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Fares and tolls in a competitive system with transit and highway: the case with two groups of commuters

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Abstract

This paper deals with pricing and modal split in a competitive mass transit/highway system with heterogeneous commuters. Two groups of commuters that differ in their disutility from travel time, schedule delay and transit crowding, select the transit or auto mode for traveling from a residential area to a workplace. We compare three pricing schemes: the marginal cost-based transit fare with no-toll (called 'm' for short), the average cost-based fare with no-toll ('a') and marginal cost-based fare with time-invariant toll for subsidizing transit ('s'), and derive a socially optimal combination of transit fare and road toll which minimizes the total social cost of the competitive system meanwhile ensuring no deficit to the transit side ('o'). The main findings from the analytical and numerical results are: (1) the 'o' policy generates the most total transit usage, then 's', 'm' and 'a' in order; (2) the total usage of each mode is independent of the demand composition when group 1 uses both modes; (3) the group 2's aversion to transit crowding does not affect total transit usage; (4) group 2 has relatively larger welfare gains from some changes in pricing policy, such as changing 'm' to 's' or to 'o'; (5) the a-policy results in the highest total social cost, then 'm', 's' and 'o' in that order. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

The simplest traffic bottleneck model studies commuting congestion in a highway with a single bottleneck between a residential area and a workplace, and investigates the effects of various road tolls to alleviate the queue behind the bottleneck. Because the capacity of the bottleneck is finite, each commuter is confronted with a trade-off between travel time cost relating to queue length and schedule delay cost of arriving early or late at work. The travel cost experienced by a

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commuter will be determined by his or her departure time from home. Using deterministic queuing theory, Vickrey (1969) first developed an endogenous departure time choice model, which leads to an equilibrium of costs on all commuters. Subsequently, his model has been extended by many others (see a review paper by Arnott et al., 1998).

In many cities there exists an alternative mass commuting mode such as railway or subway parallel to the road with a bottleneck. We are aware of only one study of a competitive system with mass transit and highway, that is Tabuchi (1993). He studied the modal split problem under various pricing regimes. Unlike the road, the mass transit exhibits scale economics in that the greater the number of transit commuters, the lower its average cost. The average cost may affect the determination of transit fare and the fare will affect commuters' mode choices. Furthermore, the trains of a railway normally arrive on time no matter how crowded their carriages may be. Obviously, the analysis of this system will be significantly different from that of a single mode system, although the cases having parallel roads in a single mode system also provide commuters with two or more substitutable choices on routes (Arnott et al., 1990; Braid, 1996; Huang and Yang, 1996; Mahmassani and Herman, 1984).

There are several points worthy of study beyond Tabuchi's work. First, it is true that mass transit can enable people to arrive at work on time, but meanwhile it may bring people discomfort generated by the body congestion at railway stations and in carriages if they are crowded. [1] Second, the model should consider the heterogeneity of commuters (Arnott et al., 1987, 1988, 1992, 1994; Cohen, 1987; Henderson, 1974, 1981). Commuters of different types will exhibit different decision-making behavior not only on departure time choices but also on mode choices. Generally, professional and self-employed workers have high values of times but relatively flexible work hours; in contrast, assembly-line workers and clerks or support staff in white-collar jobs have high values of schedule delay times (late in arriving at work) as they have rigid work schedules. The higher-income people may be more willing to pay out-of-pocket money to avoid queuing and body congestion (if this exchange is possible) than the lower-income ones. Third, the welfare effects of transit fare and road tolls on different commuting population are worth discussing (Arnott et al., 1994; Cohen, 1987; Evans, 1992; Glazer, 1981; Small, 1983). Huang et al. (1997) extended Tabuchi's (1993) study of modal choice in two directions by introducing crowding congestion on transit and by admitting two groups of commuters. Introducing crowding congestion, to a certain extent, may relax Tabuchi's infinite capacity assumption on transit; the later implies no schedule delay because trains are very frequent. But, travelers do feel congestion in carriages when their arrival at the station does not match the trains frequency and carrying ability. However, these extensions to Tabuchi's work did not derive the socially optimal combination of transit fare and road toll that generates sufficient revenue to cover transit's fixed cost although the integrated pricing policies of this sort have been gaining favor. [2]

This paper refines the unpublished work by Huang et al. (1997) and derives the socially optimal combination of transit fare and road toll mentioned above. We examine four pricing schemes, some of them are commonly encountered in the literature and practice. Through comparing the analytical and numerical results, we try to find the relative efficiency of the four pricing schemes in

[1] The body congestion also leads to the loss of independence and privacy by transit commuters.

[2] I thank R. Lindsey who pointed out this in his reading of the working paper, Huang et al. (1997).

attracting transit commuters, changing individual travel costs and total social costs under certain conditions. The time-varying road pricing was also investigated in Huang et al. (1997), but eliminated in this paper to shorten the paper length. Extending the model to incorporate time-varying road pricing is not conceptually difficult, but the algebra is more tedious.

This paper is organized as follows. Section 2 investigates the modal splits and individual travel costs resulting from three pricing schemes, namely the marginal cost-based fare with no toll on the road, the average cost-based fare with no road toll, and a marginal cost-based fare with a time-invariant toll for subsidizing transit. Section 3 shows a socially optimal combination of transit fare and road toll, which minimizes the total social cost of the competitive system and lets the transit mode operate without deficit. Section 4 presents numerical results for comparing the modal choices, the individual travel costs and the total social costs under the four pricing policies discussed in this paper. Sensitivity analysis is also made in this section. Section 5 concludes the paper.

2. Modal splits under three pricing policies

Consider a simplified corridor network which contains two modes to provide transportation service between H (a residential area or home) and W (a workplace). Mode 1 represents a mass transit (e.g., railway) with an assumed infinite capacity; mode 2 represents a highway with a single bottleneck which is located at the entering point of highway and has a deterministic capacity of s commuters per unit time. To keep the analysis manageable we limit consideration to two groups of commuters, i.e., we divide all commuters who can either travel on highway by car (i.e., auto mode, one person per car) or on railway by transit from H to W, into two groups which have different unit costs of crowding or discomfort on transit (θ_1, θ_2), travel time (α_1, α_2) and schedule delay time (β_1 and β_2 for time-early, γ_1 and γ_2 for time-late). We assume that $\theta_1 > \theta_2$, $\alpha_1/\beta_1 > \alpha_2/\beta_2, \gamma_1/\beta_1 = \gamma_2/\beta_2 = \eta$, [3] and all commuters have the same official work start time. These assumptions indicate that group 1 dislikes even more the discomfort resulted from body congestion in transit and has higher ratio of travel time cost to schedule delay cost than group 2. Hence, group 1 is more likely to comprise relatively highly paid white-collar workers, with flexible work hours but high values of time, group 2 likely consists of blue-collar workers and clerks or support staff in white-collar jobs, with rigid work schedules but relatively lower values of time.

More general cases which may approach closer to reality have been considered by others. Newell (1987) dealt with a continuous distribution of commuters differing in work start time, costs of travel time and costs of schedule delay. Vickrey (1969) assumed that the work start times are uniformly distributed in the commuting population over a one-hour interval. This assumption was adopted by Cohen (1987). Arnott et al. (1994) treated more than two groups of commuters indexed in order of increasing β/α and having the same γ/β and work start time. In this paper, we consider the case in which two groups differ in three parameters (θ, α, β). This case is representative of the studies of bottleneck models, as explained by Arnott et al. (1992).

