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ABSTRACT

This paper uses a new approach to determine the fraction of truly skilled managers among the universe of U.S. domestic-equity mutual funds over the 1975 to 2006 period. We develop a simple technique that properly accounts for "false discoveries," or mutual funds which exhibit significant alphas by luck alone. We use this technique to precisely separate actively managed funds into those having (1) unskilled, (2) zero-alpha, and (3) skilled fund managers, net of expenses, even with cross-fund dependencies in estimated alphas. This separation into skill groups allows several new insights. First, we find that the majority of funds (75.4%) pick stocks well enough to cover their trading costs and other expenses, producing a zero alpha, consistent with the equilibrium model of Berk and Green (2004). Further, we find a significant proportion of skilled (positive alpha) funds prior to 1995, but almost none by 2006, accompanied by a large increase in unskilled (negative alpha) fund managers–due both to a large reduction in the proportion of fund managers with stockpicking skills and to a persistent level of expenses that exceed the value generated by these managers. Finally, we show that controlling for false discoveries substantially improves the ability to find funds with persistent performance.

Investors and academic researchers have long searched for outperforming mutual fund managers. Although several researchers document negative average fund alphas, net of expenses and trading costs (e.g., Jensen (1968), Lehman and Modest (1987), Elton et al. (1993), and Carhart (1997)), recent papers show that some fund managers have stock-selection skills. For instance, Kosowski, Timmermann, Wermers, and White (2006; KTWW) use a bootstrap technique to document outperformance by some funds, while Baks, Metrick, and Wachter (2001), Pastor and Stambaugh (2002b), and Avramov and Wermers (2006) illustrate the benefits of investing in actively-managed funds from a Bayesian perspective. While these papers are useful in uncovering whether, on the margin, outperforming mutual funds exist, they are not particularly informative regarding their prevalence in the entire fund population. For instance, it is natural to wonder how many fund managers possess true stockpicking skills, and where these funds are located in the cross-sectional *estimated* alpha distribution. From an investment perspective, precisely locating skilled funds maximizes our chances of achieving persistent outperformance.[1]

Of course, we cannot observe the *true* alpha of each fund in the population. Therefore, a seemingly reasonable way to estimate the prevalence of skilled fund managers is to simply count the number of funds with sufficiently high estimated alphas, $\widehat{\alpha}$. In implementing such a procedure, we are actually conducting a multiple (hypothesis) test, since we simultaneously examine the performance of several funds in the population (instead of just one fund).[2] However, it is clear that this simple count of significant-alpha funds does not properly adjust for luck in such a multiple test setting–many of the funds have significant estimated alphas by luck alone (i.e., their true alphas are zero). To illustrate, consider a population of funds with skills just sufficient to cover trading costs and expenses (zero-alpha funds). With the usual chosen significance level of 5%, we should expect that 5% of these zero-alpha funds will have significant estimated alphas–some of them will be unlucky ($\widehat{\alpha} < 0$) while others are lucky ($\widehat{\alpha} > 0$), but all will be "false discoveries"–funds with significant *estimated* alphas, but zero *true* alphas.

This paper implements a new approach to controlling for false discoveries in such a multiple fund setting. Our approach much more accurately estimates (1) the proportions of unskilled and skilled funds in the population (those with *truly* negative and positive

[1] From an investor perspective, "skill" is manager talent in selecting stocks sufficient to generate a positive alpha, net of trading costs and fund expenses.

[2] This multiple test should not be confused with the joint hypothesis test with the null hypothesis that all fund alphas are equal to zero in a sample. This test, which is employed by several papers (e.g., Grinblatt and Titman (1989, 1993)), addresses only whether at least one fund has a non-zero alpha among several funds, but is silent on the prevalence of these non-zero alpha funds.

alphas, respectively), and (2) their respective locations in the left and right tails of the cross-sectional *estimated* alpha (or estimated alpha t-statistic) distribution. One main virtue of our approach is its simplicity–to determine the proportions of unlucky and lucky funds, the only parameter needed is the proportion of zero-alpha funds in the population, π_0. Rather than arbitrarily imposing a prior assumption on π_0, our approach estimates it with a straightforward computation that uses the p-values of individual fund estimated alphas–no further econometric tests are necessary. A second advantage of our approach is its accuracy. Using a simple Monte-Carlo experiment, we demonstrate that our approach provides a much more accurate partition of the universe of mutual funds into zero-alpha, unskilled, and skilled funds than previous approaches that impose an a priori assumption about the proportion of zero-alpha funds in the population.[3]

Another important advantage of our approach to multiple testing is its robustness to cross-sectional dependencies among fund estimated alphas. Prior literature has indicated that such dependencies, which exist due to herding and other correlated trading behaviors (e.g., Wermers (1999)), greatly complicate performance measurement in a group setting. However, Monte Carlo simulations show that our simple approach, which requires only the (alpha) p-value for each fund in the population–and not the estimation of the cross-fund covariance matrix–is quite robust to such dependencies.

We apply our novel approach to the monthly returns of 2,076 actively managed U.S. open-end, domestic-equity mutual funds that exist at any time between 1975 and 2006 (inclusive), and revisit several important themes examined in the previous literature. We start with an examination of the long-term (lifetime) performance (net of trading costs and expenses) of these funds. Our decomposition of the population reveals that 75.4% are zero-alpha funds–funds having managers with some stockpicking abilities, but that extract all of the rents generated by these abilities through fees. Among remaining funds, only 0.6% are skilled (true $\alpha > 0$), while 24.0% are unskilled (true $\alpha < 0$). While our empirical finding that the majority are zero-alpha funds is supportive of the long-run equilibrium theory of Berk and Green (2004), it is surprising that we find so many truly negative-alpha funds–those that overcharge relative to the skills of their managers. Indeed, we find that such unskilled funds underperform for long time periods, indicating that investors have had some time to evaluate and identify them as underperformers.

We also find some notable time trends in our study. Examining the evolution of

[3] The reader should note the difference between our approach and that of KTWW (2006). Our approach simultaneously estimates the prevalence and location of outperforming funds in a group, while KTWW test for the skills of a single fund that is chosen from the universe of alpha-ranked funds. As such, our approach examines fund performance from a more general perspective, with a richer set of information about active fund manager skills.

each skill group between 1990 and 2006, we observe that the proportion of skilled funds dramatically decreases from 14.4% to 0.6%, while the proportion of unskilled funds increases sharply from 9.2% to 24.0%. Thus, although the number of actively managed funds has dramatically increased, skilled managers (those capable of picking stocks well enough to overcome their trading costs and expenses) have become increasingly rare.

Motivated by the possibility that funds may outperform over the short-run, before investors compete away their performance with inflows, we conduct further tests over five-year subintervals–treating each five-year fund record as a separate "fund." Here, we find that the proportion of skilled funds equals 2.4%, implying that a small number of managers have "hot hands" over short time periods. These skilled funds are located in the extreme right tail of the cross-sectional estimated alpha distribution, which indicates that a *very* low p-value is an accurate signal of short-run fund manager skill (relative to pure luck). Across the investment subgroups, Aggressive Growth funds have the highest proportion of managers with short-term skills, while Growth & Income funds exhibit no skills.

The concentration of skilled funds in the extreme right tail of the estimated alpha distribution suggests a natural way to choose funds in seeking out-of-sample persistent performance. Specifically, we form portfolios of right-tail funds that condition on the frequency of "false discoveries"–during years when our tests indicate higher proportions of lucky, zero-alpha funds in the right tail, we move further to the extreme tail to decrease false discoveries. Forming such a false discovery controlled portfolio at the beginning of each year from January 1980 to 2006, we find a four-factor alpha of 1.45% per year, which is statistically significant. Notably, we show that this luck-controlled strategy outperforms prior persistence strategies used by Carhart (1997) and others, where constant top-decile portfolios of funds are chosen with no control for luck.

Our final tests examine the performance of fund managers before expenses (but after trading costs) are subtracted. Specifically, while fund managers may be able to pick stocks well enough to cover their trading costs, they usually do not exert direct control over the level of fund expenses and fees–management companies set these expenses, with the approval of fund directors. We find evidence that indicates a very large impact of fund fees and other expenses. Specifically, on a pre-expense basis, we find a much higher incidence of funds with positive alphas–9.6%, compared to our above-mentioned finding of 0.6% after expenses. Thus, almost all outperforming funds appear to capture (or waste through operational inefficiencies) the entire surplus created by their portfolio managers. It is also noteworthy that the proportion of skilled managers (before expenses) declines substantially over time, again indicating that portfolio managers with skills have become increasingly rare. We also observe a large reduction in the proportion of unskilled funds when we move from net alphas to pre-expense alphas (from 24.0% to 4.5%), indicating a big role for excessive fees (relative to manager stockpicking skills) in underperforming funds. Although industry sources argue that competition among funds has reduced fees and expenses substantially since 1980 (Rea and Reid (1998)), our study indicates that a large subgroup of investors appear to either be unaware that they are being overcharged (Christoffersen and Musto (2002)), or are constrained to invest in high-expense funds (Elton, Gruber, and Blake (2007)).

The remainder of the paper is as follows. The next section explains our approach to separating luck from skill in measuring the performance of asset managers. Section 2 presents the performance measures, and describes the mutual fund data. Section 3 contains the results of the paper, while Section 4 concludes.

I The Impact of Luck on Mutual Fund Performance

A Overview of the Approach

A.1 Luck in a Multiple Fund Setting

Our objective is to develop a framework to precisely estimate the fraction of mutual funds in a large group that truly outperform their benchmarks. To begin, suppose that a population of M actively managed mutual funds is composed of three distinct performance categories, where performance is due to stock-selection skills. We define such performance as the ability of fund managers to generate superior model alphas, net of trading costs as well as all fees and other expenses (except loads and taxes). Our performance categories are defined as follows:

- **Unskilled funds**: funds having managers with stockpicking skills insufficient to recover their trading costs and expenses, creating an "alpha shortfall" ($\alpha < 0$),
- **Zero-alpha funds**: funds having managers with stockpicking skills sufficient to just recover trading costs and expenses ($\alpha = 0$),
- **Skilled funds**: funds having managers with stockpicking skills sufficient to provide an "alpha surplus," beyond simply recovering trading costs and expenses ($\alpha > 0$).

Note that our above definition of skill is one that is relative to expenses, and not in an absolute sense. This definition is driven by the idea that consumers look for mutual funds that deliver surplus alpha, net of all expenses.[4]

[4]However, perhaps a manager exhibits skill sufficient to more than compensate for trading costs, but the fund management company overcharges fees or inefficiently generates other services (such as administrative services, e.g., record-keeping)–costs that the manager usually has little control over. In

Of course, we cannot observe the true alphas of each fund in the population. Therefore, how do we best infer the prevalence of each of the above skill groups from performance estimates for individual funds? First, we use the t-statistic, $\widehat{t}_i = \widehat{\alpha}_i / \widehat{\sigma}_{\widehat{\alpha}_i}$, as our performance measure, where $\widehat{\alpha}_i$ is the estimated alpha for fund i, and $\widehat{\sigma}_{\widehat{\alpha}_i}$ is its estimated standard deviation–KTWW (2006) show that the t-statistic has superior properties relative to alpha, since alpha estimates have differing precision across funds with varying lives and portfolio volatilities. Second, after choosing a significance level, γ (e.g., 10%), we observe whether $\widehat{t}_i$ lies outside the thresholds implied by γ (denoted by t_γ^- and t_γ^+), and label it "significant" if it is such an outlier. This procedure, simultaneously applied across all funds, is a multiple-hypothesis test:

$$\begin{aligned} H_{0,1} &: \alpha_1 = 0, \quad H_{A,1} : \alpha_1 \neq 0, \\ ... &: ... \\ H_{0,M} &: \alpha_M = 0, \quad H_{A,M} : \alpha_M \neq 0. \end{aligned} \tag{1}$$

To illustrate the difficulty of controlling for luck in this multiple test setting, Figure 1 presents a simplified hypothetical example that borrows from our empirical findings (to be presented later) over the last five years of our sample period. In Panel A, individual funds within the three skill groups–unskilled, zero alpha, and skilled–are assumed to have true annual four-factor alphas of -3.2%, 0%, and 3.8%, respectively (the choice of these values is explained in Appendix B).[5] The individual fund t-statistic distributions shown in the panel are assumed to be normal for simplicity, and are centered at -2.5, 0, and 3.0 (which correspond to the prior-mentioned assumed true alphas; see Appendix B).[6] The t-distribution shown in Panel B is the cross-section that (hypothetically) would be observed by a researcher. This distribution is a mixture of the three skill-group distributions in Panel A, where the weight on each distribution is equal to the proportion of zero-alpha, unskilled, and skilled funds, respectively, in the population of mutual funds (specifically, $\pi_0 = 75\%$, $\pi_A^- = 23\%$, and $\pi_A^+ = 2\%$; see Appendix B).

Please insert Figure 1 here

a later section (III.D.1), we redefine stockpicking skill in an absolute sense (net of trading costs only) and revisit some of our basic tests to be described.

[5] Individual funds within a given skill group are assumed to have identical true alphas in this illustration. In our empirical section, our approach makes no such assumption.

[6] The actual t-statistic distributions for individual funds are non-normal for most U.S. domestic equity funds (KTWW (2006)). Accordingly, in our empirical section, we use a bootstrap approach to more accurately estimate the distribution of t-statistics for each fund (and their associated p-values).

To illustrate further, suppose that we choose a significance level, γ, of 10% (corresponding to $t_\gamma^- = -1.65$ and $t_\gamma^+ = 1.65$). With the test shown in Equation (1), the researcher would expect to find 5.4% of funds with a positive and significant t-statistic.[7] This proportion, denoted by $E(S_\gamma^+)$, is represented by the shaded region in the right tail of the cross-sectional t-distribution (Panel B). Does this area consist merely of skilled funds, as defined above? Clearly not, because some funds are just lucky; as shown in the shaded region of the right tail in Panel A, zero-alpha funds can exhibit positive and significant estimated t-statistics. By the same token, the proportion of funds with a negative and significant t-statistic (the shaded region in the left-tail of Panel B) overestimates the proportion of unskilled funds, because it includes some unlucky zero-alpha funds (the shaded region in the left tail in Panel A). Note that we have not considered the possibility that skilled funds could be *very* unlucky, and exhibit a negative and significant t-statistic. In our example of Figure 1, the probability that the estimated t-statistic of a skilled fund is lower than $t_\gamma^- = -1.65$ is less than 0.001%. This probability is negligible, so we ignore this pathological case. The same applies to unskilled funds that are very lucky.

The message conveyed by Figure 1 is that we measure performance with a limited sample of data, therefore, unskilled and skilled funds cannot easily be distinguished from zero-alpha funds. This problem can be worse if the cross-section of actual skill levels has a complex distribution (and not all fixed at the same levels, as assumed by our simplified example), and is further compounded if a substantial proportion of skilled fund managers have low levels of skill, relative to the error in estimating their t-statistics. To proceed, we must employ a procedure that is able to precisely account for "false discoveries," i.e., funds that falsely exhibit significant estimated alphas (i.e., their true alphas are zero) in the face of these complexities.

