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Far-Field Multicast Beamforming for Uniform Linear Antenna Arrays

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***Abstract*—The problem of transmit beamforming to multiple cochannel multicast groups is considered for the important special case when the channel vectors are Vandermonde. This arises when a uniform linear antenna antenna (ULA) array is used at the transmitter under far-field line-of-sight propagation conditions, as provisioned in 802.16e and related wireless backhaul scenarios. Two design approaches are pursued: i) minimizing the total transmitted power subject to providing at least a prescribed received signal-to-interference-plus-noise-ratio (SINR) to each intended receiver; and ii) maximizing the minimum received SINR under a total transmit power budget. Whereas these problems have been recently shown to be NP-hard, in general, it is proven here that for Vandermonde channel vectors, it is possible to recast the optimization in terms of the autocorrelation sequences of the sought beamvectors, yielding an equivalent convex reformulation. This affords efficient optimal solution using modern interior point methods. The optimal beamvectors can then be recovered using spectral factorization. Robust extensions for the case of partial channel state information, where the direction of each receiver is known to lie in an interval, are also developed. Interestingly, these also admit convex reformulation. The various optimal designs are illustrated and contrasted in a suite of pertinent numerical experiments.**

***Index Terms*—Broadcasting, convex optimization, downlink beamforming, multicasting, semidefinite relaxation.**

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I. Introduction

As network technology evolves towards seamless interconnection and *triple-play*[1] services, multicasting techniques become increasingly important in delivering batch updates and streaming media content. Multicasting is a network layer issue for wired and optical networks, where multicast routing has received considerable attention, and associated tools (e.g., MBONE) have long been available for the Internet.

In recent years, there is a clear trend and emerging consensus that wireless is the access method of choice for the last hop, or even the last few hops. This is partially due to accessibility and cost issues, but, perhaps more important, for ease of use and mobility considerations. This is evident in the proliferation of wireless local area and wireless backhaul solutions, in addition to the convergence of cellular phones and wireless-enabled handheld computers.

Wireless is an inherently broadcast medium, thus opening the door to multicasting *at the physical layer*, in addition to multicast routing at the network layer. Access points nowadays are typically equipped with antenna arrays. Baseband beamforming can be used to create suitable beampatterns to serve multiple multicast groups simultaneously over the same bandwidth. Physical-layer multicasting can yield significant rate, energy, and latency advantages over network-layer multicast routing, in which duplicate transmissions are unavoidable. However, wireless multicasting requires some level of physical channel state information (CSI) to be effective, and it obviously cannot be employed over the optical or wired backbone. Thus, physical-layer multicasting and network-layer multicast routing are complementary techniques.

The first reference to consider the concept of physical-layer multicasting was the Ph.D. dissertation of Lopez [2], who posed the problem of designing a beamformer that maximizes the average received signal power across a user population, under a transmitted power constraint. The drawback of such an approach is that it does not guarantee a minimum multicast rate. The next step was taken in [3] and [4], which formulated the problem of minimizing transmitted power under a received signal power constraint for each of the users, as well as a fair approach maximizing the smallest received signal power under a transmitted power constraint. Surprisingly, both problems were shown to be NP-hard in general; however, computationally efficient approximate solutions were also developed, based on semidefinite relaxation (SDR) ideas [3], [4].

[1]Triple-play is voice, Internet, video on-demand, video broadcasting/multicasting.

In follow-up work [5], [6], the general problem of simultaneously designing beamformers for several cochannel multicast groups was considered under quality of service (QoS), i.e., minimum attained signal-to-interference-plus-noise ratio (SINR) at each receiver, and max–min fair criteria. Again, these problems are NP-hard in general; however, computationally efficient, high-quality approximate solutions can be derived using SDR coupled with randomization and multicast power control. Interestingly, in extensive numerical experiments, the resulting algorithms were shown to attain close to optimal performance for both simulated and measured channel data [6].

In [5] and [6], the multicast power control problem was recast as a linear program and solved using interior-point methods. Building on [4] and [5], iterative multicast power control algorithms based on the concept of interference functions were proposed in [7]. However, unlike [5] and [6], the multicast power control algorithms in [7] do not converge when the power control problem is infeasible, as it often happens during randomization.

While both formulations are NP-hard in general, numerical findings in [5] and [6] suggest that, for Vandermonde channel vectors, exact solutions are often generated with remarkable consistency. Vandermonde channel vectors arise when a uniform linear antenna array (ULA) is used at the transmitter under far-field, line-of-sight propagation conditions. Such conditions are quite realistic in wireless backhaul scenarios, such as the line-of-sight mode of 802.16e. In this paper, we prove that, indeed, the aforementioned design problems are convex (and thus "easy" to solve exactly) under such conditions. We also show that the natural (Lagrange bi-dual) SDR of both problems is tight for Vandermonde channel vectors. Furthermore, we depart from the perfect CSI assumption and allow the user angles to be known only within a certain tolerance. We formulate robust design problems under both QoS and fair service criteria and show that these too are convex problems that can be optimally and efficiently solved using modern interior point methods. We conclude the paper by providing several illustrative simulation results for all algorithms considered.

II. Quality of Service Multicast Beamforming

Consider a communication scenario where an access point employing an antenna array of N elements is used to feed content, simultaneously and over the same frequency channel, to M single-antenna[2] receivers. Each receiver listens to a single multicast stream $k \in \{1,\ldots,G\}$, where $1 \leq G \leq M$ is the total number of multicasts. Each multicast group, denoted by $\mathcal{G}_k$, is formed by the indexes of the participating receivers; these sets are nonoverlapping and collectively contain all the users, i.e., $\mathcal{G}_k \cap \mathcal{G}_\ell = \emptyset$, $\ell \neq k$, and $\cup_k \mathcal{G}_k = \{1,\ldots,M\}$. Note that $G=1$ corresponds to the case of (selective) broadcasting [4], whereas $G=M$ corresponds to the case of individual information transmission to each receiver (the *multiuser downlink* problem; see, e.g., [8]).

The channel from each transmit element to the receive antenna of user $i \in \{1,\ldots,M\}$ is considered frequency-flat quasi-static, and the $N \times 1$ complex vector $\mathbf{h}_i$ models the respective propagation loss and phase shift. In the beamforming designs presented herein, optimization is performed with respect to the $N \times 1$ complex weight vectors $\{\mathbf{w}_k\}_{k=1}^G$, applied to the transmit antenna elements to generate the spatial channel for transmission to each multicast group. The temporal information-bearing signal $s_k(t)$ directed to receivers in multicast group k is assumed zero-mean, temporally white with unit variance, and the waveforms $\{s_k(t)\}_{k=1}^G$ mutually uncorrelated. Then, the signal vector transmitted from the antenna array is $\sum_{k=1}^G \mathbf{w}_k^* s_k(t)$, and the total radiated power is equal to $\sum_{k=1}^G \|\mathbf{w}_k\|_2^2$.

The joint design of transmit beamformers can be posed as the QoS problem of minimizing the total radiated power subject to meeting prescribed SINR constraints γ_i at each of the M intended mobile receivers

$$\mathcal{Q}: \quad \min_{\{\mathbf{w}_k \in \mathbb{C}^N\}_{k=1}^G} \sum_{k=1}^G \|\mathbf{w}_k\|_2^2$$
$$\text{s.t.}: \quad \frac{\left|\mathbf{w}_k^H \mathbf{h}_i\right|^2}{\sum_{\ell \neq k} \left|\mathbf{w}_\ell^H \mathbf{h}_i\right|^2 + \sigma_i^2} \geq \gamma_i$$
$$\forall i \in \mathcal{G}_k \quad \forall k, \ell \in \{1,\ldots,G\}.$$

Problem $\mathcal{Q}$ was considered in [5] and [6], where it was found to be NP-hard for general channel vectors, based on arguments in earlier work [4]. Therefore, a two-step approach was proposed and shown to yield high-quality approximate solutions at manageable complexity cost. In the first step, the original nonconvex quadratically constrained quadratic programming problem $\mathcal{Q}$ is relaxed to a suitable semidefinite program (SDP). This relaxation can be interpreted and motivated [9] as the Lagrange bi-dual of $\mathcal{Q}$ and also derived by changing the optimization variables to $\{\mathbf{X}_k := \mathbf{w}_k \mathbf{w}_k^H\}_{k=1}^G$ and dropping the associated nonconvex constraints $\{\text{rank}(\mathbf{X}_k) = 1\}_{k=1}^G$. Using $\mathbf{Q}_i := \mathbf{h}_i \mathbf{h}_i^H$ and introducing M real nonnegative "slack" variables $\{s_i\}_{i=1}^M$, the resulting relaxation is

$$\mathcal{Q}_r: \quad \min_{\{\mathbf{X}_k \in \mathbb{C}^{N\times N}\}_{k=1}^G, \{s_i \in \mathbb{R}\}_{i=1}^M} \sum_{k=1}^G \text{tr}(\mathbf{X}_k)$$
$$\text{s.t.}: \quad \text{tr}(\mathbf{Q}_i \mathbf{X}_k) - \gamma_i \sum_{\ell \neq k} \text{tr}(\mathbf{Q}_i \mathbf{X}_\ell) - s_i = \gamma_i \sigma_i^2$$
$$\forall i \in \mathcal{G}_k \; \forall k, \ell \in \{1,\ldots,G\},$$
$$s_i \geq 0 \; \forall i \in \{1,\ldots,M\},$$
$$\mathbf{X}_k \succeq \mathbf{0} \; \forall k \in \{1,\ldots,G\}$$

where $\text{tr}()$ denotes the trace operator. In the second step, a randomization procedure is employed to generate candidate beamforming vectors from the solution of $\mathcal{Q}_r$. For each candidate set of vectors, a multigroup power control linear programming problem is solved to ensure that the constraints of the original problem $\mathcal{Q}$ are met. The final solution is the set of feasible beamforming vectors yielding the smallest power control objective. The overall complexity of the algorithm is manageable, since the SDP and linear programming problems can be solved efficiently using interior point methods and the randomization procedure is designed so that its computational cost is negligible.

