{\prime} / \phi_{1}$, which is equivalent to

$$
\frac{\frac{1}{\Delta_{1}-\frac{1}{2} \Theta_{1}^{\prime}}-1}{\frac{1}{\Delta_{1}+\frac{1}{2} \Theta_{1}^{\prime}}-1}<\frac{\frac{1}{\Delta_{1}-\frac{1}{2} \Theta_{1}}-1}{\frac{1}{\Delta_{1}+\frac{1}{2} \Theta_{1}}-1}
$$

which is violated. Hence, $\phi_{1}^{\prime} t^{\prime} \leq \phi_{1} t$, which is equivalent to $\Lambda_{12}^{\prime}(x) \geq \Lambda_{12}(x)$. Hence, $\Delta_{1}^{\prime}=\Delta_{1}$, $\Delta_{2}^{\prime}=\Delta_{2}$ and $\Theta_{1}^{\prime} \geq \Theta_{1}$ imply $\vartheta_{1}^{\prime} \geq \vartheta_{1}$.

We now use Steps 1 and 2 to conclude the proof of Proposition 5. Here we compare across two specifications, one in which entry is exogenous and one in which entry is endogenous, where the endogenous entry case is denoted by ${ }^{\prime}$. We proceed by contradiction. Suppose that $\Theta_{1} \geq \Theta_{1}^{\prime}>0$, $\Delta_{1}=\Delta_{1}^{\prime}>0$, and $\Delta_{2}=\Delta_{2}^{\prime}>0$. Using similar logic that lead to Condition (40), $\Theta_{1} \geq \Theta_{1}^{\prime}$ implies

$$
\begin{equation*}
\frac{M_{1}^{\prime}(y) M_{2}^{\prime}(x)}{M_{1}^{\prime}(x) M_{2}^{\prime}(y)}\left[\frac{v_{1}^{\prime}(x) / v_{1}^{\prime}(y)}{v_{2}^{\prime}(x) / v_{2}^{\prime}(y)}\right]^{\frac{\eta \gamma-\eta+1}{\eta-1}} \geq \frac{M_{1}(y) M_{2}(x)}{M_{1}(x) M_{2}(y)}\left[\frac{v_{1}(x) / v_{1}(y)}{v_{2}(x) / v_{2}(y)}\right]^{\frac{\eta \gamma-\eta+1}{\eta-1}} \tag{53}
\end{equation*}
$$

Moreover, Proposition 3 and $\Theta_{1} \geq \Theta_{1}^{\prime}>0$ imply

$$
\frac{v_{1}(x) / v_{1}(y)}{v_{2}(x) / v_{2}(y)} \geq \frac{v_{1}^{\prime}(x) / v_{1}^{\prime}(y)}{v_{2}^{\prime}(x) / v_{2}^{\prime}(y)} .
$$

Hence, Condition (53) requires

$$
\begin{equation*}
\frac{M_{2}^{\prime}(x) / M_{2}^{\prime}(y)}{M_{2}(x) / M_{2}(y)} \geq \frac{M_{1}^{\prime}(x) / M_{1}^{\prime}(y)}{M_{1}(x) / M_{1}(y)} . \tag{54}
\end{equation*}
$$

Imposing $M_{i}(j)=M_{i}$ for $j=x, y$ in the exogenous entry case, Condition (54) is equivalent to

$$
\begin{equation*}
\frac{M_{2}^{A \prime}(x)}{M_{2}^{A \prime}(y)} \frac{M_{2}^{\prime}(x) / M_{2}^{\prime}(y)}{M_{2}^{A \prime}(x) / M_{2}^{A \prime}(y)} \geq \frac{M_{1}^{\prime}(x) / M_{1}^{\prime}(y)}{M_{1}^{A \prime}(x) / M_{1}^{A \prime}(y)} \frac{M_{1}^{A^{\prime}}(x)}{M_{1}^{A \prime}(y)} \tag{55}
\end{equation*}
$$

With $\Theta_{1}^{\prime}>0$, Steps 1 and 2 imply $\frac{M_{1}^{\prime}(x) / M_{1}^{\prime}(y)}{M_{1}^{A^{\prime}}(x) / M_{1}^{A^{\prime}}(y)}>1$ and $\frac{M_{2}^{\prime}(x) / M_{2}^{\prime}(y)}{M_{2}^{A^{\prime}}(x) / M_{2}^{A^{\prime}}(y)}<1$. Hence, Condition (55) requires

$$
\begin{equation*}
M_{2}^{A \prime}(x) / M_{2}^{A \prime}(y)>M_{1}^{A \prime}(x) / M_{1}^{A \prime}(y) \tag{56}
\end{equation*}
$$

We know that $M_{i}^{A}(x) / M_{i}^{A}(y)=v_{i}^{A}(y) / v_{i}^{A}(x)$, so that Condition (56) is equivalent to

$$
v_{2}^{A^{\prime}}(y) / v_{2}^{A^{\prime}}(x)>v_{1}^{A^{\prime}}(y) / v_{1}^{A^{\prime}}(x)
$$

which implies that country 2 has a comparative advantage in $x$ and contradicts $\Theta_{1}^{\prime}>0$. Hence, we must have $\Theta_{1}^{\prime}>\Theta_{1}>0$. Proposition 3 then implies that the between effect is stronger with endogenous entry. QED.

