

Factoring the strong CP problem

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ABSTRACT: We present a new mechanism to solve the strong CP problem using $N \geq 2$ axions, each dynamically relaxing part of the $\bar{\theta}$ parameter. At high energies $M \gg \Lambda_{\text{QCD}}$ the $\text{SU}(3)_c$ group becomes the diagonal subgroup of an $\text{SU}(3)^N$ gauge group, and the non-perturbative effects in each individual $\text{SU}(3)$ factor generate a potential for the corresponding axion. The vacuum is naturally aligned to ensure $\bar{\theta} = 0$ at low energies, and the masses of these axions can be much larger than for the standard QCD axion. This mechanism avoids the introduction of a discrete Z_2 symmetry and associated ‘mirror’ copies of the SM fermions, and also avoids the introduction and stabilization of new light colored states to modify the running of the QCD gauge coupling found in other heavy axion models. This strengthens the motivation for axion-like particles solving the strong CP problem at points beyond the standard QCD axion curve in the (m_a, f_a) plane.

KEYWORDS: Anomalies in Field and String Theories, Beyond Standard Model, CP violation, Solitons Monopoles and Instantons

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1 Introduction

The Standard Model (SM) describes physics remarkably well at the smallest scales probed so far. With the discovery of the Higgs boson, the SM can be consistently extrapolated to very high energies. However, there are a number of puzzles in the SM that remain. The strong CP problem is one such long standing problem. The QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}^a G^{a,\mu\nu} - \frac{g^2\theta}{32\pi^2}G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} + \bar{q}Mq \tag{1.1}$$

in general violates CP, with the CP violation encoded by

$$\bar{\theta} = \theta + \arg \det M. \tag{1.2}$$

Given that CP is badly broken by the weak interactions, we expect $\bar{\theta} \sim 1$. However, the QCD θ -angle is constrained to be very small, $\bar{\theta} \lesssim 10^{-10}$, from limits on the electric dipole moment of the neutron/Hg [1–3].

The solutions to the strong CP problem can be classified roughly into two categories. One class of solutions imposes discrete symmetries [4–6], like CP as in the Nelson-Barr mechanism [7–10] (see [11] for a recent discussion) or parity (P) [9, 12, 13] that forbid the θ -angle. In Nelson-Barr models there are generally no low-energy states, while parity models often require new light colored particles [14].

The second class of solutions invokes a global U(1) Peccei-Quinn symmetry [15, 16]. This U(1)_{PQ} has a mixed anomaly with QCD, such that QCD non-perturbative dynamics break PQ explicitly and the θ -angle is dynamically relaxed to zero. The most economical version of such a solution is the massless (up-) quark solution [17], in which case the θ -angle

becomes unphysical (or equivalently is relaxed dynamically by the η' meson). However, lattice calculations indicate that all quarks are massive [18, 19], excluding the simplest model with a massless up-quark. Another elegant model that uses the PQ symmetry is the QCD axion [15, 16, 20, 21] (see [22] for a review). The $U(1)_{PQ}$ is spontaneously broken in the UV, giving rise to a light pseudo-Nambu Goldstone boson, the axion. The axion makes the θ angle dynamical, and has a potential with a minimum at $\bar{\theta} = 0$ [23].

In our paper we focus on this axion solution to the strong CP problem. Given the QCD axion decay constant f_a , the mass of the axion is determined by QCD dynamics [20],

$$m_{a,\text{QCD}}^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{f_\pi^2 m_\pi^2}{f_a^2} \simeq \frac{(75.5 \text{ MeV})^4}{f_a^2} \equiv \frac{\Lambda_0^4}{f_a^2}. \quad (1.3)$$

The axion couples to the standard model through dimension-5 operators suppressed by f_a , and the precise mass relationship equation (1.3) has motivated many existing and future experimental efforts (see ref. [24] for a review). In this work, we study an extension of the QCD axion framework where instead of a single axion relaxing the vacuum to $\bar{\theta} = 0$, two or more axions naturally cooperate to solve the strong CP problem. Each of these axions can have a mass much larger than the standard QCD axion, motivating searches for axions in the (m_a, f_a) plane outside of the QCD axion window given by equation (1.3).

This multi-axion solution to the strong CP problem arises in models that extend the low-energy $SU(3)_c$ gauge group to be the diagonal subgroup of a parent $SU(3) \times SU(3) \times \dots$ product gauge group, which is broken down to $SU(3)_c$ at some high scale M . All the SM quarks are taken to be charged under a single $SU(3)$ factor of the parent gauge group, and we introduce an axion for each individual $SU(3)$ factors that independently relaxes the corresponding θ -angle to 0. Each $SU(3)$ factor of the parent gauge group is more strongly coupled than the diagonal $SU(3)_c$ subgroup, and therefore the contributions to the axion potentials from UV instantons near the scale M can be larger than the non-perturbative potential generated at low energies from QCD. Even for the smallest extension, $SU(3) \times SU(3) \rightarrow SU(3)_c$, we find that the two axions can have masses significantly larger than for the standard QCD axion. In a follow-up work, ref. [25], we will describe a related family of models where each quark generation is charged under a different $SU(3)$ factor, and the strong CP problem can be solved without introducing any axions degrees of freedom in a spirit similar to the massless up quark solution.

Other mechanisms have been proposed that may increase the mass of the QCD axion. The models of refs. [26–30] also extend $SU(3)_c$ to an $SU(3) \times SU(3)$ gauge group, but with QCD living in a single factor instead of the diagonal subgroup. In these models, a discrete Z_2 symmetry is introduced, requiring an entire mirror SM matter sector. In contrast, our mechanism does not involve any new matter in the $SU(3)$ factors, and the only new low-energy states are the axions directly responsible for relaxing $\bar{\theta}$. The models of refs. [31–36] introduce extra colored matter at an intermediate scale so that QCD runs back to strong coupling at a high scale M where $SU(3)_c$ is embedded in a $SU(3+n)$ gauge group. However, the introduction of new colored matter generally leads to either new hierarchy problems for scalars or new CP phases for fermions, spoiling the solution to the strong CP problem [37].

In contrast, in our mechanism no new colored states are introduced below the scale M , and extra CP phases that would feed into the UV axion potential are naturally absent.

In section 2, we describe in detail the structure of the model and the calculation of the non-perturbative contributions to the axion potentials at the scale M . In section 3, we discuss the sensitivity of the mechanism to higher dimensional operators and the connection to the electroweak hierarchy problem. In section 4, we describe the limits on these models from axion-like particle (ALP) searches.