A.2 Measuring Luck

How do we measure the frequency of "false discoveries" in the tails of the cross-sectional (alpha) t-distribution? At a given significance level γ, it is clear that the probability that a zero-alpha fund (as defined in the last section) exhibits luck equals $\gamma/2$ (shown as the dark shaded region in Panel A of Figure 1)). If the proportion of zero-alpha funds in the population is π_0, the expected proportion of "lucky funds" (zero-alpha funds with

[7] From Panel A, the probability that the observed t-statistic is greater than $t_\gamma^+ = 1.65$ equals 5% for a zero-alpha fund and 84% for a skilled fund. Multiplying these two probabilities by the respective proportions represented by their categories (π_A^- and π_A^+) gives 5.4%.

positive and significant t-statistics) equals

$$E(F_\gamma^+) = \pi_0 \cdot \gamma/2. \tag{2}$$

Now, to determine the expected proportion of skilled funds, $E(T_\gamma^+)$, we simply adjust $E(S_\gamma^+)$ for the presence of these lucky funds:

$$E(T_\gamma^+) = E(S_\gamma^+) - E(F_\gamma^+) = E(S_\gamma^+) - \pi_0 \cdot \gamma/2. \tag{3}$$

Since the probability of a zero-alpha fund being unlucky is also equal to $\gamma/2$ (i.e., the grey and black areas in Panel A of Figure 1 are identical), $E(F_\gamma^-)$, the expected proportion of "unlucky funds," is equal to $E(F_\gamma^+)$. As a result, the expected proportion of unskilled funds, $E(T_\gamma^-)$, is similarly given by

$$E(T_\gamma^-) = E(S_\gamma^-) - E(F_\gamma^-) = E(S_\gamma^-) - \pi_0 \cdot \gamma/2. \tag{4}$$

What is the role played by the significance level, γ, chosen by the researcher? By defining the significance thresholds t_γ^- and t_γ^+, γ determines the portion of the right (or left) tail which is examined for lucky vs. skilled funds (or unlucky vs. unskilled funds), as described by Equations (3) and (4). By varying γ, we can determine the location of skilled (or unskilled) funds–by measuring the proportion of such funds in any segment of the cross-section.

This flexibility in choosing γ provides us with opportunities to make important insights into the merits of active fund management. First, by choosing a larger γ (i.e., lower t_γ^- and t_γ^+, in absolute value), we can estimate the proportions of unskilled and skilled funds in a larger portion of the left and right tails of the cross-sectional t-distribution, respectively–thus, giving us an appreciation of the proportions of unskilled and skilled funds in the entire population, π_A^- and π_A^+. That is, as we increase γ, $E(T_\gamma^-)$ and $E(T_\gamma^+)$ converge to π_A^- and π_A^+, thus minimizing Type II error (failing to locate truly unskilled or skilled funds). Alternatively, by reducing γ, we can determine the precise location of unskilled or skilled funds in the extreme tails of the t-distribution. For instance, choosing a very low γ (i.e., very large t_γ^- and t_γ^+, in absolute value) allows us to determine whether extreme tail funds are skilled or simply lucky (unskilled or simply unlucky)–information that is quite useful to investors trying to locate skilled (or avoid unskilled) managers.

A.3 Estimation Procedure

The key to our approach to measuring luck in a group setting, as shown in Equation (2), is the estimator of the proportion, π_0, of zero-alpha funds in the population. Here, we turn to a recent estimation approach developed by Storey (2002)–called the "False Discovery Rate" (FDR) approach. The FDR approach is very straightforward, as its sole inputs are the (two-sided) p-values associated with the (alpha) t-statistics of each of the M funds. By definition, zero-alpha funds satisfy the null hypothesis, $H_{0,i} : \alpha_i = 0$, and, therefore, have p-values that are uniformly distributed over the interval $[0, 1]$.[8] On the other hand, p-values of unskilled and skilled funds tend to be very small because their estimated t-statistics tend to be far from zero (see Panel A of Figure 1). We can exploit this information to estimate π_0 without knowing the exact distribution of the p-values of the unskilled and skilled funds.

To explain further, a key intuition of the FDR approach is that it uses information from the center of the cross-sectional t-distribution (which is dominated by zero-alpha funds) to correct for luck in the tails. To illustrate the FDR procedure, suppose we randomly draw 2,076 t-statistics (the number of funds in our study), each from one of the three t-distributions in Panel A of Figure 1. Each t-statistic is drawn from a given distribution with probability according to our estimates of the proportion of unskilled, zero-alpha, and skilled funds in the population, $\pi_0 = 75\%$, $\pi_A^- = 23\%$, and $\pi_A^+ = 2\%$, respectively. Thus, our draw of t-statistics comes from a known frequency of each type (23%, 75%, and 2%). Next, we apply the FDR technique to estimate these frequencies–from the sampled t-statistics, we compute two-sided p-values, $\widehat{p}_i$, for each of the 2,076 funds, then plot them in Figure 2.

Please insert Figure 2 here

The darkest grey zone near zero captures the majority of p-values of unskilled and skilled funds ($\pi_A^- + \pi_A^+$ =25%). The area below the horizontal line at 0.075 represents the true (but unknown to the researcher) proportion, π_0, of zero-alpha funds (75%), since zero-alpha funds have uniformly distributed p-values. The researcher estimates π_0 from the histogram of observed p-values as follows. If we take a sufficiently high threshold λ^* (e.g., $\lambda^* = 0.6$), we know that the vast majority of p-values higher than λ^* come from

[8] To see this, let us denote by t and p the t-statistic and p-value of the zero-alpha fund. We have $p = 1 - F(|t|)$, where $F(t) = prob(\left|\widehat{t}_i\right| < |t| \,|\, \alpha_i = 0)$. The p-value is uniformly distributed over $[0, 1]$ since its cdf, $G(p) = prob(\widehat{p}_i < p) = prob(1 - F(|t_{\widehat{p}_i}|) < p) = prob(|t_{\widehat{p}_i}| > F^{-1}(1 - p)) = 1 - F\left(F^{-1}(1 - p)\right) = p$.

zero-alpha funds. Thus, we first measure the proportion of the total area that is covered by the four lightest grey bars on the right of λ^*, $\widehat{W}(\lambda^*)/M$ (where $\widehat{W}(\lambda^*)$ denotes the number of funds having p-values exceeding λ^*). Then, we extrapolate this area over the entire interval $[0,1]$ by multiplying by $1/(1-\lambda^*)$ (e.g., if $\lambda^* = 0.6$, the area is multiplied by 2.5):[9]

$$\widehat{\pi}_0(\lambda^*) = \frac{\widehat{W}(\lambda^*)}{M} \cdot \frac{1}{(1-\lambda^*)}. \tag{5}$$

To select λ^*, we use the simple data-driven approach suggested by Storey (2002) and explained in detail in Appendix A.

Substituting the estimate $\widehat{\pi}_0$ in Equations (2), (3), and replacing $E(S_\gamma^+)$ with the observed proportion of significant funds in the right tail, $\widehat{S}_\gamma^+$, we can easily estimate $E(F_\gamma^+)$ and $E(T_\gamma^+)$ corresponding to any chosen significance level, γ. The same approach can be used in the left tail by replacing $E(S_\gamma^-)$ in Equation (4) with the observed proportion of significant funds in the left tail, $\widehat{S}_\gamma^-$. This implies the following estimates of the proportions of unlucky and lucky funds:

$$\widehat{F}_\gamma^- = \widehat{F}_\gamma^+ = \widehat{\pi}_0 \cdot \gamma/2. \tag{6}$$

Using Equation (6), the estimated proportions of unskilled and skilled funds (at the chosen significance level, γ) are, respectively, equal to

$$\begin{aligned} \widehat{T}_\gamma^- &= \widehat{S}_\gamma^- - \widehat{F}_\gamma^- = \widehat{S}_\gamma^- - \widehat{\pi}_0 \cdot \gamma/2, \\ \widehat{T}_\gamma^+ &= \widehat{S}_\gamma^+ - \widehat{F}_\gamma^+ = \widehat{S}_\gamma^+ - \widehat{\pi}_0 \cdot \gamma/2. \end{aligned} \tag{7}$$

Finally, we estimate the proportions of unskilled and skilled funds in the entire population as

$$\widehat{\pi}_A^- = \widehat{T}_{\gamma^*}^-, \qquad \widehat{\pi}_A^+ = \widehat{T}_{\gamma^*}^+, \tag{8}$$

where γ^* is a sufficiently high significance level–we choose γ^* with a simple data-driven method explained in Appendix A.

B Comparison of Our Approach with Existing Methods

The previous literature has followed two alternative approaches when estimating the proportions of unskilled and skilled funds. The "full luck" approach proposed by Jensen

[9] This estimation procedure cannot be used in a one-sided multiple test, since the null hypothesis is tested under the least favorable configuration (LFC). For instance, consider the following null hypothesis $H_{0,i}: \alpha_i \leq 0$. Under the LFC, it is replaced with $H_{0,i}: \alpha_i = 0$. Therefore, all funds with $\alpha_i \leq 0$ (i.e., drawn from the null) have inflated p-values which are not uniformly distributed over $[0,1]$.

(1968) and Ferson and Qian (2004) assumes, a priori, that all funds in the population have zero alphas, $\pi_0 = 1$. Thus, for a given significance level, γ, this approach implies an estimate of the proportions of unlucky and lucky funds equal to $\gamma/2$.[10] At the other extreme, the "no luck" approach reports the observed number of significant funds (for instance, Ferson and Schadt (1996)) without making a correction for luck.

What are the errors introduced by assuming, a priori, that π_0 equals 0 or 1, when it does not accurately describe the population? To address this question, we compare the bias produced by these two approaches relative to our FDR approach across different possible values for π_0 ($\pi_0 \in [0, 1]$) using our simple framework of Figure 1. Our procedure consists of three steps. First, for a chosen value of π_0, we create a simulated sample of 2,076 fund t-statistics (corresponding to our fund sample size) by randomly drawing from the three distributions in Panel A of Figure 1 in the proportions π_0, π_A^-, and π_A^+. For each π_0, the ratio π_A^-/π_A^+ is held fixed to 11.5 (0.23/0.02), as in Figure 1, to assure that the proportion of skilled funds remains low compared to the unskilled funds. Second, we use these sampled t-statistics to estimate the proportion of unlucky ($\alpha = 0$, $\widehat{\alpha} < 0$), lucky ($\alpha = 0$, $\widehat{\alpha} > 0$), unskilled ($\alpha < 0$, $\widehat{\alpha} < 0$), and skilled ($\alpha > 0$, $\widehat{\alpha} > 0$) funds under each of the three approaches–the "no luck," "full luck," and FDR techniques.[11] Third, under each approach, we repeat these first two steps 1,000 times to compare the average value of each estimator with its true population value.

Please insert Figure 3 here

Specifically, Panel A of Figure 3 compares the three estimators of the expected proportion of unlucky funds. The true population value, $E(F_\gamma^-)$, is an increasing function of π_0 by construction, as shown by Equation (2). While the average value of the FDR estimator closely tracks $E(F_\gamma^-)$, this is not the case for the other two approaches. Note that, by assuming that $\pi_0 = 0$, the "no luck" approach consistently underestimates $E(F_\gamma^-)$ when the true proportion of zero-alpha funds is higher ($\pi_0 > 0$). Conversely, the "full luck" approach, which assumes that $\pi_0 = 1$, overestimates $E(F_\gamma^-)$ when $\pi_0 < 1$. To illustrate the extent of the bias, consider the case where $\pi_0 = 75\%$. While the "no luck" approach substantially underestimates $E(F_\gamma^-)$ (0% instead of its true value of 7.5%), the "full luck" approach overestimates $E(F_\gamma^-)$ (10% instead of its true 7.5%). The biases for estimates of lucky funds $E(F_\gamma^+)$ shown in Panel B are exactly the same,

[10] Jensen (1968) summarizes the "full luck" approach as follows: "...if all the funds had a true α equal to zero, we would expect (merely by random chance) to find 5% of them having t values 'significant' at the 5% level."

[11] We choose $\gamma = 0.20$ to examine a large portion of the tails of the cross-sectional t-distribution, although other values for γ provide similar results.

since $E(F_\gamma^+) = E(F_\gamma^-)$.

Estimates of the expected proportions of unskilled and skilled funds ($E(T_\gamma^-)$ and $E(T_\gamma^+)$) provided by the three approaches are shown in Panels C and D, respectively. As we move to higher true proportions of zero-alpha funds (a higher value of π_0), the true proportions of unskilled and skilled funds, $E(T_\gamma^-)$ and $E(T_\gamma^+)$, decrease by construction. In both panels, our FDR estimator accurately captures this feature, while the other approaches do not fare well due to their fallacious assumptions about the prevalence of luck. For instance, when $\pi_0 = 75\%$, the "no luck" approach exhibits a large upward bias in its estimates of the total proportion of unskilled and skilled funds, $E(T_\gamma^-) + E(T_\gamma^+)$ (37.3% rather than the correct value of 22.3%). At the other extreme, the "full luck" approach underestimates $E(T_\gamma^-) + E(T_\gamma^+)$ (17.8% instead of 22.3%).

Panel D reveals that the "no luck" and "full luck" approaches also exhibit a non-sensical positive relation between π_0 and $E(T_\gamma^+)$. This result is a consequence of the low proportion of skilled funds in the population. First, as π_0 rises, the additional lucky funds drive the proportion of significant funds up, making the "no-luck" approach wrongly believe that more skilled funds are present. Second, the few skilled funds in the population cannot offset the excessive luck adjustment made by the "full luck" approach, which actually produces negative estimates of $E(T_\gamma^+)$.

In addition to the bias properties exhibited by our FDR estimators, their variability is low because of the large cross-section of funds ($M = 2,076$). To understand this, consider our main estimator $\widehat{\pi}_0$ (the same arguments apply to the other estimators). Since $\widehat{\pi}_0$ is a proportion estimator that depends on the proportion of $\widehat{p}_i > \lambda^*$, the Law of Large Numbers drives it close to its true value with our large sample size. For instance, taking $\lambda^* = 0.6$ and $\pi_0 = 75\%$, σ_{π_0} is as low as 2.5% with independent p-values ($1/30^{th}$ the magnitude of π_0).[12] In the appendix, we provide further evidence of the remarkable accuracy of our estimators using Monte-Carlo simulations.

C Estimation under Cross-Sectional Dependence among Funds

Mutual funds can have correlated residuals if they "herd" in their stockholdings (Wermers (1999)) or hold similar industry allocations. In general, cross-sectional dependence in fund estimated alphas greatly complicates performance measurement. Any inference test with dependencies becomes quickly intractable as M rises, since this requires the

[12] Specifically, $\widehat{\pi}_0 = (1-\lambda^*)^{-1} \cdot 1/M \sum_{i=1}^{M} x_i$, where x_i follows a binomial distribution with probability of success $P_{\lambda^*} = prob(\widehat{p}_i > \lambda^*) = 0.30$ (i.e., the rectangle area delimited by the horizontal black line and the vertical line at $\lambda^* = 0.6$ in Figure 2). Therefore, we have $\sigma_x = (P_{\lambda^*}(1-P_{\lambda^*}))^{\frac{1}{2}} = 0.46$, and $\sigma_{\pi_0} = (1-\lambda^*)^{-1} \cdot \sigma_x/\sqrt{M} = 2.5\%$.

estimation and inversion of an $M \times M$ residual covariance matrix. In a Bayesian framework, Jones and Shanken (2005) show that performance measurement requires intensive numerical methods when investor prior beliefs about fund alphas include cross-fund dependencies. Further, KTWW (2006) show that a complicated bootstrap is necessary to test the significance of performance of a fund located at a particular alpha rank, since this test depends on the joint distribution of all fund estimated alphas–cross-correlated fund residuals must be bootstrapped simultaneously.

An important advantage of our approach is that we estimate the p-value of each fund in isolation–avoiding the complications that arise because of the dependence structure of fund residuals. However, high cross-sectional dependencies could potentially bias our estimators. To illustrate this point with an extreme case, suppose that all funds produce zero alphas ($\pi_0 = 100\%$), and that fund residuals are perfectly correlated (perfect herding). In this case, all fund p-values would be the same, and the p-value histogram would not converge to the uniform distribution, as shown in Figure 2. Clearly, we would make serious errors no matter where we set λ^*.

In our sample, we are not overly concerned with dependencies, since we find that the average correlation between four-factor model residuals of pairs of funds is only 0.08. Further, many of our funds do not have highly overlapping return data, thus, ruling out highly correlated residuals by construction. Specifically, we find that 15% of the funds pairs do not have a single monthly return observation in common; on average, only 55% of the return observations of fund pairs is overlapping. As a result, we believe that cross-sectional dependencies are sufficiently low to allow consistent estimators (i.e., mutual fund residuals satisfy the ergodicity conditions discussed in Storey, Taylor, and Siegmund (2004)).

