# Far-field contribution of evanescent modes to the electromagnetic Green tensor

**Andrei V. Shchegrov and P. Scott Carney**

*Rochester Theory Center for Optical Science and Engineering and Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627-0171*

Received March 1, 1999; revised manuscript received May 28, 1999; accepted June 1, 1999

Understanding the behavior of the evanescent part of the electromagnetic field has important implications in many branches of modern physics, such as near-field optics. Motivated by recent disagreement in the literature, we derive an expression for the far-field asymptotic behavior of the free-space electromagnetic Green tensor that is due to the evanescent modes. © 1999 Optical Society of America [S0740-3232(99)01110-2]

*OCIS codes:* 350.5500, 260.2110.

In a series of papers[1–5] it has been argued that the evanescent modes contribute as $1/|\mathbf{x}|$ to the far-field behavior of the monochromatic, free-space electromagnetic Green tensor $G_{\alpha\beta}(\mathbf{x}, \omega)$. However, this argument leads to some unphysical results, as has been demonstrated in the literature.[6,7] The correct formula for the evanescent contribution was obtained in Ref. 7 but was left in integral form. This decomposition of the propagator is important in studies of the electromagnetic field scattered or radiated from a surface, for example in near-field microscopy.[8] Once the field and its normal derivative are known on the surface, the Green tensor allows the field to be determined elsewhere. In this communication we explicitly derive the asymptotic form of the evanescent contribution to the far-field behavior of the Green tensor.

The Green tensor can be expressed in terms of the scalar Green function $G_0(\mathbf{x}, \omega)$ for the Helmholtz equation in the form

$$G_{\alpha\beta}(\mathbf{x}, \omega) = \left(\delta_{\alpha\beta} + \frac{c^2}{\omega^2}\frac{\partial^2}{\partial x_\alpha \partial x_\beta}\right) G_0(\mathbf{x}, \omega), \tag{1}$$

where $\mathbf{x} = (x_1, x_2, x_3)$ is the position vector, $\omega$ is the frequency, $c$ is the speed of light in vacuum, and the outgoing condition at infinity is assumed. The scalar Green function may be represented as an angular spectrum of homogeneous and evanescent plane waves and is given by the expression[9]

$$G_0(\mathbf{x}, \omega) = \frac{1}{2\pi}\int \mathrm{d}^2 k_\parallel \frac{1}{\beta(k_\parallel, \omega)} \times \exp[i\mathbf{k}_\parallel \cdot \mathbf{x}_\parallel - \beta(k_\parallel, \omega)|x_3|], \tag{2}$$

where $\mathbf{x}_\parallel = (x_1, x_2, 0)$, $\mathbf{k}_\parallel = (k_1, k_2, 0)$, the integration extends over the whole $\mathbf{k}_\parallel$ plane, and

$$\beta(k_\parallel, \omega) = \begin{cases} \sqrt{k_\parallel{}^2 - \omega^2/c^2} & \text{for } k_\parallel > \omega/c \\ -i\sqrt{\omega^2/c^2 - k_\parallel{}^2} & \text{for } k_\parallel < \omega/c \end{cases}. \tag{3}$$

The contribution to integral (2) from evanescent waves corresponds to the range of integration $\omega/c < k_\parallel < \infty$. We denote this contribution $G_0{}^e(\mathbf{x}, \omega)$ and perform the angular part of the integration. Writing $x_\parallel = r\sin\theta$ and $x_3 = r\cos\theta$, with $0 \leqslant \theta \leqslant \pi$ and $r = |\mathbf{x}|$, we find that

$$G_0{}^e(\mathbf{x}, \omega) = \int_{\omega/c}^{\infty} \mathrm{d}k_\parallel \frac{k_\parallel}{\beta(k_\parallel, \omega)} J_0(k_\parallel r\sin\theta) \times \exp[-\beta(k_\parallel, \omega) r|\cos\theta|], \tag{4}$$

where $J_n$ is the $n$th-order Bessel function. To obtain the asymptotic behavior of $G_0{}^e(\mathbf{x}, \omega)$ in the far zone, $(\omega/c)r \gg 1$ (with $\theta$ fixed), we first integrate Eq. (4) by parts and obtain the exact result:

$$G_0{}^e(\mathbf{x}, \omega) = \frac{1}{r|\cos\theta|} J_0\left(\frac{\omega}{c} r\sin\theta\right) - |\tan\theta| \int_{\omega/c}^{\infty} \mathrm{d}k_\parallel J_1(k_\parallel r\sin\theta) \times \exp[-\beta(k_\parallel, \omega) r|\cos\theta|]. \tag{5}$$