2 Non-perturbative effects in product gauge groups

2.1 $SU(3) \times SU(3)$ model

We start with a simple extension of the SM where the QCD gauge group emerges from Higgsing a product group

$$SU(3)_1 \times SU(3)_2 \rightarrow SU(3)_c, \tag{2.1}$$

at a scale $M \gg \text{TeV}$. This is a similar set-up to the renormalizable coloron models of [38–41], although in our case the scales will be far out of reach of the LHC. The SM quarks are charged only under the $SU(3)_1$ gauge factor, and there are no fermions charged under $SU(3)_2$. We take the theory to have two spontaneously broken anomalous $U(1)_{PQ}$ symmetries at scales $f_i > M$, giving an axion $a_{1,2}$ in each $SU(3)$ sector:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}(G_1)_{\mu\nu}^a (G_1)^{a,\mu\nu} + \frac{g_{s1}^2}{32\pi^2} \left(\frac{a_1}{f_1} - \theta_1 \right) (\tilde{G}_1)_{\mu\nu}^a (G_1)^{a,\mu\nu} \\ & -\frac{1}{4}(G_2)_{\mu\nu}^a (G_2)^{a,\mu\nu} + \frac{g_{s2}^2}{32\pi^2} \left(\frac{a_2}{f_2} - \theta_2 \right) (\tilde{G}_2)_{\mu\nu}^a (G_2)^{a,\mu\nu} \end{aligned} \tag{2.2}$$

The gauge couplings $g_{s1,2}$ and theta angles $\theta_{1,2}$ are independent parameters not related to each other by any symmetries. The presence of two independent axion degrees of freedom will allow both physical θ -angles to be dynamically removed. An additional cross-coupling term for the axions $\propto \frac{a_1}{f_1} \tilde{G}_2 G_2$ can not be removed by a field redefinition. It leads to a mass-mixing between the axions, but does not destabilize the solution to the strong CP problem or significantly change the phenomenology of the light axion states, so we take it to vanish for simplicity. The UV couplings of $a_{1,2}$ to the electroweak topological terms $W\tilde{W}$ and $B\tilde{B}$ will only be relevant for the axion phenomenology, and will be discussed in section 4.

The theory will match to the SM at a scale M where the $SU(3)_1 \times SU(3)_2$ gauge group is Higgsed to a diagonal $SU(3)_c$. For simplicity, we take the dynamics to be a bifundamental scalar field $\Sigma_{12} : (3, \bar{3}, 1)$ of $SU(3)_1 \times SU(3)_2 \times U(1)_\Sigma$ with a renormalizable scalar potential

$$V_\Sigma = -m_\Sigma^2 \text{Tr}(\Sigma_{12} \Sigma_{12}^\dagger) + \frac{\lambda}{2} [\text{Tr}(\Sigma_{12} \Sigma_{12}^\dagger)]^2 + \frac{\kappa}{2} \text{Tr}(\Sigma_{12} \Sigma_{12}^\dagger \Sigma_{12} \Sigma_{12}^\dagger) \tag{2.3}$$

inducing a vev [38, 41–43]

$$\langle \Sigma \rangle = \frac{m_\Sigma}{\sqrt{\kappa + 3\lambda}} \mathbb{I}_3 \equiv \frac{f_\Sigma}{2} \mathbb{I}_3. \tag{2.4}$$

One combination of the SU(3) gauge bosons becomes massive with $M_V^2 \equiv M^2 = (g_{s_1}^2 + g_{s_2}^2)f_\Sigma^2$, and the unbroken SU(3)_{QCD} gauge symmetry is given by

$$G_{(\text{SM})}^\mu = \cos \gamma G_1^\mu + \sin \gamma G_2^\mu, \quad \tan \gamma = g_{s_1}/g_{s_2} \quad (2.5)$$

The additional U(1)_Σ gauge factor forbids a trilinear term in the potential, and the corresponding gauge boson absorbs the otherwise massless singlet Goldstone mode.¹

We can integrate out the heavy degrees of freedom to match to the SM at this scale. The tree-level matching condition gives the couplings of the two SU(3) factors in terms of the standard model QCD coupling evaluated at the scale M ,

$$\frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_{s_1}(\mu)} + \frac{1}{\alpha_{s_2}(\mu)}, \quad \mu = M, \quad (2.6)$$

with $\alpha = g^2/4\pi$. Each individual gauge factor is more strongly coupled than the SM QCD gauge coupling, and this will lead to enhanced non-perturbative effects compared to the SM. The SM coupling $g_s(M)$ can be obtained with the 1-loop running from the top pole, $\alpha_s(m_t) = 0.10$.

The tree-level value of the SM theta term is simply

$$\bar{\theta}_{SM} = \bar{\theta}_1 + \bar{\theta}_2$$

Because the flavor symmetries are the same as the SM, the loop-level thresholds to $\bar{\theta}$ proportional to the CKM phase are negligible [44, 45].

Since we are integrating out the heavy degrees of freedom, we must also consider short distance non-perturbative effects in the individual SU(3)₁ and SU(3)₂ factors, which will generate the shift-symmetry breaking potentials for the axions. We will be interested in the regime where each factor is still at relatively weak coupling near M , and therefore the dilute instanton gas approximation (DGA) gives a good approximation for the non-perturbative effects. The effective Lagrangian generated for the axions is [46–48]

$$\mathcal{L}_a = \Lambda_1^4 \cos\left(\frac{a_1}{f_1} - \bar{\theta}_1\right) + \Lambda_2^4 \cos\left(\frac{a_2}{f_2} - \bar{\theta}_2\right) + \frac{g_s^2}{32\pi^2} \left(\left(\frac{a_1}{f_1} - \bar{\theta}_1\right) + \left(\frac{a_2}{f_2} - \bar{\theta}_2\right) \right) G\tilde{G} \quad (2.7)$$

The axions still couple to the low energy QCD $\tilde{G}G$ term, and at the scale $\Lambda_{\text{QCD}} \sim 1$ GeV the non-perturbative SM contributions to the axion potentials will be generated. However, unlike the standard axion case, the axion potential can be dominated by the higher energy contributions to the potential with scales Λ_1 and Λ_2 generated at the scale M . Although these scales Λ_1 and Λ_2 will be suppressed by non-perturbative and chiral suppression factors, they can still greatly exceed the scale of the standard axion potential.