[3] The same relationships between the disutility parameters of travel time and schedule delay are used by Arnott et al. (1987, 1988, 1992, 1994) and Cohen (1987). In addition, the relation $\beta_i < \alpha_i < \gamma_i$, $i = 1, 2$, holds according to the estimates by Small (1982).

For sake of simplicity, in this paper we set the moving times on both modes to be zero. The moving time is the uncongested travel time from getting through the bottleneck to arriving at W for auto mode and the in-carriage time plus access (egress) time from H to the railway station (from railway station to W). Huang et al. (1998) studied the case having different moving times on both modes but with identical commuters.

We first give the main outputs of the no-tolling bottleneck model with two groups of commuters. Let the numbers of auto commuters of group 1 and group 2 be N_{A1} and N_{A2}, respectively, and set $\delta_1 = \beta_1\gamma_1/(\beta_1+\gamma_1) = \beta_1\eta/(1+\eta)$, $\delta_2 = \beta_2\gamma_2/(\beta_2+\gamma_2) = \beta_2\eta/(1+\eta)$. At a no-toll equilibrium state, all individuals in the same group must experience identical travel cost, no matter when she or he leaves home. The equilibrium individual travel costs of group 1 and group 2 are

$$C_{A1} = \delta_1 \frac{N_{A1}+N_{A2}}{s}, \tag{1}$$

and

$$C_{A2} = \delta_2 \frac{N_{A2}}{s} + \frac{\alpha_2}{\alpha_1}\delta_1 \frac{N_{A1}}{s}, \tag{2}$$

respectively. A detailed derivation of Eqs. (1) and (2) can be found in Arnott et al. (1990, 1992) and is described concisely in the attached Appendix A.

We now consider the transit mode. Let N_{Ri} be the number of transit commuters of group i $(i = 1, 2)$, and define the individual travel cost experienced by group i as

$$p_R + \theta_i g(N_R), \tag{3}$$

where p_R is the transit fare and $g(N_R)$ represents the discomfort degree generated by body congestion at railway stations and in carriages. To simplify, we use a linear function in this paper, i.e., $g(N_R) = N_R = N_{R1} + N_{R2}$ (equivalents), where 'equivalent' is the unit of the function value.

In the following subsections, we let N_{A1}, N_{A2}, N_{R1} and N_{R2} become endogenous variables to be determined in a competitive system with transit and auto modes. This paper does not consider the elasticity of travel demand by each group, so lets the total number of commuters in each group, i.e., N_1 and N_2 for group 1 and group 2, respectively, be given. We then have $N_{A1} + N_{R1} = N_1$ and $N_{A2} + N_{R2} = N_2$. Therefore, we are in fact studying the interaction between commuter heterogeneity and mode choice. A similar study was done by Arnott et al. (1992) focusing on route choice. In Arnott et al. (1992), the ratio of congestion coefficients of the linear cost functions on two routes was assumed to be identical. This assumption does not hold in our study, [4] hence the formulae derived in this paper cannot be reproduced simply by applying Arnott et al.'s (1992) reduced form equation.

[4] For group 1, the own congestion coefficient for road travel is δ_1/s, and the cross-congestion coefficient is also δ_1/s, as shown in (1). The corresponding coefficients for taking transit are both θ_1. The ratios of the road coefficients to the transit coefficients are both $\delta_1/(s\theta_1)$, and hence the same. But for group 2 the ratios are not the same, *i.e.*, the ratio of the own congestion coefficients for taking two modes is $\delta_2/(s\theta_2)$ while the ratio of the cross-congestion coefficients is $\alpha_2\delta_1/(\alpha_1 s\theta_2)$.

2.1. Marginal cost-based fare on transit with no-toll on road

Denote c as the marginal (variable) cost of transit, which mainly comprises the expenses on labor, fuel, electricity and routine materials by the transit operator. In reality, most of these expenses are independent of the number of passengers carried. We assume that the coefficient, c, used in this paper covers the portions that do vary with the load. The pricing policy considered in this subsection sets the transit fare on the marginal cost basis, i.e., $p_R = c$. The equilibrium of travel costs for group 1 individuals, if they select both modes, can be expressed by

$$c + \theta_1(N_{R1}^{\mathrm{m}} + N_{R2}^{\mathrm{m}}) = \delta_1 \frac{N_{A1}^{\mathrm{m}} + N_{A2}^{\mathrm{m}}}{s}, \quad \text{for } N_{R1}^{\mathrm{m}} > 0,\ N_{A1}^{\mathrm{m}} > 0, \tag{4}$$

similarly, the cost equilibrium for group 2 individuals is

$$c + \theta_2(N_{R1}^{\mathrm{m}} + N_{R2}^{\mathrm{m}}) = \delta_2 \frac{N_{A2}^{\mathrm{m}}}{s} + \frac{\alpha_2}{\alpha_1}\delta_1 \frac{N_{A1}^{\mathrm{m}}}{s}, \qquad \text{for } N_{R2}^{\mathrm{m}} > 0,\ N_{A2}^{\mathrm{m}} > 0. \tag{5}$$

In Eqs. (4) and (5), the superscript m represents the ‘marginal cost-based fare on transit with no-toll on road’ policy. Note that what we study here is not the classical marginal cost-based congestion pricing for transit. The classical one requires that transit fare for each group should be the difference between the marginal social cost of an individual in the group and the travel cost that the individual bears him/herself. The fare works out, for both groups, to $c + \theta_1 N_{R1} + \theta_2 N_{R2}$. [5]

Solving Eqs. (4) and (5) with the conservation conditions $N_{A1}^{\mathrm{m}} + N_{R1}^{\mathrm{m}} = N_1$ and $N_{A2}^{\mathrm{m}} + N_{R2}^{\mathrm{m}} = N_2$, we get the modal split in equilibrium

$$N_{A1}^{\mathrm{m}} = \frac{\delta_2 + \theta_2 s}{\delta_2 - (\alpha_2/\alpha_1)\delta_1}\left[\frac{cs + \theta_1 s(N_1 + N_2)}{\delta_1 + \theta_1 s} - \frac{cs + \theta_2 s(N_1 + N_2)}{\delta_2 + \theta_2 s}\right], \tag{6a}$$

$$N_{A2}^{\mathrm{m}} = \frac{cs + \theta_1 s(N_1 + N_2)}{\delta_1 + \theta_1 s} - N_{A1}^{\mathrm{m}}, \tag{6b}$$

and N_{R1}^{m} and N_{R2}^{m} calculated by conservation conditions. The total number of commuters who choose transit as their travel mode is

$$N_R^{\mathrm{m}} = N_{R1}^{\mathrm{m}} + N_{R2}^{\mathrm{m}} = [\delta_1(N_1 + N_2) - cs]/(\delta_1 + \theta_1 s), \tag{7a}$$

and the total number of auto commuters is

$$N_A^{\mathrm{m}} = N_{A1}^{\mathrm{m}} + N_{A2}^{\mathrm{m}} = [cs + \theta_1 s(N_1 + N_2)]/(\delta_1 + \theta_1 s). \tag{7b}$$

