Failure mode and effects analysis by data envelopment analysis

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ABSTRACT

Failure mode and effects analysis (FMEA) is a method that examines potential failures in products or processes and has been used in many quality management systems. One important issue of FMEA is the determination of the risk priorities of failure modes. In this paper we propose an FMEA which uses data envelopment analysis (DEA), a well-known performance measurement tool, to determine the risk priorities of failure modes. The proposed FMEA measures the maximum and minimum risks of each failure mode. The two risks are then geometrically averaged to measure the overall risks of failure modes. The risk priorities are determined in terms of overall risks rather than maximum or minimum risks only. Two numerical examples are provided and examined using the proposed FMEA to show its potential applications and benefits.

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1. Introduction

Failure mode and effects analysis (FMEA) is an engineering technique used to define, identify and eliminate known and/or potential failures, problems, and errors from the system, design, process, and/or service before they reach the customer [29]. It is also referred to as failure mode, effects and criticality analysis (FMECA) when used for a criticality analysis. When applying FMEA, a cross-functional and multidisciplinary team identifies failure modes, evaluates their risks and prioritizes them so that appropriate corrective actions can be taken. A failure mode is defined as the manner in which a component, subsystem, system, process, etc. could potentially fail to meet the design intent. A failure mode in one component can serve as the cause of a failure mode in another component. A failure cause is defined as a design weakness that may result in a failure. For each failure mode identified, the FMEA team should determine what the ultimate effect of failure will be. A failure effect is defined as the result of a failure mode on the function of the product or process as perceived by the customer. The traditional FMEA determines the risk priorities of failure modes through the risk priority number (RPN), which is determined by

$$RPN = O \times S \times D, \qquad (1)$$

where the risk factors O and S are occurrence and severity of a failure, and D is the ability to detect the failure before it reaches the customer. The three risk factors are evaluated using the ratings (also called ranks or scores) from 1 to 10, as described in Tables 1–3. Failures with

higher RPNs are viewed to be more important and should be given greater considerations.

FMEA proves to be one of the most important early preventative actions in system, design, process, or service which will prevent failures and errors from occurring and reaching the customer [29]. However, the RPN has been criticized for a variety of reasons [1,3,6,9,17,25,27], some of which are listed as follows:

- Different combinations of O, S and D may produce exactly the same value of RPN, but their hidden risk implications may be totally different. For example, two different events with the values of 2, 3, 2 and 4, 1, 3 for O, S and D, respectively, have the same RPN value of 12. However, the hidden risk implications of the two events may not necessarily be the same. This may cause a waste of resources and time, and in some cases a high risk event may go noticed.
- The relative importance among O, S and D is not taken into consideration. The three risk factors are assumed to be equally important. This may not be the case when considering a practical application of FMEA.
- The mathematical formula for calculating RPN is questionable and debatable. There is no rationale as to why O, S and D should be multiplied to produce the RPN.
- The three risk factors are difficult to be precisely evaluated.
- RPNs are not continuous with many holes and heavily distributed at the bottom of the scale from 1 to 1000. This causes problems in interpreting the meaning of the differences between different RPNs. For example, is the difference between 1 and 2 the same as or less than the difference between 900 and 1000?

To overcome the drawbacks listed above, a number of approaches have been suggested in the literature. For example, Bevilacqua et al.

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Table 1
Traditional ratings for occurrence of a failure [3,27,35].

Rating	Probability of occurrence	Failure probability
10	Very high: failure is almost inevitable	>1 in 2
9		1 in 3
8	High: repeated failures	1 in 8
7		1 in 20
6	Moderate: occasional failures	1 in 80
5		1 in 400
4		1 in 2000
3	Low: relatively few failures	1 in 15,000
2		1 in 150,000
1	Remote: failure is unlikely	<1 in 1,500,000

[2] defined RPN as the weighted sum of six parameters (safety, machine importance for the process, maintenance costs, failure frequency, downtime length, and operating conditions) multiplied by a seventh factor (machines access difficulty), where the relative importance of the six attributes was estimated using pairwise comparisons.

Sankar and Prabhu [27] presented a modified approach for prioritization of failures in a system FMEA, which used the ranks 1 through 1000, called risk priority ranks (RPRs), to represent the increasing risk of the 1000 possible severity–occurrence–detection combinations. These 1000 possible combinations were tabulated by an expert in the order of increasing risk and can be interpreted as 'if–then' rules. The failure having a higher rank was given a higher priority.

Braglia [5] proposed a multi-attribute failure mode analysis (MAFMA) based on the analytic hierarchy process (AHP) technique. The proposed MAFMA viewed the risk factors (O, S, D, and expected cost) as decision attributes, causes of failure as decision alternatives, and the selection of cause of failure as decision goal, which together with the attributes and alternatives formed a three-level hierarchical structure. Pairwise comparison matrices were used to estimate the weights of attributes and local priorities of the causes with respect to the expected cost attribute. The conventional ratings for O, S and D were normalized as the local priorities of the causes with respect to O, S, and D, respectively, and the weight aggregation technique in the AHP was used to synthesize the local priorities into global priorities, based on which the possible causes of failure were ranked.

Braglia et al. [7] also proposed an alternative multi-attribute decision-making approach, called fuzzy TOPSIS approach for FMECA, which is a fuzzy version of the technique for order preference by similarity to ideal solution (TOPSIS). The TOPSIS method is a well-known multi-attribute decision-making methodology based on the assumption that the best decision alternative should be as close as possible to the ideal solution and the farthest from the negative-ideal

Table 2
Traditional ratings for severity of a failure [3,27,35].

Rating	Effect	Severity of effect
10	Hazardous without warning	Very high severity ranking when a potential failure mode effects safe system operation without warning
9	Hazardous with warning	Very high severity ranking when a potential failure mode affects safe system operation with warning
8	Very high	System inoperable with destructive failure without compromising safety
7	High	System inoperable with equipment damage
6	Moderate	System inoperable with minor damage
5	Low	System inoperable without damage
4	Very low	System operable with significant degradation of performance
3	Minor	System operable with some degradation of performance
2	Very minor	System operable with minimal interference
1	None	No effect

Table 3
Traditional ratings for detection [3,27,35].

Rating	Detection	Likelihood of detection by design control
10	Absolute uncertainty	Design control cannot detect potential cause/mechanism and subsequent failure mode
9	Very remote	Very remote chance the design control will detect potential cause/mechanism and subsequent failure mode
8	Remote	Remote chance the design control will detect potential cause/mechanism and subsequent failure mode
7	Very low	Very low chance the design control will detect potential cause/mechanism and subsequent failure mode
6	Low	Low chance the design control will detect potential cause/mechanism and subsequent failure mode
5	Moderate	Moderate chance the design control will detect potential cause/mechanism and subsequent failure mode
4	Moderately high	Moderately high chance the design control will detect potential cause/mechanism and subsequent failure mode
3	High	High chance the design control will detect potential cause/mechanism and subsequent failure mode
2	Very high	Very high chance the design control will detect potential cause/mechanism and subsequent failure mode
1	Almost certain	Design control will detect potential cause/mechanism and subsequent failure mode

solution. The proposed fuzzy TOPSIS approach allows the risk factors O, S, and D and their relative importance to be assessed using triangular fuzzy numbers.