[2]Single-antenna receives are assumed for brevity of exposition; our designs can be generalized to account for multiantenna receivers.

When the transmitter employs a ULA under far-field line-of-sight propagation conditions, the $N \times 1$ complex vectors that model the phase shift from each transmit antenna element to the receive antenna of user $i \in \{1, \ldots, M\}$ are Vandermonde

$$\mathbf{h}_i = \mathbf{v}(\theta_i) = \left[1 \; e^{j\theta_i} \; e^{j2\theta_i} \; \cdots \; e^{j(N-1)\theta_i}\right]^T \tag{1}$$

where the angles θ_i are given by $\theta_i = -2\pi d \sin(\phi_i)/\lambda$. Here, d denotes the spacing between successive antenna elements, λ is the carrier wavelength, and the angles ϕ_i define the directions of the receivers.

In such a propagation scenario, it was observed from the simulation results of [5] and [6] that there exist configurations of users' directions for which the optimal solution blocks $\{\mathbf{X}_k^{\mathrm{opt}}\}_{k=1}^G$ of the relaxed problem $\mathcal{Q}_{\mathrm{r}}$, when feasible, turn out all being rank one. Then, the second step of the overall algorithm (comprising the randomization—multicast power control loop) is no longer needed, and the set of optimum beamforming vectors $\{\mathbf{w}_k^{\mathrm{opt}}\}_{k=1}^G$ can be formed by the principal components of the blocks $\{\mathbf{X}_k^{\mathrm{opt}}\}_{k=1}^G$. In such an occasion, problem $\mathcal{Q}_{\mathrm{r}}$ is equivalent to, and not a relaxation of, the original problem $\mathcal{Q}$. In Section II-A, this fact is proven to hold for any feasible configuration, in the case of Vandermonde channel vectors, and it suggests that the original problem $\mathcal{Q}$ is no longer NP-hard, but may be equivalently posed as a convex optimization problem. A suitable convex reformulation, in terms of the autocorrelation of the beamforming vectors, is developed in Section II-B.

A. Semidefinite Relaxation Is Tight for Vandermonde Channels

Claim 1: The SDR $\mathcal{Q} \to \mathcal{Q}_{\mathrm{r}}$ is always tight for Vandermonde channels, i.e., if $\mathcal{Q}_{\mathrm{r}}$ is feasible, then it always admits an equivalent solution whose blocks are all rank one.

Proof: Let $\mathbf{X}_k^{\mathrm{opt}} \in \mathbb{C}^{N \times N}$ be one of the G blocks comprising the optimal solution of the SDR problem $\mathcal{Q}_{\mathrm{r}}$. Let $\rho_k \geq 1$ denote the rank of $\mathbf{X}_k^{\mathrm{opt}}$, and consider the outer product decomposition[3] $\mathbf{X}_k^{\mathrm{opt}} = \sum_{\ell=1}^{\rho_k} \mathbf{w}_{k\ell}^{\mathrm{opt}} (\mathbf{w}_{k\ell}^{\mathrm{opt}})^H$. The signal power received at node i by multicast k can then be written as

$$\begin{aligned}
\mathrm{tr}\left(\mathbf{X}_k^{\mathrm{opt}}\mathbf{Q}_i\right) &= \mathrm{tr}\left(\mathbf{X}_k^{\mathrm{opt}}\mathbf{h}_i\mathbf{h}_i^H\right)\\
&= \mathrm{tr}\left[\sum_{\ell=1}^{\rho_k} \mathbf{w}_{k\ell}^{\mathrm{opt}} \left(\mathbf{w}_{k\ell}^{\mathrm{opt}}\right)^H \mathbf{h}_i\mathbf{h}_i^H\right]\\
&= \sum_{\ell=1}^{\rho_k} \mathrm{tr}\left[\mathbf{w}_{k\ell}^{\mathrm{opt}} \left(\mathbf{w}_{k\ell}^{\mathrm{opt}}\right)^H \mathbf{h}_i\mathbf{h}_i^H\right]\\
&= \sum_{\ell=1}^{\rho_k} \mathrm{tr}\left[\mathbf{h}_i^H\mathbf{w}_{k\ell}^{\mathrm{opt}} \left(\mathbf{w}_{k\ell}^{\mathrm{opt}}\right)^H \mathbf{h}_i\right]\\
&= \sum_{\ell=1}^{\rho_k} \left|\mathbf{h}_i^H\mathbf{w}_{k\ell}^{\mathrm{opt}}\right|^2\\
&= \sum_{\ell=1}^{\rho_k} \left|\mathbf{v}(\theta_i)^H\mathbf{w}_{k\ell}^{\mathrm{opt}}\right|^2
\end{aligned} \tag{2}$$

using the linearity of the trace operator and the property that $\mathrm{tr}(\mathbf{AB}) = \mathrm{tr}(\mathbf{BA})$, for any matrices $\mathbf{A}$ and $\mathbf{B}$ of appropriate dimensions. The last equality comes from the assumption that channel vectors are Vandermonde [cf. (1)].

The result of (2) is a real-valued complex trigonometric polynomial, which is nonnegative for any value of $\theta_i \in [0, 2\pi)$. Thus, according to the Riesz–Féjer theorem [10], there exists a vector $\mathbf{w}_k^{\mathrm{opt}} \in \mathbb{R} \times \mathbb{C}^{N-1}$ that is independent of θ_i such that for all θ_i

$$\begin{aligned}
\sum_{\ell=1}^{\rho_k} \left|\mathbf{v}(\theta_i)^H\mathbf{w}_{k\ell}^{\mathrm{opt}}\right|^2 &= \left|\mathbf{v}(\theta_i)^H\mathbf{w}_k^{\mathrm{opt}}\right|^2\\
&= \mathrm{tr}\left(\left|\mathbf{h}_i^H\mathbf{w}_k^{\mathrm{opt}}\right|^2\right)\\
&= \mathrm{tr}\left[\mathbf{w}_k^{\mathrm{opt}}\left(\mathbf{w}_k^{\mathrm{opt}}\right)^H\mathbf{h}_i\mathbf{h}_i^H\right]\\
&= \mathrm{tr}\left(\bar{\mathbf{X}}_k^{\mathrm{opt}}\mathbf{Q}_i\right)
\end{aligned} \tag{3}$$

where $\bar{\mathbf{X}}_k^{\mathrm{opt}} := \mathbf{w}_k^{\mathrm{opt}}(\mathbf{w}_k^{\mathrm{opt}})^H$. Combining the results of (2) and (3), we obtain

$$\mathrm{tr}\left(\mathbf{X}_k^{\mathrm{opt}}\mathbf{Q}_i\right) = \mathrm{tr}\left(\bar{\mathbf{X}}_k^{\mathrm{opt}}\mathbf{Q}_i\right) \tag{4}$$

which shows that for every optimum (generally high-rank) beamforming matrix $\mathbf{X}_k^{\mathrm{opt}}$, there exists a rank-one positive-semidefinite matrix $\bar{\mathbf{X}}_k^{\mathrm{opt}}$, which is equivalent with respect to the power received at each node. Therefore, the blocks $\{\bar{\mathbf{X}}_k^{\mathrm{opt}}\}_{k=1}^G$ form a feasible solution set of the SDR problem $\mathcal{Q}_{\mathrm{r}}$.

Integrating out θ_i in the first equality of (3) yields (cf. Parseval's theorem)

$$\begin{aligned}
\sum_{\ell=1}^{\rho_k} \left\|\mathbf{w}_{k\ell}^{\mathrm{opt}}\right\|^2 = \left\|\mathbf{w}_k^{\mathrm{opt}}\right\|^2 &\Leftrightarrow\\
\mathrm{tr}\left[\sum_{\ell=1}^{\rho_k} \mathbf{w}_{k\ell}^{\mathrm{opt}}\left(\mathbf{w}_{k\ell}^{\mathrm{opt}}\right)^H\right] = \mathrm{tr}\left[\mathbf{w}_k^{\mathrm{opt}}\left(\mathbf{w}_k^{\mathrm{opt}}\right)^H\right] &\Leftrightarrow\\
\mathrm{tr}\left(\mathbf{X}_k^{\mathrm{opt}}\right) = \mathrm{tr}\left(\bar{\mathbf{X}}_k^{\mathrm{opt}}\right).&
\end{aligned} \tag{5}$$

Hence, the feasible set of rank-one blocks $\{\bar{\mathbf{X}}_k^{\mathrm{opt}}\}_{k=1}^G$ is an optimum solution of the SDR problem $\mathcal{Q}_{\mathrm{r}}$, since it has the same objective value as $\{\mathbf{X}_k^{\mathrm{opt}}\}_{k=1}^G$. ∎

B. Convex Reformulation for Vandermonde Channels

In this subsection, we reformulate the nonconvex quadratic inequality constraints of the original QoS multicast beamforming problem $\mathcal{Q}$ in terms of the autocorrelation of the beamforming vectors. Towards this end, the signal power received at each user i by multicast k can be equivalently written, for Vandermonde channel vectors (1), as

$$\begin{aligned}
\left|\mathbf{w}_k^H\mathbf{h}_i\right|^2 &= \left(\mathbf{w}_k^H\mathbf{h}_i\right)\left(\mathbf{h}_i^H\mathbf{w}_k\right)\\
&= \sum_{n=1}^N w_{k,n}^* h_{i,n} \sum_{m=1}^N w_{k,m} h_{i,m}^*\\
&= \sum_{n=1}^N \sum_{m=1}^N w_{k,m} w_{k,n}^* h_{i,n} h_{i,m}^*\\
&= \sum_{n=1}^N \sum_{m=1}^N w_{k,m} w_{k,n}^* e^{j\theta_i(n-1)} e^{-j\theta_i(m-1)}
\end{aligned}$$