¹As an alternative to gauging the extra U(1) factor, the trilinear term $\mu \text{Det} \Sigma_{12} + \text{h.c}$ could be introduced in the potential to lift the singlet Goldstone mode. Note that this CP phase in the trilinear term can be rotated away into the Σ field, and the potential dynamically prefers a CP preserving Σ vev [41].

Crucially, and as is clear from the UV PQ symmetries of the theory, the new short-distance non-perturbative contributions to the axion potentials are exactly aligned to remove the effective θ -angle,

$$\bar{\theta}_{\text{eff}} = \left\langle \left(\frac{a_1}{f_1} + \bar{\theta}_1 \right) + \left(\frac{a_2}{f_2} + \bar{\theta}_2 \right) \right\rangle = 0. \quad (2.8)$$

The low-energy vacuum is in-fact guaranteed to align with $\bar{\theta} = 0$ by a generalization of the Vafa-Witten argument [23]. This result is very different from theories which have new *perturbative* breakings of the PQ symmetry in the UV- for example, higher-dimensional operators violating the PQ symmetry give axion potentials that are generically misaligned and would spoil the axion solution to the strong CP problem [49–51].

We can now estimate how the mass of the two axions is affected by these UV non-perturbative contributions to the potential. In the $SU(3)_2$ factor, where no colored fermions are present, the axion potential is suppressed only by the non-perturbative instanton action. The scale can be calculated as [46–48]

$$\Lambda_2^4 = \int_{\rho=0}^{\rho=1/M} 2 \frac{d\rho}{\rho^5} D[\alpha_{s_2}(1/\rho)] \quad (2.9)$$

where the dimensionless instanton density depends non-perturbatively on the running gauge coupling as

$$D[\alpha] = 0.1 \left(\frac{2\pi}{\alpha} \right)^6 e^{-\frac{2\pi}{\alpha}}, \quad (2.10)$$

with $\alpha(1/\rho)$ the running coupling evaluated at the scale $\mu = \rho^{-1}$ corresponding to the size of the instanton. Higher order corrections to the instanton density have been calculated in ref. [19]. In this work, we use the leading order result, and estimate the theoretical uncertainty by varying the scale at which the density is evaluated: $D[\alpha_s(1/\rho)] \rightarrow (D[\alpha_s(2/\rho)], D[\alpha_s(\frac{1}{2\rho})])$.

Since the instanton contribution dominantly comes from the IR, near the scale M , the instanton density in the integrand (2.9) can be approximated as

$$D[\alpha_{s_{1,2}}(1/\rho)] \approx D[\alpha_{s_{1,2}}(M)](\rho M)^{b_{1,2}}, \quad (2.11)$$

where the one-loop running coupling satisfies

$$\frac{d\alpha_{s_i}^{-1}}{d \ln \mu} = \frac{b_i}{2\pi} \quad (2.12)$$

with $b_1 = 13/2$ and $b_2 = 21/2$ (including the contributions of the bifundamental scalar Σ to the running). This gives

$$\Lambda_2^4 = \frac{4}{13} D[\alpha_{s_2}(M)] M^4 \quad (2.13)$$

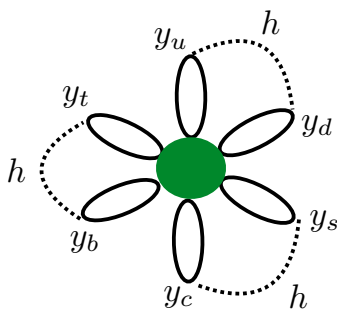


Figure 1. An instanton vacuum-diagram schematically generates a short distance contribution to the axion potential that is proportional to the breakings of the individual quark chiral U(1) factors by the Higgs Yukawa couplings. The dominant contribution at short distances is proportional to the Higgs vacuum fluctuations, corresponding to looping off the Higgs propagators.

In the SU(3)₁ factor, there is a further suppression due to the Yukawa couplings and Higgs loops, as depicted in figure 1, which can be estimated as [35, 36]

$$\Lambda_1^4 \sim K \int_{\rho=0}^{\rho=1/M} 2 \frac{d\rho}{\rho^5} D[\alpha_{s_1}(1/\rho)] \approx K \frac{4}{5} D[\alpha_{s_1}(M)] M^4. \quad (2.14)$$

Where K is a chiral suppression factor capturing the breaking of the U(1)⁶ axial symmetry of the individual quarks by the Yukawa couplings,²

$$K = \left(\frac{y_u}{4\pi}\right) \left(\frac{y_d}{4\pi}\right) \left(\frac{y_c}{4\pi}\right) \left(\frac{y_s}{4\pi}\right) \left(\frac{y_t}{4\pi}\right) \left(\frac{y_b}{4\pi}\right) \approx 10^{-23}. \quad (2.15)$$

We can compare the scales $\Lambda_{1,2}$ of the UV contributions to the scale Λ_0 of the usual non-perturbative axion potential generated around the QCD scale [20] given in equation (1.3).

The size of the non-perturbative UV potentials generated in each sector are shown in figure 2. The sensitivity to the large scale M can overcome the non-perturbative and chiral suppression factors. The relation equation (2.6) between α_{s_1} and α_{s_2} implies that Λ_1 grows as Λ_2 shrinks. In much of the parameter range, $\Lambda_1 \gg \Lambda_0 \gg \Lambda_2$ or $\Lambda_2 \gg \Lambda_0 \gg \Lambda_1$. In these cases, one axion DOF will simply decouple, and the other will behave largely like the QCD axion.

The most interesting regime is instead when $\Lambda_1, \Lambda_2 \gg \Lambda_0$ — in this case there will be no light state resembling the standard QCD axion. This regime generally corresponds to $\alpha_{s_1} > \alpha_{s_2}$ to compensate for the extra chiral suppression factor in Λ_1 . As the scale M increases, the scales $\Lambda_{1,2}$ grow — although the running of α_3 to weak coupling at higher energies leads to stronger instanton suppression factors, this is overcome by the M^4 dependence of the potential. For $M \sim 10^8$ GeV, the UV effects can become comparable

²Ref. [35] considers the case where Yukawa couplings to a scalar state of mass m_s break the chiral symmetries of the colored fermions in the theory. Their result contains an additional suppression factor of m_s^2/M^2 , which would arise in theories where a scalar mass-insertion is necessary to violate some of the chiral symmetries, e.g. a two-higgs-doublet model. In contrast, in our case there are non-decoupling effects even in the limit $m_s \rightarrow 0$.