Chang et al. [8] utilized grey theory for FMEA. They used fuzzy linguistic terms such as *Very Low*, *Low*, *Moderate*, *High* and *Very High* to evaluate the degrees of O, S and D, and grey relational analysis to determine the risk priorities of potential causes. To carry out the grey relational analysis, fuzzy linguistic terms were defuzzified as crisp values, the lowest levels of the three factors O, S and D were defined as a standard series, and the assessment information of the three factors for each potential cause was viewed as a comparative series, whose grey relational coefficients and grey relational degree with the standard series were computed in terms of the grey theory. Big grey relational degree means small effect of potential cause. In [9], they also utilized the grey theory for FMEA, but the grey relational degrees were computed using the traditional scores 1–10 for the three risk factors rather than fuzzy linguistic terms.

Bowles and Peláez [4] described a fuzzy logic based approach for prioritizing failures in a system FMEA, which uses linguistic terms to describe O, S, D, and the risks of failures. The relationships between the risks and O, S, D were characterized by fuzzy if–then rules extracted from expert knowledge and expertise. Crisp rankings for O, S, D were fuzzified to match the premise of each possible if–then rule. All the rules that have any truth in their premises were fired to contribute to a fuzzy conclusion. The fuzzy conclusion was then defuzzified by the weighted mean of maximum method (WMoM) as the ranking value of the risk priority. Similar fuzzy inference methods also appeared in [6,12,18,19,24,25,30].

Yang et al. [36] presented a novel, efficient fuzzy rule-based Bayesian reasoning (FuRBaR) approach for prioritizing failures in FMEA. The technique was specifically developed to deal with some of the drawbacks concerning the use of conventional fuzzy logic (i.e. rule-based) methods in FMEA. In their approach, subjective belief degrees were assigned to the consequent part of the rules to model the incompleteness encountered in establishing the knowledge base. A Bayesian reasoning mechanism was then used to aggregate all relevant rules for assessing and prioritizing potential failure modes. The applicability of the proposed approach was demonstrated by studying a maritime collision risk due to technical failures.

Recently, Chin et al. [13] proposed an FMEA using the group-based evidential reasoning (ER) approach to capture FMEA team members' diversity opinions and prioritize failure modes under different types of uncertainties such as incomplete assessment, ignorance and intervals. The risk priority model was developed using the group-

based ER approach, which includes assessing risk factors using belief structures, synthesizing individual belief structures into group belief structures, aggregating the group belief structures into overall belief structures, converting the overall belief structures into expected risk scores, and ranking the expected risk scores using the minimax regret approach (MRA). Similar applications for risk analysis on dynamic alliance can also be found in [21].

In spite of the fact that much effort has been paid to the improvement of RPN, the improved methods either need to specify or determine the weights of risk factors in advance or take no account of them at all. It is argued that the specification or determination of risk factor weights is not easy because different decision makers (DMs) may have distinct judgments or preferences. For example, Pillay and Wang [25] gave more emphasis on the risk factor D followed by S and less weight to O; while Braglia et al. [7] observed that a failure cause with a very high severity value but a very remote occurrence probability might be less critical than a failure cause which occurs repeatedly and therefore considered the chance of failure more important than the other factors. Obviously, the above authors' opinions are in conflict with each other.

Another reason why risk factor weights are not easy to determine is that different failure modes have different consequences. The specification or determination of a fixed set of risk factor weights for all the failure modes might be inappropriate, particularly in the case with a large number of failure modes. In other words, it might be a better choice to use different sets of risk factor weights for different failure modes when there are a large number of failure modes to be prioritized. In this aspect, Garcia et al. [16] proposed a fuzzy data envelopment analysis (DEA) approach for FMEA, which does not require specifying or determining risk factor weights subjectively. Their approach, however, was computationally very complicated and also could not produce a full ranking for the failure modes to be prioritized.

Based on the above analyses, we propose in this paper a new FMEA, which utilizes DEA, a well-known performance measurement tool, to determine the risk priorities of failure modes. The proposed FMEA takes into account the relative importance weights of risk factors, but has no need to specify or determine them subjectively, which are determined by DEA models. The weights determined by DEA models differ from one failure mode to another. The new FMEA measures the maximum and minimum risks of failure modes, which are geometrically averaged to reflect the overall risks of the failure modes, based on which the failure modes can be prioritized. Incomplete and imprecise information on the evaluation of risk factors can also be considered in the FMEA.

The rest of the paper is organized as follows. In Section 2, we give a brief description of DEA and its main mathematical models for efficiency measurement. In Section 3, we develop DEA models for FMEA. In Section 4, we discuss uncertainties such as incomplete and imprecise information related to FMEA and develop interval DEA models for prioritization of failure modes. Numerical examples are provided in Section 5 to demonstrate the potential applications of the proposed FMEA and its advantages. Section 6 concludes the paper with a brief summary.

2. DEA and DEA models for efficiency measurement

DEA, i.e. data envelopment analysis, was originally developed by Charnes et al. [11] for measuring the relative efficiencies of a group of decision-making units (DMUs) that utilize multiple inputs to produce multiple outputs. Since its development in the late 1970s, DEA has found surprising applications [14,15,20,22,33]. The traditional DEA models measure only the optimistic efficiencies of DMUs. This usually leads to more than one DMU being evaluated as efficient. The efficient DMUs are difficult to be discriminated from each other. Recently, Wang et al. [32] proposed the pessimistic efficiency model that measures the pessimistic efficiencies of DMUs and suggested a geometric average efficiency, which is the integration of the optimistic and pessimistic efficiencies of DMUs, as the overall efficiency measure of DMUs. The geometric average efficiency considers not only the optimistic efficiencies of DMUs but also their pessimistic efficiencies and is therefore more comprehensive and more convincing than both of them. By the geometric average efficiency, all the DMUs can be fully ranked and discriminated. In what follows, we briefly review the two DEA models that measure the optimistic and pessimistic efficiencies of DMUs, and the geometric average efficiency.

2.1. Optimistic efficiency — the best relative efficiency

Assume that there are n DMUs to be evaluated in terms of m inputs and s outputs. Denote by x_{ij} $(i = 1,\dots, m)$ and y_{rj} $(r = 1,\dots, s)$ the input and output values of DMU$_j$ $(j = 1,\dots, n)$, which are known and nonnegative. The efficiency of DMU$_j$ is defined as

$$\theta_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}, \tag{2}$$

where u_r and v_i are the output and input weights assigned to the rth output and the ith input. To determine the efficiency of DMU$_j$ relative to the other DMUs, Charnes et al. [11] developed the following well-known CCR model, which was named by acronym and measures the best relative efficiencies of DMUs:

$$
\begin{aligned}
\text{Maximize} \quad & \theta_0 = \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}} \\
\text{Subject to} \quad & \theta_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \ j = 1,\dots,n, \\
& u_r, v_i \geq \varepsilon, \ r = 1,\dots,s; i = 1,\dots,m,
\end{aligned}
\tag{3}
$$

where the subscript zero represents the DMU under evaluation, u_r and v_i are decision variables and ε is a very small positive number called non-Archimedean infinitesimal in the DEA literature. Through Charnes and Cooper's transformation [10], the above fractional programming is transformed into the following equivalent linear programming (LP) model:

$$
\begin{aligned}
\text{Maximize} \quad & \theta_0 = \sum_{r=1}^{s} u_r y_{r0} \\
\text{Subject to} \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \ j = 1,\dots,n, \\
& \sum_{i=1}^{m} v_i x_{i0} = 1, \\
& u_r, v_i \geq \varepsilon, \ r = 1,\dots,s; i = 1,\dots,m.
\end{aligned}
\tag{4}
$$

If there exists a set of positive weights that makes $\theta_0^* = 1$, then DMU$_0$ is referred to as optimistic efficient; otherwise, it is referred to as optimistic inefficient. For n different DMUs, there are a total of n LP models to be solved. Accordingly, there are n sets of weights available, some of which may be different. All the optimistic efficient DMUs determine an efficiency frontier.