Eq. (7a) shows the number of transit commuters is inversely proportional to θ_1. This indicates that the number of transit commuters will increase if the railway sector improves the service quality and it is recognized by higher-income people (i.e., θ_1-value decreases). Another interesting point showed by Eqs. (7a) and (7b) is that the total transit or highway usage is independent of parameter θ_2 although the modal split in each group is related to both θ_1 and θ_2. The reason is as

[5] The total social cost of the transit system is $(c + \theta_1 N_R)N_{R1} + (c + \theta_2 N_R)N_{R2}$. The marginal social costs for group 1 and group 2 are $c + \theta_1 N_R + \theta_1 N_{R1} + \theta_2 N_{R2}$ and $c + \theta_2 N_R + \theta_1 N_{R1} + \theta_2 N_{R2}$, respectively. The marginal private costs for group 1 and group 2 are $\theta_1 N_R$ and $\theta_2 N_R$, respectively.

follows: according to (4), for a given total usage $(N_1 + N_2)$ the cost equilibrium for group 1 requires a given total transit usage (N_R); this equilibrium condition is unaffected by a change in group 2's preferences (θ_2), so total transit usage cannot depend on θ_2.

The above explanation also applies to why the total transit or highway usage does not depend on the composition of demand but the total $(N_1 + N_2)$, as shown in (7a) and (7b). Certainly, so do the equilibrium individual costs. This conclusion holds only when an interior solution exists. The same phenomena will be observed in the following two subsections as the modal splits in both groups are functions of N_A^{m} and N_R^{m}. When an interior solution does not exist, the demand composition will affect the total transit usage, as demonstrated by the numerical example presented in Section 4 (see for example, Fig. 5 corresponding to case 2).

So far we have assumed that the equilibrium occurs at an interior point. Existence of an interior solution requires that $0 < N_{A1}^{\mathrm{m}} < N_1$ and $0 < N_{A2}^{\mathrm{m}} < N_2$. From (6a) and (6b), we know that these inequalities hold only when the values of parameters (including the demands in two groups) are in certain ranges. Otherwise, corner solutions, i.e., one group chooses exclusively one mode, have to be produced. For example, the modal split that group 2 chooses transit only while group 1 chooses both modes, implies

$$c + \theta_2(N_{R1}^{\mathrm{m}} + N_2) < \frac{\alpha_2}{\alpha_1}\delta_1\frac{N_{A1}^{\mathrm{m}}}{s}, \tag{8a}$$

and

$$c + \theta_1(N_{R1}^{\mathrm{m}} + N_2) = \delta_1\frac{N_{A1}^{\mathrm{m}}}{s}. \tag{8b}$$

Combining Eqs. (8a) and (8b) leads to

$$(\alpha_1 - \alpha_2)c + (N_{R1}^{\mathrm{m}} + N_2)(\theta_2\alpha_1 - \theta_1\alpha_2) < 0. \tag{9}$$

Inequality (9) may hold when $\theta_2\alpha_1 - \theta_1\alpha_2 < 0$, N_2 is sufficiently large and c is sufficiently small. In our numerical examples used in Section 4, $\theta_2\alpha_1 - \theta_1\alpha_2 < 0$. Similarly, the modal split that would lead group 1 to choose transit exclusively while group 2 chooses both modes, requires $(\delta_2 - \delta_1)c + (N_{R2}^{\mathrm{m}} + N_1)(\theta_1\delta_2 - \theta_2\delta_1) < 0$. Clearly, this inequality cannot hold when $\delta_1 < \delta_2$, then the modal split will not occur (as in our numerical examples).

2.2. Average cost-based fare on transit with no-toll on road

Let F be the fixed cost of transit, which consists of facility costs and fixed operating costs. Set the transit fare on average cost basis, i.e., $p_R^{\mathrm{a}} = c + F/(N_{R1}^{\mathrm{a}} + N_{R2}^{\mathrm{a}})$, here the superscript a represents the 'average cost-based fare on transit with no-toll on road' policy. In Eq. (4), replacing c by p_R^{a}, modal split variables by $N_R^{\mathrm{a}} = N_{R1}^{\mathrm{a}} + N_{R2}^{\mathrm{a}}$ and $N_A^{\mathrm{a}} = N_{A1}^{\mathrm{a}} + N_{A2}^{\mathrm{a}} = N_1 + N_2 - N_R^{\mathrm{a}}$, solving the revised Eq. (4), we then obtain a stable solution [6]

[6] There are two candidates for equilibrium solutions in a quadratic equation. One is unstable against small perturbations whereas the other is stable.

$$N_R^{\rm a} = \frac{N_R^{\rm m}}{2} + \sqrt{\left(\frac{N_R^{\rm m}}{2}\right)^2 - \frac{Fs}{\delta_1 + \theta_1 s}}, \tag{10a}$$

$$N_A^{\rm a} = \frac{N_1 + N_2}{2} + \frac{N_A^{\rm m}}{2} - \sqrt{\left(\frac{N_R^{\rm m}}{2}\right)^2 - \frac{Fs}{\delta_1 + \theta_1 s}}, \tag{10b}$$

where $N_R^{\rm m}$ and $N_A^{\rm m}$ are given by (7a) and (7b). From Eq. (10a) we have $N_R^{\rm a} < N_R^{\rm m}$ and then $N_A^{\rm a} > N_A^{\rm m}$, which is easy to understand because the number of transit commuters will certainly decrease with the fare increase. An interesting feature is that the number of transit commuters is still inversely proportional to θ_1-value under average cost-based fare policy. For interpreting this, we consider any two different θ_1-values with $0 < \theta_1' < \theta_1$. Eq. (10a) gives

$$\frac{N_R^{\rm a'}}{N_R^{\rm a}} = \frac{N_R^{\rm m'}}{N_R^{\rm m}} \frac{1 + \sqrt{1 - 4Fs/[(N_R^{\rm m'})^2(\delta_1 + \theta_1' s)]}}{1 + \sqrt{1 - 4Fs/[(N_R^{\rm m})^2(\delta_1 + \theta_1 s)]}}. \tag{11}$$

As shown before, $N_R^{\rm m} = [\delta_1(N_1 + N_2) - cs]/(\delta_1 + \theta_1 s)$, hence $N_R^{\rm m'} > N_R^{\rm m}$. And, $\theta_1' < \theta_1$ leads to

$$[\delta_1(N_1 + N_2) - cs)]^2/(\delta_1 + \theta_1' s) > [\delta_1(N_1 + N_2) - cs)]^2/(\delta_1 + \theta_1 s), \tag{12a}$$

which equals

$$(N_R^{\rm m'})^2(\delta_1 + \theta_1' s) > (N_R^{\rm m})^2(\delta_1 + \theta_1 s). \tag{12b}$$

Combining (11) and (12b), we then conclude $N_R^{\rm a'} > N_R^{\rm a}$. Note that $N_R^{\rm a'} > N_R^{\rm a}$ still holds if the fixed cost F rises to F' for improving service quality but $F'/F < (\delta_1 + \theta_1 s)/(\delta_1 + \theta_1' s)$. This point is important since it verifies that mass transit can still attract more commuters if the additional investment for enhancing service quality is controlled in certain content, although this investment is counted in the fare.