Proof of Lemma 1. The result in which entry is endogenous and trade shares are positive follow from Part (a) or Proposition 4 in BRS, combined with the fact that domestic cutoffs are identical across countries in autarky. The proof with exogenous entry follows.

Equation (13) implies that relative domestic cutoffs are given by

$$
\begin{equation*}
\frac{z_{i i}^{*}(x)}{z_{i i}^{*}(y)}=\frac{P_{i}(y)}{P_{i}(x)}\left(\frac{v_{i}(x)}{v_{i}(y)}\right)^{\frac{\eta}{\eta-1}} . \tag{57}
\end{equation*}
$$

The price level equation-Equation (30)—and Equation (57) imply $z_{11}^{*}(x) / z_{11}^{*}(y)<z_{22}^{*}(x) / z_{22}^{*}(y)$ if and only if

$$
\begin{equation*}
\left(\frac{v_{1}(x)}{v_{1}(y)} \frac{v_{2}(y)}{v_{2}(x)}\right)^{\frac{\gamma \eta}{\eta-1}}<\frac{v_{1}(y)^{\frac{\eta \gamma+1-\eta}{1-\eta}} t+v_{2}(y)^{\frac{\eta \gamma+1-\eta}{1-\eta}}}{v_{1}(x)^{\frac{\eta \gamma+1-\eta}{1-\eta}} t+v_{2}(x)^{\frac{\eta \gamma+1-\eta}{1-\eta}}} \times \frac{v_{1}(x)^{\frac{\eta \gamma+1-\eta}{1-\eta}}+v_{2}(x)^{\frac{\eta \gamma+1-\eta}{1-\eta}} t}{v_{1}(y)^{\frac{\eta \gamma+1-\eta}{1-\eta}}+v_{2}(y)^{\frac{\eta \gamma+1-\eta}{1-\eta}} t} \tag{58}
\end{equation*}
$$

which is equivalent to

$$
\begin{align*}
& \frac{v_{2}(x)}{v_{1}(x)} \frac{v_{2}(y)}{v_{1}(y)}\left[\left(\frac{v_{2}(x)}{v_{1}(x)}\right)^{\frac{\eta \gamma}{1-\eta}}-\left(\frac{v_{2}(y)}{v_{1}(y)}\right)^{\frac{\eta \gamma}{1-\eta}}\right] t+\left[\frac{v_{2}(y)}{v_{1}(y)}-\frac{v_{2}(x)}{v_{1}(x)}\right] t^{2}+  \tag{59}\\
& \quad\left[\left(\frac{v_{1}(y)}{v_{2}(y)}\right)^{\frac{\gamma \eta}{1-\eta}}-\left(\frac{v_{1}(x)}{v_{2}(x)}\right)^{\frac{\gamma \eta}{1-\eta}}\right] t<\left(\frac{v_{1}(y)}{v_{2}(y)} \frac{v_{2}(x)}{v_{1}(x)}\right)^{\frac{\eta \eta}{\eta-1}} \frac{v_{2}(y)}{v_{1}(y)}-\left(\frac{v_{2}(y)}{v_{1}(y)} \frac{v_{1}(x)}{v_{2}(x)}\right)^{\frac{\gamma \eta}{\eta-1}} \frac{v_{2}(x)}{v_{1}(x)}
\end{align*}
$$

Country 1's comparative advantage in sector $x$ implies $v_{2}(x) / v_{1}(x)>v_{2}(y) / v_{1}(y)$ (without factor price equalization). This implies that the left-hand side of Condition (59) is negative and that the right-hand side of Condition (59) is positive. Therefore, Condition (59) is satisfied, which is equivalent to $z_{11}^{*}(x) / z_{11}^{*}(y)<z_{22}^{*}(x) / z_{22}^{*}(y)$. QED.

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[^0]:    ${ }^{1}$ See e.g. Feenstra (2004) for a textbook presentation of this model and its implications for trade patterns and factor prices
    ${ }^{2}$ See e.g. Yeaple (2005), Matsuyama (2007), and Burstein and Vogel (2010) for the first mechanism, Epifani and Gancia (2006) for the second, and Burstein, Cravino, and Vogel (2010) and Parro (2010) for the third.
    ${ }^{3}$ The restriction that all employed efficiency units of a given factor receive a common price is relaxed in the trade and search literature; see e.g. Davidson, Matusz, and Shevchenko (2008) and Helpman, Itskhoki, and Redding (2010).

[^1]:    ${ }^{4}$ Many empirical papers use this theoretical result to quantify the effects of trade on the skill premium; see e.g. Katz and Murphy (1992) and Krugman (1995).