2.2. Pessimistic efficiency — the worst relative efficiency

Efficiency is a relative measure and can be measured within different ranges. The CCR model measures the optimistic efficiency of each DMU within the range of less than or equal to one. If the efficiency of a DMU is measured within the range of no less than one, then we have the so-called pessimistic efficiency, also referred to as the worst relative efficiency. The pessimistic efficiency of DMU_0 is measured by the following pessimistic DEA model [32]:

$$\text{Minimize} \quad \psi_0 = \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}}$$

$$\text{Subject to} \quad \psi_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \geq 1, \; j = 1, \ldots, n, \tag{5}$$

$$u_r, v_i \geq \varepsilon, \; r = 1, \ldots, s; i = 1, \ldots, m,$$

which can be further converted into the following equivalent LP model:

$$\text{Minimize} \quad \psi_0 = \sum_{r=1}^{s} u_r y_{r0}$$

$$\text{Subject to} \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \geq 0, \; j = 1, \ldots, n, \tag{6}$$

$$\sum_{i=1}^{m} v_i x_{i0} = 1,$$

$$u_r, v_i \geq \varepsilon, \; r = 1, \ldots, s; i = 1, \ldots, m.$$

When there exists a set of positive weights making $\psi_0^* = 1$, we refer to DMU_0 as pessimistic inefficient; otherwise, DMU_0 is referred to as pessimistic efficient. All the pessimistic inefficient DMUs determine an inefficiency frontier.

According to the above definitions, DMUs can be classified into three categories: optimistic efficient, pessimistic inefficient, and those that are neither optimistic efficient nor pessimistic inefficient. Obviously, optimistic inefficient units include pessimistic inefficient and part of pessimistic efficient DMUs. As such, pessimistic efficient units include optimistic efficient and part of optimistic inefficient DMUs.

2.3. Geometric average efficiency — the overall efficiency measurement

It is a common knowledge that optimistic efficiency and pessimistic efficiency should form an interval when measured under the same constraints such as $\alpha \leq \sum_{r=1}^{s} u_r y_{rj} / \sum_{i=1}^{m} v_i x_{ij} \leq 1$ with $0 < \alpha < \min_{j \in \{1,\ldots,n\}} \{\theta_j^* / \psi_j^*\}$ and $j = 1, \ldots, n$. The efficiency interval of DMU_j could accordingly be expressed as $[\alpha \psi_j^*, \theta_j^*]$ if α value is small enough. To avoid the difficulty in determining the value of α, Wang et al. [32] suggested a geometric average efficiency, determined by

$$\phi_j^* = \sqrt{\psi_j^* \theta_j^*}, \; j = 1, \ldots, n, \tag{7}$$

where θ_j^* and ψ_j^* are respectively the optimistic and pessimistic efficiencies of DMU_j ($j = 1, \ldots, n$). The geometric average efficiency considers not only the optimistic efficiency of a DMU, but also its pessimistic efficiency. It measures the overall efficiency of a DMU and considers both sides of a coin. The integration of two extreme efficiencies, optimistic and pessimistic, into a geometric average efficiency is undoubtedly more meaningful and more comprehensive than the use of either of the two efficiencies.

When efficiency intervals $[\alpha \psi_j^*, \theta_j^*]$ ($j = 1, \ldots, n$) are compared through their geometric midpoints $\sqrt{\alpha \psi_j^* \cdot \theta_j^*}$, the rankings among the n DMUs depend only upon their geometric average efficiencies $\phi_j^* = \sqrt{\psi_j^* \theta_j^*}$ ($j = 1, \ldots, n$) and have nothing to do with the value of α. This is the reason why geometric average rather than arithmetic average or another average is used for ranking. This good property enables the decision maker (DM) not to worry about how to determine the value of α. He/She can therefore leave it alone and compare directly the geometric average efficiencies of the n DMUs to determine their overall performances and rankings. Interested readers are referred to Wang et al. [32] for further theoretical justifications on geometric average efficiency.

3. DEA models for FMEA

Suppose there are n failure modes denoted by FM_i ($i = 1, \ldots, n$) to be prioritized, each being evaluated against m risk factors denoted by RF_j ($j = 1, \ldots, m$). Let r_{ij} ($i = 1, \ldots, n; j = 1, \ldots, m$) be the ratings of FM_i on RF_j and w_j be the weight of risk factor RF_j ($j = 1, \ldots, m$). Since the RPN defined as the product of three risk factors O, S and D has been largely criticized for its mathematical formula and equal treatment of the risk factors, we define in this paper the risks of failures with a different mathematical form, which can be either of the following:

$$R_i = \sum_{j=1}^{m} w_j r_{ij}, \; i = 1, \ldots, n, \tag{8}$$

$$R_i = \prod_{j=1}^{m} r_{ij}^{w_j}, \; i = 1, \ldots, n. \tag{9}$$

Eq. (8) defines the risk of each failure mode as the weighted sum of m risk factors, whereas Eq. (9) as the weighted product of m risk factors. For convenience to distinguish between the two risks, we refer to the risk determined by Eq. (8) as additive risk and the risk by Eq. (9) as multiplicative risk, respectively. It is worthwhile to point out that the definition for additive risks was first proposed by Braglia et al. [6], who defined the RPN as the weighted sum of O, S and D, whereas the definition for multiplicative risks was first proposed by Wang et al [34], who defined the RPN as the fuzzy weighted geometric mean of the three risk factors O, S, and D, which they referred to as fuzzy risk priority number (FRPN). Both of the papers require the weights of the risk factors to be specified or determined subjectively, but in this paper the risk factor weights will be determined by DEA models automatically.

The traditional DEA often assigns too many zeros to input and output weights, leading to optimistic efficiency being unreasonably high and pessimistic efficiency being extraordinarily low. To avoid this from happening in FMEA, we consider imposing a constraint on the ratio of maximum weight to minimum weight. According to Saaty's AHP [26], the maximum value, as a ratio of the comparative importance of a criterion over another, can assume to be 9. We therefore constrain the ratio of maximum weight to minimum weight within the range of one and nine. That is

$$1 \leq \frac{\max\{w_1, \ldots, w_m\}}{\min\{w_1, \ldots, w_m\}} \leq 9. \tag{10}$$

The DM can also set a specific but different upper bound for the ratio of maximum weight to minimum weight if necessary. The main reasons for us to set the maximum ratio as 9 are based on the following observations:

- The pairwise comparison matrices in the AHP are the most widely used approaches for estimating the relative importance weights of decision attributes or criteria, in which the maximum ratio scale between the importance of two attributes or criteria are usually not greater than 9.

- Risk factors O, S and D are all evaluated using the ratings between 1 and 10, where 1 represents no risk. Accordingly, their relative importance should also be evaluated using similar ratings. Due to the fact that "no importance" makes no sense, the ratings used for evaluating the relative importance of risk factors should therefore be defined as 1–9 rather than 1–10. As a result, the maximum ratio between the importance of two risk factors is less than or equal to 9.
- Five-point Likert scale is also widely used in practice for measuring the relative importance of risk factors. We may conduct a sensitivity analysis to the risk priority ranking obtained under the maximum ratio scale of 9 to test the robustness of the ranking.