Using (5) with the substitution of average cost for marginal cost, and (10) that we just obtained, we get the modal split by groups:

$$N_{R1}^{\rm a} = \frac{1}{\delta_2 - (\alpha_2/\alpha_1)\delta_1}\left[cs + \frac{Fs}{N_R^{\rm a}} + (\delta_2 + \theta_2 s)N_R^{\rm a} - \left(\delta_2 N_2 + \frac{\alpha_2}{\alpha_1}\delta_1 N_1\right)\right], \tag{13a}$$

$$N_{A1}^{\rm a} = N_1 - N_{R1}^{\rm a}, \tag{13b}$$

$$N_{R2}^{\rm a} = N_R^{\rm a} - N_{R1}^{\rm a}, \tag{13c}$$

$$N_{A2}^{\rm a} = N_2 - N_{R2}^{\rm a}. \tag{13d}$$

Denote $C_1^{\rm m}$ and $C_2^{\rm m}$ as the individual costs of group 1 and group 2, respectively, under the marginal cost-base fare policy studied in Section 2.1, $C_1^{\rm a}$ and $C_2^{\rm a}$ as that under current a-policy, we have

$$C_1^{\rm a} - C_1^{\rm m} = \frac{\delta_1}{s}\left(\frac{N_R^{\rm m}}{2} - \sqrt{\left(\frac{N_R^{\rm m}}{2}\right)^2 - \frac{Fs}{\delta_1 + \theta_1 s}}\right) > 0, \tag{14a}$$

$$C_2^{\mathrm{a}} - C_2^{\mathrm{m}} = \left(\frac{\delta_1}{s} + \theta_1 - \theta_2\right)\left(\frac{N_R^{\mathrm{m}}}{2} - \sqrt{\left(\frac{N_R^{\mathrm{m}}}{2}\right)^2 - \frac{Fs}{\delta_1 + \theta_1 s}}\right) > 0. \tag{14b}$$

Eqs. (14a) and (14b) show the individual costs of both groups increase, but the rising size of group 2 is larger than that of group 1 (since $\theta_1 > \theta_2$). This implies group 2's utility loss is more than group 1's from the action of increasing transit fares.

Up to now, we summarize that the marginal cost-base fare policy attracts more commuters than the average cost-based fare policy, results in lower individual costs on both groups, but yields a deficit with the value of F on transit side. It is thus of strong interest to investigate the policy that uses the revenue generated by a road toll to subsidize mass transit when transit fare is set on the marginal cost-based fare. This is done in the following section.

2.3. Marginal cost-based fare with uniform toll for subsidizing transit

Assume that there exists a time-invariant or uniform road-use toll which can generate revenue to cover the fixed cost of transit, such a toll should equal F/N_{A}. The uniform road-use toll does not change the distribution of auto commuters' departure time choices and their costs relating to waiting times, so the individual costs in equilibrium, referring to (1) and (2), become $\delta_1 N_A^{\mathrm{s}}/s + F/N_A^{\mathrm{s}}$ for group 1 and $\delta_2 N_{A2}^{\mathrm{s}}/s + (\alpha_2/\alpha_1)\delta_1 N_{A1}^{\mathrm{s}}/s + F/N_A^{\mathrm{s}}$ for group 2, respectively, here the superscript s represents the 'marginal cost-based fare with uniform toll for subsidizing transit' policy. Denote the modal split as $(N_R^{\mathrm{s}}, N_A^{\mathrm{s}}) = (N_{R1}^{\mathrm{s}} + N_{R2}^{\mathrm{s}}, N_{A1}^{\mathrm{s}} + N_{A2}^{\mathrm{s}})$, the equation describing the equilibrium of group 1's individual costs is

$$c + \theta_1 N_R^{\mathrm{s}} = \delta_1 \frac{N_A^{\mathrm{s}}}{s} + \frac{F}{N_A^{\mathrm{s}}}, \quad \text{for } N_R^{\mathrm{s}} > 0,\ N_A^{\mathrm{s}} > 0, \tag{15}$$

with $N_R^{\mathrm{s}} + N_A^{\mathrm{s}} = N_1 + N_2$. We solve this equation with the following stable solution:

$$N_R^{\mathrm{s}} = \frac{N_1 + N_2}{2} + \frac{N_R^{\mathrm{m}}}{2} - \sqrt{\left(\frac{N_A^{\mathrm{m}}}{2}\right)^2 - \frac{Fs}{\delta_1 + \theta_1 s}}, \tag{16a}$$

$$N_A^{\mathrm{s}} = \frac{N_A^{\mathrm{m}}}{2} + \sqrt{\left(\frac{N_A^{\mathrm{m}}}{2}\right)^2 - \frac{Fs}{\delta_1 + \theta_1 s}}, \tag{16b}$$

where N_R^{m} and N_A^{m} are given by (7a) and (7b). Eq. (16b) shows that $N_A^{\mathrm{s}} < N_A^{\mathrm{m}}$ and then $N_R^{\mathrm{s}} > N_R^{\mathrm{m}}$. So, we get an important result about the total usage of each mode under the three kinds of pricing policies discussed in this section

$$N_R^{\mathrm{s}} > N_R^{\mathrm{m}} > N_R^{\mathrm{a}} \text{ and } N_A^{\mathrm{s}} < N_A^{\mathrm{m}} < N_A^{\mathrm{a}}. \tag{17a}$$

We also find

$$C_1^{\mathrm{s}} - C_1^{\mathrm{m}} = \theta_1(N_R^{\mathrm{s}} - N_R^{\mathrm{m}}) > 0 \quad \text{and} \quad C_2^{\mathrm{s}} - C_2^{\mathrm{m}} = \theta_2(N_R^{\mathrm{s}} - N_R^{\mathrm{m}}) > 0, \tag{17b}$$

where C_1^{s} and C_2^{s} are the individual travel costs of group 1 and group 2 generated by current s-policy. Note now that the rising size of group 2's individual cost from m- to s-policy is less than that of group 1 since $\theta_2 < \theta_1$. However, the analytical comparisons between C_i^{s} and C_i^{a} $(i = 1, 2)$

cannot be carried out because of their too complex formulae. Section 4 will demonstrate this by numerical results.