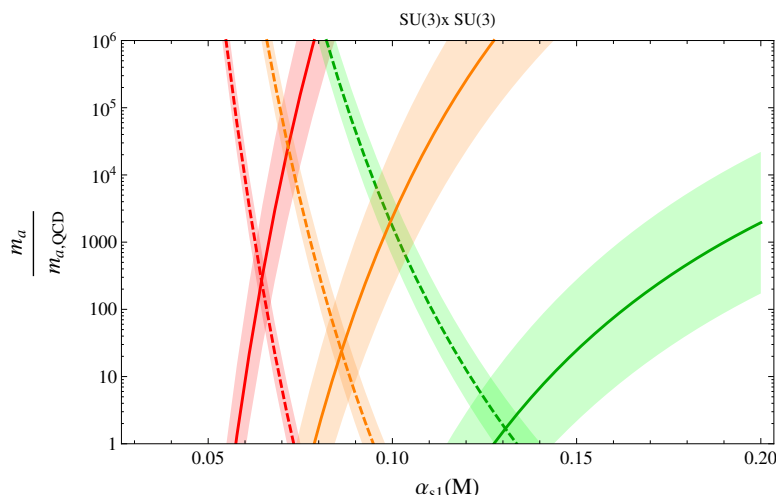


Figure 2. The scale of the UV contribution to the axion mass as a function of α_{s_1} , compared to the mass $m_{a,QCD}$ of a standard QCD axion given the same value of f_a . The solid lines give m_{a_1} and the dashed lines give m_{a_2} , with α_{s_2} determined from α_{s_1} by the matching condition equation (2.6). The results are shown for a range of values of strong coupling scales, from left to right $M = 10^{14} \text{ GeV}, 10^{11} \text{ GeV}, 10^8 \text{ GeV}$ (red, orange, green). The parameter space of interest occurs when both $m_{a_1}/m_{a,QCD} \gg 1$ and $m_{a_2}/m_{a,QCD} \gg 1$ simultaneously, so that neither state has properties similar to the standard QCD axion.

in size to the IR potential, with $\Lambda_1 \sim \Lambda_2 \sim \Lambda_0$. For $M \sim 10^{14} \text{ GeV}$, effects as large as $\Lambda_1 \sim \Lambda_2 \sim 30\Lambda_0$ can be realized. This realizes a model with two axions, each ~ 1000 times heavier than a standard QCD axion with the same decay constant f , with a potential aligned to dynamically set $\bar{\theta} = 0$. Note that although the non-perturbative effects have a large effect on the axion masses in this regime, the individual gauge factors are still reasonably weakly coupled at the scale M , with α_{s_1} far away from the chiral-symmetry breaking phase which would be expected to occur at $\alpha_{s_1} \gtrsim 0.7 - 1$ [52, 53].

So far we have considered the case $f_a \gtrsim M$ and assumed that all states in the spontaneous PQ-breaking sector are decoupled. If some of these states are lighter than M , the instanton effects can be suppressed. In a KSVZ axion model [54, 55] with N_f flavors of vector-like quarks of mass $M_\psi \lesssim M$, there will be a suppression in the instanton calculation by a factor of $\sim (M_\psi/\Lambda)^{N_f}$. We will take the benchmark $M_\psi = f$ and $N_f = 1$ for the phenomenological studies in section 4, giving

$$m_{a_i}^2 \approx \begin{cases} \frac{\Lambda_i^4}{f_{a_i}^2} & f_{a_i} > M \\ \frac{\Lambda_i^3}{f_{a_i}} & f_{a_i} < M \end{cases} . \tag{2.16}$$

Other benchmarks like a KSVZ axion with $N_f > 1$ or a DFSZ axion [56, 57] would typically have more severe suppressions for the regime $f_{a_i} < M$.

2.2 $SU(3)^N$ products

Although each gauge factor in the $SU(3) \times SU(3)$ model is more strongly coupled than the SM $SU(3)_{QCD}$, they remain relatively weakly coupled, and the additional short distance

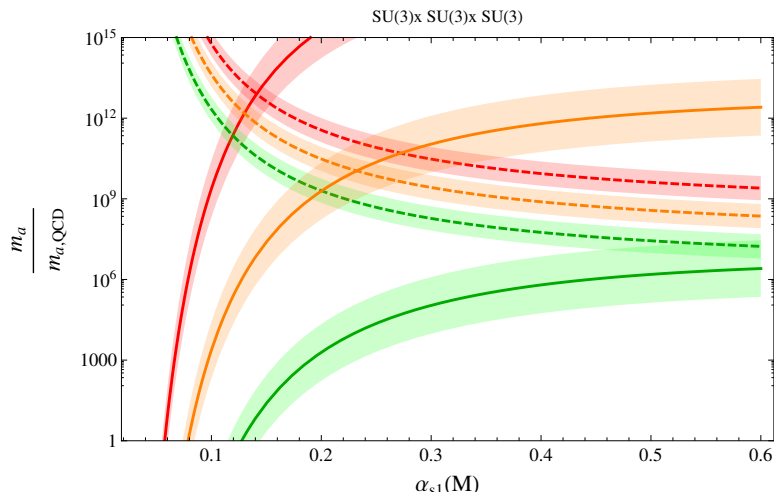


Figure 3. The scale of the UV contribution to the axion mass as a function of α_{s_1} , compared to the mass $m_{a,QCD}$ of a standard QCD axion given the same value of f_a . The solid lines give m_{a_1} and the dashed lines give $m_{a_{2,3}}$, with $\alpha_{s_{2,3}}$ determined from α_{s_1} by the matching condition equation (2.6) taking $\alpha_{s_2} = \alpha_{s_3}$. The results are shown for a range of values of strong coupling scales, from top to bottom $M = 10^{14}$ GeV, 10^{11} GeV, 10^8 GeV (red, orange, green). The parameter space of interest occurs when both $m_{a_1}/m_{a,QCD} \gg 1$ and $m_{a_{2,3}}/m_{a,QCD} \gg 1$ simultaneously, so that neither state has properties similar to the standard QCD axion.

non-perturbative effects still suffer a significant exponential suppression. Extending the gauge group to $SU(3)^N$, with the $SU(3)_{QCD}$ emerging from the diagonal subgroup allows the gauge coupling in each individual factor to be substantially increased,

$$\frac{1}{\alpha_s}(\mu) = \sum_{i=1}^N \frac{1}{\alpha_{s_i}(\mu)}, \quad \mu = M. \tag{2.17}$$