However, in order to explicitly verify the properties of our estimators, we run a Monte-Carlo simulation. In order to closely reproduce the actual pairwise correlations between funds in our dataset, we estimate the residual covariance matrix directly from the data, then use these dependencies in our simulations. In further simulations, we impose other types of dependencies, such as residual block correlations or residual factor dependencies, as in Jones and Shanken (2005). In all simulations, we find both that average estimates (for all of our estimators) are very close to their true values, and that confidence intervals for estimates are comparable to those that result from simulations where independent residuals are assumed. These results, as well as further details on the simulation experiment are discussed in Appendix B.

II Performance Measurement and Data Description

A Asset Pricing Models

To compute fund performance, our baseline asset pricing model is the four-factor model proposed by Carhart (1997):

$$r_{i,t} = \alpha_i + b_i \cdot r_{m,t} + s_i \cdot r_{smb,t} + h_i \cdot r_{hml,t} + m_i \cdot r_{mom,t} + \varepsilon_{i,t}, \tag{9}$$

where $r_{i,t}$ is the month t excess return of fund i over the riskfree rate (proxied by the monthly T-bill rate); $r_{m,t}$ is the month t excess return on the value-weighted market portfolio; and $r_{smb,t}$, $r_{hml,t}$, and $r_{mom,t}$ are the month t returns on zero-investment factor-mimicking portfolios for size, book-to-market, and momentum obtained from Kenneth French's website.

We also implement a conditional four-factor model to account for time-varying exposure to the market portfolio (Ferson and Schadt (1996)),

$$r_{i,t} = \alpha_i + b_i \cdot r_{m,t} + s_i \cdot r_{smb,t} + h_i \cdot r_{hml,t} + m_i \cdot r_{mom,t} + B^{'} \left(z_{t-1} \cdot r_{m,t}\right) + \varepsilon_{i,t}, \tag{10}$$

where z_{t-1} denotes the $J \times 1$ vector of centered predictive variables, and B is the $J \times 1$ vector of coefficients. The four predictive variables are the one-month T-bill rate; the dividend yield of the CRSP value-weighted NYSE/AMEX stock index; the term spread, proxied by the difference between yields on 10-year Treasurys and three-month T-bills; and the default spread, proxied by the yield difference between Moody's Baa-rated and Aaa-rated corporate bonds. We have also computed fund alphas using the CAPM and the Fama-French (1993) models. These results are summarized in Section III.D.2.

To compute each fund t-statistic, we use the Newey-West (1987) heteroscedasticity and autocorrelation consistent estimator of the standard deviation, $\widehat{\sigma}_{\widehat{\alpha}_i}$. Further, KTWW (2006) find that the finite-sample distribution of $\widehat{t}$ is non-normal for approximately half of the funds. Therefore, we use a bootstrap procedure (instead of asymptotic theory) to compute fund p-values. In order to estimate the distribution of $\widehat{t}_i$ for each fund i under the null hypothesis $\alpha_i = 0$, we use a residual-only bootstrap procedure, which draws with replacement from the regression estimated residuals $\{\widehat{\varepsilon}_{i,t}\}$.[13] For each

[13] To determine whether assuming homoscedasticity and temporal independence in individual fund residuals is appropriate, we have checked for heteroscedasticity (White test), autocorrelation (Ljung-Box test), and Arch effects (Engle test). We have found that only a few funds present such regularities. We have also implemented a block bootstrap methodology with a block length equal to $T^{\frac{1}{5}}$ (proposed by Hall, Horowitz, and Jing (1995)), where T denotes the length of the fund return time-series. All of our results to be presented remain unchanged.

fund, we implement 1,000 bootstrap replications. The reader is referred to KTWW (2006) for details on this bootstrap procedure.

B Mutual Fund Data

We use monthly mutual fund return data provided by the Center for Research in Security Prices (CRSP) between January 1975 and December 2006 to estimate fund alphas. Each monthly fund return is computed by weighting the net return of its component shareclasses by their beginning-of-month total net asset values. The CRSP database is matched with the Thomson/CDA database using the MFLINKs product of Wharton Research Data Services (WRDS) in order to use Thomson fund investment-objective information, which is more consistent over time. Wermers (2000) provides a description of how an earlier version of MFLINKS was created. Our original sample is free of survivorship bias, but we further select only funds having at least 60 monthly return observations in order to obtain precise four-factor alpha estimates. These monthly returns need not be contiguous. However, when we observe a missing return, we delete the following-month return, since CRSP fills this with the cumulated return since the last non-missing return. In unreported results, we find that reducing the minimum fund return requirement to 36 months has no material impact on our main results, thus, we believe that any biases introduced from the 60-month requirement are minimal.

Our final universe has 2,076 open-end, domestic equity mutual funds existing for at least 60 months between 1975 and 2006. Funds are classified into three investment categories: Growth (1,304 funds), Aggressive Growth (388 funds), and Growth & Income (384 funds). If an investment objective is missing, the prior non-missing objective is carried forward. A fund is included in a given investment category if its objective corresponds to the investment category for at least 60 months.

Table I shows the estimated annualized alpha as well as factor loadings of equally-weighted portfolios within each category of funds. The portfolio is rebalanced each month to include all funds existing at the beginning of that month. Results using the unconditional and conditional four-factor models are shown in Panels A and B, respectively.

Please insert Table I here

Similar to results previously documented in the literature, we find that unconditional estimated alphas for each category is negative, ranging from -0.45% to -0.60% per annum. Aggressive Growth funds tilt towards small capitalization, low book-to-market, and momentum stocks, while the opposite holds for Growth & Income funds. Introducing time-varying market betas provides similar results (Panel B). In tests available upon request from the authors, we find that all results to be discussed in the next section are qualitatively similar whether we use the unconditional or conditional version of the four-factor model. For brevity, we present only results from the unconditional four-factor model.

III Empirical Results

A Impact of Luck on Long-Term Performance

We begin our empirical analysis by measuring the impact of luck on long-term mutual fund performance, measured as the lifetime performance of each fund (over the period 1975-2006) using the four-factor model of Equation (9). Panel A of Table II shows estimated proportions of zero-alpha, unskilled, and skilled funds in the population ($\widehat{\pi}_0$, $\widehat{\pi}_A^-$, and $\widehat{\pi}_A^+$), as defined in Section I.A.1, with standard deviations of estimates in parentheses. These point estimates are computed using the procedure described in Section I.A.3, while standard deviations are computed using the method of Genovese and Wasserman (2004)–which is described in the appendix.

Please insert Table II here

Among the 2,076 funds, we estimate that the majority–75.4%–are zero-alpha funds. Managers of these funds exhibit stockpicking skills just sufficient to cover their trading costs and other expenses (including fees). These funds, therefore, capture all of the economic rents that they generate–consistent with the long-run prediction of Berk and Green (2004).

Further, it is quite surprising that the estimated proportion of skilled funds is statistically indistinguishable from zero (see “Skilled” column). This result may seem surprising in light of some prior studies, such as Ferson and Schadt (1996), which find that a small group of top mutual fund managers appear to outperform their benchmarks, net of costs. However, a closer examination–in Panel B–shows that our adjustment for luck is key in understanding the difference between our study and prior research.

To be specific, Panel B shows the proportion of significant alpha funds in the left and right tails ($\widehat{S}_\gamma^-$ and $\widehat{S}_\gamma^+$, respectively) at four different significance levels ($\gamma = 0.05$, 0.10, 0.15, 0.20). Similar to past research, there are many significant alpha funds in the right tail–$\widehat{S}_\gamma^+$ peaks at 8.2% of the total population (170 funds) when $\gamma = 0.20$. However, of course, “significant alpha” does not always mean “skilled fund manager.”

Illustrating this point, the right side of Panel B decomposes these significant funds into the proportions of lucky zero-alpha funds and skilled funds ($\widehat{F}_{\gamma}^{+}$ and $\widehat{T}_{\gamma}^{+}$, respectively). Clearly, we cannot reject that all of the right tail funds are merely lucky outcomes among the large number of zero-alpha funds (1,565), and that none of these right-tail funds have truly skilled fund managers.

It is interesting (Panel A) that 24% of the population (499 funds) are truly unskilled fund managers–unable to pick stocks well enough to recover their trading costs and other expenses.[14] In untabulated results, we find that left-tail funds, which are overwhelmingly comprised of unskilled (and not merely unlucky) funds, have a relatively long fund life–12.7 years, on average. And, these funds generally perform poorly over their entire lives, making their survival puzzling. Perhaps, as discussed by Elton, Gruber, and Busse (2003), such funds exist if they are able to attract a sufficient number of unsophisticated investors, who are also charged higher fees (Christoffersen and Musto (2002)).

The bottom of Panel B presents characteristics of the average fund in each segment of the tails. Although the average estimated alpha of right-tail funds is somewhat high (between 4.8% and 6.5% per year), this is simply due to very lucky outcomes for a small proportion of the 1,565 zero-alpha funds in the population. It is also interesting that expense ratios are higher for left-tail funds, which likely explains some of the underperformance of these funds (we will revisit this issue when we examine pre-expense returns in a later section). Turnover does not vary systematically among the various tail segments, but left-tail funds are much smaller than right-tail funds, presumably due to the combined effects of outflows and poor investment returns. Results for the three investment-objective subgroups (Aggressive Growth, Growth, and Growth & Income) are similar–these results are available upon request from the authors.

As mentioned earlier, the universe of U.S. domestic equity mutual funds has expanded substantially since 1990. Accordingly, we next examine the evolution of the proportions of unskilled and skilled funds over time. To accomplish this, at the end of each year from 1989 to 2006, we estimate the proportions of unskilled and skilled funds using the entire return history for each fund up to that point in time–this would correspond to the entire history of fund returns (starting in 1975) observed by a researcher for the universe of domestic equity funds at that point in time. For instance, our initial estimates, on December 31, 1989, cover the first 15 years of the sample, 1975-89, while our final estimates, on December 31, 2006, are based on the entire 32 years of the sample,

[14] This minority of funds is the driving force explaining the negative average estimated alpha that is widely documented in the literature (e.g., Jensen (1968), Carhart (1997), Elton et al. (1993), and Pastor and Stambaugh (2002a)).

1975-2006 (i.e., these are the estimates shown in Panel A of Table II).[15] The results in Panel A of Figure 4 show that the proportion of funds with non-zero alphas (equal to the sum of the proportions of skilled and unskilled funds) remains fairly constant over time. However, there are dramatic changes in the relative proportions of unskilled and skilled funds: from 1989 to 2006. Specifically, the proportion of skilled funds declines from 14.4% to 0.6%, while the proportion of unskilled funds rises from 9.2% to 24.0% of the entire universe of funds. These changes are also reflected in the population average estimated alpha (shown in Panel B), which drops from 0.16% to -0.97% per year over the same period.

Please insert Figure 4 here

Panel B also displays the yearly count of funds included in the estimated proportions of Panel A. From 1996 to 2005, there are more than 100 additional actively managed domestic-equity mutual funds per year.[16] Interestingly, this coincides with the time trend in unskilled and skilled funds shown in Panel A–the huge increase in numbers of actively managed mutual funds has resulted in a much larger proportion of unskilled funds, at the expense of skilled funds. Either the growth of the fund industry has coincided with greater levels of stock market efficiency, making stockpicking a more difficult and costly endeavor, or the large number of new managers simply have inadequate skills. It is also interesting that, during our period of analysis, many fund managers with good track records left the sample to manage hedge funds (as shown by Kostovetski (2007)), and that indexed investing increased substantially.

B Impact of Luck on Short-Term Performance

Our above results indicate that funds do not achieve superior long-term alphas, perhaps because flows compete away any alpha surplus. However, we might find evidence of funds with superior short-term alphas, before investors become fully aware of such outperformers due to search costs.

To test for short-run mutual fund performance, we partition our data into six non-overlapping subperiods of five years, beginning with 1977-1981 and ending with 2002-2006. For each subperiod, we include all funds having 60 monthly return observations, then compute their respective alpha p-values–in other words, we treat each fund during

[15] To be included at the end of a given year, a fund must have at least 60 monthly return observations before that date, although these observations need not be contiguous.

[16] Since we require 60 monthly observations to measure fund performance, this rise reflects the massive entry of new funds over the period 1993-2001.

each five-year period as a separate "fund."[17] We pool these five-year records together across all time periods to represent the average experience of an investor in a randomly chosen fund during a randomly chosen five-year period. After pooling, we obtain a total of 3,311 p-values from which we compute our different estimators. Results for the entire population (All Funds) are shown in Table III, while results for Growth, Aggressive Growth, and Growth & Income funds are displayed in Panels A, B, and C of Table IV, respectively.

Please insert Table III here

First, Panel A of Table III shows that a small fraction of funds (2.4% of the population) exhibit skill over the short-run (with a standard deviation of 0.7%). Thus, short-term superior performance is rare, but does exist, as opposed to long-term performance. Second, these skilled funds are located in the extreme right tail of the cross-sectional t-distribution. Panel B of Table III shows that, with a γ of only 10%, we capture almost all skilled funds, as $\widehat{T}_{\gamma}^{+}$ reaches 2.3% (close to its maximum value of 2.4%). Proceeding toward the center of the distribution (by increasing γ to 0.10 and 0.20) produces almost no additional skilled funds and almost entirely additional zero-alpha funds that are lucky ($\widehat{F}_{\gamma}^{+}$). Thus, skilled fund managers, while rare, may be somewhat easy to find, since they have extremely high t-statistics (extremely low p-values)–we will use this finding in our next section, where we attempt to find funds with out-of-sample skills. It is notable that we find evidence of short-term outperformance of some funds here, but no evidence of long-term outperformance in the prior section of this paper. This is consistent with Berk and Green (2004), where outperforming funds exist only until investors are successfully able to locate them.

In the left tail, we observe that the great majority of funds are unskilled, and not merely unlucky zero-alpha funds. For instance, in the extreme left tail (at $\gamma = 0.05$), the proportion of unskilled funds, $\widehat{T}_{\gamma}^{-}$, is roughly five times the proportion of unlucky funds, $\widehat{F}_{\gamma}^{-}$ (9.4% versus 1.8%). Here, the short-term results are similar to the prior-discussed long-term results–the great majority of left-tail funds are truly unskilled. It is also interesting that true skills seem to be inversely related to turnover, as indicated by the substantially higher levels of turnover of left-tail funds (which are mainly unskilled funds). Unskilled managers apparently trade frequently to appear skilled, which ultimately hurts their performance. Perhaps poor governance of some funds explains why they end up in the left tail (net of expenses), in the short-run–they overexpend on both

[17] Note that reducing the number of observations comes at a cost: it increases the standard deviation of the estimated alphas, making the p-values of non-zero alpha funds harder to distinguish from those of zero-alpha funds.

trading costs (through high turnover) and other expenses relative to their skills.

Table IV shows results for investment-objective subgroups. Panel A shows that the proportions of skilled Growth funds in various segments of the right tail are similar to those of the entire universe (from Table III). However, Aggressive-Growth funds (Panel B) exhibit somewhat higher skills. For instance, at $\gamma = 0.05$, 73% of significant Aggressive-Growth funds are truly skilled (3.1/4.9). On the other hand, Panel C shows that no Growth & Income funds are truly skilled, but that a substantial proportion of them are unskilled. The long-term existence of this category of actively-managed funds, which includes "value funds" and "core funds" is remarkable in light of these poor results.

Please insert Table IV here

C Performance Persistence

Our previous analysis reveals that only 2.4% of the funds are skilled over the short-term. Can we detect these skilled funds over time, in order to capture their superior alphas? Ideally, we would like to form a portfolio containing only the truly skilled funds in the right tail; however, since we only know which segment of the tails in which they lie, but not their identities, such an approach is not feasible.