[3]Such decomposition is not unique, but this is irrelevant for our purposes; we simply use one such decomposition.

$$= \sum_{n=1}^{N}\sum_{m=1}^{N} w_{k,m}w_{k,n}^{*}e^{-j\theta_i(m-n)}$$
$$= \sum_{m=1}^{N}\sum_{\ell=m-1}^{m-N} w_{k,m}w_{k,m-\ell}^{*}e^{-j\theta_i\ell}$$
$$= \sum_{\ell=-(N-1)}^{N-1}\sum_{m=\max(1+\ell,1)}^{\min(N+\ell,N)} w_{k,m}w_{k,m-\ell}^{*}e^{-j\theta_i\ell}$$
$$= \sum_{\ell=-(N-1)}^{N-1} r_{k,\ell}e^{-j\theta_i\ell} \quad (6)$$

where

$$\mathbf{w}_k := [w_{k,1}, w_{k,2}, \dots, w_{k,N}]^T$$

and

$$\mathbf{h}_i := [h_{i,1}, h_{i,2}, \dots, h_{i,N}]^T. \quad (7)$$

In (6), we have denoted $\ell := m - n$ and for all $\ell \in \{-N+1, \dots, N-1\}$

$$r_{k,\ell} := \sum_{m=\max(1+\ell,1)}^{\min(N+\ell,N)} w_{k,m}w_{k,m-\ell}^{*}. \quad (8)$$

It is easy to see that $r_{k,\ell}$ is conjugate-symmetric about the origin, which allows us to rewrite the received signal power in terms of $\{r_{k,\ell}\}_{\ell=0}^{N-1}$ only

$$\left|\mathbf{w}_k^H\mathbf{v}(\theta_i)\right|^2 = r_{k,0} + \sum_{\ell=1}^{N-1}\left(r_{k,\ell}e^{-j\theta_i\ell} + r_{k,-\ell}e^{j\theta_i\ell}\right)$$
$$= r_{k,0} + \sum_{\ell=1}^{N-1}\left(r_{k,\ell}e^{-j\theta_i\ell} + r_{k,\ell}^{*}e^{j\theta_i\ell}\right)$$
$$= r_{k,0} + 2\sum_{\ell=1}^{N-1}\mathrm{Re}[r_{k,\ell}e^{-j\theta_i\ell}]$$
$$= \mathrm{Re}\left[\mathbf{v}(\theta_i)^H\tilde{\mathbf{I}}\mathbf{r}_k\right] \quad (9)$$

where we have defined the autocorrelation vectors $\mathbf{r}_k \in \mathbb{R} \times \mathbb{C}^{(N-1)}$ $\forall k \in \{1, \dots, G\}$ as

$$\mathbf{r}_k := [r_{k,0}, r_{k,1}, \dots, r_{k,N-1}]^T \quad (10)$$

and the $N \times N$ diagonal matrix

$$\tilde{\mathbf{I}} = \begin{bmatrix} 1 & 0 \\ 0 & 2\mathbf{I}_{N-1} \end{bmatrix} \quad (11)$$

where $\mathbf{I}_{N-1}$ denotes the $(N-1) \times (N-1)$ identity matrix. Furthermore, note that $r_{k,0} = \sum_{m=1}^{N} w_{k,m}w_{k,m}^{*} = \|\mathbf{w}_k\|_2^2$.

It therefore follows that problem $\mathcal{Q}$ can be equivalently written as

$$\mathcal{Q}': \quad \min_{\{\mathbf{r}_k \in \mathbb{R}\times\mathbb{C}^{(N-1)}\}_{k=1}^{G}} \sum_{k=1}^{G} r_{k,0}$$
$$\text{s.t.}: \frac{\mathrm{Re}\left[\mathbf{v}(\theta_i)^H\tilde{\mathbf{I}}\mathbf{r}_k\right]}{\sum_{\ell\neq k}\mathrm{Re}\left[\mathbf{v}(\theta_i)^H\tilde{\mathbf{I}}\mathbf{r}_\ell\right] + \sigma_i^2} \geq \gamma_i$$
$$\forall i \in \mathcal{G}_k \; \forall k, \ell \in \{1, \dots, G\},$$
$$\mathbf{r}_k \text{ is an autocorrelation vector } \forall k \in \{1, \dots, G\}.$$

Each of the M inequality constraints can be written as

$$\mathrm{Re}\left[\mathbf{v}(\theta_i)^H\tilde{\mathbf{I}}\mathbf{r}_k\right] - \gamma_i\sum_{\ell\neq k}\mathrm{Re}\left[\mathbf{v}(\theta_i)^H\tilde{\mathbf{I}}\mathbf{r}_\ell\right] \geq \gamma_i\sigma_i^2 \Leftrightarrow$$
$$\mathrm{Re}\left[\mathbf{v}(\theta_i)^H\tilde{\mathbf{I}}\left(\mathbf{r}_k - \gamma_i\sum_{\ell\neq k}\mathbf{r}_\ell\right)\right] \geq \gamma_i\sigma_i^2 \Leftrightarrow$$
$$\mathrm{Re}\left[\mathbf{v}(\theta_i)^H\tilde{\mathbf{I}}\mathbf{A}_i\mathbf{r}\right] \geq \gamma_i\sigma_i^2 \Leftrightarrow$$
$$\mathbf{v}(\theta_i)^H\tilde{\mathbf{I}}\mathbf{A}_i\mathbf{r} + j\xi_i + s_i = \gamma_i\sigma_i^2 \quad (12)$$

where in the first step, the fact that the terms in the denominator are all nonnegative was taken into account. In the third step, the $N \times GN$ matrix $\mathbf{A}_i = \mathbf{a}_i \otimes \mathbf{I}_N$ was introduced, where $\mathbf{a}_i = (\gamma_i + 1)\mathbf{e}_k^T - \gamma_i\mathbf{1}_G^T$ is the $1 \times G$ vector whose kth element is equal to 1, whereas all others are set to $-\gamma_i$. Here, $\mathbf{1}_G$ is the $G \times 1$ all-1 vector, $\mathbf{e}_k$ is the $G \times 1$ vector indicating the multicast group k that user i belongs to, and $\otimes$ denotes the Kronecker product. Furthermore, the autocorrelation vectors are stacked in the $GN \times 1$ optimization variable vector $\mathbf{r} = [\mathbf{r}_1^T \cdots \mathbf{r}_G^T]^T$. In the last step, the real (unconstrained sign) "slack" variables $\{\xi_i \in \mathbb{R}\}_{i=1}^{M}$ were inserted to compensate the terms $\mathrm{Im}[\mathbf{v}(\theta_i)^H\tilde{\mathbf{I}}\mathbf{A}_i\mathbf{r}]$. Finally, the real nonnegative "slack" variables $\{s_i \in \mathbb{R}\}_{i=1}^{M}$ were introduced to convert the linear inequalities to equalities.

Each of the G autocorrelation constraints admits an equivalent linear matrix inequality representation (see, e.g., [11] and [12]). Specifically, $r_{k,\ell}$ $\forall \ell \in \{0, \dots, N-1\}$ belongs to the set of finite autocorrelation sequences if and only if

$$r_{k,\ell} = \mathrm{tr}\left[\left(\mathbf{E}_N^{\ell}\right)^T\mathbf{X}_k\right] \quad \forall \ell \in \{0, \dots, N-1\} \quad (13)$$

for some positive-semidefinite matrix $\mathbf{X}_k \in \mathbb{C}^{N\times N}$, where $\mathbf{E}_N^{\ell}$ is the $N \times N$ Toeplitz matrix with 1's in the ℓth lower subdiagonal and zeros elsewhere (note that $\mathbf{E}_N^0 = \mathbf{I}_N$). Thus, introducing G positive-semidefinite $N \times N$ auxiliary matrices $\mathbf{X}_k$, one for each autocorrelation vector $\mathbf{r}_k$, the G autocorrelation constraints are equivalently converted to GN linear equality constraints plus G positive-semidefinite constraints. Substituting the constraints of problem $\mathcal{Q}'$, with the equivalent representations of (12) and (13), the QoS multicast beamforming problem is reformulated to

$$\mathcal{Q}_c: \min_{\{\mathbf{r}_k\}_{k=1}^G,\,\{\mathbf{X}_k\}_{k=1}^G,\,\{s_i\in\mathbb{R}\}_{i=1}^M,\,\{\xi_i\in\mathbb{R}\}_{i=1}^M} [\mathbf{1}_G\otimes\mathbf{e}_1]^T\mathbf{r}$$
$$\text{s.t.}: \quad \mathbf{v}(\theta_i)^H\tilde{\mathbf{I}}\mathbf{A}_i\mathbf{r}+j\xi_i+s_i=\gamma_i\sigma_i^2, \quad \forall i\in\mathcal{G}_k\ \forall k\in\{1,\ldots,G\},$$
$$r_{k,\ell}-\mathrm{vec}\left(\mathbf{E}_N^\ell\right)^T\mathrm{vec}(\mathbf{X}_k)=0, \quad \forall \ell\in\{0,\ldots,N-1\}\ \forall k\in\{1,\ldots,G\},$$
$$s_i\geq 0\ \forall i\in\{1,\ldots,M\},$$
$$\mathbf{X}_k\succeq\mathbf{0}\ \forall k\in\{1,\ldots,G\},$$

where $\mathrm{vec}()$ denotes the "vectorization" operator, which stacks the columns of a matrix to form a vector.