The left-hand-side of Eq. (10) is trivial and holds always. Its right-hand-side is equivalent to the following:

$$\max\left\{\frac{w_j}{w_k}\Big|j,k=1,...m;k\neq j\right\}\leq 9,\tag{11}$$

which can be further rewritten as

$$w_j-9w_k\leq 0,\ j,k=1,...,m;k\neq j.\tag{12}$$

According to the DEA models introduced in Section 2, we are now able to build FMEA models for measuring the maximum and minimum risks of each failure mode, as shown below:

$$R_0^{\max}=\text{Maximize}\,R_0\tag{13}$$
$$\text{Subject to}\begin{cases}R_i\leq 1, & i=1,...,n,\\ w_j-9w_k\leq 0, & j,k=1,...,m;k\neq j,\end{cases}$$

$$R_0^{\min}=\text{Minimize}\,R_0\tag{14}$$
$$\text{Subject to}\begin{cases}R_i\geq 1, & i=1,...,n,\\ w_j-9w_k\leq 0, & j,k=1,...,m;k\neq j,\end{cases}$$

where R_0 is the risk of the failure mode under evaluation. The overall risk of each failure mode is defined by Eq. (7) as the geometric average of the maximum and minimum risks of the failure mode. That is

$$\overline{R}_i=\sqrt{R_i^{\max}\cdot R_i^{\min}},\ i=1,...,n.\tag{15}$$

This definition enables us not to worry about how to determine an interval for the risk of each failure mode. The bigger the geometric average risk, the higher the risk priority. The n failure modes FM_i $(i=1,...,n)$ can be easily prioritized by their geometric average risks $\overline{R}_i$ $(i=1,...,n)$.

The above models (13) and (14) are developed for additive risks. For multiplicative risks defined by Eq. (9), the maximum and minimum risk models can be built in the same way, but the ratings and risks need to be transformed into logarithmic scales for linearity. The two models are constructed as follows:

$$\ln R_0^{\max}=\text{Maximize}\,\ln R_0\tag{16}$$
$$\text{Subject to}\begin{cases}\ln R_i\leq 1, & i=1,...,n,\\ w_j-9w_k\leq 0, & j,k=1,...,m;k\neq j,\end{cases}$$

$$\ln R_0^{\min}=\text{Minimize}\,\ln R_0\tag{17}$$
$$\text{Subject to}\begin{cases}\ln R_i\geq 1, & i=1,...,n,\\ w_j-9w_k\leq 0, & j,k=1,...,m;k\neq j.\end{cases}$$

Accordingly, the geometric average risk is defined as

$$\overline{R}_i=\sqrt{EXP(\ln R_i^{\max})\cdot EXP(\ln R_i^{\min})},\ i=1,...,n.\tag{18}$$

where $EXP(.)$ is the exponential function.

Through the solution of models (13) and (14) or (16) and (17) for each failure mode, the maximum and minimum risks of all the n failure modes can be obtained. Their geometric average risks can then be computed by using Eqs. (15) or (18), based on which the n failure modes can be prioritized.

4. Interval DEA models for FMEA

FMEA is a team function. Different FMEA team members may sometimes behave very differently and provide distinct assessment information, some of which may be incomplete or imprecise. For example, an FMEA team member may provide an interval rating for a risk factor such as 4–5 which refers to the rating between 4 and 5, or an incomplete distribution such as 4 at 40% and 5 at 50% which means the failure mode under evaluation is assessed to have a risk rating of 4 at 40% confidence and a risk rating of 5 at 50% confidence. Since the total confidence $40\%+50\%=90\%<100\%$, the above assessment is said to be incomplete. The missing 10% $(=100\%-90\%)$ confidence is called local ignorance in the terminology of Dempster–Shafer theory of evidence [28] and could be assigned to any rating between 1 and 10. The above incomplete assessment can be transformed into an expectation interval, whose lower and upper bound values are respectively computed as

$$4\times 40\%+5\times 50\%+1\times(100\%-40\%-50\%)=4.2,$$
$$4\times 40\%+5\times 50\%+10\times(100\%-40\%-50\%)=5.1.$$

As a result, the maximum, minimum and geometric average risks will also become intervals.

Let $[r_{ij}^L, r_{ij}^U]$ be the interval ratings of FM_i on RF_j, which are the weighted sum of individual expectation intervals of FMEA team members, namely,

$$[r_{ij}^L, r_{ij}^U]=\sum_{k=1}^K\lambda_k E(r_{ij}^{(k)}),\ i=1,...,n;j=1,...,m,\tag{19}$$

where λ_k $(k=1,...,K)$ are the relative importance weights of FMEA team members satisfying $\lambda_k>0$ and $\sum_{k=1}^K\lambda_k=1$, and $E(r_{ij}^{(k)})=[r_{ijk}^L, r_{ijk}^U]$ are the expectation intervals provided by K team members. In the case of interval ratings, the additive and multiplicative risks defined by Eqs. (8) and (9) can be rewritten as

$$\left[R_i^L, R_i^U\right]=\left[\sum_{j=1}^m w_j r_{ij}^L, \sum_{j=1}^m w_j r_{ij}^U\right],\ i=1,...,n,\tag{20}$$

$$\left[R_i^L, R_i^U\right]=\left[\prod_{j=1}^m\left(r_{ij}^L\right)^{w_j}, \prod_{j=1}^m\left(r_{ij}^U\right)^{w_j}\right],\ i=1,...,n.\tag{21}$$

Accordingly, the maximum and minimum risk models for additive risks can be expressed as

$$\left[(R_0^{\max})^L, (R_0^{\max})^U\right]=\text{Maximize}\left[R_0^L, R_0^U\right]$$
$$\text{Subject to}\begin{cases}\left[R_i^L, R_i^U\right]\leq 1, & i=1,...,n,\\ \\ w_j-9w_k\leq 0,\ j,k=1,...,m;k\neq j,\end{cases}\tag{22}$$

$$\left[\left(R_0^{\min}\right)^L, \left(R_0^{\min}\right)^U\right] = \text{Minimize} \left[R_0^L, R_0^U\right]$$

$$\text{Subject to} \begin{cases} \left[R_i^L, R_i^U\right] \geq 1, & i = 1, \ldots, n, \\ w_j - 9w_k \leq 0, & j, k = 1, \ldots, m; k \neq j, \end{cases} \tag{23}$$

which are further broken down into four LP models, as shown below:

$$\left(R_0^{\max}\right)^L = \text{Maximize} \, R_0^L$$

$$\text{Subject to} \begin{cases} R_i^U \leq 1, & i = 1, \ldots, n, \\ w_j - 9w_k \leq 0, & j, k = 1, \ldots, m; k \neq j, \end{cases} \tag{24}$$

$$\left(R_0^{\max}\right)^U = \text{Maximize} \, R_0^U$$

$$\text{Subject to} \begin{cases} R_i^U \leq 1, & i = 1, \ldots, n, \\ w_j - 9w_k \leq 0, & j, k = 1, \ldots, m; k \neq j, \end{cases} \tag{25}$$

$$\left(R_0^{\min}\right)^L = \text{Minimize} \, R_0^L$$

$$\text{Subject to} \begin{cases} R_i^L \geq 1, & i = 1, \ldots, n, \\ w_j - 9w_k \leq 0, & j, k = 1, \ldots, m; k \neq j, \end{cases} \tag{26}$$

$$\left(R_0^{\min}\right)^U = \text{Minimize} \, R_0^U$$

$$\text{Subject to} \begin{cases} R_i^L \geq 1, & i = 1, \ldots, n, \\ w_j - 9w_k \leq 0, & j, k = 1, \ldots, m; k \neq j. \end{cases} \tag{27}$$