Nonetheless, the reader should recall from the last section that skilled funds are located in the extreme right tail. By forming portfolios containing all funds in this extreme tail, we have a greater chance of capturing the superior alphas of the truly skilled ones. For instance, Panel B of Table III shows that, at $\gamma = 0.05$, the proportion of skilled funds among all significant funds, $\widehat{T}_\gamma^+/\widehat{S}_\gamma^+$, is about 50%, which is much higher than the proportion of skilled funds in the entire universe, 2.4%.

To select a portfolio of funds, we use the False Discovery Rate in the right tail, FDR^+. At a given significance level, γ, the FDR^+ is defined as the expected proportion of lucky funds among all significant funds in this tail:

$$FDR_\gamma^+ = E\left(\frac{F_\gamma^+}{S_\gamma^+}\right). \tag{11}$$

The FDR_γ^+ provides a simple portfolio formation rule.[18] When we set a low FDR^+ target, we allow only a small proportion of lucky funds ("false discoveries") in the chosen

[18] Our new measure, FDR_γ^+, is an extension of the traditional FDR introduced in the statistical literature (e.g., Benjamini and Hochberg (1995), Storey (2002)), since the latter does not distinguish between bad and good luck. The traditional measure is $FDR_\gamma = E\left(F_\gamma/S_\gamma\right)$, where $F_\gamma = F_\gamma^+ + F_\gamma^-$, $S_\gamma = S_\gamma^+ + S_\gamma^-$.

portfolio. Specifically, we set a sufficiently low significance level, γ, so as to include skilled funds along with a small number of zero-alpha funds that are extremely lucky. Conversely, increasing the FDR^+ target has two opposing effects on a portfolio. First, it decreases the portfolio's expected future performance, since the proportion of lucky funds in the portfolio is higher. However, it also increases its diversification, since more funds are selected–reducing the volatility of the portfolio's out-of-sample performance. Accordingly, we examine five FDR^+ target levels in our persistence test: 10%, 30%, 50%, 70%, and 90%.

The construction of the portfolios proceeds as follows. At the end of each year, we estimate the alpha p-values of each existing fund using the previous five-year period. Using these p-values, we estimate the FDR^+_γ over a range of chosen significance levels (γ =0.01, 0.02,..., 0.60). Following Storey (2002) and Storey and Tibshirani (2003), we implement the following straightforward estimator of the FDR^+_γ :

$$\widehat{FDR}^+_\gamma = \frac{\widehat{F}^+_\gamma}{\widehat{S}^+_\gamma} = \frac{\widehat{\pi}_0 \cdot \gamma/2}{\widehat{S}^+_\gamma}, \tag{12}$$

where $\widehat{\pi}_0$ is the estimator of the proportion of zero-alpha funds described in Section I.A.3. For each FDR^+ target level, we determine the significance level, γ^P, that provides an $\widehat{FDR}^+_{\gamma^P}$ as close as possible to this target. Then, only funds with p-values smaller than γ^P are included in an equally-weighted portfolio. This portfolio is held for one year, after which the selection procedure is repeated. If a selected fund does not survive after a given month during the holding period, its weight is reallocated to the remaining funds during the rest of the year to mitigate survival bias. The first portfolio formation date is December 31, 1979 (after five years of returns have been observed), while the last is December 31, 2005.

In Panel A of Table V, we show the FDR level ($\widehat{FDR}^+_{\gamma^P}$) of the five portfolios, as well as the proportion of funds in the population that they include ($\widehat{S}^+_{\gamma^P}$) during the five-year formation period, averaged over the 27 formation periods (ending from 1979 to 2005)–and, their respective distributions. First, we observe (as expected) that the achieved FDR increases with the FDR target assigned to a portfolio. However, the average $\widehat{FDR}^+_{\gamma^P}$ does not always match its target. For instance, FDR10% achieves an average of 41.5%, instead of the targeted 10%–during several formation periods, the proportion of skilled funds in the population is too low to achieve a 10% FDR target.[19]

[19] For instance, the minimum achievable FDR at the end of 2003 and 2004 is equal to 47.0% and 39.1%, respectively. If we look at the $\widehat{FDR}^+_{\gamma^P}$ distribution for the portfolio FDR 10% in Panel A,we observe that in 6 years out of 27, the $\widehat{FDR}^+_{\gamma^P}$ is higher than 70%.

Of course, a higher FDR target means an increase in the proportion of funds included in a portfolio–as shown in the rightmost columns of Panel A–since our selection rule becomes less restrictive.

In Panel B, we present the average out-of-sample performance (during the following year) of these five false discovery controlled portfolios, starting January 1, 1980 and ending December 31, 2006. We compute the estimated annualized alpha, $\widehat{\alpha}$, along with its bootstrapped p-value; annualized residual standard deviation, $\widehat{\sigma}_{\varepsilon}$; information ratio, IR$=\widehat{\alpha}/\widehat{\sigma}_{\varepsilon}$; four-factor model loadings; annualized mean return (minus T-bills); and annualized time-series standard deviation of monthly returns. The results reveal that our FDR portfolios successfully detect funds with short-term skills. For example, the portfolios FDR10% and 30% produce out-of-sample alphas (net of expenses) of 1.45% and 1.15% per year (significant at the 5% level). As the FDR target rises to 90%, the proportion of funds in the portfolio increases, which improves diversification ($\widehat{\sigma}_{\varepsilon}$ falls from 4.0% to 2.7%). However, we also observe a sharp decrease in the alpha (from 1.45% to 0.39%), reflecting the large proportion of lucky funds contained in the FDR90% portfolio.

Please insert Table V here

Panel C examines portfolio turnover–we determine the proportion of funds which are still selected using a given false discovery rule 1, 2, 3, 4, and 5 years after their initial inclusion. The results sharply illustrate the short-term nature of truly outperforming funds. After 1 year, 40% or fewer funds remain in portfolios FDR10% and 30%, while after 3 years, these percentages drop below 6%.

Finally, we examine, in Figure 5, how the estimated alpha of the portfolio FDR10% evolves over time using expanding windows. The initial value, on December 31, 1989, is the yearly out-of-sample alpha, averaged over the period 1980 to 1989, while the final value, on December 31, 2006, is the yearly out-of-sample alpha, averaged over the entire period 1980-2006 (i.e., this is the estimated alpha shown in Panel B of Table V). Again, these are the entire history of persistence results that would be observed by a researcher at the end of each year. The similarity with Figure 4 is striking. While the alpha accruing to the FDR10% portfolio is impressive at the beginning of the 1990s, it consistently declines thereafter. As the proportion, π_A^+, of skilled funds falls, the FDR approach moves much further to the extreme right tail of the cross-sectional t-distribution (from 5.7% of all funds in 1990 to 0.9% in 2006) in search of skilled managers. However, this change is not sufficient to prevent the performance of FDR10% from dropping substantially.

Please insert Figure 5 here

It is important to note the differences between our approach to persistence and that of the previous literature (e.g., Hendricks, Patel, and Zeckhauser (1993), Elton, Gruber, and Blake (1996), Carhart (1997)). These prior papers generally classify funds into fractile portfolios based on their past performance (past returns, estimated alpha, or alpha t-statistic) over a previous ranking period (one to three years). The size of fractile portfolios (e.g., deciles) are held fixed, with no regard to the changing proportion of lucky funds within these fixed fractiles. As a result, the signal used to form portfolios is likely to be noisier than our FDR approach. To compare these approaches with ours, Figure 5 displays the performance evolution of two top decile portfolios which are formed based on ranking funds by their alpha t-statistic, estimated over the previous one and three years, respectively.[20] Over most years, the FDR approach performs much better, consistent with the idea that it much more precisely detects skilled funds. However, this performance advantage declines during later years, when the proportion of skilled funds decreases substantially, making them much tougher to locate. Therefore, we find that the superior performance of the FDR portfolio is tightly linked to the prevalence of skilled funds in the population.

D Additional Results

D.1 Performance Measured with Pre-Expense Returns

In our baseline framework described previously, we define a fund as skilled if it generates a positive alpha net of trading costs, fees, and other expenses. Alternatively, skill could be defined, in an absolute sense, as the manager's ability to produce a positive alpha before expenses are deducted. Measuring performance on a pre-expense basis allows one to disentangle the manager's stockpicking skills, net of trading costs, from the fund's expense policy–which may be out of the control of the fund manager. To address this issue, we add monthly expenses (1/12 times the most recent reported annual expense ratio) to net returns for each fund, then revisit the long-term performance of the mutual fund industry.[21]

Panel A of Table VI contains the estimated proportions of zero-alpha, unskilled, and skilled funds in the population ($\widehat{\pi}_0$, $\widehat{\pi}_A^-$, and $\widehat{\pi}_A^+$), on a pre-expense basis. Comparing

[20] We use the t-statistic to be consistent with the rest of our paper, but the results are qualitatively similar when we rank on the estimated alpha.

[21] We discard funds which do not have at least 60 pre-expense return observations over the period 1975-2006. This leads to a small reduction in our sample from 2,076 to 1,836 funds.

these estimates with those shown in Table II, we observe a striking reduction in the proportion of unskilled funds–from 24.0% to 4.5%. This result indicates that only a small fraction of fund managers have stockpicking skills that are insufficient to at least compensate for their trading costs. Instead, mutual funds produce negative net-of-expense alphas chiefly because they charge excessive fees, in relation to the selection abilities of their managers. In Panel B, we further find that the average expense ratio across funds in the left tail is lower when performance is measured prior to expenses (1.3% versus 1.5% per year), indicating that high fees (potentially charged to unsophisticated investors) are a chief reason why funds end up in the extreme left tail, net of expenses. In addition, turnover seems to have no relation to pre-expense performance, as with the long-term net-of-expense results of Table II.

Please insert Table VI here

In the right tail, we find that 9.6% of fund managers have stockpicking skills sufficient to more than compensate for trading costs (Panel A). Consistent with Berk and Green (2004), the rents stemming from their skills are extracted through fees and other expenses, driving the proportion of net-expense skilled funds to zero.

Since 75.4% of funds produce zero net-expense alphas, it seems surprising that that we do not find more pre-expense skilled funds. However, this is due to the relatively small impact of expense ratios on fund performance in the center of the cross-sectional t-distribution. Adding back these expenses leads only to a marginal increase in the alpha t-statistic, making the power of the tests rather low.[22]

Finally, in untabulated tests, we find that the proportion of skilled funds in the population decreases from 27.5% to 10% between 1996 and 2006. This implies that the decline in net-expense skills noted in Figure 4 is mostly driven by a reduction in stockpicking skills over time (as opposed to an increase in expenses for (pre-expense) skilled funds).

On the contrary, the proportion of pre-expense unskilled funds remains equal to zero until the end of 2003. Thus, poor stockpicking skills (net of trading costs) cannot explain the large increase in the proportion of unskilled funds (net of both trading costs and expenses) from 1996 onwards. This increase is likely to be due to rising expenses

[22] The average expense ratio across funds with $|\widehat{\alpha}_i| < 1\%$ is approximately 10 bp per month. Adding back these expenses to a fund with zero net-expense alpha only increases its t-statistic mean from 0 to 0.9 (based on $T^{\frac{1}{2}}\alpha_A/\sigma_\varepsilon$, with $T = 384$, and $\sigma_\varepsilon = 0.021$). It implies that the null and alternative t-statistic distributions are extremely difficult to distinguish (for a fund with a (pre-expense) t-statistic mean of 0.9, the probability of observing a negative (pre-expense) t-statistic is equal to 18%!).

charged by funds with weak stock-selection abilities, or the introduction of new funds with high expense ratios and little stockpicking skills.

D.2 Performance Measured with Other Asset Pricing Models

Our estimation of the proportions of unskilled and skilled funds, $\widehat{\pi}_A^-$ and $\widehat{\pi}_A^+$, obviously depends on the choice of the asset pricing model. To examine the sensitivity of our results, we repeat the long-term (net of expense) performance analysis using the (unconditional) CAPM and Fama-French models. Based on the CAPM, we find that $\widehat{\pi}_A^-$ and $\widehat{\pi}_A^+$ are equal to 14.3% and 8.6% respectively, which is much more supportive of active management skills, compared to Section III.A.1. However, this result may be due to the omission of the size, book-to-market, and momentum factors. This conjecture is confirmed in Panel A of Table VII: the funds located in the right tail (according to the CAPM) have substantial loadings on the size and the book-to-market factors, which carry positive risk premia over our sample period (3.7% and 5.4% per year, respectively).

Please insert Table VII here

Turning to the Fama-French (1993) model, we find that $\widehat{\pi}_A^-$ and $\widehat{\pi}_A^+$ amount to 25.0% and 1.7%, respectively. These proportions are very close to those obtained with the four-factor model, since only one factor is omitted. As expected, the 1.1% difference in the estimated proportion of skilled funds between the two models (1.7%-0.6%) can be explained by the momentum factor. As shown in Panel B, the funds located in the right tail (according to the Fama-French model) have substantial loadings on the momentum factor, which carries a positive risk premium over the period (9.4% per year).

D.3 Bayesian Interpretation

Although we operate in a classical frequentist framework, our new FDR measure, FDR^+, also has a natural Bayesian interpretation.[23] To see this, we denote, by H_i, a random variable which takes the value of -1 if fund i is unskilled, 0 if it has zero alpha, and +1 if it is skilled. The prior probabilities for the three possible values (-1, 0, +1) are given by the proportion of each skill group in the population, π_A^-, π_0, and π_A^+. The Bayesian version of our FDR^+ measure, denoted by fdr_γ^+, is defined as the posterior probability that fund i has a zero alpha given that its t-statistic, $\widehat{t}_i$, is positive and significant: $fdr_\gamma^+ = prob\left(H_i = 0 \middle| \widehat{t}_i \in \Gamma_A^+(\gamma)\right)$, where $\Gamma_A^+(\gamma) = \left(t_\gamma^+, +\infty\right)$. Using

[23] Our demonstration follows from the arguments used by Efron and Tibshirani (2002) and Storey (2003) for the traditional FDR, defined as $FDR_\gamma = E\left(F_\gamma / S_\gamma\right)$, where $F_\gamma = F_\gamma^+ + F_\gamma^-$, $S_\gamma = S_\gamma^+ + S_\gamma^-$.

Bayes theorem, we have:

$$fdr_\gamma^+ = \frac{prob\left(\widehat{t}_i \in \Gamma_A^+(\gamma) \middle| H_i = 0\right) \cdot prob(H_i = 0)}{prob\left(\widehat{t}_i \in \Gamma_A^+(\gamma)\right)} = \frac{\gamma/2 \cdot \pi_0}{E(S_\gamma^+)}. \qquad (13)$$

Stated differently, the fdr_γ^+ indicates how the investor changes his prior probability that fund i has a zero alpha ($H_i = 0$) after observing that its t-statistic is significant. In light of Equation (13), our estimator $\widehat{FDR}_\gamma^+ = (\gamma/2 \cdot \widehat{\pi}_0)/\widehat{S}_\gamma^+$ can therefore be interpreted as an empirical Bayes estimator of fdr_γ^+, where π_0 and $E(S_\gamma^+)$ are directly estimated from the data.[24]

In the recent Bayesian literature on mutual fund performance (e.g., Baks, Metrick, and Wachter (2001) and Pastor and Stambaugh (2002a)), attention is given to the posterior distribution of the fund alpha, α_i, as opposed to the posterior distribution of H_i. Interestingly, our approach also provides some relevant information for modeling the fund alpha prior distribution in an empirical Bayes setting. The parameters of the prior can be specified based on the relative frequency of the three fund skill groups (zero-alpha, unskilled, and skilled funds). In light of our estimates, an empirically-based alpha prior distribution is characterized by a point mass at $\alpha = 0$, reflecting the fact that 75.4% of the funds yield zero alphas, net of expenses. Since $\widehat{\pi}_A^-$ is higher than $\widehat{\pi}_A^+$, the prior probability of observing a negative alpha is higher than that of observing a positive alpha. These empirical constraints yield an asymmetric prior distribution. A tractable way to model the left and right parts of this distribution is to exploit two truncated normal distributions in the same spirit as in Baks, Metrick, and Wachter (2001). Further, we estimate that 9.6% of the funds have an alpha greater than zero, before expenses. While Baks, Metrick, and Wachter (2001) set this probability to 1% in order to examine the portfolio decision made by a skeptical investor, our analysis reveals that this level represents an overly skeptical belief.