Problem $\mathcal{Q}_c$ is an SDP problem, expressed in the primal standard form. It can therefore be efficiently solved by any general-purpose SDP solver, such as SeDuMi [13], by means of interior point methods. Problem $\mathcal{Q}_c$ consists of G vector variables of size $N\times 1$, G matrix variables of size $N\times N$, and $M+GN$ linear constraints. Interior point methods will take $O[\sqrt{GN}\log(1/\epsilon)]$ iterations, with each iteration requiring at most $O[G^3N^6+(M+GN)GN^2]$ arithmetic operations [14], where the parameter ϵ represents the desired solution accuracy at the algorithm's termination. Actual runtime complexity will usually scale far slower with G, N, and M than this worst-case bound.[4] Once the optimum autocorrelation sequences $\{\mathbf{r}_k^{\mathrm{opt}}\}_{k=1}^G$ are found, they can be factored to obtain the respective optimum beamforming vectors $\{\mathbf{w}_k^{\mathrm{opt}}\}_{k=1}^G$ using spectral factorization techniques (see, e.g., [15]).

The QoS multicast beamforming problem $\mathcal{Q}$ for Vandermonde channels can thus be solved equivalently in two distinct ways. First, by the principal components of the optimum solution blocks of the SDP problem $\mathcal{Q}_r$, when these turn out all being rank one. Second, by spectral factorization of the optimum autocorrelation sequences solving the SDP problem $\mathcal{Q}_c$. However, problem $\mathcal{Q}_r$ is not guaranteed to *consistently* yield rank-one solutions for Vandermonde channel vectors; Claim 1 only proves the existence of such a solution, and counter examples in which SDP yields higher-rank solutions do arise in practice. Postprocessing via spectral factorization is needed in such cases in order to obtain an equivalent rank-one solution. The first approach (via $\mathcal{Q}_r$) is computationally cheaper when general-purpose interior point SDP software is used, because $\mathcal{Q}_c$ involves a higher number of optimization variables and associated constraints. However, the dual of $\mathcal{Q}_c$ involves significantly fewer variables and can be solved via application-specific interior point methods, which can drop the arithmetic operations per iteration by two to three orders of magnitude (see, e.g., [11]). Finally, and perhaps most importantly, the reformulation of the QoS constraints in terms of autocorrelation sequences as inequalities on (the real part of) trigonometric polynomials [cf. (12)] enables us to extend the multicast beamforming problem to the case where there is partial knowledge of the angles θ_i determining the Vandermonde channel vectors. The respective robust design is considered in Section IV-A.

[4] This is true for all problems considered here.

III. Joint Max–Min Fair Multicast Beamforming

In this section, we consider the related problem of maximizing the minimum received SINR, subject to an upper bound P on the total transmitted power. Specifically, the joint max–min fair (JMMF) transmit beamforming design can be formulated as

$$\mathcal{F}: \max_{\{\mathbf{w}_k\in\mathbb{C}^N\}_{k=1}^G,\, t\in\mathbb{R}} t$$
$$\text{s.t.}: \quad \frac{\left|\mathbf{w}_k^H\mathbf{h}_i\right|^2}{\sum_{\ell\neq k}\left|\mathbf{w}_\ell^H\mathbf{h}_i\right|^2+\sigma_i^2}\geq t,$$
$$\forall i\in\mathcal{G}_k\ \forall k,\ell\in\{1,\ldots,G\},$$
$$\textstyle\sum_{k=1}^G\|\mathbf{w}_k\|_2^2=P, \quad \text{and} \quad t\geq 0.$$

Problem $\mathcal{F}$ was considered in [6], and it was found to be NP-hard in the case of general channel vectors, based on arguments in earlier work [4]. Therefore, a two-step approach was proposed and shown to yield high-quality approximate solutions at manageable complexity cost. In the first step, the original nonconvex problem $\mathcal{F}$ is relaxed to

$$\mathcal{F}_r: \max_{\{\mathbf{X}_k\in\mathbb{C}^{N\times N}\}_{k=1}^G,\, t\in\mathbb{R}} t$$
$$\text{s.t.}: \quad \mathrm{tr}(\mathbf{Q}_i\mathbf{X}_k)-t\left[\sum_{\ell\neq k}\mathrm{tr}(\mathbf{Q}_i\mathbf{X}_\ell)+\sigma_i^2\right]\geq 0,$$
$$\forall i\in\mathcal{G}_k\ \forall k,\ell\in\{1,\ldots,G\},$$
$$\textstyle\sum_{k=1}^G\mathrm{tr}(\mathbf{X}_k)=P,$$
$$\mathbf{X}_k\succeq\mathbf{0}\ \forall k\in\{1,\ldots,G\}, \quad \text{and} \quad t\geq 0$$

by changing the optimization variables, as for the QoS problem presented in Section II, to $\{\mathbf{X}_k:=\mathbf{w}_k\mathbf{w}_k^H\}_{k=1}^G$ and discarding the associated nonconvex constraints $\{\mathrm{rank}(\mathbf{X}_k)=1\}_{k=1}^G$. Again, such relaxation can be motivated [9] as the Lagrange bi-dual of problem $\mathcal{F}$. A solution to the relaxed problem $\mathcal{F}_r$ can be found by means of bisection, for the nonnegative real variable t, over SDP feasibility problems. In the second step, postprocessing of the relaxed solution is needed when the optimum solution matrices $\{\mathbf{X}_k^{\mathrm{opt}}\}_{k=1}^G$ do not turn out being all rank one, so as to yield an approximate solution to the original JMMF problem $\mathcal{F}$. This can be accomplished by using a combined randomization—joint power control procedure.

As we have already noted in Section II, when a ULA array is used for far-field line-of-sight transmit beamforming, the channel vectors are Vandermonde [cf. (1)]. In such a scenario, Claim 1, proven in Section II-A for the QoS formulation, holds for the JMMF formulation as well: the relaxation $\mathcal{F}\to\mathcal{F}_r$ is always tight. The proof is similar to the one for the QoS case. Using (4) and (5), it is seen that the set of rank-one blocks $\{\bar{\mathbf{X}}_k^{\mathrm{opt}}\}_{k=1}^G$ form a feasible solution of problem $\mathcal{F}_r$, giving the same objective value as the (generally higher-rank) optimum solution set $\{\mathbf{X}_k^{\mathrm{opt}}\}_{k=1}^G$.

In the following, an approach similar to the one in Section II-B is followed to reformulate the original JMMF multicast beamforming problem $\mathcal{F}$ in terms of the autocorrelation of the beamforming vectors. Using (9) for the signal power received at each user by each multicast, problem $\mathcal{F}$ may be equivalently written as

$$\begin{aligned}
&\mathcal{F}': \\
&\max_{\{\mathbf{r}_k \in \mathbb{R}\times\mathbb{C}^{N-1}\}_{k=1}^G,\ t\in\mathbb{R}} \quad t \\
&\text{s.t.}: \quad \frac{\mathrm{Re}\left[\mathbf{v}(\theta_i)^H \tilde{\mathbf{I}}\mathbf{r}_k\right]}{\sum_{\ell\neq k} \mathrm{Re}\left[\mathbf{v}(\theta_i)^H \tilde{\mathbf{I}}\mathbf{r}_\ell\right] + \sigma_i^2} \geq t, \\
&\forall i \in \mathcal{G}_k\ \forall k, \ell \in \{1,\ldots,G\}, \\
&\mathbf{r}_k \text{ is an autocorrelation vector } \forall k \in \{1,\ldots,G\}, \\
&\textstyle\sum_{k=1}^G r_{k,0} = P, \quad \text{and} \quad t \geq 0.
\end{aligned}$$

The resulting JMMF multicast beamforming problem $\mathcal{F}'$ comprises a linear cost, M nonlinear inequality constraints, G autocorrelation constraints, a linear equality constraint, and a nonnegativity constraint. The autocorrelation constraints can be recast as linear matrix inequalities, introducing G positive-semidefinite $N \times N$ auxiliary matrix variables $\mathbf{X}_k$, as in Section II-B. Hence, except for the first M nonlinear inequality constraints, the JMMF problem is an SDP problem expressed in the standard primal form.

Each of the nonlinear inequalities can be equivalently written as

$$\mathbf{v}(\theta_i)^H \tilde{\mathbf{I}}\mathbf{A}_i(t)\mathbf{r} + j\xi_i + s_i = t\sigma_i^2 \tag{14}$$

following the same steps as in (12) of Section II-B. The sole difference is that the value γ_i is replaced by the variable t so that the auxiliary vectors $\mathbf{a}_i(t) = (t+1)\mathbf{e}_k^T - t\mathbf{1}_G^T$ and the auxiliary matrices $\mathbf{A}_i(t) = \mathbf{a}_i(t) \otimes \mathbf{I}_N$ are now functions of t. Hence, contrary to the QoS case, the equalities of (14) are nonlinear. The key here is that i) by fixing t the equalities become linear in the remaining variables; and ii) the objective is to maximize t. It follows that $\mathcal{F}'$ can be solved by bisection over SDP problems.[5] Specifically, let $[L, U]$ denote the interval containing the optimum value t^* of problem $\mathcal{F}'$. Due to the nonnegativity of t^* and the Cauchy–Schwartz inequality, we may set $L = 0$ and $U = \min_{i\in\{1,\ldots,M\}} PN/\sigma_i^2$ for the lower and the upper bound, respectively. Given $[L, U]$, the SDP feasibility problem $\mathcal{F}_\mathrm{f}$, shown in the box below, is solved at the midpoint $t = (L+U)/2$ of the interval. If problem $\mathcal{F}_\mathrm{f}$ is feasible for the given choice of t, we set $L := t$; otherwise $U := t$. Thus, in each iteration, the interval is halved. This bisection algorithm terminates when the interval length becomes smaller than the desired accuracy.

$$\begin{aligned}
&\mathcal{F}_\mathrm{f}: \\
&\quad \text{find } \mathbf{x} \\
&\text{s.t.}: \quad \mathbf{v}(\theta_i)^H \tilde{\mathbf{I}}\mathbf{A}_i(t)\mathbf{r} + j\xi_i + s_i = t\sigma_i^2, \\
&\qquad \forall i \in \mathcal{G}_k\ \forall k \in \{1,\ldots,G\}, \\
&\quad \textstyle\sum_{k=1}^G r_{k,0} = P, \\
&\quad r_{k,\ell} - \mathrm{vec}\left(\mathbf{E}_N^\ell\right)^T \mathrm{vec}(\mathbf{X}_k) = 0, \\
&\qquad \forall \ell \in \{0,\ldots,N-1\}\ \forall k \in \{1,\ldots,G\}, \\
&\quad \mathbf{X}_k \succeq \mathbf{0}\ \forall k \in \{1,\ldots,G\}
\end{aligned}$$

[5]Bisection is a standard trick in this context, and it has also been used in [7].