We next analyze the second term on the right-hand side of Eq. (5) and find by standard asymptotic methods[10] that the dominant contribution to this integral comes from the vicinity of the boundary, i.e., the point $k_\parallel = \omega/c$. Explicitly, we obtain the asymptotic expression

$$G_0{}^e(\mathbf{x}, \omega) \sim \frac{1}{r|\cos\theta|} J_0\left(\frac{\omega}{c} r\sin\theta\right) - \frac{\sin\theta}{(\omega/c) r^2 |\cos^3\theta|} J_1\left(\frac{\omega}{c} r\sin\theta\right) + O(r^{-7/2}), \quad (\omega/c)r \gg 1. \tag{6}$$

To verify the correctness of asymptotic expression (6), we evaluated it and the integral in Eq. (4) numerically and found them to be in excellent agreement for large values of $(\omega/c)r$.

 

On the $x_3$ axis ($\theta = 0$, or $\pi$) Eq. (6) gives the exact result, *viz.*,

$$G_0{}^e(\mathbf{x}, \omega)|_{\theta=0,\pi} = 1/r, \tag{7}$$

as is well known in the scalar theory.[11] In contrast to the conclusions reached in Refs. 1–4, it is also clear from Eq. (6) that the $1/r$ behavior found on the $x_3$ axis does not hold for arbitrary directions. The directions specified by $\theta = 0$ and $\theta = \pi$ are special, as elaborated in Refs. 6 and 11. The off-axis behavior of $J_0[(\omega/c)r \sin\theta]$ in Eq. (6) for large $r$ is $r^{-1/2}$. Therefore the off-axis behavior of $G_0{}^e(\mathbf{x}, \omega)$ is $r^{-3/2}$. This clearly indicates that the evanescent waves do not contribute to the radiated energy in the far zone.

Finally, using Eq. (1), we give the leading term in the far-field contribution from the evanescent modes to the Green tensor:

$$G_{\alpha\beta}{}^e(\mathbf{x}, \omega) \sim \delta_{\alpha\beta}\frac{1}{r|\cos\theta|} J_0\left(\frac{\omega}{c} r \sin\theta\right) + O(r^{-5/2}),$$

$$(\omega/c)r \gg 1. \tag{8}$$

As in the scalar case, $G_{\alpha\beta}{}^e(\mathbf{x}, \omega)$ decays off axis in the far zone as $r^{-3/2}$.

In conclusion, the authors hope that this communication resolves the controversy in the literature regarding the far-field asymptotic behavior of the evanescent part of the electromagnetic Green tensor.

## ACKNOWLEDGMENT

This research was supported by the National Science Foundation and the New York State Foundation for Science and Technology.

## REFERENCES

1. M. Xiao, "A study of resolution limit in optical microscopy: near and far field," Opt. Commun. **132**, 403–409 (1996).
2. M. Xiao, "Evanescent field coupling of dipole to a surface: configurational resonance at long distances," Chem. Phys. Lett. **258**, 363–368 (1996).
3. M. Xiao, "On near-field scanning optical microscopy. Homogeneous and evanescent radiation," J. Mod. Opt. **44**, 327–344 (1997).
4. M. Xiao, "Two-point optical resolution with homogeneous, evanescent and self field: resolution criterion in near field imaging," J. Mod. Opt. **44**, 1609–1615 (1997).
5. M. Xiao, "Evanescent fields do contribute to the far field," J. Mod. Opt. **46**, 729–733 (1999).
6. E. Wolf and J. T. Foley, "Do evanescent waves contribute to the far field?" Opt. Lett. **23**, 16–18 (1998).
7. T. Setälä, M. Kaivola, and A. T. Friberg, "Decomposition of the point-dipole field into homogeneous and evanescent parts," Phys. Rev. E **59**, 1200–1206 (1999).
8. D. Courjon and C. Bannier, "Near field microscopy and near field optics," Rep. Prog. Phys. **57**, 989–1028 (1994).
9. H. Weyl, "Ausbreitung elektromagnetischer Wellen über einem ebenen Leiter," Ann. Phys. (Leipzig) **60**, 481–500 (1919). See also A. Baños, *Dipole Radiation in the Presence of a Conducting Half-Space* (Pergamon, New York, 1966), Eq. (2.19).
10. N. Bleistein and R. A. Handelsman, *Asymptotic Expansions of Integrals* (Holt, Rinehart & Winston, New York, 1975), Chap. 3.
11. G. C. Sherman, J. J. Stamnes, A. J. Devaney, and É Lalor, "Contribution of the inhomogeneous waves in angular-spectrum representations," Opt. Commun. **8**, 271–274 (1973).