The gauge group can be Higgsed to the diagonal subgroup at a scale M by including multiple Higgs link fields $\Sigma_{12}, \dots, \Sigma_{N-1,N}$. An axion in each sector a_1, \dots, a_N can remove each $\theta_1, \dots, \theta_N$. As for the $SU(3) \times SU(3)$ case, we take all of the SM quarks charged under $SU(3)_1$, and assume no additional colored fermions are present in the other $SU(3)$ factors. Even for $N = 3$, the scale of the axion potential can now be dramatically increased compared to the standard QCD axion. Figure 3 shows the $N = 3$ case taking $\alpha_{s_2} = \alpha_{s_3}$. For example for $M = 10^{14}$ GeV, a model can be realized with 3 axions all with masses $\sim 10^{12}$ times larger than a standard QCD axion with the same decay constant f , again with a potential aligned to dynamically set $\bar{\theta} = 0$.

It is interesting to ask how large the axion mass can be made if an even larger number of product factors are introduced. For $N \sim 10$, the couplings of the individual $SU(3)$ factors can be made sufficiently strong to generate $\mathcal{O}(1)$ non-perturbative effects, giving $N - 1$ axions with masses $m_{a_i} \sim M^2/f_i$, while the mass of the axion in the $SU(3)_1$ sector will still be suppressed by the chiral suppression factor, giving $m_{a_1} \sim 10^{-12} M^2/f_1$.

3 UV sensitivity

There are several interesting questions about the sensitivity of this multi-axion solution to UV physics above the scale M . First, as is familiar from the standard QCD axion picture, Planck-suppressed operators may spoil some of the PQ symmetries [49–51], naively leading to a misalignment of the axion potential by

$$\Delta\bar{\theta} \sim \frac{f_a^{(d-2)}}{M_{\text{pl}}^{(d-4)} m_a^2}, \tag{3.1}$$

with the dimension $d \geq 5$ of the operator depending on the details of the axion model. In our case, this problem is significantly alleviated compared to the standard QCD axion because smaller values of f_a and larger values of m_a can be realized. Figure 4 shows that large parts of parameter space remain open even assuming the most dangerous $d = 5$ operators are present.

In our model there are also PQ-preserving higher dimensional operators that can be dangerous. Focusing for simplicity on the $SU(3) \times SU(3)$ case, at $D = 6$ the operator

$$\frac{\Sigma_{12} \tilde{G}_2 \Sigma_{12}^\dagger G_1}{\Lambda_{\text{UV}}^2} \tag{3.2}$$

can introduce a shift $\Delta\bar{\theta} \sim M^2/\Lambda_{\text{UV}}^2$ that is not canceled at the axion minima. Requiring $\bar{\theta} \lesssim 10^{-10}$ then requires a hierarchy $M \lesssim (10^{14} \text{ GeV}) \left(\frac{\Lambda_{\text{UV}}}{M_{\text{pl}}}\right)^2$.

Another possibility is that non-perturbative effects may not decouple sufficiently rapidly to protect the theory from CP violation at scales far above M . For example, if there are new unsuppressed sources of CP violation in the theory at the scale Λ_{UV} , then non-perturbative effects at this scale can generate additional misaligned contributions to the axion potential. Because each gauge factor is asymptotically free, the non-perturbative effects decouple at high energies as α_{s_i} run to weak coupling. The $SU(3)_1$ factor contains the SM quarks, and therefore runs to weak coupling the most slowly. From equations (2.14), (2.11), the scale of the potential generated for a_1 at Λ_{UV} is suppressed compared to the potential generated at M by a factor

$$\left(\frac{M}{\Lambda_{\text{UV}}}\right)^{b_1-4} = \left(\frac{M}{\Lambda_{\text{UV}}}\right)^{5/2} \tag{3.3}$$

Assuming an $\mathcal{O}(1)$ misalignment of the potential generated at Λ_{UV} , the limit $\bar{\theta} \lesssim 10^{-10}$ requires a hierarchy $M \lesssim (10^{15} \text{ GeV}) \left(\frac{\Lambda_{\text{UV}}}{M_{\text{pl}}}\right)^{5/2}$.

Therefore without additional assumptions about the nature of CP violation at the scale Λ_{UV} , there must be a substantial hierarchy in scales $M \ll \Lambda_{\text{UV}}$ to protect against higher dimensional operators and sufficiently suppress non-perturbative effects. In the simple model we have presented, M is set by an elementary scalar field Σ_{12} breaking the gauge group, and it seems that maintaining this hierarchy reintroduces as severe of a tuning as the tuning in $\bar{\theta}$ we have set out to address. Two well-known solutions exist — this hierarchy can be protected by supersymmetry (SUSY) or by a technicolor-like mechanism.

In a SUSY model, the scale M can be protected if $m_{\text{soft}} \lesssim M$. When $m_{\text{soft}} \ll M$, the instanton-generated potential will be suppressed by a factor of at least $(m_{\text{soft}}/M)^5$ due to insertions of gaugino masses and the PQ-breaking $bH_u H_d$ soft terms [36], so it is desirable to stay in the regime $m_{\text{soft}} \sim M$. Since $m_{\text{soft}} \sim M \gg 100$ GeV, it is not possible to protect the weak scale with SUSY in this model, but at least the hierarchy $M \ll \Lambda_{\text{UV}}$ can be maintained. However, the soft SUSY breaking introduces new CP phases into the theory, and the alignment of the axion potential which gives $\bar{\theta} = 0$ is no longer guaranteed. As is familiar from low-scale SUSY, we must assume that the soft terms are communicated in a way that does not introduce new CP violating phases (for a review, see ref. [58]), since these would lead radiatively to misalignment of the short-distance axion potential generated near M (the standard QCD axion mechanism is similarly sensitive to the CP violation in the soft sector when $m_{\text{soft}} \lesssim 100\text{TeV}$ [59]). Ref. [11] points out a similar sensitivity to SUSY-breaking CP violation in Nelson-Barr models.

In a technicolor-like solution, the Σ can be made a composite of elementary fermions under a new strong group, with the scale M generated dynamically. This resembles asymptotically free completions of the dimensional deconstruction framework [60]. However, because new fermions charged under $SU(3)_1$ and $SU(3)_2$ will be introduced, we must take care that their additional chiral symmetries are broken in a way that does not suppress the non-perturbative axion potential or re-introduce new CP phases. An example of such a model is given in appendix A.