In the above, $\mathbf{x}$ denotes the optimization variable vector

$$\mathbf{x} = \left[\mathbf{r}_1^T \cdots \mathbf{r}_G^T\ \boldsymbol{\xi}^T\ \mathbf{s}^T\ \mathrm{vec}(\mathbf{X}_1)^T \cdots \mathrm{vec}(\mathbf{X}_G)^T\right]^T \tag{15}$$

where the vectors $\mathbf{s} \in \mathbb{R}^M$, and $\boldsymbol{\xi} \in \mathbb{R}^M$ contain the "slack" variables. The solution vector obtained by the last feasible iteration of the bisection algorithm contains the sought autocorrelation sequences $\{\mathbf{r}_k^{\mathrm{opt}}\}_{k=1}^G$. The respective beamforming vectors $\{\mathbf{w}_k^{\mathrm{opt}}\}_{k=1}^G$ can then be found using spectral factorization techniques (see, e.g., [15]).

For each iteration of the bisection, the algorithm tries to find a solution to the feasibility problem $\mathcal{F}_\mathrm{f}$, which is an SDP problem expressed in the standard primal form. Problems of this form can be efficiently solved by any standard interior point SDP solver, such as SeDuMi [13]. The use of interior point methods is convenient, because they not only yield a solution to problem $\mathcal{F}_\mathrm{f}$ when the latter is feasible, but also provide a certificate of infeasibility otherwise. Similar to problem $\mathcal{Q}_\mathrm{c}$, problem $\mathcal{F}_\mathrm{f}$ consists of G vector variables of size $N \times 1$, G matrix variables of size $N \times N$, and $M + 1 + GN$ linear constraints. Computing an optimal solution of tolerance ϵ using an interior point method will have an overall iteration count of $O[\sqrt{GN}\log(1/\epsilon)]$, with each iteration costing $O[G^3N^6 + (M+GN)GN^2]$ in the worst case [14].

As for the QoS problem considered in Section II, we have two algorithms to solve the JMMF multicast beamforming problem for Vandermonde channels, both employing bisection over SDP problems. Again, going through $\mathcal{F}_\mathrm{r}$ entails lower complexity when using a general-purpose interior point solver; however, the dual of $\mathcal{F}_\mathrm{f}$ is cheaper to solve using custom interior point methods developed specifically for problems involving autocorrelation constraints (see, e.g., [11]). Furthermore, problem $\mathcal{F}_\mathrm{f}$ forms the basis for the robust extension considered in Section IV-B.

IV. Robust Multicast Beamforming for Vandermonde Channels

In the multicast beamforming problems presented so far in Sections II and III, the beamformers are designed under the assumption that full CSI is available at the transmitter. For the case of Vandermonde channel vectors, considered throughout this paper, full CSI boils down to exact knowledge of users' directions $\{\phi_i\}_{i=1}^M$ (equivalently, of the angles $\{\theta_i\}_{i=1}^M$). In a more realistic scenario, only partial CSI is available at the design center, due to errors in the estimation of the angles $\{\theta_i\}_{i=1}^M$. It is often reasonable to assume that errors are bounded, in which case θ_i lies in some interval $[\alpha_i, \beta_i]$. In the following, robust designs for the QoS and JMMF multicast beamforming problems are presented, in the sense that the constraints are still met for all possible channel vectors $\{\mathbf{v}(\theta_i)\}_{i=1}^M$, where $\theta_i \in [\alpha_i, \beta_i]$.

A. QoS Problem Formulation

A robust extension of the QoS multicast beamforming problem $\mathcal{Q}_\mathrm{c}$, considered in Section II-B, would be to jointly design the transmit beamformers so that the received SINR targets are reached (or exceeded) for all possible values of the angles $\theta_i \in [\alpha_i, \beta_i]$, which determine the Vandermonde channel vectors. In such a scenario, the QoS SINR constraints are posed [cf. (12)] as

$$\mathrm{Re}\left[\mathbf{v}(\theta_i)^H \tilde{\mathbf{I}}\mathbf{A}_i\mathbf{r}\right] \geq \gamma_i\sigma_i^2 \quad \forall\theta_i \in [\alpha_i, \beta_i] \tag{16}$$

$\forall i \in \{1,\ldots,M\}$. An interpretation of these constraints is that they require (the real part of) certain trigonometric polynomials to be nonnegative over a segment of the unit circle. As proved in [12], constraints of this form can be equivalently reformulated to the linear matrix inequality constraints

$$\tilde{\mathbf{I}}\mathbf{A}_i\mathbf{r} + j\xi_i\mathbf{e}_1 - \gamma_i\sigma_i^2\mathbf{e}_1 = \mathbf{p}(\mathbf{Y}_i) + \mathbf{q}(\mathbf{Z}_i;\alpha_i,\beta_i), \tag{17}$$

$\forall i \in \{1,\ldots,M\}$. Here $\mathbf{Y}_i \in \mathbb{C}^{N\times N} \succeq \mathbf{0}$, $\mathbf{Z}_i \in \mathbb{C}^{(N-1)\times(N-1)} \succeq \mathbf{0}$, $\xi_i \in \mathbb{R}$ is unconstrained in sign, and $\mathbf{e}_1$ denotes the $N \times 1$ indicator vector whose first element is 1 and all others are 0. The linear operator $\mathbf{p}(\mathbf{Y}) = [p_1, p_2, \ldots, p_N]^T \in \mathbb{R}\times\mathbb{C}^{N-1}$ is defined [12] by the equations

$$\begin{aligned} p_1 &:= \left\langle \mathbf{E}_N^0, \mathbf{Y}\right\rangle, \\ p_{\ell+1} &:= 2\left\langle \mathbf{E}_N^\ell, \mathbf{Y}\right\rangle \quad \forall\ell \in \{1,\ldots,N-1\} \end{aligned} \tag{18}$$

where the inner product between two (generally complex) matrices $\mathbf{A}$ and $\mathbf{B}$ is defined as

$$\langle \mathbf{A}, \mathbf{B}\rangle := \mathrm{tr}(\mathbf{A}^H\mathbf{B}) = \mathrm{vec}(\mathbf{A})^H\mathrm{vec}(\mathbf{B}). \tag{19}$$

The linear operator $\mathbf{q}(\mathbf{Z};\alpha,\beta) = [q_1, q_2, \ldots, q_N]^T \in \mathbb{C}^N$ is defined [12] by

$$\begin{aligned} q_1 &:= d_1(\alpha,\beta)\left\langle \mathbf{E}_{N-1}^0, \mathbf{Z}\right\rangle + d_2^*(\alpha,\beta)\left\langle \mathbf{E}_{N-1}^1, \mathbf{Z}\right\rangle \\ q_{\ell+1} &:= 2d_1(\alpha,\beta)\left\langle \mathbf{E}_{N-1}^\ell, \mathbf{Z}\right\rangle + d_2(\alpha,\beta)\left\langle \mathbf{E}_{N-1}^{\ell-1}, \mathbf{Z}\right\rangle \\ &\quad + d_2^*(\alpha,\beta)\left\langle \mathbf{E}_{N-1}^{\ell+1}, \mathbf{Z}\right\rangle \; \forall\ell \in \{1,\ldots,N-3\} \\ q_{N-1} &:= 2d_1(\alpha,\beta)\left\langle \mathbf{E}_{N-1}^{N-2}, \mathbf{Z}\right\rangle + d_2(\alpha,\beta)\left\langle \mathbf{E}_{N-1}^{N-3}, \mathbf{Z}\right\rangle \\ q_N &:= d_2(\alpha,\beta)\left\langle \mathbf{E}_{N-1}^{N-2}, \mathbf{Z}\right\rangle \end{aligned} \tag{20}$$

where, for given $\alpha, \beta \in [0, 2\pi)$, the vector $\mathbf{d}(\alpha,\beta) = [d_1(\alpha,\beta)\; d_2(\alpha,\beta)]^T \in \mathbb{R}\times\mathbb{C}$ is defined as

$$\mathbf{d}(\alpha,\beta) := \begin{bmatrix} \cos\alpha + \cos\beta - \cos(\beta-\alpha) - 1 \\ [1-\exp(j\alpha)]\,[\exp(j\beta)-1] \end{bmatrix}. \tag{21}$$