IV Conclusion

In this paper, we apply a new method for measuring the skills of fund managers in a group setting. Specifically, the "False Discovery Rate" (FDR) approach provides a simple and straightforward method to estimate the proportion of funds within a population

[24] A full Bayesian estimation of fdr_γ^+ requires to posit prior distributions for the proportions π_0, π_A^-, and π_A^+, and for the distribution parameters of $\widehat{t}_i$ for each skill group. This method based on additional assumptions (including independent p-values) as well as intensive numerical methods is illustrated by Tang, Ghosal, and Roy (2007) in the case of the traditional FDR.

that have stockpicking skills. In Monte Carlo simulations, we show that our novel approach gives very accurate estimates of the proportion of skilled funds (those providing a positive alpha, net of trading costs and expenses), zero-alpha funds, and unskilled funds (those providing a negative alpha) in the entire population. Further, we can use these estimates to provide accurate counts of skilled funds within various intervals in the right tail of the cross-sectional (estimated) alpha distribution, as well as unskilled funds within segments of the left tail.

We also apply the FDR technique to show that the proportion of skilled fund managers has diminished rapidly over the past 20 years. On the contrary, unskilled fund managers have increased substantially in the population over this period. Further analysis of pre-expense alphas reveals that the increase in unskilled fund managers (net of expenses) is due to an increase in the number of funds who charge high fees while possessing no particular stockpicking skills.

Our paper focuses the long-standing puzzle of actively managed mutual fund underperformance on the minority of truly underperforming funds. Most actively managed funds provide either positive or zero net-of-expense alphas, putting them at least on par with passive funds. Still, it is puzzling why investors seem to increasingly tolerate the existence of a large minority of funds that produce negative alphas, when an increasing array of passively managed funds have become available (such as ETFs). Perhaps a class of unsophisticated or inattentive investors remain shareholders in funds after they have clearly demonstrated (over time) their inferior returns. Or, as Elton, Gruber, and Blake (2007) discuss, maybe investors are forced to make constrained rational decisions–since these authors document that many 401(k) plans offer inefficient choices of mutual funds.

While our paper focuses on mutual fund performance, our approach has potentially wide applications in finance. It can be used in any setting in which a multiple hypothesis test is run and a large sample is available. We list two illustrative examples. First, technical trading can be implemented with a myriad of trading rules (e.g., Sullivan, Timmermann, and White (1999)). Our estimators can be used to determine the impact of luck on the performance of all these trading rules simultaneously. Second, testing the presence of commonality in liquidity boils down to regressing an individual stock liquidity measure on the market liquidity measure (e.g., Chordia, Roll, and Subrahmanyam (2000)). Since this regression is run for each individual stock, we are dealing with multiple testing. As a result, a correct measurement of commonality in liquidity necessitates a proper adjustment for luck. Because our approach only requires the estimation of π_0, controlling for luck in multiple testing is trivial: the only input required is a vector of p-values, one for each stock.

V Appendix

A Estimation Procedure

A.1 Determining the Value for λ^* from the Data

We use the bootstrap procedure proposed by Storey (2002) and Storey, Taylor, and Siegmund (2004). This resampling approach chooses λ from the data such that an estimate of the Mean-Squared Error (MSE) of $\widehat{\pi}_0(\lambda)$ is minimized. First, we compute $\widehat{\pi}_0(\lambda)$ using Equation (5) across a range of λ values ($\lambda = 0.30, 0.35, ..., 0.70$). Second, for each possible value of λ, we form 1,000 bootstrap replications of $\widehat{\pi}_0(\lambda)$ by drawing with replacement from the $M \times 1$ vector of fund p-values. These are denoted by $\widehat{\pi}_0^b(\lambda)$, for $b = 1, ..., 1,000$. Third, we compute the estimated MSE for each possible value of λ:

$$\widehat{MSE}(\lambda) = \frac{1}{1,000} \sum_{b=1}^{1,000} \left[\widehat{\pi}_0^b(\lambda) - \min_{\lambda} \widehat{\pi}_0(\lambda) \right]^2. \tag{14}$$

We choose λ^* such that $\lambda^* = \arg\min_{\lambda} \widehat{MSE}(\lambda)$. In unreported results (available upon request), we find that fixing λ to 0.5 or 0.6 yields similar results to those obtained with the bootstrap procedure (see also Storey (2002)). Still, the main advantage of the bootstrap approach is that it is entirely data-driven.

A.2 Determining the Value for γ^* from the Data

Similar to the approach used to determine λ^*, we use a bootstrap procedure which minimizes the estimated MSE of $\widehat{\pi}_A^-(\gamma)$ and $\widehat{\pi}_A^+(\gamma)$. First, we compute $\widehat{\pi}_A^-(\gamma)$ using Equation (8) across a range of γ ($\gamma = 0.10, 0.15, ..., 0.50$). Second, we form 1,000 bootstrap replications of $\widehat{\pi}_A^-(\gamma)$ for each possible value of γ. These are denoted by $\widehat{\pi}_A^{b-}(\gamma)$, for $b = 1, ..., 1,000$. Third, we compute the estimated MSE for each possible value of γ:

$$\widehat{MSE}^-(\gamma) = \frac{1}{1,000} \sum_{b=1}^{1,000} \left[\widehat{\pi}_A^{b-}(\gamma) - \max_{\gamma} \widehat{\pi}_A^-(\gamma) \right]^2. \tag{15}$$

We choose γ^- such that $\gamma^- = \arg\min_{\gamma} \widehat{MSE}^-(\gamma)$. We use the same data-driven procedure for $\widehat{\pi}_A^+(\gamma)$ to determine $\gamma^+ = \arg\min_{\gamma} \widehat{MSE}^+(\gamma)$. If $\min_{\gamma} \widehat{MSE}^-(\gamma) < \min_{\gamma} \widehat{MSE}^+(\gamma)$, we set $\widehat{\pi}_A^-(\gamma^*) = \widehat{\pi}_A^-(\gamma^-)$. To preserve the equality $1 = \pi_0 + \pi_A^+ + \pi_A^-$, we set $\widehat{\pi}_A^+(\gamma^*) = 1 - \widehat{\pi}_0 - \widehat{\pi}_A^-(\gamma^*)$. Otherwise, we set $\widehat{\pi}_A^+(\gamma^*) = \widehat{\pi}_A^+(\gamma^+)$ and $\widehat{\pi}_A^-(\gamma^*) = 1 - \widehat{\pi}_0 - \widehat{\pi}_A^+(\gamma^*)$.

A.3 Determining the Standard Deviation of the Estimators

We rely on the large-sample theory proposed by Genovese and Wasserman (2004). The essential idea is to recognize that the estimators $\widehat{\pi}_0(\lambda^*)$, $\widehat{S}_\gamma^+$, $\widehat{F}_\gamma^+$, $\widehat{T}_\gamma^+$, $\widehat{S}_\gamma^-$, $\widehat{F}_\gamma^-$, and $\widehat{T}_\gamma^-$ are all stochastic processes indexed by λ^* or γ which converge to a Gaussian process when the number of funds, M, goes to infinity. Proposition 3.2 of Genovese and Wasserman (2004) shows that $\widehat{\pi}_0(\lambda^*)$ is asymptotically normally distributed when $M \to \infty$, with standard deviation $\widehat{\sigma}_{\widehat{\pi}_0(\lambda^*)} = \left(\frac{\widehat{W}(\lambda^*)(M-\widehat{W}(\lambda^*))}{M^3(1-\lambda^*)^2}\right)^{\frac{1}{2}}$, where $\widehat{W}(\lambda^*)$ denotes the number of funds having p-values exceeding λ^*. Similarly, we have $\widehat{\sigma}_{\widehat{F}_\gamma^+} = (\gamma/2)\,\widehat{\sigma}_{\widehat{\pi}_0(\lambda^*)}$, $\widehat{\sigma}_{\widehat{S}_\gamma^+} = \left(\frac{\widehat{S}_\gamma^+(1-\widehat{S}_\gamma^+)}{M}\right)^{\frac{1}{2}}$, and $\widehat{\sigma}_{\widehat{T}_\gamma^+} = \left(\widehat{\sigma}^2_{\widehat{S}_\gamma^+} + (\gamma/2)^2\,\widehat{\sigma}^2_{\widehat{\pi}_0(\lambda^*)} + 2\frac{(\gamma/2)}{1-\lambda^*}\widehat{S}_\gamma^+\frac{\widehat{W}(\lambda^*)}{M}\right)^{\frac{1}{2}}$ (using the equality $\widehat{S}_\gamma^+ = \widehat{F}_\gamma^+ + \widehat{T}_\gamma^+$). Standard deviations for the estimators in the left tail $\left(\widehat{S}_\gamma^-,\ \widehat{F}_\gamma^-,\ \widehat{T}_\gamma^-\right)$ are obtained by simply replacing $\widehat{S}_\gamma^+$ with $\widehat{S}_\gamma^-$ in the above formulas.

Finally, if $\gamma^* = \gamma^+$, the standard deviation of $\widehat{\pi}_A^+$ and $\widehat{\pi}_A^-$ are respectively given by $\widehat{\sigma}_{\widehat{\pi}_A^+} = \widehat{\sigma}_{\widehat{T}_{\gamma^*}^+}$, and $\widehat{\sigma}_{\widehat{\pi}_A^-} = \left(\widehat{\sigma}^2_{\widehat{\pi}_A^+} + \widehat{\sigma}^2_{\widehat{\pi}_0(\lambda^*)} - 2\left(\frac{1}{1-\lambda^*}\right)\widehat{S}_{\gamma^*}^+\frac{\widehat{W}(\lambda^*)}{M} - 2\,(\gamma^*/2)\,\widehat{\sigma}^2_{\widehat{\pi}_0(\lambda^*)}\right)^{\frac{1}{2}}$ (using the equality $\widehat{\pi}_A^+ = 1 - \widehat{\pi}_0^+ - \widehat{\pi}_A^-$). Otherwise if $\gamma^* = \gamma^-$, we just reverse the superscripts $+/-$ in the two formulas above.

B Monte-Carlo Analysis

B.1 Under Cross-Sectional Independence

We use Monte-Carlo simulations to examine the performance of all estimators used in the paper: $\widehat{\pi}_0$, $\widehat{\pi}_A^-$, $\widehat{\pi}_A^+$, $\widehat{S}_\gamma^-$, $\widehat{F}_\gamma^-$, $\widehat{T}_\gamma^-$, and $\widehat{S}_\gamma^+$, $\widehat{F}_\gamma^+$, $\widehat{T}_\gamma^+$. We generate the $M \times 1$ vector of fund monthly excess returns, r_t, according to the four-factor model (market, size, book-to-market, and momentum factors):

$$\begin{aligned} r_t &= \alpha + \beta F_t + \varepsilon_t, \qquad t = 1, ..., T, \\ F_t &\sim N(0, \Sigma_F), \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2 I), \end{aligned} \tag{16}$$

where α denotes the $M \times 1$ vector of fund alphas, and β is the $M \times 4$ matrix of factor loadings. The 4×1 vector of factor excess returns, F_t, is normally distributed with covariance matrix Σ_F. ε_t is the $M \times 1$ vector of normally distributed residuals. We initially assume that the residuals are cross-sectionally independent and have the same variance σ_ε^2, so that the covariance matrix of ε_t can simply be written as $\sigma_\varepsilon^2 I$, where I is the $M \times M$ identity matrix.

Our estimators are compared with their respective true population values defined as follows. The parameters π_0, π_A^-, and π_A^+ denote the true proportions of zero-alpha, unskilled, and skilled funds. The expected proportions of unlucky and lucky funds, $E(F_\gamma^-)$ and $E(F_\gamma^+)$, are both equal to $\pi_0 \cdot \gamma/2$. To determine the expected proportions of unskilled and skilled funds, $E(T_\gamma^-)$ and $E(T_\gamma^+)$, we use the fact that, under the alternative hypothesis $\alpha_i \neq 0$, the fund t-statistic follows a non-central student distribution with $T-5$ degrees of freedom and a noncentrality parameter equal to $T^{\frac{1}{2}}\alpha_A/\sigma_\varepsilon$ (Davidson and MacKinnon (2004), p. 169):

$$\begin{aligned} E(T_\gamma^-) &= \pi_A^- \cdot prob\left(t < t_{T-5,\gamma/2}\,|H_A, \alpha_A < 0\right), \\ E(T_\gamma^+) &= \pi_A^+ \cdot prob\left(t > t_{T-5,1-\gamma/2}\,|H_A, \alpha_A > 0\right), \end{aligned} \quad (17)$$

where $t_{T-5,\gamma/2}$ and $t_{T-5,1-\gamma/2}$ denote the quantiles of probability level $\gamma/2$ and $1-\gamma/2$, respectively (these quantiles correspond to the thresholds t_γ^- and t_γ^+ used in the text). Finally, we have $E(S_\gamma^-) = E(F_\gamma^-) + E(T_\gamma^-)$, and $E(S_\gamma^+) = E(F_\gamma^+) + E(T_\gamma^+)$.

To compute these population values, we need to set values for the (true) proportions π_0, π_A^-, π_A^+, as well as for the means of the non-central student distributions (required to compute Equation (17)). In order to set realistic values, we estimate π_0, π_A^-, and π_A^+ at the end of each of the final five years of our sample (2002-2006) using the entire return history for each fund up to that point in time. These estimates are then averaged to produce values that reflect the recent trend observed in Figure 4: $\pi_0 = 75\%$, $\pi_A^- = 23\%$, and $\pi_A^+ = 2\%$. To determine the means of the t-statistic distributions of the unskilled and skilled funds, we use a simple calibration method. We first compute the average $\widehat{T}_\gamma^-$ and $\widehat{T}_\gamma^+$ (at $\gamma = 0.20$) over the final 5 years of our sample (2002-2006). Inserting these values along with $\pi_A^- = 23\%$ and $\pi_A^+ = 2\%$ in Equation (17), we can determine what are the means of the distributions which satisfy both equalities. The resulting values are -2.5 and 3, and correspond to an annual four-factor alpha of -3.2% and 3.8%, respectively (using the equality $t_A = T^{\frac{1}{2}}\alpha_A/\sigma_\varepsilon$).

The total number of funds, M, used in the simulation is equal to 1,400.[25] The input for β is equal to the empirical loadings of a random draw of 1,400 funds (among the total population of 2,076 funds). Consistent with our database, we set $T = 384$ (months), $\sigma_\varepsilon = 0.021$ (equal to the empirical average across the 1,400 funds), and proxy Σ_F by its empirical counterpart. To build the vector of fund alphas, α, we need to determine the identity of the unskilled and skilled funds. This is done by randomly choosing 322 funds (i.e., 23% of the entire population) to which we assign a negative alpha (-3.2% per

[25] We use this sample size to allow for comparison with the dependence case (described hereafter), which uses a sample of 1,400 correlated fund returns. Since our original sample of funds is larger than 1,400 ($M = 2,076$), our assessment of the precision of the estimators in this section is conservative.

year), and 28 funds (2% of the population), to which we assign a positive alpha (3.8% per year).

After randomly drawing F_t and ε_t ($t = 1, ..., 384$), we construct the fund return time-series according to Equation (14), and compute their t-statistics by regressing the fund returns on the four-factor model. To determine the alpha p-values, we use the fact that the fund t-statistic follows a Student distribution with $T - 5$ degrees of freedom under the null hypothesis $\alpha_i = 0$. Then, we compute $\widehat{\pi}_0$, $\widehat{\pi}_A^-$, and $\widehat{\pi}_A^+$ using Equations (5) and (8). $\widehat{S}_\gamma^-$ and $\widehat{S}_\gamma^+$ correspond to the observed number of significant funds with negative and positive alphas, respectively. $\widehat{F}_\gamma^-$ and $\widehat{F}_\gamma^+$ are computed with Equation (6). $\widehat{T}_\gamma^-$ and $\widehat{T}_\gamma^+$ are given in Equation (7). We repeat this procedure 1,000 times.