4 Phenomenology

Laboratory experiments, cosmology, astrophysics, beam dump, and collider experiments can all be sensitive probes of new light pseudoscalars, also known as axion-like particles. Their reach can be parameterized in the space of masses m_a and the dimension-5 couplings of the new states, which scale as $\frac{1}{f_a}$. Although our models contain multiple axion-like states, the direct couplings in-between these states are phenomenologically unimportant and we can treat the limits on each axion-like state independently.

Since we have not specified the UV details of the spontaneous PQ-breaking, the heavy axions in our models may realize dimension-5 couplings to the electroweak gauge bosons $aW\tilde{W}, aB\tilde{B}$, and light fermions $\partial^\mu a(f^\dagger \gamma_5 \gamma_\mu f)$ in addition to the gluon coupling $a\tilde{G}G$. When $m_a < m_{\eta'} \sim \text{GeV}$, these additional couplings to the mass eigenstates are inevitable at low energies since the axions mix with the η', η , and π_0 mesons [29], as in the standard QCD axion model [20, 21, 61]. When the axion is light enough that hadronic decay channels are closed, $m_a \lesssim 3m_\pi$, the phenomenology is primarily determined by the low-energy axion-photon coupling, and this ALP case has been well studied.

For $m_a \gtrsim 1\text{GeV}$, hadronic decay channels open and typically dominate, and studies focused on an axion decaying dominantly to electroweak gauge bosons no longer apply. Limits depending on these couplings are even further weakened when UV couplings to the electroweak gauge bosons are absent, since the mixing-induced couplings become rapidly suppressed as QCD runs to weak coupling.

To study the limits on our models, we take as a benchmark an effective low-energy photon coupling of

$$\mathcal{L}_{a\gamma\gamma} = \frac{e^2}{32\pi^2} \frac{a_i}{f_i} F\tilde{F} \tag{4.1}$$

throughout the parameter space (for $m_a \gg m_{\eta'}$, the photon coupling will be entirely due to the UV physics, while for $m_a \lesssim m_{\eta'}$ it is a result of both the UV coupling and the mixing with the neutral mesons). We use this simplified approximation as a rough guide to the phenomenology and a more realistic treatment would generically include couplings to the Z as well as to other SM particles.

We show existing limits and future projections for axion experiments as a function of m_a, f_a in figure 4. We also include two benchmark models in which the axions are lifted to masses much larger than the standard QCD axion.

The first benchmark model is the $SU(3) \times SU(3)$ model with $\alpha_{s_1}(M) = 0.063$ and $M = 10^{14}$ GeV. This model contains two axions, each corresponding to a separate curve in the (m_a, f_a) plane. In principle, f_1 and f_2 can occur at very disparate scales, so each axion could appear at any point on its curve, although it might be appealing to connect the scales f_1 and f_2 in a detailed model of the spontaneous PQ-breaking sector. At small values of f_a , both Λ_1 and Λ_2 become less than Λ_0 . In this region one linear combination behaves as the usual QCD axion, and the orthogonal combination is light and has suppressed couplings to QCD.

The second benchmark model is the $SU(3) \times SU(3) \times SU(3)$ model with $\alpha_{s_1}(M) = 0.1$, $\alpha_{s_2} = \alpha_{s_3}$, and $M = 10^{14}$ GeV. Of particular interest is a new region in parameter space for a $f_a \sim \text{TeV}$ weak-scale axion that appears to be open in this model when $m_a \gtrsim \text{GeV}$ — it would be interesting to study in further detail the limits on such a hadronically decaying state.

We have chosen these two specific benchmarks to illustrate the parameter space and the correlations between the scales of the multiple axions present in each model. We note however, that the parameter space of these models motivates axions spanning the entire (m_a, f_a) plane above the standard QCD axion line.

We briefly describe the constraints and projections shown in figure 4. Current constraints are shown as shaded regions. The light blue shaded region are independent of a dark matter interpretation of the axion or its cosmology. The yellow shaded regions assume that the reheating temperature was large enough to have thermally produced axions, and that there was no large entropy dump to the SM after axion decoupling. The red shaded regions assume that the axion makes up all of the dark matter.

Misalignment and Ω_{DM} . In parts of our parameter space, axions can make up all of the dark matter. In our model the mass of the axion goes to its zero temperature value much earlier than the time it starts oscillating, making the computation of relic abundance much simpler than the standard QCD axion case. In our case the relic abundance for a

single axion with mass m_a and decay constant f_a is given as,

$$\Omega_a h^2 = \frac{m_a^2 f_a^2 \theta_0^2}{\rho_{\text{crit}}/h^2} \left(\frac{a(T_f)}{a_0} \right)^3 \simeq 0.12 \left(\frac{f_a}{10^{12} \text{ GeV}} \right)^2 \left(\frac{m_a}{0.01 \text{ eV}} \right)^{1/2} \theta_0^2 \quad (4.2)$$

where $a(T_f)$ is the scale factor at the temperature T_f where the axion starts oscillating, $H(T_f) \simeq m_a/3$, θ_0 is the initial misalignment angle, and $\rho_{\text{crit}} = 3.96 \times 10^{-47} \text{ GeV}^4$ is the critical density of the Universe. This estimate ignores the thermal production of axions, which can become important for higher masses as well. We show the part of the parameter space in our benchmarks where heavy axions could make up the dark matter by solid lines in figure 4. It is interesting to note that in our models there is a new region of the m_a - f_a parameter space where axions solve the strong CP problem and constitute all of the dark matter for generic initial conditions ($\theta_0 \sim 1$).

Cosmology constraints. The yellow region is ruled out by a combination of cosmological constraints, assuming that the reheat temperature was sufficiently high to produce an equilibrium thermal population of axions [62] which dominates over misalignment production in this region [63]. Future CMB observations can also put significant constraints on thermally produced axions from measurements of ΔN_{eff} that constrains extra relativistic degrees of freedom present during recombination. We show the sensitivity of the CMB-S4 assuming a reheat temperature of $T_R = 10^{10} \text{ GeV}$ [64] with a horizontal gray line.

If the axion makes up dark matter, then its early universe values can be large enough to affect BBN [65]. This bound is superseded by the solar bound mentioned below.