Hence, introducing M positive-semidefinite $N\times N$ auxiliary matrices $\mathbf{Y}_i$, M positive-semidefinite $(N-1)\times(N-1)$ auxiliary matrices $\mathbf{Z}_i$, and replacing the SINR constraints of (16) with the representation of (17), the robust QoS multicast beamforming problem is equivalently written as

$$\begin{aligned} \mathcal{QR}: &\\ &\min_{\{\mathbf{r}_k\}_{k=1}^G,\,\{\mathbf{X}_k\}_{k=1}^G,\,\{\mathbf{Y}_i\}_{i=1}^M,\,\{\mathbf{Z}_i\}_{i=1}^M,\,\{\xi_i\}_{i=1}^M} [\mathbf{1}_G\otimes\mathbf{e}_1]^T\mathbf{r} \\ \text{s.t.}: \;& \tilde{\mathbf{I}}\mathbf{A}_i\mathbf{r} + j\xi_i\mathbf{e}_1 - \mathbf{p}(\mathbf{Y}_i) - \mathbf{q}(\mathbf{Z}_i;\alpha_i,\beta_i) = \gamma_i\sigma_i^2\mathbf{e}_1, \\ &\quad \forall i \in \mathcal{G}_k\; \forall k \in \{1,\ldots,G\}, \\ & r_{k,\ell} - \mathrm{vec}\left(\mathbf{E}_N^\ell\right)^T \mathrm{vec}(\mathbf{X}_k) = 0, \\ &\quad \forall\ell \in \{0,\ldots,N-1\},\; \forall k \in \{1,\ldots,G\}, \\ & \mathbf{X}_k \succeq \mathbf{0}\; \forall k \in \{1,\ldots,G\}, \\ & \mathbf{Y}_i \succeq \mathbf{0}\; \forall i \in \{1,\ldots,M\}, \\ & \mathbf{Z}_i \succeq \mathbf{0}\; \forall i \in \{1,\ldots,M\}. \end{aligned}$$

Problem $\mathcal{QR}$ is therefore an SDP problem, expressed in the standard primal form, since it consists of a linear cost, $MN + GN$ linear equality constraints, and $G + 2M$ positive-semidefinite constraints. Standard SDP solvers, such as SeDuMi [13], can be used to efficiently solve problem $\mathcal{QR}$, by means of general-purpose interior point methods. A solution of tolerance ϵ entails $O[\sqrt{(G+2M)N}\log(1/\epsilon)]$ iterations, each of complexity $O[(G+2M)^3N^6 + (M+G)(G+2M)N^3]$ [14]. More efficient solutions of significantly lower complexity can be found by means of application-specific interior point methods (see, e.g., [11]). The optimum beamforming vectors $\{\mathbf{w}_k^{\mathrm{opt}}\}_{k=1}^G$ can be computed from the solution of problem $\mathcal{QR}$ using spectral factorization techniques (see, e.g., [15]).

It is interesting to note that the robust version of the QoS multicast beamforming problem developed in this subsection is still convex and of the same form as the original problem $\mathcal{Q}_\mathrm{c}$. The price paid for the extension to the partial CSI case is higher computational complexity due to the larger size of the optimization variable vector and the larger number of constraints.

B. JMMF Problem Formulation

A robust extension of the JMMF multicast beamforming problem $\mathcal{F}'$ can be found in a manner similar to the respective QoS problem considered in the previous subsection. The inequality constraints that must be fulfilled here are posed as

$$\mathrm{Re}\left[\mathbf{v}(\theta_i)^H \tilde{\mathbf{I}}\mathbf{A}_i(t)\mathbf{r}\right] \geq t\sigma_i^2 \quad \forall\theta_i \in [\alpha_i, \beta_i] \tag{22}$$

$\forall i \in \{1,\ldots,M\}$, where $\mathbf{A}_i(t)$ is a function of t, defined in Section III. Exactly as in (17), these inequalities can be equivalently represented [12] as

$$\tilde{\mathbf{I}}\mathbf{A}_i(t)\mathbf{r} + j\xi_i\mathbf{e}_1 - t\sigma_i^2\mathbf{e}_1 = \mathbf{p}(\mathbf{Y}_i) + \mathbf{q}(\mathbf{Z}_i;\alpha_i,\beta_i) \tag{23}$$

$\forall i \in \{1,\ldots,M\}$. Fixing the value of the variable t, (23) represents linear matrix inequality constraints. Thus, the robust JMMF multicast beamforming problem can be efficiently solved, by means of bisection over the following SDP feasibility problem $\mathcal{FR}_\mathrm{f}$. The optimization variable vector $\mathbf{y}$ is defined by

$$\begin{aligned} \mathbf{y} = \big[\tilde{\mathbf{x}}^T\; \mathrm{vec}(\mathbf{Y}_1)^T \;\cdots\; \mathrm{vec}(\mathbf{Y}_M)^T \\ \mathrm{vec}(\mathbf{Z}_1)^T \;\cdots\; \mathrm{vec}(\mathbf{Z}_M)^T\big]^T \end{aligned} \tag{24}$$

where the vector $\tilde{\mathbf{x}}$ is equal to the vector $\mathbf{x}$, defined by (15), excluding from its contents the vector $\mathbf{s}$, as follows:

$$\begin{aligned} \mathcal{FR}_\mathrm{f}: &\\ &\text{find } \mathbf{y} \\ \text{s.t.}: \;& \tilde{\mathbf{I}}\mathbf{A}_i(t)\mathbf{r} + j\xi_i\mathbf{e}_1 - \mathbf{p}(\mathbf{Y}_i) - \mathbf{q}(\mathbf{Z}_i;\alpha_i,\beta_i) = t\sigma_i^2\mathbf{e}_1, \\ &\quad \forall i \in \mathcal{G}_k,\; \forall k \in \{1,\ldots,G\}, \\ & \textstyle\sum_{k=1}^G r_{k,0} = P, \\ & r_{k,\ell} - \mathrm{vec}\left(\mathbf{E}_N^\ell\right)^T \mathrm{vec}(\mathbf{X}_k) = 0, \\ &\quad \forall\ell \in \{0,\ldots,N-1\},\; \forall k \in \{1,\ldots,G\}, \\ & \mathbf{X}_k \succeq \mathbf{0}\; \forall k \in \{1,\ldots,G\}, \\ & \mathbf{Y}_i \succeq \mathbf{0}\; \forall i \in \{1,\ldots,M\}, \\ & \mathbf{Z}_i \succeq \mathbf{0}\; \forall i \in \{1,\ldots,M\}. \end{aligned}$$

TABLE I
MULTICAST TRANSMIT BEAMFORMING

Fig.	N	M	G	P	γ [dB]	Time [s]
1	6	30	3	28.32	10	0.26
2	12	30	3	10.44	10	0.52
3	12	30	3	12.35	10	12
4	6	44	2	9.56	10/6	0.31
5	12	44	2	6.03	10/6	0.45
6	6	44	2	10.82	10/6	3.3
7	8	22	2	10	9.45	0.20
8	8	22	2	10	7.97	0.26
9	8	22	2	10	7.49	2.34

Feasibility problem $\mathcal{FR}_\mathrm{f}$ is an SDP problem, expressed in the standard primal form, consisting of $MN + 1 + GN$ linear equality constraints and $G + 2M$ positive-semidefinite constraints. As in the QoS case, it is of the same form as the original problem $\mathcal{F}_\mathrm{f}$, where full CSI is available, but now of higher dimensionality. Standard SDP solvers, such as SeDuMi [13], can be used to efficiently solve problem $\mathcal{FR}_\mathrm{f}$, by means of general-purpose interior point methods. A solution of tolerance ϵ entails $O[\sqrt{(G+2M)N}\log(1/\epsilon)]$ iterations, each of complexity $O[(G+2M)^3N^6 + (M+G)(G+2M)N^3]$ [14]. More efficient solutions, of significantly lower complexity, can again be found by means of application-specific interior point methods (see, e.g., [11]), and the optimum beamforming vectors $\{\mathbf{w}_k^{\mathrm{opt}}\}_{k=1}^G$ can be computed using spectral factorization techniques (see, e.g., [15]).

V. NUMERICAL RESULTS

In this section, we provide some representative numerical results illustrating and contrasting the various transmit beamformer design methods presented in Sections II–IV. For each design, the resulting optimized transmit beam pattern in the plane of the ULA is plotted in linear scale. All patterns are symmetric with respect to the vertical axis, due to the inherent radiation symmetry of the ULA. The Vandermonde channel vector of each user is calculated by plugging the respective angle in (1). The directions of the users comprising each multicast group are shown in the caption of each plot, for ease of reference. The noise variance of all channels is set to $\sigma^2 = 1$, except for the scenario in Fig. 8. The basic parameters for all simulation configurations considered are gathered in Table I. Columns two, three, and four contain the number N of transmit antenna elements (spaced $d = \lambda/2$ apart), the total number M of users served, and the number G of multicasts, respectively. Column five contains the minimum (over all intended users) received SINR (in decibels). For the QoS problems in Figs. 1–6, the value in column five is the SINR constraint (input parameter), whereas for the JMMF problems in Figs. 7–9, it is the attained objective value. Column six reports the total transmitted power, which is the objective value (constraint) for the QoS (respectively, JMMF) problems. The last column lists the time (in seconds) spent on a typical desktop computer to solve the core SDP problem for each design, using SeDuMi [13]. The reported values are averages values over ten problem instances.