Using the equilibrium condition on group 2's individual costs, i.e.,

$$c+\theta_2 N_R^{\rm s}=\delta_2\frac{N_{A2}^{\rm s}}{s}+\frac{\alpha_2}{\alpha_1}\delta_1\frac{N_{A1}^{\rm s}}{s}+\frac{F}{N_A^{\rm s}},\quad \text{for } N_{A1}^{\rm s}>0,\ N_{A2}^{\rm s}>0, \tag{18}$$

as well as (16a) and (16b), we get the modal split by groups

$$N_{A1}^{\rm s}=\frac{1}{\delta_2-(\alpha_2/\alpha_1)\delta_1}\left[\delta_2 N_A^{\rm s}-\theta_2 s N_R^{\rm s}-\left(cs-\frac{Fs}{N_A^{\rm s}}\right)\right], \tag{19a}$$

$$N_{R1}^{\rm s}=N_1-N_{A1}^{\rm s}, \tag{19b}$$

$$N_{A2}^{\rm s}=N_A^{\rm s}-N_{A1}^{\rm s}, \tag{19c}$$

$$N_{R2}^{\rm s}=N_2-N_{A2}^{\rm s}. \tag{19d}$$

The uniform toll is

$$w_A^{\rm s}=F/(N_{A1}^{\rm s}+N_{A2}^{\rm s}). \tag{20}$$

3. Optimal combination of transit fare and road toll

We now derive a socially optimal combination of transit fare and road toll which minimizes the total social cost of the competitive system while ensuring no deficit to the transit side. The road-use tolling is still time-invariant as assumed in Section 2.3. Denote $p_R^{\rm o}$ and $w_A^{\rm o}$ as the transit fare and road toll, respectively, here the superscript o represents the 'optimal combination subject to the mentioned breakeven constraint' policy. We construct a minimization model for the title problem of this section

$$\begin{aligned}\min\ \mathrm{TSC}(N_{R1}^{\rm o},N_{A1}^{\rm o},N_{R2}^{\rm o},N_{A2}^{\rm o},p_R^{\rm o},w_A^{\rm o}) = {}& N_{A1}^{\rm o}\frac{\delta_1}{s}(N_{A1}^{\rm o}+N_{A2}^{\rm o})+N_{A2}^{\rm o}\left(\frac{\delta_2}{s}N_{A2}^{\rm o}+\frac{\alpha_2}{\alpha_1}\frac{\delta_1}{s}N_{A1}^{\rm o}\right)\\ &+N_{R1}^{\rm o}[\theta_1(N_{R1}^{\rm o}+N_{R2}^{\rm o})+c]\\ &+N_{R2}^{\rm o}[\theta_2(N_{R1}^{\rm o}+N_{R2}^{\rm o})+c]+F,\end{aligned} \tag{21}$$

subject to

$$p_R^{\rm o}+\theta_1(N_{R1}^{\rm o}+N_{R2}^{\rm o})=\frac{\delta_1}{s}(N_{A1}^{\rm o}+N_{A2}^{\rm o})+w_A^{\rm o}, \tag{22a}$$

$$p_R^{\rm o}+\theta_2(N_{R1}^{\rm o}+N_{R2}^{\rm o})=\frac{\delta_2}{s}N_{A2}^{\rm o}+\frac{\alpha_2}{\alpha_1}\frac{\delta_1}{s}N_{A1}^{\rm o}+w_A^{\rm o}, \tag{22b}$$

$$(p_R^{\rm o}-c)(N_{R1}^{\rm o}+N_{R2}^{\rm o})+w_A^{\rm o}(N_{A1}^{\rm o}+N_{A2}^{\rm o})=F, \tag{22c}$$

$$N_{R1}^{\rm o}+N_{A1}^{\rm o}=N_1, \tag{22d}$$

$$N^{o}_{R2} + N^{o}_{A2} = N_2, \tag{22e}$$

and all variables are non-negative. The total social cost of the transit/highway system is given by the objective function (21) in which the first two terms are the total social cost of the highway side and the last three terms are the total social cost of transit. Constraints (22a) and (22b) represent the equilibrium of individual costs for group1 and group 2, respectively. Eq. (22c) means the total revenue generated by transit fares and road tolls minus the total variable cost of transit, equals the fixed cost of transit. The revenue made by road tolls is used to subsidize transit. The conservation conditions of demands are described by (22d) and (22e).

The solution of the model is

$$N^{o}_{R} = N^{o}_{R1} + N^{o}_{R2} = \frac{\delta_1(N_1+N_2) - cs}{\delta_1 + \theta_1 s} + \frac{(\theta_1 - \theta_2)N_2 s + cs}{2(\delta_1 + \theta_1 s)}, \tag{23a}$$

$$N^{o}_{A} = N^{o}_{A1} + N^{o}_{A2} = \frac{(\theta_1 + \theta_2)N_2 s + 2\theta_1 N_1 s + cs}{2(\delta_1 + \theta_1 s)}, \tag{23b}$$

$$N^{o}_{A1} = \frac{(\delta_1 + \delta_2)N^{o}_{A} - s[\theta_1(N_1 - N^{o}_{A}) + \theta_2(N_2 + N^{o}_{R}) + c]}{\delta_2 - (\alpha_2/\alpha_1)\delta_1}, \tag{23c}$$

$$p^{o}_{R} = \frac{F + cN^{o}_{R} + (\delta_1/s)(N^{o}_{A})^2 - \theta_1(N^{o}_{R}N^{o}_{A})}{N_1 + N_2}, \tag{23d}$$

$$w^{o}_{A} = \frac{F + cN^{o}_{R} - (\delta_1/s)(N^{o}_{R}N^{o}_{A}) + \theta_1(N^{o}_{R})^2}{N_1 + N_2}, \tag{23e}$$

and N^{o}_{R1}, N^{o}_{A2} andN^{o}_{R2} computed by Eqs. (22d)–(23c). Comparing (23a) and $N^{m}_{R} = [\delta_1(N_1+N_2) - cs]/(\delta_1 + \theta_{1s})$ obtained in Section 2.1, we see that $N^{o}_{R} > N^{m}_{R}$. But, we do not know whether N^{o}_{R} is larger than N^{s}_{R}. Also, we are unable to compare the individual costs resulted from current policy with that by other pricing schemes analytically. So, we investigate these by numerical analyses in next section.

4. Numerical examples

Now, we use numerical results to support our analyses made above and to gain some insights into the characteristics of the pricing polices that the analyses cannot provide. The basic parameters are: $(\gamma_1, \alpha_1, \beta_1) = (3.0, 1.2, 0.6)$ (HK\$/min), $(\gamma_2, \alpha_2, \beta_2) = (3.5, 1.0, 0.7)$ (HK\$/min), $(\theta_1, \theta_2) = (0.02, 0.01)$ (HK\$/discomfort equivalent), $s = 5$ (veh/min), $c = 6.0$ (HK\$/commuter) and $F = 100$ (HK\$). It should be pointed out that these data do not correspond to reality but illustrate the logic required by the models presented in this paper. Arnott et al. (1990) in their numerical example used a group of data, which is relatively close to reality. We investigate the modal splits, individual travel costs and the total social costs resulted from the four pricing polices presented in this paper in three cases. Case 1 allows the total number of commuters to change from 150 to 250 while keeping the relative shares of the two groups unchanged, i.e.,

$N_1 = N_2 = 0.5(N_1 + N_2)$. Case 2 lets the portion of group 1 in the total number of commuters change from 0.1 to 0.9, the total is set to be 250. In Case 3, the relative strength of aversion to transit crowding by the two groups, measured by θ_1/θ_2, varies from 1.1 to 2.0 with given $\theta_2 = 0.01$ and $N_1 + N_2 = 250$. For Case 1, all numerical results concerning modal split, individual travel cost, total social cost and pricing level are shown in Figs. 1–4, respectively. For cutting down the paper length, we only investigate the modal splits for Case 2 and Case 3, as shown in Figs. 5 and 6.