Astrophysical bounds. Some of the strongest constraints on the axion parameter space arise from stellar physics, where axion coupling to photons is constrained by evolution of horizontal branch stars. For lower masses of axions supernova constraints and constraints from conversion of X-ray photons to axions in cluster magnetic fields [66–68] impose a stronger bound. Axion helioscopes such as CAST [69] also put constraints on the couplings of axions with photons, with the future IAXO helioscope [70] improving the reach. The light blue region includes these astrophysical constraints (adapted from [71]). We note that these astrophysical bounds off the QCD line are usually computed only for axion-like particles, i.e. with no nucleon coupling included. It will be interesting to recast these limits to the case where the axion does have a coupling to nucleons.

Axions which lie above the QCD line in figure 4 need a tuned negative contribution to their mass to partially cancel the mass from QCD. Their potential can then flip its sign due to finite density effects in astrophysical objects, which then source the axion field. This observation was used by ref. [72] to put bounds on this parameter space by dense astrophysical objects sourcing axion fields. In figure 4 we show the current bounds arising from the Sun and projections from neutron star observations (purple), which may be able to probe most of this tuned region of parameter space. We use $m_a = 0.1 m_{a,QCD}$ as a conservative projection from LIGO. Black hole superradiance [73] puts constraints on very light axions.

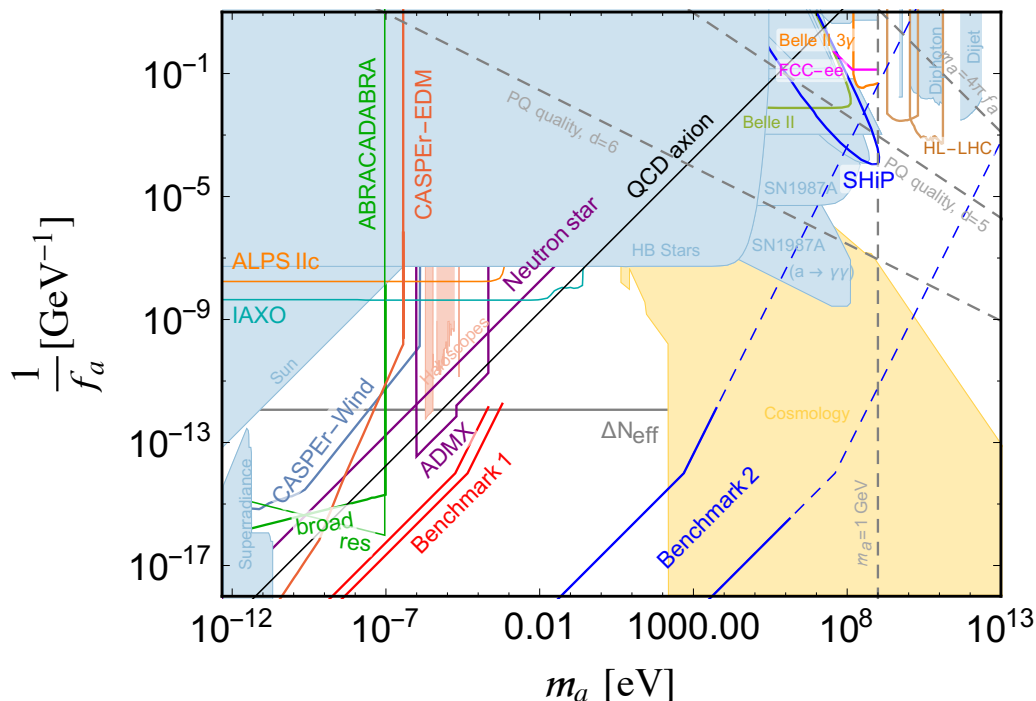


Figure 4. Limits on axions in the (m_a, f_a) plane for an axion with couplings to gluons and photons given by equations (2.7) and (4.1). The black solid line is the standard QCD axion, with mass given by equation (1.3). Parallel to this line from left to right are the lines for a_1 and a_2 (red) in the $SU(3) \times SU(3)$ benchmark parameters described in the text, and $a_{2,3}$ and a_1 (blue) for the $SU(3) \times SU(3) \times SU(3)$ benchmark parameters. The solid segments on these lines indicate where they could be realized as dark matter from the misalignment mechanism. The shaded regions show current constraints, and the regions bounded by solid lines are sensitivities of future experiments (see text for details).

Collider constraints. The possibility of heavier QCD axions also opens up weak-scale decay constants, making these axions visible at colliders. We show constraints from monojet searches at the LHC [74] and from LHC dijet [75–77] and diphoton searches [78–82] for pseudoscalar states produced in gluon fusion [83]. Also shown are the constraints from by monophoton and beam dump experiments (adapted from [84]). The Belle II experiment [85] will be sensitive to heavier axion-like particles at the weak scale [84]. The SHiP beam dump experiment [86] can cover intermediate mass scale axions which are not yet excluded by current experiments. Future e^+e^- colliders on the Z mass peak can also improve on axion couplings. We show projections [71] for the FCC-ee collider [87].

Some of the analyses above were carried out for axion-like particles which only have a coupling to photons, and hence do not directly apply to our case when decays to hadrons become important. As a crude fix we have cut off all such limits at $m_a = 1$ GeV. We have also included constraints from ref. [88] which do take a gluon as well as a photon coupling into account.

Laboratory experiments. Laboratory experiments such as the haloscope ADMX constrains the QCD axion in the region where it can be the dark matter. We show regions of

parameter space that can be covered by future experiments. The upgraded cavity experiment ADMX2 [89] will cover a larger range of QCD axion masses with higher sensitivity. The CASPER experiments [90–92] propose using NMR techniques to measure time-varying EDMs induced by the axion, which will be sensitive for low-mass axions. The ABRA-CADABRA [93] experiment can also cover the low mass axion parameter space for axion-photon couplings. These experiments all rely on the axion being all of dark matter. The upgraded light-shining-through-wall experiment ALPS II [94] will be sensitive to larger couplings of axion-like particles with the photon.

We have included in figure 4 some dashed gray lines motivated by theoretical considerations. The line in the upper right corner signals the invalidity of the effective axion theory when $m_a \gtrsim 4\pi f_a$. The reduced sensitivity to quality of the PQ symmetry through Planck suppressed higher dimensional operators as described by equation (3.1) are also shown. In regions above these lines, generic dimension-5 and -6 operators do not spoil the solution to the strong CP problem. The vertical dashed gray line at $m_a = 1$ GeV shows the scale where hadronic decays begin to dominate and axion mixings with the neutral mesons begin to be suppressed.

