

Factorizations of Algebraic Integers, Block Monoids and Additive Number Theory

Scott Chapman

*Sam Houston State University
Department of Mathematics and Statistics
Huntsville, TX 77340*

Abstract

Let \mathcal{O}_K be the ring of integers in a finite extension of the rationals. Fundamental problems involving the factorizations of elements in \mathcal{O}_K into irreducibles appear early in the Abstract Algebra curriculum. At a basic level, the usual technique for attacking such problems involves using the norm function. There is a much deeper connection between factorizations of elements in \mathcal{O}_K and the class group of \mathcal{O}_K . We will explore this connection and show that it easily generalizes to Dedekind and Krull domains. Implicit in this discussion is the introduction of a structure known as a Block Monoid. A well-known theorem of Geroldinger constructs a monoid homomorphism from \mathcal{O}_K to an appropriately chosen Block Monoid \mathcal{B} which preserves lengths of factorizations of elements into products of irreducibles. The analysis of the factorization properties of Block Monoids leads to the study of two well-known arithmetic constants from Additive Number Theory, the Davenport Constant and the Cross Number.

Email address: `scott.chapman@shsu.edu` (Scott Chapman).