Fig. 1 shows the number of commuters joining the transit mode versus the total number of commuters under various pricing policies (the four pricing policies are indicated by m, a, s and o in the legend, respectively). For each policy, the total transit usage increases with the total demand (this has been proved by the analytical solutions) and approaches the same point at higher levels of congestion. This convergence is because F is held fixed, hence its influence wanes as total usage and congestion grows. The four pricing regimes will yield identical solutions if the fixed costs are zero. Arranging the four pricing schemes in order of inducing total number of transit commuters, we have o, s, m and a, from the most to the fewest. Such order is still kept for group 1.

Fig. 2 shows the individual travel costs of the two groups versus the total demand. We first note that this figure coincides with the results given by (14a), (14b) and (17b), and all curves ascend monotonously with total demand except that of the a-policy. For each group, the largest individual cost is caused by a-policy; but the lowest is by m-policy for group 1 and by o-policy for group 2. From m to s, the rising size of group 1's individual cost is a bit larger than that of group 2, so group 1 loses more than group 2 in this policy change. From m to o, group 1's individual cost increases while group 2's decreases. Therefore, group 2, which likely consists of blue-collar workers, would welcome these policy changes, such as from m to s or to o. Note that in this figure the individual travel costs in regime m do not include the fixed cost, F.

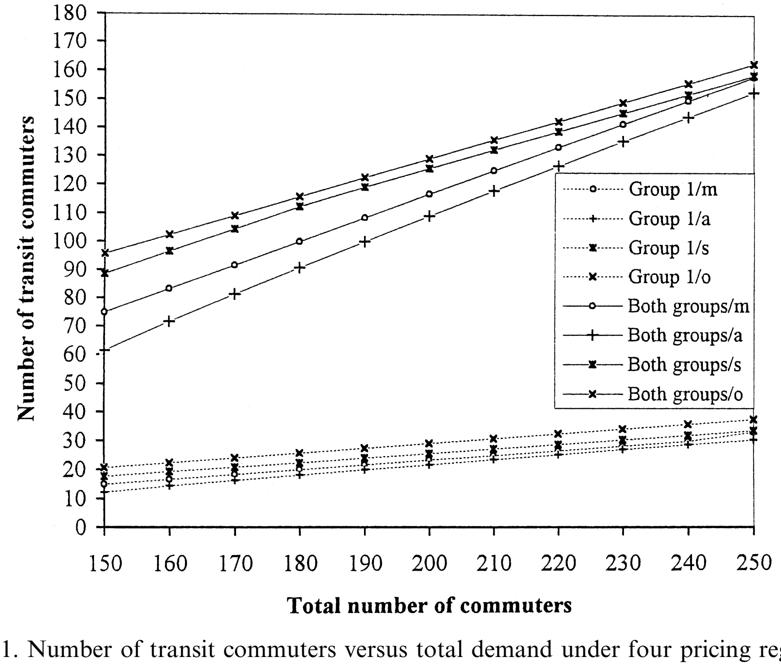

Fig. 1. Number of transit commuters versus total demand under four pricing regimes.

Fig. 3 shows the total social cost versus the total demand. The total social cost defined in this paper is the sum of all costs borne by transit operator and all commuters, but excluding fares and tolls. This figure shows that the a-policy generates the highest total social cost as it induces the

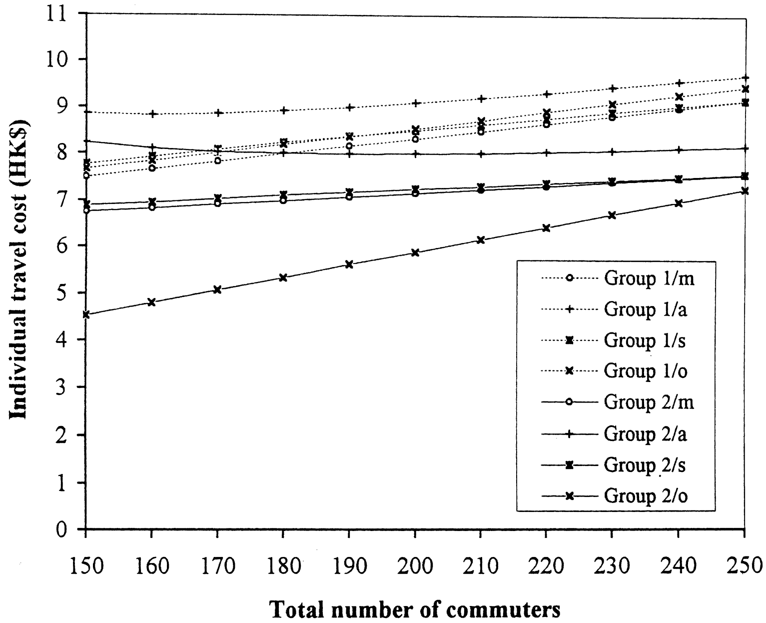

Fig. 2. Individual travel cost versus total demand under four pricing regimes.

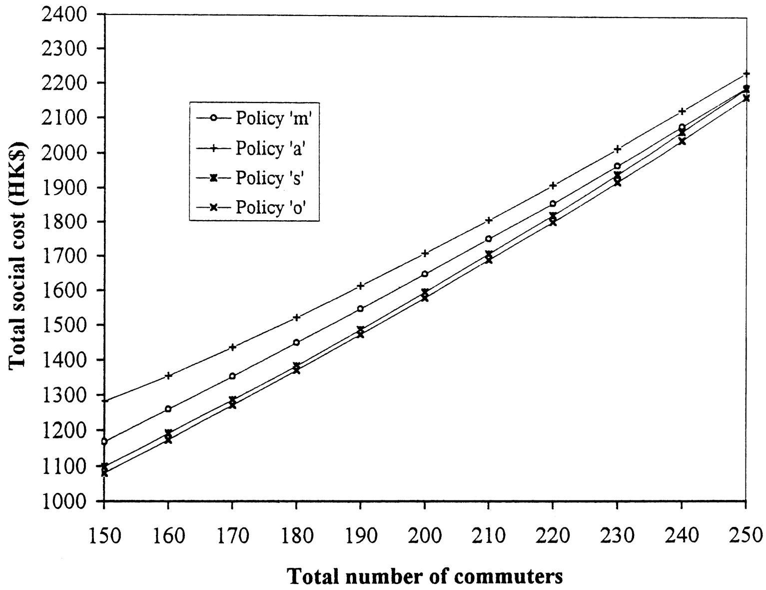

Fig. 3. Total social cost versus total demand under four pricing regimes.

fewest transit commuters (see Fig. 1). The o-policy generates the lowest total social cost as it induces the most transit commuters. Fig. 4 depicts the transit fares and road tolls corresponding to the four pricing schemes. The fare under a-policy and the toll under s-policy decreases with the