In this case, the geometric average risk defined by Eq. (15) can be determined by interval arithmetic [23] as

$$\left[\overline{R}_i^L, \overline{R}_i^U\right] = \left[\sqrt{\left(R_i^{\max}\right)^L \cdot \left(R_i^{\min}\right)^L}, \sqrt{\left(R_i^{\max}\right)^U \cdot \left(R_i^{\min}\right)^U}\right], \ i = 1, \ldots, n. \tag{28}$$

For multiplicative risks defined by Eq. (21), the maximum and minimum risk models are reconstructed as follows:

$$\ln\left(R_0^{\max}\right)^L = \text{Maximize} \, \ln R_0^L$$

$$\text{Subject to} \begin{cases} \ln R_i^U \leq 1, & i = 1, \ldots, n, \\ w_j - 9w_k \leq 0, & j, k = 1, \ldots, m; k \neq j, \end{cases} \tag{29}$$

$$\ln\left(R_0^{\max}\right)^U = \text{Maximize} \, \ln R_0^U$$

$$\text{Subject to} \begin{cases} \ln R_i^U \leq 1, & i = 1, \ldots, n, \\ w_j - 9w_k \leq 0, & j, k = 1, \ldots, m; k \neq j, \end{cases} \tag{30}$$

$$\ln\left(R_0^{\min}\right)^L = \text{Minimize} \, \ln R_0^L$$

$$\text{Subject to} \begin{cases} \ln R_i^L \geq 1, & i = 1, \ldots, n, \\ w_j - 9w_k \leq 0, & j, k = 1, \ldots, m; k \neq j, \end{cases} \tag{31}$$

$$\ln\left(R_0^{\min}\right)^U = \text{Minimize} \, \ln R_0^U$$

$$\text{Subject to} \begin{cases} \ln R_i^L \geq 1, & i = 1, \ldots, n, \\ w_j - 9w_k \leq 0, & j, k = 1, \ldots, m; k \neq j. \end{cases} \tag{32}$$

The geometric average risk in this case is given by

$$\left[\overline{R}_i^L, \overline{R}_i^U\right] = \left[\sqrt{EXP\left(\ln(R_i^{\max})^L\right) \cdot EXP\left(\ln(R_i^{\min})^L\right)}, \right.$$
$$\left. \sqrt{EXP\left(\ln(R_i^{\max})^U\right) \cdot EXP\left(\ln(R_i^{\min})^U\right)}\right], \ i = 1, \ldots, n, \tag{33}$$

Since the geometric average risks are intervals, an appropriate ranking approach for interval numbers is thus needed to prioritize failure modes. In Appendix A, we provide the minimax regret approach (MRA) for ranking interval numbers developed by Wang et al. [31], which will be used in this paper for comparing and ranking interval-valued geometric average risks.

5. Illustrations

In this section, we provide two numerical examples to illustrate the potential applications and benefits of the proposed FMEA in industry. Example 1 is taken from Pillay and Wang [25] and considers no imprecise and incomplete assessment information. Example 2 incorporates imprecise and incomplete information into FMEA and is a revised version of Example 1.

Example 1. Consider an application of FMEA to a fishing vessel. The FMEA for the fishing vessel investigates four different systems which are structure, propulsion, electrical, and auxiliary systems. Each system is considered for different failure modes that could lead to an accident with undesired consequences. The effects of each failure mode on the system and vessel are studied along with the provisions that are in place or available to mitigate or reduce risk. For each of the failure modes, the system is investigated for any alarms or condition monitoring arrangements, which are in place. Tables 4 and 5 show respectively the 21 identified failure modes and their ratings on the three risk factors O, S, and D. The traditional RPN does not consider the relative importance of the three risk factors and is therefore unable to discriminate failure mode 11 from failure modes 1, 2, and 16, and failure mode 7 from failure mode 15.

We now examine the failure modes using DEA. By solving DEA models (13) and (14) as well as (16) and (17) for each failure mode, respectively, we get the maximum and minimum risks of all the 21 failure modes, which are shown in Table 6 together with their geometric average risks computed by Eqs. (15) and (18), respectively, and the risk priority rankings of the 21 failure modes. Based on the results in Table 6, we have the following observations:

- There is not much difference between the two sets of risk priority rankings. The Spearman's rank-correlation coefficient between the two sets of rankings is as high as 0.9818, which means the choice of whether to use the additive risk model in Eq. (8) or the multiplicative risk model in Eq. (9) has no significant impact on the final risk priority ranking of the 21 failure modes. Major difference between the two sets of rankings happens at failure modes 3 and 19. The former has a difference of two ranking places, while the latter has a difference in ranking places of three. All the other failure modes are either ranked in the same places or have a very small gap of one.
- Failure mode 11 is successfully distinguished from failure modes 1, 2 and 16 no matter whether their risks are defined as additive or

Table 4
Failure modes for a fishing vessel [25].

Item	Failure mode	Description	Component	Failure effect (system)	Failure effect (vessel)	Alarm	Provision
1	Seizure	Structure	Rudder bearing	Rudder jam	No steering ctrl	No	Stop vessel
2	Breakage	Structure	Rudder bearing	Rudder loose	Reduced steering ctrl	No	Stop vessel
3	Structural failure	Structure	Rudder structure	Function loss	Reduced steering	No	Use beams
4	Loss of output	Propulsion	Main engine	Loss of thrust	Loss of speed	Yes	None
5	Auto shutdown	Propulsion	Main engine	M/E stops	Loss of speed	Yes	Anchor
6	Shaft breakage	Propulsion	Shaft and propeller	Loss of thrust	Loss of speed	No	Anchor
7	Shaft seizure	Propulsion	Shaft and propeller	Loss of thrust	Loss of speed	Yes	Anchor
8	Gearbox seizure	Propulsion	Shaft and propeller	Loss of thrust	Loss of speed	Yes	Anchor
9	Hydraulic failure	Propulsion	Shaft and propeller	Cannot reduce thrust	Cannot reduce speed	No	Anchor
10	Prop. blade failure	Propulsion	Shaft and propeller	Loss of thrust	Loss of speed	No	Slow steaming
11	No start air press.	Air services	Air receiver	Cannot start M/E	No propulsion	Yes	Recharge receiver
12	Generator fail	Electrical sys.	Power generation	No elec. power	Some system failures	Yes	Use stand-by generators
13	Complete loss	Electrical sys.	Main switch	Loss of main supply	No battery charging	Yes	Use emergency 24 V
14	Complete loss	Electrical sys.	Emergency S/B	Loss of emer. supp.	No emergency supp.	No	Use normal supply
15	Loss of output	Electrical sys.	Main batteries	Loss of main 24 V	Loss of main low volt	Yes	Use emergency 24 V
16	Loss of output	Electrical sys.	Emer. batteries	Loss of emer. supp.	No emergency supp.	No	Use normal supply
17	Contamination	Auxiliary sys.	Fuel system	M/E and gen. stop	Vessels stops	Yes	Anchor
18	No fuel to M/E	Auxiliary sys.	Fuel system	M/E stops	Vessel stops	No	Anchor
19	No cooling water	Auxiliary sys.	Water system	Engine overheat	M/E auto cut-out	Yes	Use stand-by pump
20	System loss	Auxiliary sys.	Hydraulic	No hydraulics	No steering	Yes	Stop vessel
21	Loss of pressure	Auxiliary sys.	Lube oil system	Low pressure cut-off	M/E stops	Yes	Use stand-by pump

multiplicative. Failure mode 7 is also discriminated from failure mode 15.