In Table AI, we compare the average value of each estimator (over the 1,000 replications) with the true values. The figures in parentheses denote the lower and upper bounds of the estimator 90%-confidence interval. We set γ equal to 0.05 and 0.20. In all cases, the simulation results reveal that the average values of our estimators closely match the true values, and that their 90%-confidence intervals are narrow. This result is not surprising in light of the large cross-section of funds available in our sample.

Please insert Table AI here

B.2 Under Cross-Sectional Dependence

The return-generating process is the same as the one shown in Equation (16), except that the fund residuals are cross-correlated:

$$\varepsilon_t \sim N(0, \Sigma), \tag{18}$$

where Σ denotes the $M \times M$ residual covariance matrix. The main constraint imposed on Σ is that it must be positive semi-definite. To achieve this, we select all funds with 60 valid return observations over the final five years (2002-2006), which is the period over which we have the largest possible cross-section of funds existing simultaneously–898 funds, whose covariance matrix, Σ_1, is directly estimated from the data.[26] To assess the precision of our estimators, we also need to account for the non-overlapping returns observed in the long-term fund data due to funds that do not exist at the same time. To address this issue, we introduce 502 uncorrelated funds, and write the covariance matrix

[26] The 25%, 50%, and 75% pairwise correlation quantiles are -0.09, 0.05, and 0.19, respectively.

for the resulting 1,400 funds as follows:[27]

$$\Sigma = \begin{pmatrix} \Sigma_1 & 0 \\ 0 & \sigma_\varepsilon^2 I \end{pmatrix}. \tag{19}$$

An an input for β, we use the empirical factor loadings of the 898 funds, along with the loadings of a random draw of the 502 remaining funds. The vector of fund alphas, α, is built by randomly choosing the identity of the unskilled and skilled funds, as in the independence case. The results in Table AII indicate that all estimators remain nearly unbiased ($\widehat{\pi}_0$, $\widehat{\pi}_A^-$, and $\widehat{\pi}_A^+$ exhibit small biases). Looking at the 90% confidence intervals, we logically observe that the dispersion of the estimators widens under cross-sectional dependence. However, the performance of the estimators is still very good.

Please insert Table AII here

Apart from this baseline dependence scenario, we also examine three other cases (the results are available upon request). First, we introduce correlation by block among each skill group (zero-alpha, unskilled, and skilled funds) to account for their possible similar bets. Inside each block (representing 10% of each skill group), we set the pairwise correlation equal to 0.15 or 0.30. Second, we use the residual factor specification proposed by Jones and Shanken (2005) in order to capture the role of non-priced factors. We assume that all fund residuals depend on a common residual factor, and that the unskilled and skilled funds are affected by specific residual factors. In the two cases, the results show that the precision of the estimators remain very close to those obtained under the independence case. Finally, we consider the extreme dependence case where the fund population only consists of the 898 correlated funds. We find that all estimators remain unbiased as shown in Tables AI and AII. But unsurprisingly, the confidence intervals widen slightly compared to those shown in Table AII (on average 2% are added on each side of the interval).

[27] The total number of fund pairs, P, is given by $M(M-1)/2$, where $M = 1,400$. If there are X uncorrelated funds in the population, the total number of uncorrelated fund pairs, I, equals $X \cdot (M - X) + X(X-1)/2$. In our data, 15% of the funds pairs do not have any return observations in common, and 55% of the observations are common to the remaining pairs (85%). Therefore, we estimate that the proportion of uncorrelated pairs is equal to 53% ($15\% + 85\% \cdot 45\%$). With 502 uncorrelated funds, I/P amounts to 58%, and is very close to the ratio observed in the data.

Table AI

Monte-Carlo Analysis under Cross-Sectional Independence

We examine the average value and the 90%-confidence interval (in parentheses) of the different estimators based on 1,000 replications. For each replication, we generate monthly fund returns for 1,400 funds and 384 periods using the four-factor model (market, size, book-to-market, and momentum factors). Fund residuals are independent from one another. The true parameter values for the proportions of zero-alpha, unskilled, and skilled funds (π_0, π_A^-, and π_A^+) are set to 75%, 23%, and 2%. We set the true four-factor annual alpha equal to -3.2% for the unskilled funds and +3.8% for the skilled ones. In each tail (left and right), we assess the precision of the different estimators at two significance levels (γ=0.05 and 0.20).

Fund Proportion	True	Estimator (90% interval)
Zero-alpha funds (π_0)	75.0	75.1 (71.7,78.6)
Unskilled funds (π_A^-)	23.0	22.9 (19.7,25.9)
Skilled funds (π_A^+)	2.0	2.0 (0.3,3.8)

	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
Left Tail	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E\left(S_\gamma^-\right)$	18.1	18.1 (16.4,19.7)	27.9	27.9 (26.1,30.0)
Unlucky funds $E\left(F_\gamma^-\right)$	1.8	1.8 (1.8,1.9)	7.5	7.5 (7.1,7.9)
Unskilled funds $E\left(T_\gamma^-\right)$	16.2	16.2 (14.6,17.9)	20.4	20.4 (18.2,22.7)

	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
Right Tail	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E\left(S_\gamma^+\right)$	3.6	3.6 (2.8,4.4)	9.4	9.4 (8.2,10.8)
Lucky funds $E\left(F_\gamma^+\right)$	1.8	1.8 (1.8,1.9)	7.5	7.5 (7.1,7.9)
Skilled funds $E\left(T_\gamma^+\right)$	1.7	1.7 (0.9,2.5)	1.9	1.9 (0.5,3.3)

Table AII

Monte-Carlo Analysis under Cross-Sectional Dependence

We examine the average value and the 90%-confidence interval (in parentheses) of the different estimators based on 1,000 replications. For each replication, we generate monthly fund returns for 1,400 funds and 384 periods using the four-factor model (market, size, book-to-market, and momentum factors). We assume that funds are cross-sectionally correlated and use the emprical covariance matrix of the fund residuals as the true covariance matrix. The true parameter values for the proportions of zero-alpha, unskilled, and skilled funds (π_0, π_A^-, and π_A^+) are set to 75%, 23%, and 2%. We set the true four-factor annual alpha equal to -3.2% for the unskilled funds and +3.8% for the skilled ones. In each tail (left and right), we assess the precision of the different estimators at two significance levels (γ=0.05 and 0.20).

Fund Proportion	True	Estimator (90% interval)		
Zero-alpha funds (π_0)	75.0	75.2 (69.5,80.8)		
Unskilled funds (π_A^-)	23.0	22.8 (17.0,28.9)		
Skilled funds (π_A^+)	2.0	1.9 (0.0,6.5)		
	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
Left Tail	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E\left(S_\gamma^-\right)$	18.1	18.1 (15.3,20.7)	27.9	27.9 (24.3,32.3)
Unlucky funds $E\left(F_\gamma^-\right)$	1.8	1.8 (1.6,2.1)	7.5	7.6 (6.6,8.3)
Unskilled funds $E\left(T_\gamma^-\right)$	16.2	16.2 (13.4,19.1)	20.4	20.4 (16.3,24.6)
	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
Right Tail	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E\left(S_\gamma^+\right)$	3.5	3.6 (2.4,5.3)	9.4	9.4 (6.6,12.6)
Lucky funds $E\left(F_\gamma^+\right)$	1.8	1.8 (1.6,2.1)	7.5	7.6 (6.6,8.3)
Skilled funds $E\left(T_\gamma^+\right)$	1.7	1.7 (0.5,3.8)	1.9	1.9 (0.1,5.6)

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Table I

Performance of the Equally-Weighted Portfolio of Funds

Results for the unconditional and conditional four-factor models are shown in Panels A and B for the entire fund population (All funds), as well as Growth, Aggressive Growth, and Growth & Income funds. The regressions are based on monthly data between January 1975 and December 2006. Each Panel contains the estimated annualized alpha ($\widehat{\alpha}$), the estimated exposures to the market ($\widehat{b}_m$), size ($\widehat{b}_{smb}$), book-to-market ($\widehat{b}_{hml}$), and momentum factors ($\widehat{b}_{mom}$), as well as the adjusted R^2 of an equally-weighted portfolio that includes all funds that exist at the beginning of each month. Figures in parentheses denote the Newey-West (1987) heteroskedasticity and autocorrelation consistent estimates of p-values, under the null hypothesis that the regression parameters are equal to zero.

Panel A Unconditional Four-Factor Model

	$\widehat{\alpha}$	$\widehat{b}_m$	$\widehat{b}_{smb}$	$\widehat{b}_{hml}$	$\widehat{b}_{mom}$	R^2
All (2,076)	-0.48%	0.95	0.17	-0.01	0.02	98.0%
	(0.12)	(0.00)	(0.00)	(0.38)	(0.09)	
Growth (1,304)	-0.45%	0.95	0.16	-0.03	0.02	98.0%
	(0.16)	(0.00)	(0.00)	(0.15)	(0.07)	
Aggressive	-0.53%	1.04	0.43	-0.17	0.09	95.8%
Growth (388)	(0.22)	(0.00)	(0.00)	(0.00)	(0.00)	
Growth &	-0.47%	0.87	-0.04	0.17	-0.03	98.2%
Income (384)	(0.09)	(0.00)	(0.02)	(0.00)	(0.01)	

Panel B Conditional Four-Factor Model

	$\widehat{\alpha}$	$\widehat{b}_m$	$\widehat{b}_{smb}$	$\widehat{b}_{hml}$	$\widehat{b}_{mom}$	R^2
All (2,076)	-0.60%	0.96	0.17	-0.02	0.02	98.2%
	(0.09)	(0.00)	(0.00)	(0.23)	(0.08)	
Growth (1,304)	-0.59%	0.96	0.16	-0.03	0.03	98.2%
	(0.10)	(0.00)	(0.00)	(0.08)	(0.05)	
Aggressive	-0.49%	1.05	0.43	-0.19	0.08	96.2%
Growth (388)	(0.24)	(0.00)	(0.00)	(0.00)	(0.00)	
Growth &	-0.58%	0.87	-0.04	0.16	-0.03	98.3%
Income (384)	(0.05)	(0.00)	(0.02)	(0.00)	(0.02)	

Table II

Impact of Luck on Long-Term Performance

Panel A displays the estimated proportions of zero-alpha, unskilled, and skilled funds in the entire fund population (2,076 funds). We measure fund performance with the unconditional four-factor model over the entire period 1975-2006. Panel B counts the proportions of significant funds in the left and right tails of the cross-sectional t-statistic distribution ($\widehat{S}_{\gamma}^{-}$, $\widehat{S}_{\gamma}^{+}$) at four significance levels (γ=0.05, 0.10, 0.15, 0.20). In the leftmost columns, the significant group in the left tail, $\widehat{S}_{\gamma}^{-}$, is decomposed into unlucky and unskilled funds ($\widehat{F}_{\gamma}^{-}$, $\widehat{T}_{\gamma}^{-}$). In the rightmost columns , the significant group in the right tail, $\widehat{S}_{\gamma}^{+}$, is decomposed into lucky and skilled funds ($\widehat{F}_{\gamma}^{+}$, $\widehat{T}_{\gamma}^{+}$). Figures in parentheses denote the standard deviation of the different estimators. The bottom of Panel B also presents the characteristics of each significant group ($\widehat{S}_{\gamma}^{-}$, $\widehat{S}_{\gamma}^{+}$): the average estimated alpha (% per year), expense ratio (% per year), turnover (% per year), and median size measured by the total net asset under management (millions USD).

Panel A Proportion of Unskilled and Skilled Funds

	Zero alpha($\widehat{\pi}_0$)	Non-zero alpha	Unskilled($\widehat{\pi}_A^{-}$)	Skilled($\widehat{\pi}_A^{+}$)
Proportion	75.4 (2.5)	24.6	24.0 (2.3)	0.6 (0.8)
Number	1,565	511	499	12

Panel B Impact of Luck in the Left and Right Tails

	Left Tail				Right Tail				
Signif. level(γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level(γ)
Signif. $\widehat{S}_{\gamma}^{-}$(%)	11.6	17.2	21.5	25.4	8.2	6.0	4.2	2.2	Signif. $\widehat{S}_{\gamma}^{+}$(%)
	(0.7)	(0.8)	(0.9)	(0.9)	(0.6)	(0.5)	(0.4)	(0.3)	
Unlucky $\widehat{F}_{\gamma}^{-}$(%)	1.9	3.8	5.6	7.6	7.6	5.6	3.8	1.9	Lucky $\widehat{F}_{\gamma}^{+}$(%)
	(0.0)	(0.1)	(0.2)	(0.3)	(0.3)	(0.2)	(0.1)	(0.0)	
Unskilled $\widehat{T}_{\gamma}^{-}$(%)	9.8	13.6	16.1	18.2	0.6	0.4	0.4	0.3	Skilled $\widehat{T}_{\gamma}^{+}$(%)
	(0.7)	(0.9)	(1.0)	(1.1)	(0.7)	(0.6)	(0.5)	(0.3)	
Alpha(% year)	-5.5	-5.0	-4.7	-4.6	4.8	5.2	5.6	6.5	Alpha(% year)
Exp.(% year)	1.6	1.5	1.5	1.5	1.2	1.2	1.2	1.2	Exp.(% year)
Turn.(% year)	100	99	98	96	126	95	95	105	Turn.(% year)
Size(million $)	81	88	86	84	731	985	888	745	Size(million $)

Table III

Impact of Luck on Short-Term Performance

Panel A displays the estimated proportions of zero-alpha, unskilled, and skilled funds in the entire fund population (3,311 funds). We measure fund performance with the unconditional four-factor model over non-overlapping 5-year periods between 1977-2006. Panel B counts the proportions of significant funds in the left and right tails of the cross-sectional t-statistic distribution ($\widehat{S}_\gamma^-$, $\widehat{S}_\gamma^+$) at four significance levels (γ=0.05, 0.10, 0.15, 0.20). In the leftmost columns, the significant group in the left tail, $\widehat{S}_\gamma^-$, is decomposed into unlucky and unskilled funds ($\widehat{F}_\gamma^-$, $\widehat{T}_\gamma^-$). In the rightmost columns , the significant group in the right tail, $\widehat{S}_\gamma^+$, is decomposed into lucky and skilled funds ($\widehat{F}_\gamma^+$, $\widehat{T}_\gamma^+$). Figures in parentheses denote the standard deviation of the different estimators. The bottom of Panel B also presents the characteristics of each significant group ($\widehat{S}_\gamma^-$, $\widehat{S}_\gamma^+$): the average estimated alpha (% per year), expense ratio (% per year), turnover (% per year), and median size measured by the total net asset under management (millions USD).