[^2]:    ${ }^{5}$ A large literature studies Vanek's (1968) prediction that each country is a net exporter of the services of its abundant factors, using the FCT; see e.g. Trefler (1993 and 1995) and Davis and Weinstein (2001). Our results are related to Helpman and Krugman (1985) and Trefler and Zhu (2010), who show that Vanek's prediction holds across a wide range of models.
    ${ }^{6}$ Ho (2010) uses a similar framework to study the implications of idiosyncratic distortions on betweensector factor allocation, the skill premium, and welfare, while Lu (2010) uses it to study how export market participation decisions of Chinese firms vary across sectors.
    ${ }^{7}$ One standard assumption under which each efficiency unit of a factor receives a common price is that factors are perfectly mobile across producers within a country. At this point we do not require an assumption regarding the mobility of factors across countries nor do we need to distinguish between cases in which $L_{k, i}$ is in fixed supply or not.

[^3]:    ${ }^{8}$ If factor intensities vary across highly disaggregated industries, then constructing the FCT in practice requires highly disaggregated data. See Feenstra and Hanson (2000) for an analysis of the bias in measuring the FCT using aggregated industry data.

[^4]:    ${ }^{9}$ An alternative assumption that simplifies equation (7) is that countries are symmetric. In this case, trade is balanced sector-by-sector, so that $\omega_{i}(j)=0$ for all $i$ and $j$, and $F C T_{k, i}=0$ for all $k$ and $i$. Hence, changes in relative factor prices only depend on changes in factor supplies and relative payments for domestic absorption.
    ${ }^{10}$ While some paper treat technological change as exogenous, there is a large literature on endogenous factor-biased technological change; see, e.g. Acemoglu (2002). The effect of such changes on relative factor wages operate through changes in relative factor intensities (changes in $\alpha$ 's).
    ${ }^{11}$ Parro (2010) considers a model similar to Burstein, Cravino, and Vogel (2010) in which factor intensities vary across sectors.

[^5]:    ${ }^{12}$ Epifani and Gancia (2008) consider an alternative model of international trade an monopolistic competition. In their model, changes in relative factor wages are driven by changes in the FCT and sectoral expenditures. Trade raises expenditures, $E_{i}(j)$, in the skill-intensive sector relative to the unskill-intensive sector in all countries, which tends to increase the skill premium in all countries, as is evident in equation (6).

[^6]:    ${ }^{13}$ Under this specification, our results remain unchanged if market access costs are stricty greater than zero and all firms sell in all markets.

[^7]:    ${ }^{14}$ This assumption implies that there are only two independent sources of comparative advantage: relative endowments and sectoral productivities. Alternatively, we could combine differences in $M_{i}(j)$ and $A_{i}(j)$ into a single parameter, $\widetilde{A}_{i}(j)$. The parameter $a$ in this case would be defined using the $\widetilde{A}_{i}(j)$ s.

[^8]:    ${ }^{15}$ After solving for an equilibrium assuming that the model is either with selection or without selection, one must verify that all cutoffs are either greater than one or equal to one, respectively, using equation (13).

[^9]:    ${ }^{16}$ If we were to hold trade costs rather than trade shares fixed, then some of our comparative static results would be ambiguous. For example, an increase in technological dispersion, i.e. a reduction in $\gamma$, would increase total trade. This could offset the direct effect of $\gamma$ discussed in Proposition 4 below. Note also that for given trade shares, the partial elasticity of trade flows with respect to variable trade costs does not fully determine the implications of trade liberalization for the skill premium and factor allocation.
    ${ }^{17}$ Based on economic intuition and many numerical examples, we believe that in the case of endogenous entry Propositions 4 and 6 continue to hold even if we do not impose mirror symmetry. However, we have not yet been successful proving this more general result.
    ${ }^{18}$ Had we not imposed $M_{i}(j)=M_{i}$, then increasing the relative mass of entrants in a sector would have identical implications as increasing the relative sectoral productivity in that sector.

[^10]:    ${ }^{19}$ With endogenous entry, the relative mass of entrants also plays a role in this result.

[^11]:    ${ }^{20}$ In Lemma 1 , we impose $M_{i}(j)=1$ with exogenous entry and $A_{i}(j)=1$ so that factor endowment differences are the unique source of exogenous comparative advantage, as in BRS.

[^12]:    ${ }^{21}$ Another result in BRS-that the export cutoff relative to the domestic cutoff is relatively lower in each country's comparative advantage sector, e.g. $z_{12}^{*}(x) / z_{11}^{*}(x)<z_{12}^{*}(y) / z_{11}^{*}(y)$ —holds both with endogenous entry (as considered in BRS) and exogenous entry.

[^13]:    ${ }^{22}$ Of course, if $M_{i}(x) \neq M_{i}(y)$ in the case of exogenous entry, then we can always include the $M$ 's into the $a$ term, as described in Section 3 and the proof remains unchanged.