- Failure mode 11 is ranked far behind failure modes 1, 2 and 16 because it has a very small severity rating in comparison with failure modes 1, 2, and 16. This is also true for failure modes 7 and 15. The former has a very high severity rating and is therefore ranked higher than the latter.
- Except for failure modes 14 and 20, the risk priority rankings of the other 19 failure modes obtained by their geometric average risks are all different from those by their RPNs. This shows the fact that the proposed FMEA is totally different from the traditional FMEA. The biggest difference among the three sets of risk priority rankings in Tables 4 and 5 happens at failure modes 11 and 19, which have a difference of up to four or five ranking places by the two different FMEA priority methods. The Spearman's rank-correlation coefficients between the priority rankings of the 21 failure modes by the RPNs and the two geometric average risks are 0.9195 and 0.9340 for additive and multiplicative risks, respectively, which are less than that between the rankings by the two geometric average risks.

Table 5
FMEA for the fishing vessel by RPN [25].

Failure mode	O	S	D	RPN	Priority ranking
1	1	8	3	24	14
2	1	8	3	24	14
3	2	8	4	64	10
4	8	8	5	320	2
5	6	8	6	288	3
6	2	8	1	16	19
7	2	9	2	36	12
8	1	4	3	12	20
9	3	2	3	18	18
10	1	2	4	8	21
11	4	2	3	24	14
12	9	3	7	189	4
13	8	3	6	144	7
14	3	7	4	84	9
15	3	3	4	36	12
16	1	8	3	24	14
17	4	8	5	160	6
18	2	7	7	98	8
19	7	2	4	56	11
20	9	8	9	648	1
21	9	3	6	162	5

The above observations show the applicability and potentials of the new FMEA and its advantages over the traditional FMEA. To test the robustness of the risk priority rankings in Table 6, we conduct a sensitivity analysis to the weight restriction of the ratio of maximum weight to minimum weight. Table 7 shows the risk priority rankings of the 21 failure modes under different weight restrictions, where the five-point Likert scale means the weight restriction of $1 \leq \max\{w_1,...,w_m\}/\min\{w_1,...,w_m\} \leq 5$, whereas $+\infty$ represents no restriction to the weights of risk factors. In the case that there is no weight restriction, some of the risk factors may be assigned a zero weight by the DEA models. From Table 7 the following observations have been made:

- Failure modes can also be prioritized by DEA models without imposing any weight restriction, but this will result in difficulties in explaining the risks of failure modes due to the presence of zero weight for some of the risk factors and may also result in some ties for risk priority ranking. So, imposing a weight restriction is of benefit to the risk prioritizations of failure modes and their explanations.
- For additive risks, no matter whether the weight restriction is five, seven or nine, the risk priority rankings of the 21 failure modes are nearly the same with only a small ranking difference of one for some of the failure modes such as failure modes 8, 11, 12, 17, 18 and 21.
- For multiplicative risks, there is seemingly a big difference of three between the rankings of failure mode 15, but in fact, it is only the difference that the failure mode 15 is ranked higher or lower than failure modes 1, 2 and 16, which have exactly the same ratings on the three risk factors and can therefore be viewed as one failure mode or DMU. In this sense, the differences between the rankings under different weight restrictions are still very small and are of no more than one.

From the above observations it can be concluded that the priority rankings in Table 6 are stable and robust, and will not be significantly changed when a different five-point Likert scale is imposed as the weight restriction on the ratio of maximum weight to minimum weight.

Example 2. The above example assumes that FMEA team members reach a consensus on the ratings of each failure mode. In reality, however, different team members may have different opinions and provide distinct ratings, some of which may be incomplete or imprecise. Tables 8–10 show hypothetical ratings of the 21 failure modes on O, S, and D provided by five FMEA team members, where incomplete and imprecise ratings are shaded for the sake of clarity. The relative importance weights of the five FMEA team members are

Table 6
FMEA for the fishing vessel by DEA.

Failure mode	Additive risks				Multiplicative risks			
	Maximum risk	Minimum risk	Geometric average risk	Priority ranking	Maximum risk	Minimum risk	Geometric average risk	Priority ranking
1	0.84	1.12	0.97	12	2.36	2.95	2.64	13
2	0.84	1.12	0.97	12	2.36	2.95	2.64	13
3	0.87	1.60	1.18	9	2.46	5.46	3.66	7
4	0.94	2.16	1.43	3	2.64	7.27	4.38	3
5	0.93	2.32	1.47	2	2.62	7.73	4.51	2
6	0.83	1.00	0.91	16	2.32	2.72	2.51	17
7	0.94	1.23	1.08	11	2.50	4.06	3.18	11
8	0.44	1.00	0.67	18	1.80	2.72	2.21	19
9	0.33	1.00	0.57	21	1.64	2.72	2.11	21
10	0.40	1.00	0.63	20	1.73	2.72	2.17	20
11	0.42	1.02	0.65	19	1.81	2.76	2.24	18
12	0.93	1.85	1.31	5	2.58	5.43	3.75	6
13	0.83	1.74	1.20	8	2.46	5.11	3.54	10
14	0.78	1.60	1.12	10	2.38	5.43	3.59	9
15	0.43	1.27	0.74	17	1.84	3.75	2.63	16
16	0.84	1.12	0.97	12	2.36	2.95	2.64	13
17	0.90	1.96	1.33	4	2.56	6.69	4.14	4
18	0.80	2.05	1.28	6	2.41	6.82	4.05	5
19	0.70	1.22	0.93	15	2.26	3.22	2.70	12
20	1.00	3.16	1.78	1	2.72	10.00	5.21	1
21	0.92	1.78	1.28	7	2.57	5.14	3.63	8

assumed to be 0.3, 0.3, 0.2, 0.1, and 0.1, respectively. That is, $(\lambda_1,\ldots,\lambda_5) = (0.3, 0.3, 0.2, 0.1, 0.1)$.

Evidently, existing FMEA methods can handle fuzzy information, but have no way to deal with incomplete and imprecise information. To prioritize the failure modes with incomplete and imprecise assessment information, we first synthesize individual ratings given by the five FMEA team members into group ratings using Eq. (19). The results are shown in Table 10, from which it can be seen that quite a number of the ratings are intervals rather than precise values due to the presence of incomplete and imprecise assessment information.

For the group ratings in Table 11, we view precise ratings as special cases of intervals and solve interval DEA models (24)–(27) and (29)–(32) for each of the 21 failure modes to get their maximum and minimum risks. Tables 12 and 13 show the results that are obtained from the solution of the interval DEA models. The geometric average

risks of the 21 failure modes are then computed by Eqs. (28) and (33). The corresponding results are shown in the fourth column of Tables 12 and 13, respectively. By comparing their geometric average risks using the minimax regret approach detailed in Appendix A, the 21 failure modes are finally prioritized. Their risk priority rankings are presented in the last column of Tables 12 and 13.