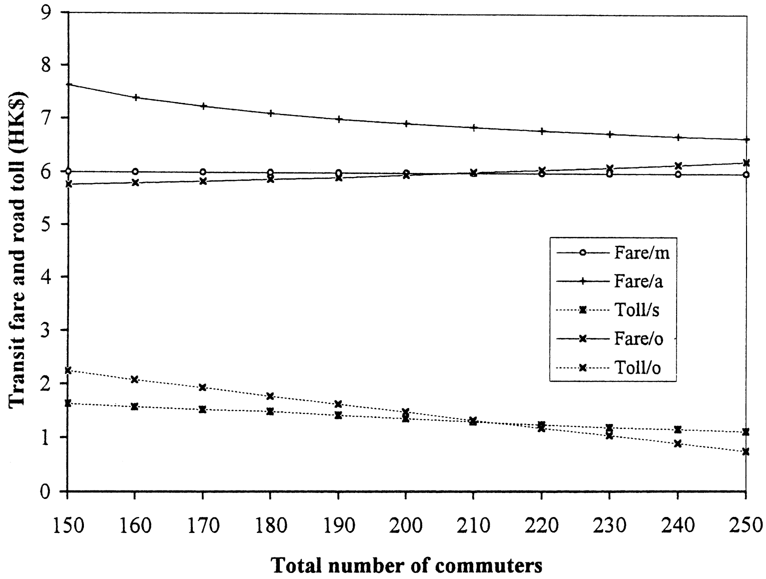

Fig. 4. Transit fare and road toll versus the total demand.

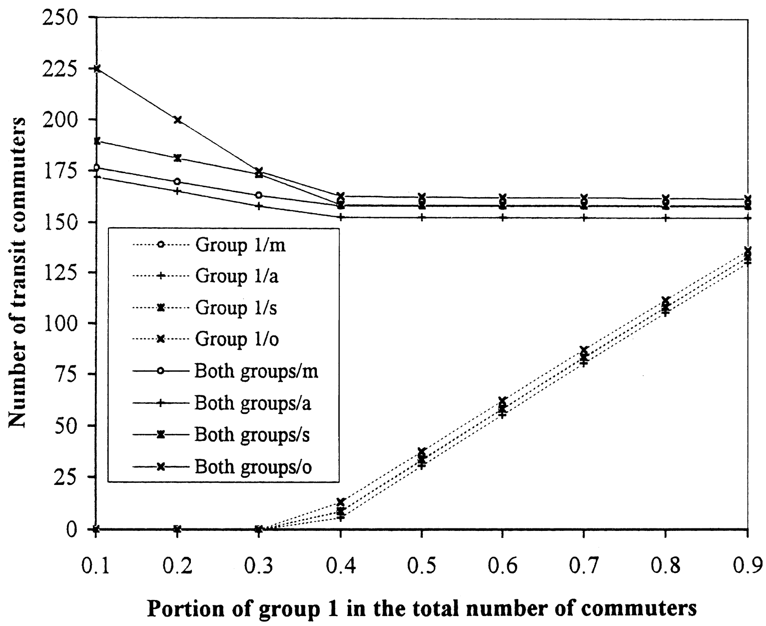

Fig. 5. Number of transit commuters versus the proportion of group 1 in total demand.

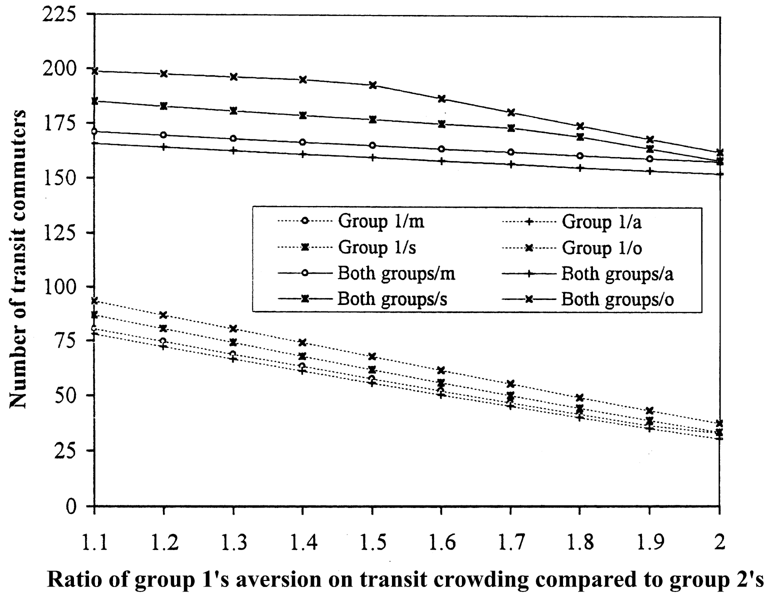

Fig. 6. Number of transit commuters compared to group 1 versus group 2's aversion to transit crowding.

total demand, as the fixed cost F is limited. Under the o-policy, the transit fare goes up gradually and the road toll comes down along with the total demand. Although this phenomenon is difficult to be analytically explained because of the complex expressions given by Eqs. (23d) and (23e), we try to give an intuitive explanation as below. The total level of pricing (fare plus toll, for example) should come down along with the total demand since the o-policy is subject to a breakeven constraint associated with a given fixed cost F. We note that the s-policy is subject to such a constraint too, by ensuring no deficit to transit, and Fig. 4 shows 'a' more quickly decreasing along with the total demand on road toll by o than by s. Hence, the transit fare by o must go up gradually in order to make up an increasing part of total revenue because the revenue generated from road tolls cannot cover the whole fixed cost of transit.

In Fig. 5, an obvious fact is that corner solutions appear when the portion of group 1 in total demand is less than 0.3, i.e., no commuters of group 1 select mass transit as their travel mode. When the portion exceeds 0.4, the total transit usage remains unchanged except that by the o-policy. This has been explained in Section 2.1. The total number of transit commuters induced by the o-policy falls as group 1 grows in relative size; this is simply because $\theta_1 > \theta_2$ and the total demand is fixed. In addition, according to the analytical solutions derived before, we can easily show that under the m-, a-, or s-policy, the derivative of the transit usage in group 1 to N_1 equals 1.0 when the total demand is given. This is because the highway usage of group 1 depends on the total value of the demand rather than its composition, as shown in (6a) for example, and the transit usage equals N_1 minus the highway usage. So, we see in Fig. 5 that the transit usage in group 1 increases linearly with the portion (>0.4), where the slope is 250 in fact.

Fig. 6 shows that for each pricing policy and with a fixed value of θ_2, the total transit usage decreases as the value of θ_1/θ_2 rises. This is what we predict. In addition, the figure also shows that group 1's transit usage is declining faster than the total transit usage. This is because θ_1 is rising while θ_2 is held fixed, with the result that group 2's transit usage is actually increasing.

5. Conclusions

In this article we studied pricing and modal split in a competitive mass transit/highway system with heterogeneous commuters. Two groups of commuters were considered that differ in their disutility from travel time, schedule delay and transit crowding. We investigated four pricing schemes, namely the marginal cost-based fare with no-toll (m), the average cost-based fare with no-toll (a), the marginal cost-based fare with time-invariant toll for subsidizing transit (s) and the socially optimal combination of transit fare and road toll subject to a breakeven constraint associated with transit's fixed cost (o).