Panel A Proportion of Unskilled and Skilled Funds

	Zero alpha($\widehat{\pi}_0$)	Non-zero alpha	Unskilled($\widehat{\pi}_A^-$)	Skilled($\widehat{\pi}_A^+$)
Proportion	72.2 (2.0)	27.8	25.4 (1.7)	2.4 (0.7)
Number	2,390	921	841	80

Panel B Impact of Luck in the Left and Right Tails

	Left Tail				Right Tail				
Signif. level(γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level(γ)
Signif. $\widehat{S}_\gamma^-$(%)	11.2	16.8	21.4	24.9	9.6	7.8	5.9	3.5	Signif. $\widehat{S}_\gamma^+$(%)
	(0.5)	(0.6)	(0.7)	(0.8)	(0.5)	(0.5)	(0.4)	(0.3)	
Unlucky $\widehat{F}_\gamma^-$(%)	1.8	3.6	5.4	7.2	7.2	5.4	3.6	1.8	Lucky $\widehat{F}_\gamma^+$(%)
	(0.0)	(0.0)	(0.1)	(0.2)	(0.2)	(0.1)	(0.0)	(0.0)	
Unskilled $\widehat{T}_\gamma^-$(%)	9.4	13.2	16.0	17.7	2.4	2.4	2.3	1.7	Skilled $\widehat{T}_\gamma^+$(%)
	(0.6)	(0.7)	(0.8)	(0.8)	(0.6)	(0.5)	(0.4)	(0.3)	
Alpha(% year)	-6.5	-5.9	-5.5	-5.3	6.7	7.0	7.2	7.5	Alpha(% year)
Exp.(% year)	1.4	1.3	1.3	1.3	1.2	1.2	1.2	1.2	Exp.(% year)
Turn.(% year)	98	105	100	99	80	81	83	81	Turn.(% year)
Size(million \$)	242	244	252	244	623	628	664	749	Size(million \$)

Table IV

Short-Term Performance across Investment Categories

The impact of luck in the left and right tails for three investment categories (Growth, Aggressive Growth, and Growth & Income funds) is presented in Panels A, B, and C, respectively. We measure fund performance with the unconditional four-factor model over non-overlapping 5-year periods between 1977-2006. For each panel, we count the proportions of significant funds in the left and right tails of the cross-sectional t-statistic distribution ($\widehat{S}_{\gamma}^{-}$, $\widehat{S}_{\gamma}^{+}$) at four significance levels (γ=0.05, 0.10, 0.15, 0.20). In the leftmost columns, the significant group in the left tail, $\widehat{S}_{\gamma}^{-}$, is decomposed into unlucky and unskilled funds ($\widehat{F}_{\gamma}^{-}$, $\widehat{T}_{\gamma}^{-}$). In the rightmost columns , the significant group in the right tail, $\widehat{S}_{\gamma}^{+}$, is decomposed into lucky and skilled funds ($\widehat{F}_{\gamma}^{+}$, $\widehat{T}_{\gamma}^{+}$). Figures in parentheses denote the standard deviation of the different estimators. The bottom of Panel B also presents the characteristics of each significant group ($\widehat{S}_{\gamma}^{-}$, $\widehat{S}_{\gamma}^{+}$): the average estimated alpha (% per year), expense ratio (% per year), turnover (% per year), and median size measured by the total net asset under management (millions USD).

Panel A Growth funds

	Left Tail				Right Tail				
Signif. level(γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level(γ)
Signif. $\widehat{S}_{\gamma}^{-}$(%)	11.3	16.6	21.4	25.2	9.9	8.1	5.9	3.5	Signif. $\widehat{S}_{\gamma}^{+}$(%)
	(0.7)	(0.8)	(0.9)	(1.0)	(0.7)	(0.6)	(0.5)	(0.4)	
Unlucky $\widehat{F}_{\gamma}^{-}$(%)	1.8	3.6	5.5	7.3	7.3	5.5	3.6	1.8	Lucky $\widehat{F}_{\gamma}^{+}$ (%)
	(0.0)	(0.1)	(0.2)	(0.2)	(0.2)	(0.2)	(0.1)	(0.0)	
Unskilled $\widehat{T}_{\gamma}^{-}$(%)	9.5	13.0	15.9	17.9	2.6	2.6	2.3	1.7	Skilled $\widehat{T}_{\gamma}^{+}$ (%)
	(0.7)	(0.9)	(1.0)	(1.1)	(0.8)	(0.7)	(0.6)	(0.4)	
Alpha(% year)	-6.0	-5.6	-5.2	-5.1	6.8	6.8	6.8	7.3	Alpha(% year)
Exp.(% year)	1.4	1.3	1.3	1.3	1.2	1.2	1.2	1.2	Exp.(% year)
Turn.(% year)	101	113	110	107	79	79	79	75	Turn.(% year)
Size(million $)	253	253	253	252	589	593	621	580	Size(million $)

Table IV

Short-Term Performance across Investment Categories (Continued)

Panel B Aggressive Growth funds

	Left Tail				Right Tail				
Signif. level(γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level(γ)
Signif. $\widehat{S}_{\gamma}^{-}$(%)	12.0	16.0	19.4	22.2	11.2	9.4	7.1	4.9	Signif. $\widehat{S}_{\gamma}^{+}$(%)
	(1.1)	(1.4)	(1.6)	(1.8)	(1.3)	(1.1)	(1.0)	(0.8)	
Unlucky $\widehat{F}_{\gamma}^{-}$(%)	1.8	3.6	5.4	7.2	7.2	5.4	3.6	1.8	Lucky $\widehat{F}_{\gamma}^{+}$(%)
	(0.1)	(0.2)	(0.3)	(0.4)	(0.4)	(0.3)	(0.2)	(0.1)	
Unskilled $\widehat{T}_{\gamma}^{-}$(%)	10.2	12.3	14.0	15.0	4.0	4.0	3.5	3.1	Skilled $\widehat{T}_{\gamma}^{+}$(%)
	(1.3)	(1.6)	(1.7)	(1.9)	(1.4)	(1.2)	(1.1)	(0.9)	
Alpha(% year)	-9.3	-8.6	-8.1	-7.6	8.5	8.8	9.7	9.7	Alpha(% year)
Exp.(% year)	1.5	1.5	1.5	1.5	1.3	1.3	1.3	1.3	Exp.(% year)
Turn.(% year)	127	123	119	117	105	104	107	104	Turn.(% year)
Size(million $)	154	187	160	192	1,014	949	1,021	1,073	Size(million $)

Panel C Growth & Income funds

	Left Tail				Right Tail				
Signif. level(γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level(γ)
Signif. $\widehat{S}_{\gamma}^{-}$(%)	11.5	17.4	22.5	26.8	7.3	5.5	3.7	1.8	Signif. $\widehat{S}_{\gamma}^{+}$(%)
	(1.1)	(1.4)	(1.5)	(1.6)	(1.0)	(0.8)	(0.7)	(0.5)	
Unlucky $\widehat{F}_{\gamma}^{-}$ (%)	1.8	3.7	5.5	7.3	7.3	5.5	3.7	1.8	Lucky $\widehat{F}_{\gamma}^{+}$(%)
	(0.1)	(0.2)	(0.3)	(0.4)	(0.4)	(0.3)	(0.2)	(0.1)	
Unskilled $\widehat{T}_{\gamma}^{-}$ (%)	9.8	13.7	17.0	19.5	0.0	0.0	0.0	0.0	Skilled $\widehat{T}_{\gamma}^{+}$(%)
	(1.2)	(1.5)	(1.7)	(1.8)	(1.1)	(0.9)	(0.8)	(0.5)	
Alpha(% year)	-4.9	-4.5	-4.2	-4.0	4.9	5.3	5.1	4.9	Alpha(% year)
Exp.(% year)	1.3	1.2	1.2	1.2	1.1	1.1	1.0	1.0	Exp.(% year)
Turn.(% year)	69	68	66	64	56	57	54	45	Turn.(% year)
Size(million $)	295	346	348	337	492	482	473	1,787	Size(million $)

Table V

Performance Persistence Based on the False Discovery Rate

For each of the five FDR targets (10%, 30%, 50%, 70%, and 90%), Panel A contains descriptive statistics on the FDR level ($\widehat{FDR}^+_{\gamma^P}$) achieved by each portfolio, as well as the proportion of funds in the population that it includes ($\widehat{S}^+_{\gamma^P}$). The panel shows the average values of $\widehat{FDR}^+_{\gamma^P}$ and $\widehat{S}^+_{\gamma^P}$ over the 27 annual formation dates (from December 1979 to 2005), as well as their respective distributions. Panel B displays the performance of each portfolio over the period 1980-2006. We estimate the annual four-factor alpha ($\widehat{\alpha}$) with its bootstrap p-value, its annual residual standard deviation ($\widehat{\sigma}_\varepsilon$), its annual information ratio (IR=$\widehat{\alpha}/\widehat{\sigma}_\varepsilon$), its loadings on the market ($\widehat{b}_m$), size ($\widehat{b}_{smb}$), book-to-market ($\widehat{b}_{hml}$), and momentum factors ($\widehat{b}_{mom}$), and its annual excess mean, and standard deviation. In Panel C, we examine the turnover of each portfolio. We compute the proportion of funds that are still included in the portfolio 1, 2, 3, 4, and 5 years after their initial selection.

Panel A Portfolio Statistics

	Achieved False Discovery Rate $\left(\widehat{FDR}^+_{\gamma^P}\right)$					Included proportion of funds($\widehat{S}^+_{\gamma^P}$)				
	Mean	10-30	30-50	50-70	>70%	Mean	0-6	6-12	12-24	>24%
FDR10%	41.5%	14	6	1	6	3.0%	25	2	0	0
FDR30%	47.5%	8	12	1	6	8.2%	15	7	3	2
FDR50%	60.4%	0	14	7	6	20.9%	5	7	4	11
FDR70%	71.3%	0	4	12	11	29.7%	1	5	5	16
FDR90%	75.0%	0	4	9	14	33.7%	0	3	4	20

Panel B Performance Analysis

	$\widehat{\alpha}$(p-value)	$\widehat{\sigma}_\varepsilon$	IR	$\widehat{b}_m$	$\widehat{b}_{smb}$	$\widehat{b}_{hml}$	$\widehat{b}_{mom}$	Mean	Std dev
FDR10%	1.45%(0.04)	4.0%	0.36	0.93	0.16	-0.04	-0.02	8.3%	15.4%
FDR30%	1.15%(0.05)	3.3%	0.35	0.94	0.17	-0.02	-0.03	8.1%	15.4%
FDR50%	0.95%(0.10)	2.9%	0.33	0.96	0.20	-0.06	-0.01	8.1%	16.1%
FDR70%	0.68%(0.15)	2.7%	0.25	0.97	0.19	-0.06	-0.01	7.9%	16.1%
FDR90%	0.39%(0.30)	2.7%	0.14	0.97	0.19	-0.05	-0.00	7.8%	16.0%

Panel C Portfolio Turnover

	Proportion of funds remaining in the portfolio...				
	After 1 year	After 2 years	After 3 years	After 4 years	After 5 years
FDR10%	36.7	12.8	3.4	0.8	0.0
FDR30%	40.0	14.7	5.1	1.7	1.3
FDR50%	48.8	23.5	12.3	4.7	2.6
FDR70%	52.2	29.0	17.4	9.5	6.3
FDR90%	55.9	33.8	20.4	13.0	8.5

Table VI

Impact of Luck on Long-Term Pre-Expense Performance

Panel A displays the estimated proportions of zero-alpha, unskilled, and skilled funds in the entire fund population on a pre-expense basis (1,836 funds). We add the monthly expenses to net return of each fund, and measure performance with the unconditional four-factor model over the entire period 1975-2006. Panel B counts the proportions of significant funds in the left and right tails of the cross-sectional t-statistic distribution ($\widehat{S}_{\gamma}^{-}$, $\widehat{S}_{\gamma}^{+}$) at four significance levels (γ=0.05, 0.10, 0.15, 0.20). In the leftmost columns, the significant group in the left tail, $\widehat{S}_{\gamma}^{-}$, is decomposed into unlucky and unskilled funds ($\widehat{F}_{\gamma}^{-}$, $\widehat{T}_{\gamma}^{-}$). In the rightmost columns , the significant group in the right tail, $\widehat{S}_{\gamma}^{+}$, is decomposed into lucky and skilled funds ($\widehat{F}_{\gamma}^{+}$, $\widehat{T}_{\gamma}^{+}$). Figures in parentheses denote the standard deviation of the different estimators. The bottom of Panel B also presents the characteristics of each significant group ($\widehat{S}_{\gamma}^{-}$, $\widehat{S}_{\gamma}^{+}$): the average estimated alpha prior to expenses (in % per year), expense ratio (in % per year), turnover (in % per year), and median size measured by the total net asset under management (in millions of dollars).

Panel A Proportion of Unskilled and Skilled Funds

	Zero alpha($\widehat{\pi}_0$)	Non-zero alpha	Unskilled($\widehat{\pi}_A^{-}$)	Skilled($\widehat{\pi}_A^{+}$)
Proportion	85.9 (2.7)	14.1	4.5 (1.0)	9.6 (1.5)
Number	1,577	259	176	83

Panel B Impact of Luck in the Left and Right Tails

	Left Tail				Right Tail				
Signif. level(γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level(γ)
Signif. $\widehat{S}_{\gamma}^{-}$(%)	4.3	7.5	10.2	12.8	17.3	13.1	9.3	5.8	Signif. $\widehat{S}_{\gamma}^{+}$(%)
	(0.5)	(0.6)	(0.7)	(0.8)	(0.9)	(0.8)	(0.7)	(0.5)	
Unlucky $\widehat{F}_{\gamma}^{-}$(%)	2.1	4.3	6.4	8.6	8.6	6.4	4.3	2.1	Lucky $\widehat{F}_{\gamma}^{+}$(%)
	(0.0)	(0.1)	(0.1)	(0.2)	(0.2)	(0.1)	(0.1)	(0.0)	
Unskilled $\widehat{T}_{\gamma}^{-}$(%)	2.2	3.2	3.8	4.2	8.7	6.6	5.0	3.6	Skilled $\widehat{T}_{\gamma}^{+}$(%)
	(0.5)	(0.6)	(0.8)	(0.9)	(1.0)	(0.9)	(0.7)	(0.5)	
Pre Expense									Pre Expense
Alpha(% year)	-5.9	-5.2	-4.8	-4.5	4.4	4.8	5.0	5.3	Alpha(% year)
Exp.(% year)	1.5	1.4	1.3	1.3	1.3	1.3	1.3	1.3	Exp.(% year)
Turn.(% year)	105	108	108	108	106	111	122	84	Turn.(% year)
Size(million $)	81	90	93	90	430	578	948	1,000	Size(million $)

Table VII

Loadings on Omitted Factors

We determine the proportions of significant funds in the left and right tails ($\widehat{S}_{\gamma}^{-}$, $\widehat{S}_{\gamma}^{+}$) at four significance levels (γ=0.05, 0.10, 0.15, 0.20) according to each asset-pricing model over the period 1975-2006. For each of these significant groups, we compute their average loadings on the omitted factors from the four-factor model: size ($\widehat{b}_{smb}$), book-to-market ($\widehat{b}_{hml}$), and momentum ($\widehat{b}_{mom}$). Panel A shows the results obtained with the unconditional CAPM, while Panel B repeats the same procedure with the unconditional Fama-French model.

Panel A Unconditional CAPM

	Left Tail				Right Tail				
Signif. level(γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level(γ)
Size($\widehat{b}_{smb}$)	0.06	0.07	0.09	0.09	0.27	0.28	0.28	0.36	Size($\widehat{b}_{smb}$)
Book($\widehat{b}_{hml}$)	-0.14	-0.14	-0.13	-0.14	0.34	0.35	0.36	0.37	Book($\widehat{b}_{hml}$)
Mom.($\widehat{b}_{mom}$)	0.00	0.00	0.00	0.01	-0.01	-0.01	-0.02	-0.01	Mom.($\widehat{b}_{mom}$)

Panel B Unconditional Fama-French model

	Left Tail				Right Tail				
Signif. level(γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level(γ)
Mom.($\widehat{b}_{mom}$)	-0.02	-0.03	-0.02	-0.03	0.09	0.10	0.11	0.12	Mom.($\widehat{b}_{mom}$)

Figure 1

Outcome of the Multiple Performance Test

Panel A shows the distribution of the fund t-statistic across the three skill groups (zero-alpha, unskilled, and skilled funds). We set the true four-factor alpha equal to -3.2% and +3.8% per year for the unskilled and skilled funds (implying that the t-statistic distributions are centered at -2.5 and +3). Panel B displays the cross-sectional t-statistic distribution. It is a mixture of the three distributions in Panel A, where the weight on each distribution depends on the proportion of zero-alpha, unskilled, and skilled funds in the population (π_0, π_A^-, and π_A^+). In this example, we set $\pi_0 = 75\%$, $\pi_A^- = 23\%$, and $\pi_A^+ = 2\%$ to match our average estimated values over the final 5 years of our sample.