From the risk priority rankings in Tables 12 and 13, it is observed that the two sets of risk priority rankings are still highly correlated and their Spearman's rank-correlation coefficient is 0.9766. The biggest difference between the two sets of risk priority rankings happens at failure modes 16 and 19, followed by failure modes 12, 15 and 18. The former two failure modes have a difference of three

Table 7
Risk priority rankings under different weight restrictions.

Failure mode	Additive risk				Multiplicative risk			
	Five-point scale	Seven-point scale	Nine-point scale	$+\infty$	Five-point scale	Seven-point scale	Nine-point scale	$+\infty$
1	12	12	12	12	14	14	13	12
2	12	12	12	12	14	14	13	12
3	9	9	9	8	8	7	7	6
4	3	3	3	3	3	3	3	3
5	2	2	2	2	2	2	2	2
6	16	16	16	15	17	17	17	16
7	11	11	11	11	11	11	11	11
8	19	18	18	18	19	19	19	18
9	21	21	21	21	21	21	21	21
10	20	20	20	19	20	20	20	19
11	18	19	19	19	18	18	18	19
12	4	4	5	6	6	6	6	8
13	8	8	8	9	10	10	10	10
14	10	10	10	10	9	9	9	7
15	17	17	17	17	13	13	16	15
16	12	12	12	12	14	14	13	12
17	5	5	4	4	4	4	4	4
18	7	7	6	5	5	5	5	5
19	15	15	15	16	12	12	12	17
20	1	1	1	1	1	1	1	1
21	6	6	7	6	7	8	8	8

Table 8
Occurrence assessment by FMEA team members.

Failure mode	FMEA team member				
	1	2	3	4	5
1	1	1	1–2	1	1
2	1: 50% 2: 50%	1	1	1	1
3	2	2: 90%	2	2	2
4	8	8	8: 80% 9: 20%	8	8
5	6	6	6	6	6
6	2	2	2	2	2–3
7	2	2	2	2	2
8	1	1: 75% 2: 25%	1	1	1
9	3–4	3	3	3	3
10	1: 80% 2: 20%	1	1	1	2: 85% 3: 15%
11	4	4	4	4: 75% 5: 25%	4
12	9	9	9	9	9
13	8	8: 80%	8	8	8
14	3	3	4	3	3
15	3	3	3	3	3: 70% 4: 30%
16	1	1	1	1	1
17	4: 90% 5: 10%	4	4	4	4
18	2	2	2: 90%	2	2
19	7	7	7	7	7: 80%
20	9	9	9	8: 30% 9: 70%	9
21	9	8–9	9	9	9

Table 9
Severity assessment by FMEA team members.

Failure mode	FMEA team member				
	1	2	3	4	5
1	7: 20% 8: 80%	8	8	7: 50% 8: 50%	8
2	8	8	8	8	7–9
3	7	8	7	8	8
4	8	8	8	8	7–9
5	8	9: 90%	8	8	8
6	8	8	8	6–8	8
7	9: 75% 8: 25%	9	9	9	9
8	4	4	4	4: 50% 5: 50%	3: 25% 4: 75%
9	2	2	2	2	2
10	2	2	2: 60% 3: 40%	2	2
11	2	2–3	2	2	2
12	3	3	3: 60% 4: 40%	3	3
13	2–3: 80% 3–4: 20%	3	3	3	3
14	7	8	7	7	7
15	3	3	3	3	3
16	8	8	8	8	8
17	8	7	8	8	8
18	7	7	7	7	7
19	2	2: 75% 3: 25%	2	2	2
20	8	8	8–9	8	8
21	3	3	3	3	3

ranking places between the two risk definitions, while the latter three failure modes have a difference of two. These differences are considered as small or not significant.

The above example shows the capability of the proposed FMEA in prioritization of failure modes under incomplete and imprecise information. This is its biggest advantage over existing FMEA methods.

Table 10
Detection assessment by FMEA team members.

Failure mode	FMEA team member				
	1	2	3	4	5
1	3	3	3	3	3: 90%
2	3	3	3	3	3
3	4	4	4	4	3–4: 80% 5–6: 20%
4	5	5	5	5	5
5	6	6	6: 85% 7: 15%	6	6
6	1	1: 85% 2: 15%	2	1–2	1
7	3	2	2	2: 75% 3: 25%	2
8	3	3	3: 80% 4: 20%	3	3
9	3	3	3: 60% 4: 40%	3	3
10	4	4	4	4	4
11	3: 70% 5: 30%	3	3	3	4
12	7	7	7	7	7
13	6	6	6	5	6
14	4	4	4	4	4
15	4	4: 95%	4	4	4
16	3	3	3	3	3
17	5	5	5	5	5
18	7	6–8	7	7	7
19	4	4	4	4: 90%	4
20	9: 95%	9	9	9	9
21	6	6	4–6	6	6: 25% 7: 75%

Table 11
Group ratings aggregated from FMEA team members.

Failure mode	O	S	D
1	1–1.2	7.89	2.98–3.07
2	1.15	7.9–8.1	3
3	1.97–2.24	7.5	3.94–4.04
4	8.04	7.9–8.1	5
5	6	8.06–8.33	6.03
6	2–2.1	7.8–8	1.245–1.345
7	2	8.925	2.325
8	1.075	4.025	3.04
9	3–3.3	2	3.08
10	1.175	2.08	4
11	4.025	2–2.3	3.28
12	9	3.08	7
13	7.58–8.12	2.76–3.06	5.9
14	3.2	7.3	4
15	3.03	3	3.955–4.09
16	1	8	3
17	4.03	7.7	5
18	1.98–2.16	7	6.7–7.3
19	6.89–7.16	2.075	3.97–4.06
20	8.97	8–8.2	8.88–9.015
21	8.7–9	3	5.675–6.075

6. Conclusions

As an improvement to the traditional RPN, we proposed in this paper an FMEA by data envelopment analysis. By defining the risks of failure modes as the weighted sum or weighted product of risk factors, we developed DEA models for measuring the maximum and minimum risks of failure modes. Their geometric averages measure the overall risk of each failure mode and are therefore used for prioritizing failure modes. Considering the fact that FMEA may involve incomplete and imprecise assessment information, we also developed interval DEA models for FMEA. The proposed FMEA was examined with two numerical examples and proved to be useful and effective.

In comparison with the traditional RPN and its various improvements such as fuzzy FMEA, the proposed FMEA has the following advantages:

- The relative importance weights of risk factors are considered and determined by DEA models with a weight restriction on the ratio of maximum weight to minimum weight to avoid the relative importance of any risk factors from being under- or overestimated.

Table 12
Additive risk assessment and risk priority.