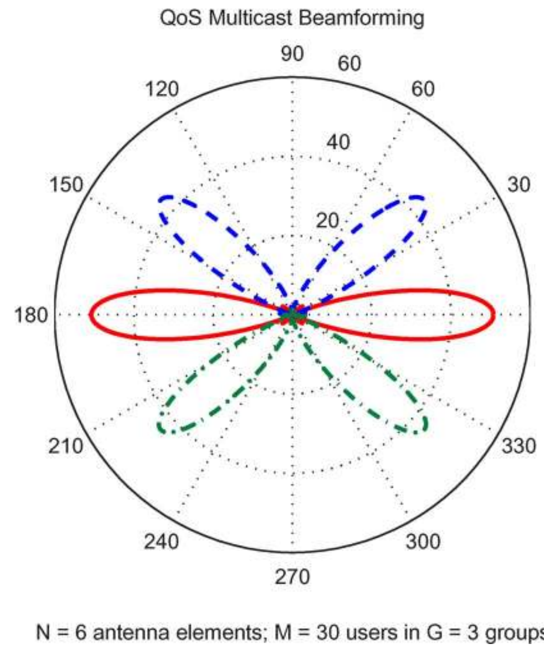

Fig. 1. QoS, $\gamma = 10$ dB, $N = 6$; $\mathcal{G}_1 = \{26^\circ : 4^\circ : 62^\circ\}$, $\mathcal{G}_2 = \{-18^\circ : 4^\circ : 18^\circ\}$, $\mathcal{G}_3 = \{-62^\circ : 4^\circ : -26^\circ\}$.

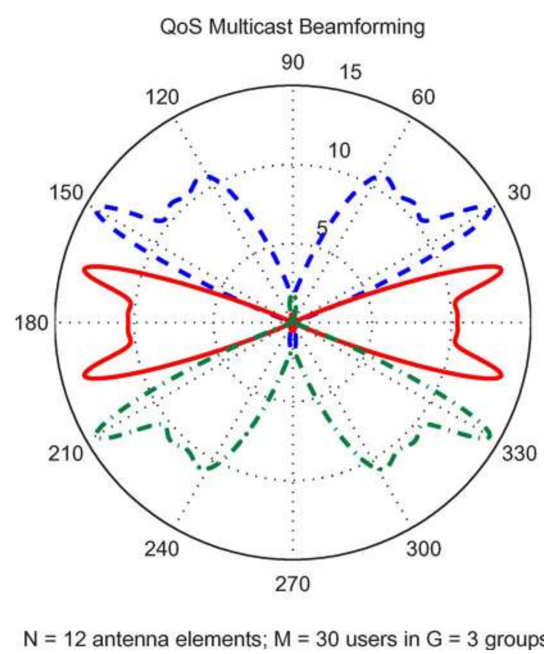

Fig. 2. QoS, $\gamma = 10$ dB, $N = 12$; $\mathcal{G}_1 = \{26^\circ : 4^\circ : 62^\circ\}$, $\mathcal{G}_2 = \{-18^\circ : 4^\circ : 18^\circ\}$, $\mathcal{G}_3 = \{-62^\circ : 4^\circ : -26^\circ\}$.

In the simplest configuration considered (Fig. 1), the transmit ULA consists of six antenna elements, and 30 intended receivers are clustered in three multicast groups. For the first multicast group, ten users are evenly distributed in the range 26°–62° at 4° apart. This is henceforth compactly denoted as $\{26^\circ : 4^\circ : 62^\circ\}$. The users of the second group are placed at $\{-18^\circ : 4^\circ : 18^\circ\}$, while the third group is the reflection of the first, with respect to

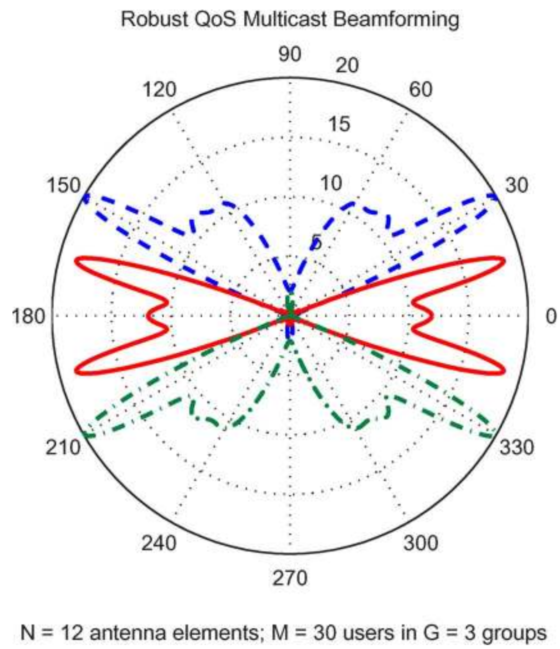

Fig. 3. Robust QoS, $\delta = 1^\circ$, $\gamma = 10$ dB, $N = 12$; $\mathcal{G}_1 = \{26^\circ : 4^\circ : 62^\circ\}$, $\mathcal{G}_2 = \{-18^\circ : 4^\circ : 18^\circ\}$, $\mathcal{G}_3 = \{-62^\circ : 4^\circ : -26^\circ\}$.

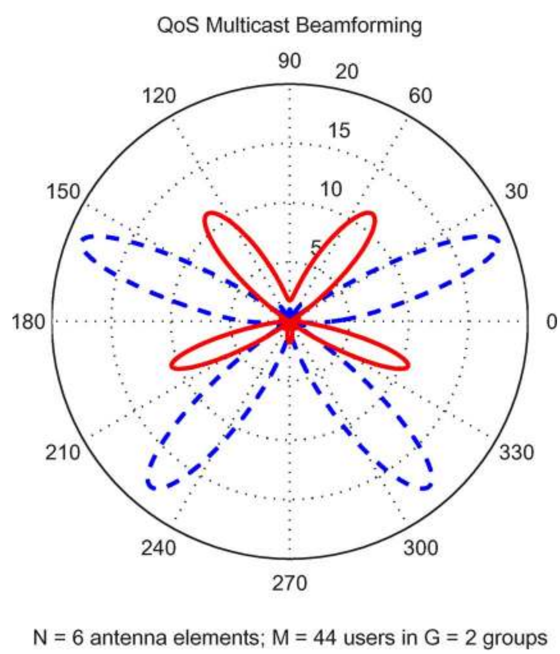

Fig. 4. QoS, $\gamma = \{10{,}6\}$ dB for $\{\mathcal{G}_1, \mathcal{G}_2\}$, $N = 6$; $\mathcal{G}_1 = \{-60^\circ : 2^\circ : -40^\circ, 10^\circ : 2^\circ : 30^\circ\}$, $\mathcal{G}_2 = \{-30^\circ : 2^\circ : -10^\circ, 40^\circ : 2^\circ : 60^\circ\}$.

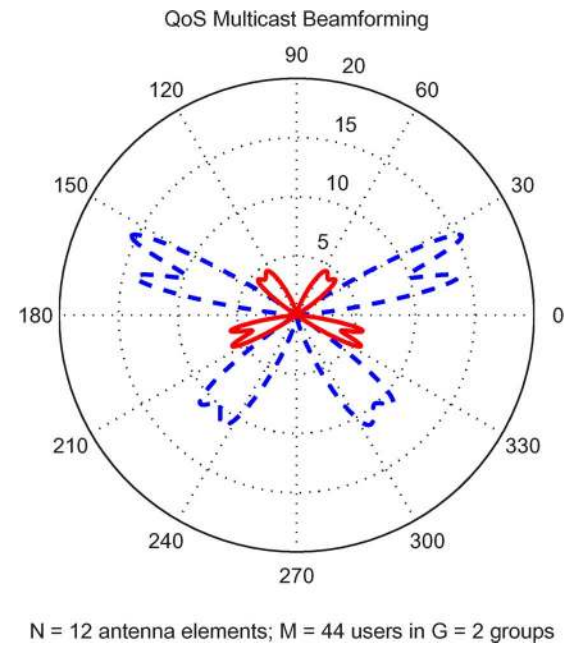

Fig. 5. QoS, $\gamma = \{10{,}6\}$ dB for $\{\mathcal{G}_1, \mathcal{G}_2\}$, $N = 12$; $\mathcal{G}_1 = \{-60^\circ : 2^\circ : -40^\circ, 10^\circ : 2^\circ : 30^\circ\}$, $\mathcal{G}_2 = \{-30^\circ : 2^\circ : -10^\circ, 40^\circ : 2^\circ : 60^\circ\}$.

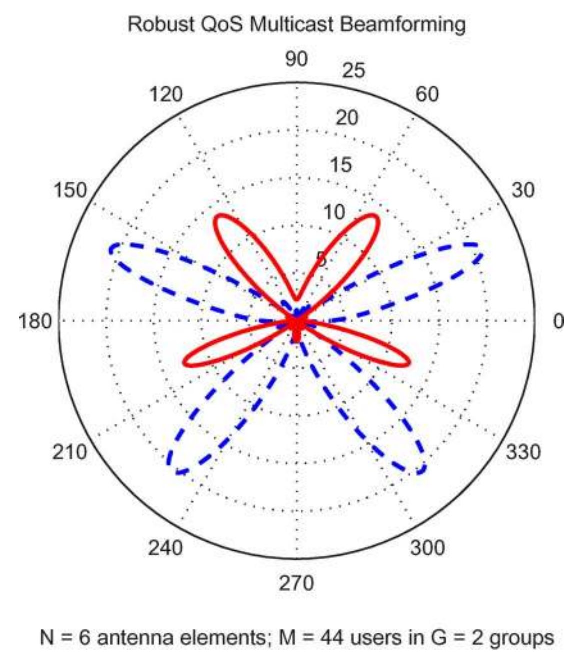

Fig. 6. Robust QoS, $\delta = 0.5^\circ$, $\gamma = \{10{,}6\}$ dB for $\{\mathcal{G}_1, \mathcal{G}_2\}$, $N = 6$; $\mathcal{G}_1 = \{-60^\circ : 2^\circ : -40^\circ, 10^\circ : 2^\circ : 30^\circ\}$, $\mathcal{G}_2 = \{-30^\circ : 2^\circ : -10^\circ, 40^\circ : 2^\circ : 60^\circ\}$.

the horizontal axis. Exact knowledge of all user directions is assumed at the design center (the transmitter). An SINR threshold of 10 dB is prescribed for all users. This is a typical scenario (the angles of the users listening to the same multicast are close and the number of transmit antenna elements is small) under which each beamvector forms a single main lobe to serve all users in the respective multicast group. Then, it is natural to expect that the optimum solution blocks of $\mathcal{Q}_\mathrm{r}$ will be rank one, and this is indeed the case. The algorithm proposed in [5] and [6] (principal components of the optimum rank-one solution blocks of the SDR problem $\mathcal{Q}_\mathrm{r}$) and the algorithm developed in Section II-B (spectral factorization of the autocorrelation sequences that solve problem $\mathcal{Q}_\mathrm{c}$) yields equivalent solutions, i.e., the beam pattern of Fig. 1.