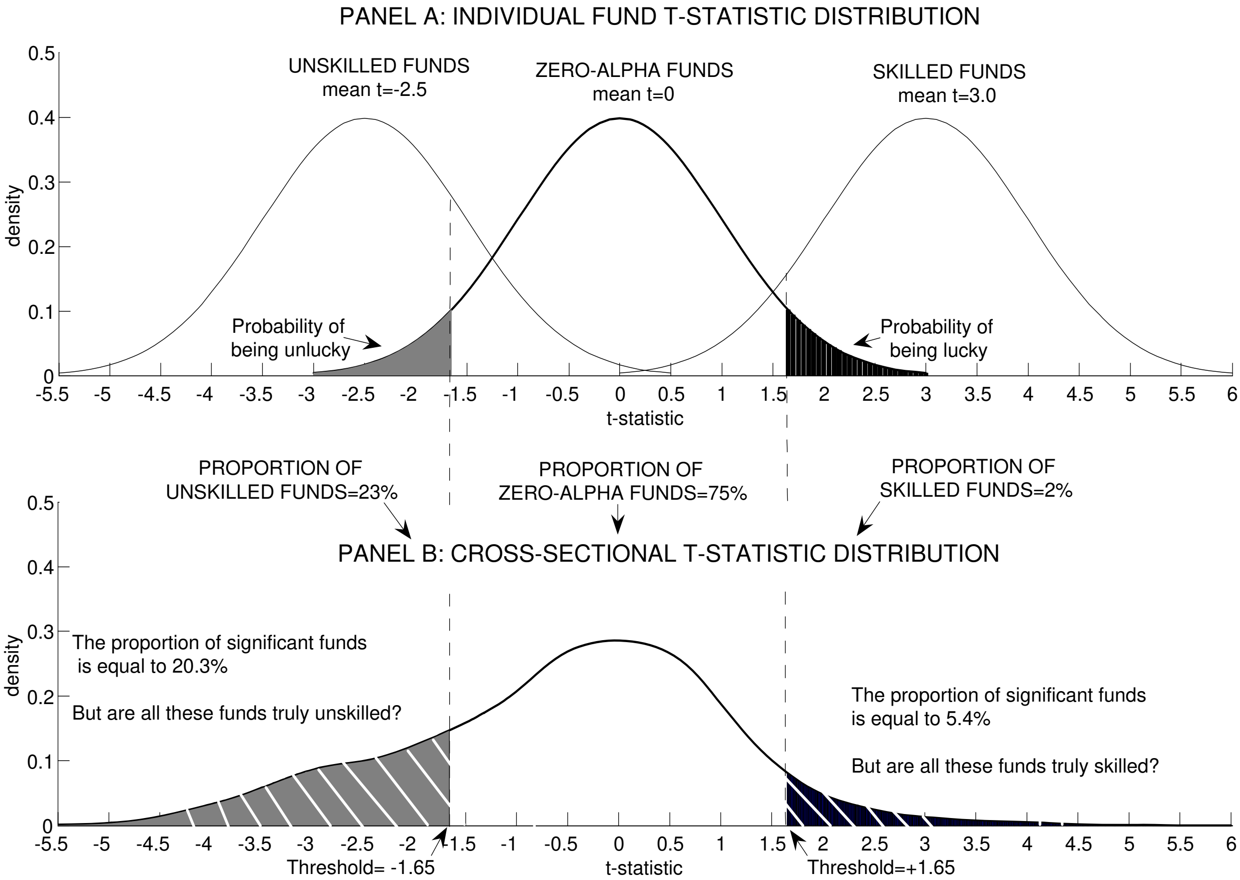


Figure 2
Histogram of Fund p-values

This figure represents the p-value histogram of M=2,076 funds (as in our database). For each fund, we draw its t-statistic from one of the distributions in Figure 1 (Panel A) according to the proportion of zero-alpha, unskilled, and skilled funds in the population (π_0, π_A^-, and π_A^+). In this example, we set $\pi_0 = 75\%$, $\pi_A^- = 23\%$, and $\pi_A^+ = 2\%$ to match our average estimated values over the final 5 years of our sample. Then, we compute the two-sided p-values of each fund from its respective t-statistic, and plot them in the histogram.

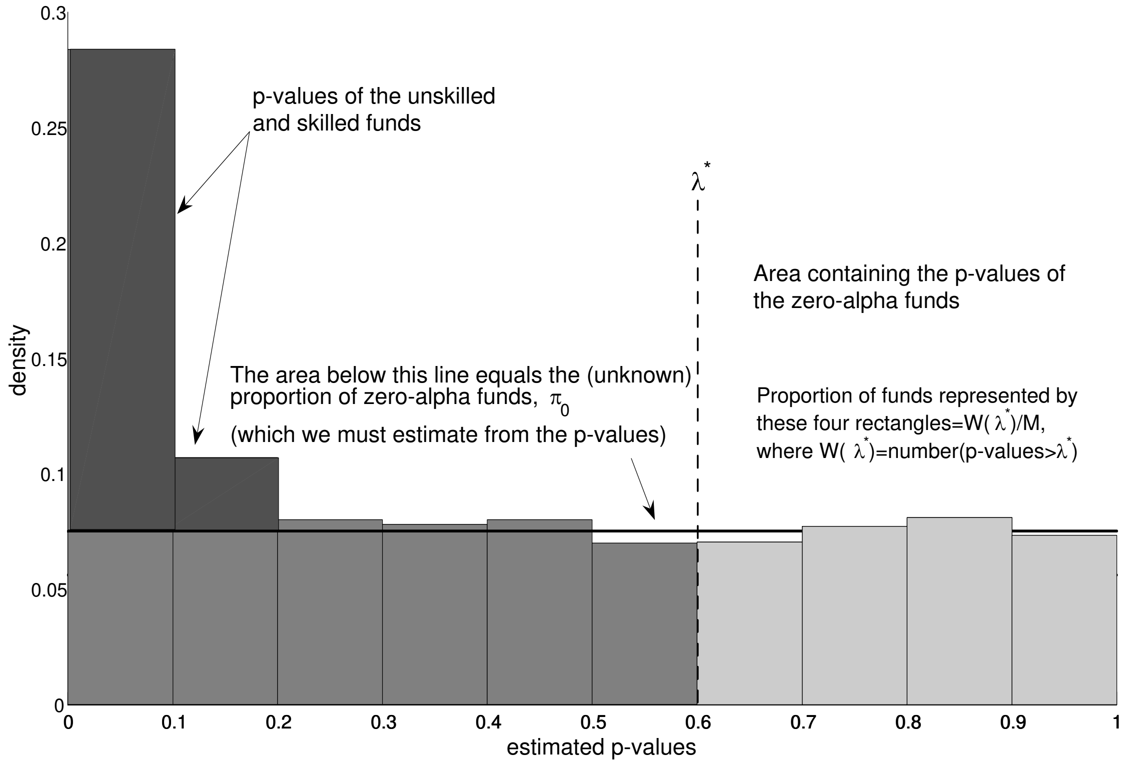
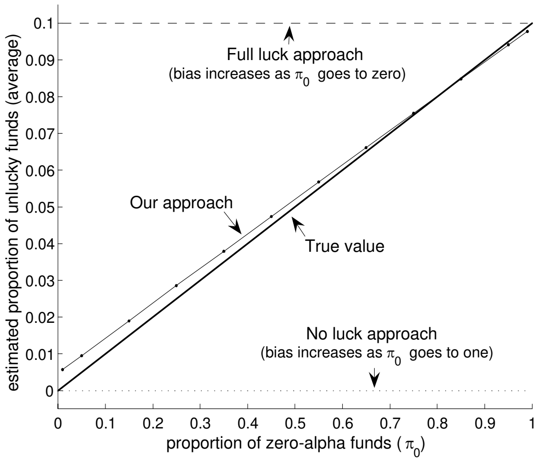


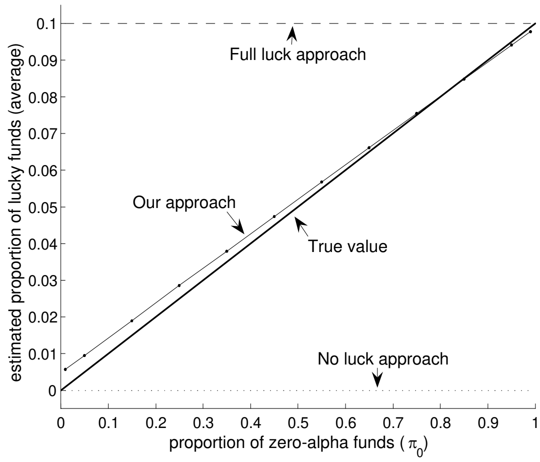
Figure 3

Measuring Luck: Comparison with Existing Approaches

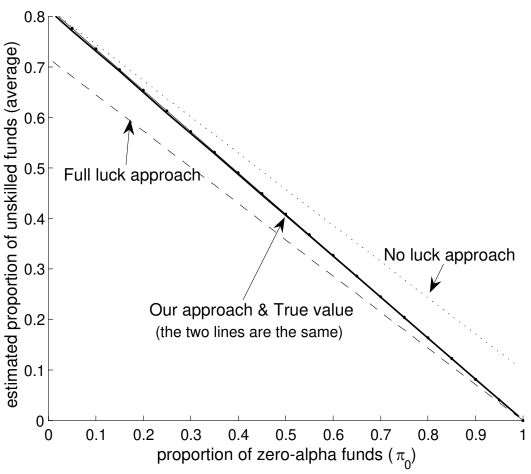
This figure examines the bias of different estimators produced by the three approaches ("no luck", "full luck", and "our approach") as a function of the proportion of zero-alpha funds, π_0. We examine the estimators of the proportions of unlucky, lucky, unskilled, and skilled funds in Panel A, B, C, and D, respectively. The "no luck" approach assumes that π_0=0, the "full luck" approach assumes that π_0=1, while "our approach" estimates π_0 directly from the data. For each approach, we compare the average estimator value (over 1,000 replications) with the true population value. For each replication, we draw the t-statistic for each fund i (i=1,...,2,076) from one of the distributions in Figure 1 (Panel A) according to the weights π_0, π_A^-, and π_A^+, and compute the different estimators at the significance level $\gamma = 0.20$. For each π_0, the ratio π_A^- over π_A^+ is held fixed to 11.5 (0.23/0.02) as in Figure 1.



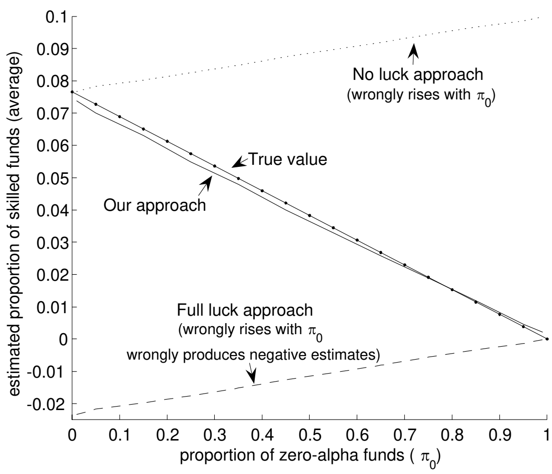
(A) Unlucky funds (left tail)



(B) Lucky funds (right tail)



(C) Unskilled funds (left tail)



(D) Skilled funds (right tail)

Figure 4

Evolution of Mutual Fund Performance over Time

Panel A plots the evolution of the estimated proportions of unskilled and skilled funds ($\widehat{\pi}_A^-$ and $\widehat{\pi}_A^+$) between 1989 and 2006. At the end of each year, we measure $\widehat{\pi}_A^-$ and $\widehat{\pi}_A^+$ using the entire fund return history up to that point. The initial estimates at the end of 1989 cover the period 1975-1989, while the last ones in 2006 use the period 1975-2006. The performance of each fund is measured with the unconditional four-factor model. Panel B displays the growth in the mutual fund industry (proxied by the total number of funds used to compute $\widehat{\pi}_A^-$ and $\widehat{\pi}_A^+$ over time), as well as its average alpha (in % per year).

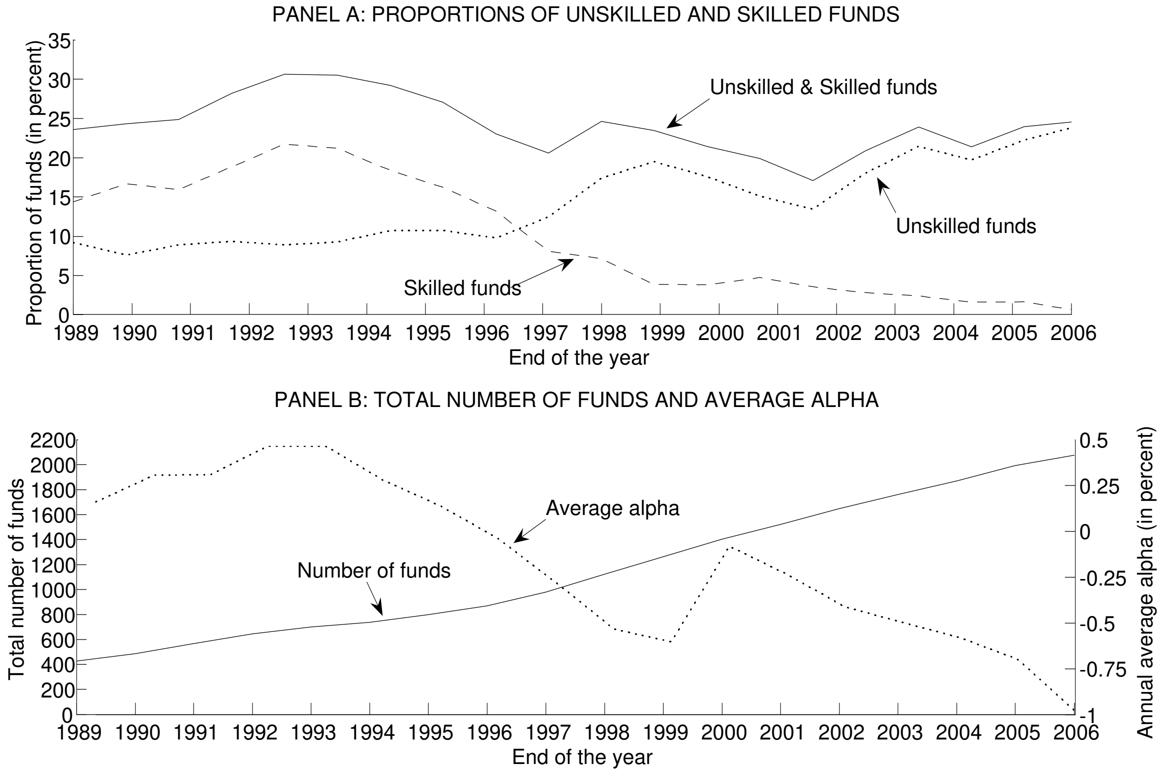


Figure 5

Performance of the Portfolios FDR10% over Time

The graph plots the evolution of the estimated annual four-factor alpha of the portfolio $FDR10\%$. This portfolio contains funds located in the right tail of the cross-sectional t-statistic distribution such that the targeted proportion of lucky funds included in the portfolio is equal to 10%. At the end of each year from 1989 to 2006, the portfolio's alpha is estimated using the portfolio return history up to that point. The initial estimates cover the period 1980-1989 (the first five years are used for the initial portfolio formation on December 31, 1979), while the last ones use the entire portfolio history from 1980 up to 2006. For comparison purposes, we also show the performance of top decile portfolios formed according to a t-statistic ranking, where the t-statistic is estimated over the prior one and three years, respectively.