Failure mode	Additive risks			Priority ranking
	Maximum risk	Minimum risk	Geometric average risk	
1	[0.817, 0.820]	[1.082, 1.152]	[0.940, 0.972]	13
2	[0.820, 0.840]	[1.125, 1.131]	[0.961, 0.974]	12
3	[0.800, 0.804]	[1.515, 1.556]	[1.101, 1.118]	10
4	[0.917, 0.936]	[2.092, 2.108]	[1.385, 1.405]	3
5	[0.921, 0.948]	[2.285, 2.306]	[1.451, 1.478]	2
6	[0.800, 0.822]	[1.000, 1.043]	[0.895, 0.926]	16
7	[0.922, 0.922]	[1.246, 1.246]	[1.072, 1.072]	11
8	[0.440, 0.440]	[1.000, 1.000]	[0.663, 0.663]	19
9	[0.334, 0.355]	[1.000, 1.006]	[0.578, 0.598]	21
10	[0.399, 0.399]	[1.000, 1.000]	[0.632, 0.632]	20
11	[0.424, 0.427]	[1.051, 1.102]	[0.668, 0.686]	18
12	[0.930, 0.930]	[1.827, 1.827]	[1.304, 1.304]	4
13	[0.785, 0.838]	[1.600, 1.731]	[1.121, 1.204]	8
14	[0.794, 0.794]	[1.606, 1.606]	[1.129, 1.129]	9
15	[0.423, 0.434]	[1.248, 1.271]	[0.727, 0.744]	17
16	[0.828, 0.828]	[1.089, 1.089]	[0.949, 0.949]	14
17	[0.853, 0.853]	[1.900, 1.900]	[1.274, 1.274]	5
18	[0.781, 0.798]	[1.888, 2.022]	[1.214, 1.271]	7
19	[0.695, 0.721]	[1.228, 1.243]	[0.928, 0.947]	15
20	[0.997, 1.000]	[3.075, 3.121]	[1.751, 1.767]	1
21	[0.888, 0.920]	[1.721, 1.753]	[1.236, 1.270]	6

Table 13
Multiplicative risk assessment and risk priority.

Failure mode	Multiplicative risks			Priority ranking
	Maximum risk	Minimum risk	Geometric average risk	
1	[2.325, 2.346]	[2.718, 3.257]	[2.514, 2.765]	14
2	[2.341, 2.363]	[3.089, 3.098]	[2.689, 2.706]	13
3	[2.375, 2.391]	[4.764, 4.922]	[3.364, 3.431]	10
4	[2.600, 2.626]	[6.669, 6.737]	[4.164, 4.206]	3
5	[2.609, 2.642]	[7.213, 7.304]	[4.338, 4.393]	2
6	[2.297, 2.332]	[2.718, 2.875]	[2.499, 2.589]	16
7	[2.485, 2.485]	[3.909, 3.909]	[3.117, 3.117]	11
8	[1.800, 1.800]	[2.718, 2.718]	[2.212, 2.212]	19
9	[1.644, 1.686]	[2.718, 2.733]	[2.114, 2.147]	21
10	[1.742, 1.742]	[2.718, 2.718]	[2.176, 2.176]	20
11	[1.821, 1.832]	[2.841, 3.039]	[2.275, 2.359]	18
12	[2.586, 2.586]	[5.432, 5.432]	[3.748, 3.748]	6
13	[2.397, 2.470]	[4.645, 5.008]	[3.337, 3.517]	9
14	[2.402, 2.402]	[5.125, 5.125]	[3.508, 3.508]	8
15	[1.832, 1.855]	[3.602, 3.663]	[2.569, 2.607]	15
16	[2.338, 2.338]	[2.735, 2.735]	[2.529, 2.529]	17
17	[2.500, 2.500]	[6.064, 6.064]	[3.894, 3.894]	4
18	[2.370, 2.421]	[6.064, 6.417]	[3.791, 3.941]	5
19	[2.248, 2.283]	[3.240, 3.269]	[2.699, 2.732]	12
20	[2.714, 2.718]	[9.301, 9.461]	[5.024, 5.071]	1
21	[2.528, 2.568]	[4.879, 5.063]	[3.512, 3.606]	7

- Risk factors are aggregated in the ways that are different from the RPN, which aggregates the risk factors by simple product and has been subject to significant criticism.
- Failure modes can be better ranked and well distinguished from each other.
- Incomplete and imprecise assessment information can be considered and handled if any.
- More risk factors can be included if necessary. The proposed FMEA is not limited to O, S and D, but applicable to any number of risk factors.
- Unlike fuzzy FMEA, there is no need to build any fuzzy if–then rules, which prove to be highly subjective and costly. Different experts may make different judgments, leading to different rules.

Finally, we point out that DEA-computed weights are not fixed for failure modes. They vary from one failure mode to another. If the DMs have preference structures on the relative importance of risk factors, they could be added as constraints to the developed DEA models to form an assurance region (AR) on the relative importance weights. It is expected that the proposed FMEA as a decision-making tool can find more applications in quality and reliability engineering in the future.

Acknowledgements

Appendix A. The minimax regret approach for ranking interval numbers

The minimax regret approach (MRA) developed by Wang et al. [31] is a method for comparing and ranking interval numbers and is briefly summarized below for the purpose of application. Let $u_i = [u_i^L, u_i^U] = <c_i, d_i>$ $(i = 1,..., N)$ be N intervals, where $c_i = \frac{1}{2}(u_i^L + u_i^U)$ and $d_i = \frac{1}{2}(u_i^U - u_i^L)$ are their midpoints and widths. Without loss of generality, suppose $u_i = [u_i^L, u_i^U]$ is chosen as the biggest interval. Let

$v = \max_{j \neq i}(u_j^U)$. If $u_i^L < v$, then the DM may regret due to the loss of opportunity that other interval numbers might be ranked higher than u_i. The maximum loss the DM may suffer from is given by

$$\text{Max}(r_i) = v - u_i^L = \max_{j \neq i}\left(u_j^U\right) - u_i^L.$$

If $u_i^L \geq v$, the DM will definitely suffer from no loss of opportunity and thus will not regret. In this situation, the DM's regret is defined as zero, i.e. $r_i = 0$. Combining the above two situations, we get

$$\text{Max}(r_i) = \max\left[\max_{j \neq i}\left(u_j^U\right) - u_i^L, 0\right].$$

The minimax regret criterion will choose the interval satisfying the following condition as the best or most desirable:

$$\min_i\{\max(r_i)\} = \min_i\left\{\max\left[\max_{j \neq i}\left(u_j^U\right) - u_i^L, 0\right]\right\}.$$

Based upon the above analysis, Wang et al. [31] gave the following definition for comparing and ranking interval numbers.

Definition 1. Let $u_i = [u_i^L, u_i^U] = <c_i, d_i>$ $(i = 1,..., N)$ be N intervals. The maximum regret value (MRV) of each interval u_i is defined as

$$R(u_i) = \max\left[\max_{j \neq i}\left(u_j^U\right) - u_i^L, 0\right], i = 1, ..., N. \quad (38)$$

The interval with the smallest MRV should be chosen as the best interval. In order to generate a full ranking for the N intervals, the following eliminating process was suggested by Wang et al.

Step 1. Calculate the MRVs of the N intervals and choose the interval with the smallest MRV as the best one. Suppose u_{i_1} is selected, where $1 \leq i_1 \leq N$.

Step 2. Eliminate u_{i_1} from further consideration and recalculate the MRVs of the remaining $(N-1)$ intervals, from which choose the one with the smallest MRV as the second best interval. Suppose u_{i_2} is chosen, where $1 \leq i_2 \leq N$, but $i_2 \neq i_1$.

Step 3. Eliminate u_{i_2} from further consideration and recalculate the MRVs of the remaining $(N-2)$ intervals, from which choose the one with the smallest MRV as the third best interval.

Step 4. Repeat the above eliminating process until only one interval u_{i_N} is left. The final ranking is $u_{i_1} > u_{i_2} > \cdots > u_{i_N}$.

By the above MRA, interval-valued geometric average risks can be compared and ranked. The ranking will serve as the risk priority of the n failure modes when incomplete or imprecise information is available.

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