A few general comments which follow from our survey of the phenomenology:

- In our plot, most of the current and future observations only probe the region above the QCD line. As noted above, this region is tuned such that there is a negative mass contribution to the axion, with a minimum which is highly aligned with the QCD minimum, and a size comparable to the QCD contribution. However, experiments which probe the axion coupling to photons can extend below the QCD line in our plot if the coupling of the axion to photons is much larger than the coupling assumed here (as in [95–97]). With such an enhanced couplings these experiments will also be sensitive to heavier QCD axions as considered in this paper.
- When hadronic channels are open, the branching ratio to photons is tiny. The collider signals of a heavy QCD axion therefore differ from the more-often studied case of ALPs which only have a photon coupling. Our work shows that the combination of gluon and photon couplings is very well motivated even for heavy collider-observable axions and deserve further study.
- We also note that the phenomenology of this realization of a heavy axion is substantially different from the phenomenology in a Z_2 heavy axion model, where the axion may decay to light mirror-sector particles [29], and is generically not expected to appear with multiple copies.
- A combination of heavy axions can make up the dark matter density through the misalignment mechanism in a new part of m_a - f_a plane, motivating searches in this region. A resonance search strategy for laboratory experiments might be less optimal if the dark matter density is not dominantly stored in axions at one mass.

5 Conclusions

We have described a novel mechanism for solving the strong CP problem. In the standard QCD axion mechanism, a spontaneously broken anomalous $U(1)_{PQ}$ symmetry results in an axion with a potential generated by non-perturbative effects near the scale Λ_{QCD} . In the absence of any UV sources of perturbative PQ violation, the strong CP problem is solved dynamically when the axion relaxes to its CP preserving minimum. In our mechanism, we embed QCD in a $SU(3)^N$ product group at a scale $M \gg \Lambda_{\text{QCD}}$, and use a separate spontaneously broken PQ symmetry in each individual $SU(3)$ factor to dynamically relax each individual $\bar{\theta}$ angle. Because each individual $SU(3)$ factor is more strongly coupled than the SM QCD at the scale M , the non-perturbative contributions to the axion potentials at the scale M can be much larger than those generated near Λ_{QCD} for the standard QCD axion. After integrating out physics above the scale M , the theory is just the standard model with N axions, each with a PQ-violating potential that arose from non-perturbative effects near the scale M . Although the PQ symmetries are explicitly violated in this low-energy effective theory, the non-perturbative origin of the axion potentials in the full theory guarantees that the minimum relaxes the low-energy $\bar{\theta}$ angle while generating a mass for each axion that is much larger than the standard QCD axion mass relationship.

Our assumptions about the existence of spontaneously broken anomalous PQ symmetries are on the same footing as the standard QCD axion, and it is encouraging that in general a large number of axions are expected in string theory models [98]. In fact, in our model, the possibility of solving the strong CP problem with a small value of f_a and large value of m_a can resolve the problem of maintaining a sufficient PQ quality in the presence of quantum-gravity corrections that plague the standard axion solution [49–51].

The main obstacle to the model we have proposed is the conflict with solutions to the electroweak hierarchy problem. Solving the electroweak hierarchy problem in this class of model suggests $M \sim \text{TeV}$, while generating large axion masses requires $M \gg \text{TeV}$. While we have shown that it is possible to stabilize at least the hierarchy $M \ll M_{\text{pl}}$, it requires further model building to ensure that additional sources of CP violation do not spoil the mechanism.

Phenomenologically, the model motivates exploring the whole (m_a, f_a) range of axion-like particles, including the range $m_a \gtrsim 3m_\pi$ where hadronic decays will dominate. While the model requires the existence of multiple heavy axion states, it would be difficult to directly verify their connection to the strong CP problem, as the precision mass-coupling relationships predicted for the standard QCD-axion are no longer realized. Because there is the possibility of decoupling the heavy axions to scales $m_a \gg M_W$, this work can also be viewed as an interesting new theoretical example of a completely decoupling solution to the strong CP problem.

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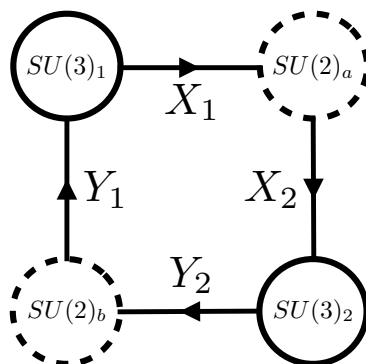


Figure 5. A moose model giving composite link fields to stabilize the hierarchy $M \ll \Lambda_{UV}$.

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A Composite link fields

A model with composite link fields can stabilize the hierarchy $M \ll \Lambda_{UV}$. For example, the $SU(3)_1 \times SU(3)_2$ model can be extended to a $SU(3)_1 \times SU(2)_a \times SU(3)_2 \times SU(2)_b$ moose theory as shown in figure 5, with condensation of elementary fermion bilinears $\langle X_1 X_2 \rangle$ and $\langle Y_1 Y_2 \rangle$ in the $SU(2)$ factors breaking $SU(3)_1 \times SU(3)_2 \rightarrow SU(3)_c$, making the scale M dynamical.

In addition to the new gauge interactions, gauge invariant 4-fermion interactions of the form

$$\mathcal{L}_\chi = \frac{\lambda_1}{M} (X_1 X_1) (Y_1 Y_1) + \frac{\lambda_2}{M} (X_2 X_2) (Y_2 Y_2) \quad (\text{A.1})$$

are necessary to break the additional chiral symmetries of the model. The instantons in $SU(3)_1$ and $SU(3)_2$ sectors will be suppressed by the coefficients λ_1 and λ_2 , so this operator must either be generated at a nearby scale (without generating additional dangerous CP violating operators or a new hierarchy problem), or the theory must enter a conformal regime in the UV where these 4-fermion operators have large negative anomalous dimension.

Field redefinitions can be used to choose $\lambda_{1,2}$ real, and one remaining anomalous $U(1)$ can be used to remove one combination of the θ -angles in the $SU(2)_a$ and $SU(2)_b$ sectors. The remaining new θ angle may be removed by coupling an additional axion degree of freedom to the $SU(2)_a$ or $SU(2)_b$ sector, which will obtain a large mass $\sim \lambda_1 \lambda_2 M^2 / f$.

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