It is apparent from the shape of the beams in Fig. 1 that the SINR constraint is oversatisfied for all users except the ones on the edges of the lobes. Note that, in such scenarios, the important design parameter is the direction span of each multicast

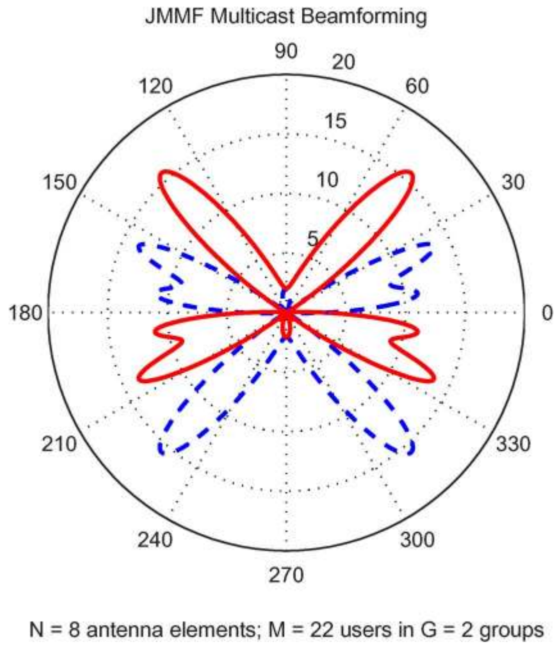

Fig. 7. JMMF, $P = 10, N = 8; \mathcal{G}_1 = \{-60^\circ : 5^\circ : -40^\circ, 5^\circ : 5^\circ : 30^\circ\}$, $\mathcal{G}_2 = \{-30^\circ : 5^\circ : -5^\circ, 40^\circ : 5^\circ : 60^\circ\}$.

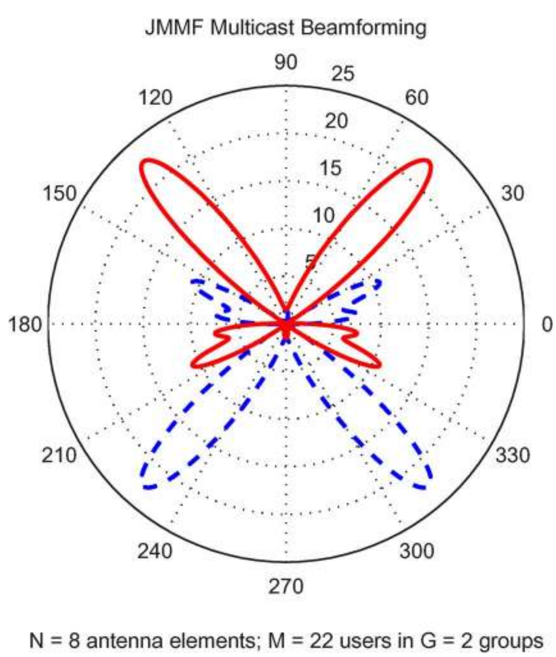

Fig. 8. JMMF, $P = 10, N = 8; \mathcal{G}_1 = \{-60^\circ : 5^\circ : -40^\circ, 5^\circ : 5^\circ : 30^\circ\}$, $\mathcal{G}_2 = \{-30^\circ : 5^\circ : -5^\circ, 40^\circ : 5^\circ : 60^\circ\}; \sigma^2 = 2$ for directions other than $\{-30^\circ : 30^\circ\}$.

group and not the actual number of users in the group. More subscribers can be added within the span of a given group without modifying the design. Repeating the design with the same parameters but this time using $N = 12$, the resulting beam pattern is plotted in Fig. 2. Due to the extra degrees of freedom, far less power is wasted in oversatisfying the constraints in the middle of the lobes. For proper interpretation of the results, it is important to note that the scale of the polar plots varies from figure to

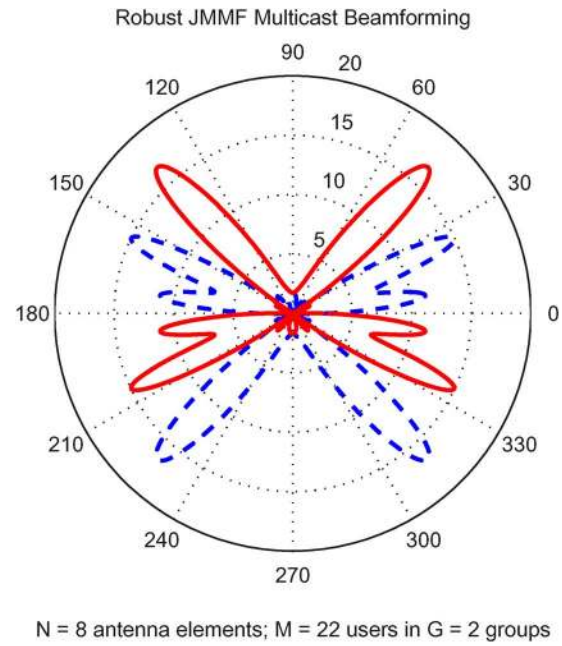

Fig. 9. Robust JMMF, $\delta = 2^\circ, P = 10, N = 8; \mathcal{G}_1 = \{-60^\circ : 5^\circ : -40^\circ, 5^\circ : 5^\circ : 30^\circ\}, \mathcal{G}_2 = \{-30^\circ : 5^\circ : -5^\circ, 40^\circ : 5^\circ : 60^\circ\}$.

figure for better visualization. The same SINR threshold is guaranteed to all users with smaller cost in terms of total transmitted power (cf. lines 1 and 2 of Table I), at the expense of additional hardware and complexity, because of the larger number of design variables. Fig. 3 plots the robust QoS design (presented in Section IV-A) for the same parameters and $N = 12$. The tolerance in the user directions is $\delta = 1^\circ$. Compared with Fig. 2, the beams are broader in Fig. 3, where higher total transmission power is required to assure the same minimum SINR level to wider ranges of directions. The runtime is also higher due to the additional auxiliary variables and constraints.

A more challenging scenario is considered in Figs. 4–6. The are two multicast groups, and the users in each group are split in two separate direction spans. Furthermore, the SINR threshold is 10 and 6 dB for the users listening to the first and the second multicast, respectively. Figs. 4 and 5 depict the optimum beam patterns for N equal to 6 and 12, respectively, and perfect CSI. Due to the interleaving of direction spans of the two groups, two main lobes are formed to serve each group. As expected, more power is transmitted towards the users of the first group which demand higher assured SINR. Again, the availability of more antenna elements at the transmitter results in less total radiated power. The respective robust design for $N = 6$ and maximum ambiguity $\delta = 0.5^\circ$ is illustrated in Fig. 6. A comparison with Fig. 4 supports the findings discussed in the previous scenario.

Figs. 7–9 illustrate an example of the JMMF beamformer design for $N = 8$, $G = 2$, and two clusters of users per multicast group. The power budget is set to 10, and the relative accuracy of the bisection to 10^{-3}, resulting in 13 iterations. Figs. 7 and 8 show the optimized beam patterns of the JMMF problem (presented in Section III) under the assumption of perfect CSI. In Fig. 8, the noise variance is doubled ($\sigma^2 = 2$), for the users in directions 40° or higher. Note that more power is transmitted towards the users who suffer from larger noise variance (or, equivalently, from larger path loss) to ensure fairness. The respective robust design (presented in Section IV-B) for $\delta = 2^\circ$, is shown in Fig. 9. Relative to Fig. 7, the SINR level assured to all users is smaller (cf. lines 7 and 9 of Table I), since wider direction spans are served with the same power budget.

VI. Conclusion

We considered problem of far-field multicast beamforming of transmit ULAs, under line-of-sight propagation conditions. We adopted both QoS (minimum SINR)-oriented and max–min fair design criteria, and exploited the Vandermonde structure of the channel vectors to derive equivalent convex reformulations, which are amenable to exact and efficient solution using modern interior point methods. This proves that, whereas the general multicast beamforming problem is NP-hard, the important special case considered here is not. In addition, we showed that the natural (Lagrange bi-dual) SDR of the problem is tight in the case of Vandermonde channel vectors. The key tool behind these developments is spectral factorization and the representation of finite autocorrelation sequences via linear matrix inequalities. Departing from the idealized assumption of perfect CSI, we also considered the situation where the receiver directions are (only) known to lie in a certain interval. We formulated robust design problems for this case, under both QoS and max–min fair criteria and showed how these can also be reformulated as SDP problems. The proposed designs have potential for practical use in a number of current and emerging wireless systems.

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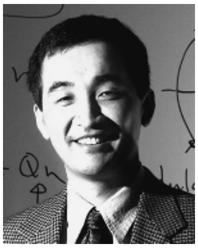

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