Through this study, we understand how different charging policies affect the mode choice behavior of the commuters in different groups, the efficiency gains or losses of individuals and the total social cost. We gained some insights from the sensitivity analyses with the aid of numerical tests. Our findings may highly benefit the design and operations of transit fare and road toll in a competitive transportation system for some purposes such as encouraging transit mode's use. The analytical solutions and simulations reported in the paper lead us to the following conclusions:

1. According to the total number of transit commuters generated, the four pricing schemes are arranged in order of o, s, m and a, from the most to the fewest.
2. When group 1 uses both modes, the total usage of each mode is independent of the demand composition.
3. The total transit usage is independent of group 2's aversion to transit crowding.
4. The individual travel cost, from highest to lowest, ranks the four pricing polices as: a, s, o and m for group 1, and a, s, m, o for group 2. Group 2, which likely consists of blue-collar workers, would have relatively larger welfare gains from some changes in pricing policy, such as changing m to s or to o.
5. The a-policy results in the largest total social cost, then m, s and o in that order.

The above results rely heavily on numerical solutions for specific functional forms and parameter values. Hence, the generality of the results is restricted to some extent. However, our study provides a starting point for evaluating various pricing schemes, which are associated with a competitive system with transit and highway. The current research may be extended by including time-varying road pricing as mentioned before, more than two groups, work start time window, and stochastic mode choices.

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Appendix A

The individual travel cost of group i which is determined by the waiting time in queue and schedule delay (time-early or time-late in arriving at W), without road-use tolling, can be expressed as

$$\begin{aligned} C_{Ai}(t) &= \alpha_i[q(t)/s] + \beta_i[t^* - (t + q(t)/s)] \quad \text{for } t \in [t_q, t_0], \\ &= \alpha_i[q(t)/s] + \gamma_i[(t + q(t)/s) - t^*] \quad \text{for } t \in [t_0, t_{q'}], \end{aligned} \tag{A.1}$$

where $i = 1, 2$. In Eq. (A.1), $q(t)$ is the queue length at departure time t, s the capacity of the bottleneck, t^* the official work starting time, t_0 the departure time at which an individual can arrive at the workplace on time, and $[t_q, t_{q'}]$ is the rush hour to be determined. Eq. (A.1) employs a linear form of the schedule delay costs; a non-linear and more flexible form was used by Braid (1996). An equilibrium means that the individuals in the same group, no matter when she or he leaves home, must experience identical travel cost, hence $\mathrm{d}C_{Ai}(t)/\mathrm{d}t = 0$. This yields

$$\mathrm{d}q(t)/\mathrm{d}t = \begin{cases} \beta_i s/(\alpha_i - \beta_i) & \text{for } t \in (t_q, t_0), \\ -\gamma_i s/(\alpha_i + \gamma_i) & \text{for } t \in (t_0, t_{q'}), i = 1, 2. \end{cases} \tag{A.2}$$

The queue length evolves according to Eq. (A.2) only when group i is departing.

From Eq. (A.2), we know that the queue length is a piecewise linear function of the departure time t. There exist *three* slope turning points between t_q and $t_{q'}$, one at $t_{12} \in [t_q, t_0]$, one at $t_{21} \in [t_0, t_{q'}]$ and one at t_0, since $\beta_1 s/(\alpha_1 - \beta_1) \neq \beta_2 s/(\alpha_2 - \beta_2), -\gamma_1 s/(\alpha_1 + \gamma_1) \neq -\gamma_2 s/(\alpha_2 + \gamma_2)$ and $\beta_i s/(\alpha_i - \beta_i) \neq -\gamma_i s/(\alpha_i + \gamma_i)$. Due to $\beta_1 s/(\alpha_1 - \beta_1) < \beta_2 s/(\alpha_2 - \beta_2)$ and $\gamma_1 s/(\alpha_1 + \gamma_1) < \gamma_2 s/(\alpha_2 + \gamma_2)$ associated with the assumptions $\alpha_1/\beta_1 > \alpha_2/\beta_2$ and $\gamma_1/\beta_1 = \gamma_2/\beta_2 = \eta$, to minimize the individual costs, group 1 should have commuting at the beginning of the rush hour and again at the end while group 2 should travel at the middle of the rush hour. In other words, group 1 leaves home during $[t_q, t_{12}]$ and $[t_{21}, t_{q'}]$, and group 2 during $[t_{12}, t_{21}]$.

The first and last individuals of group 1 face no queuing time and have the same schedule delay costs, i.e., $\beta_1(t^* - t_q) = \gamma_1(t_{q'} - t^*)$. The bottleneck operates at capacity s during rush hour, so $t_{q'} - t_q = (N_{A1} + N_{A2})/s$. These facts lead to

$$t_q = t^* - \delta_1(N_{A1} + N_{A2})/(\beta_1 s) = t^* - \eta(N_{A1} + N_{A2})/((1 + \eta)s), \tag{A.3}$$

$$t_{q'} = t^* + \delta_1(N_{A1} + N_{A2})/(\gamma_1 s) = t^* + (N_{A1} + N_{A2})/((1 + \eta)s), \tag{A.4}$$

and the individual cost of group 1

$$C_{A1} = \delta_1 \frac{N_{A1} + N_{A2}}{s}. \tag{A.5}$$

It remains to find the values of t_{12} and t_{21}, and the individual cost of group 2.

Let the queue lengths at time t_{12} and t_{21} be $q(t_{12})$ and $q(t_{21})$, respectively. An individual of group 1 leaving home at time t_{12} has cost $C_{A1}(t_{12}) = \alpha_1 q(t_{12})/s + \beta_1(t^* - t_{12} - q(t_{12})/s)$, and cost $C_{A1}(t_{21}) = \alpha_1 q(t_{21})/s + \gamma_1(t_{21} + q(t_{21})/s - t^*) = C_{A1}(t_{12})$ at time t_{21}. Similarly, for individuals of group 2 leaving home at time t_{12} and time t_{21}, they have costs $C_{A2}(t_{12}) = \alpha_2 q(t_{12})/s + \beta_2(t^* - t_{12} - q(t_{12})/s) = C_{A2}(t_{21}) = \alpha_2 q(t_{21})/s + \gamma_2(t_{21} + q(t_{21})/s - t^*)$. Subtracting equation $C_{A1}(t_{12})/\beta_1 = C_{A1}(t_{21})/\beta_1$ from equation $C_{A2}(t_{12})/\beta_2 = C_{A2}(t_{21})/\beta_2$ leads to $q(t_{12}) = q(t_{21})$. This fact shows that $t_{21} - t_{12} = N_{A2}/s$. Utilizing the slopes of the queue length curve given by (A.2), we have $q(t_{12}) = (t_{12} - t_q)\beta_1 s/(\alpha_1 - \beta_1) = q(t_{21}) = (t_{q'} - t_{21})\gamma_1 s/(\alpha_1 + \gamma_1)$. Combining these relations and the definition of t_0, i.e., $t_0 + q(t_0)/s = t^*$, we obtain

$$t_{12} = t^* - \frac{\eta}{1+\eta}\frac{N_{A2}}{s} - \frac{\beta_1}{\alpha_1}\frac{\eta}{1+\eta}\frac{N_{A1}}{s}, \tag{A.6}$$

$$t_{21} = t^* + \frac{1}{1+\eta}\frac{N_{A2}}{s} - \frac{\beta_1}{\alpha_1}\frac{\eta}{1+\eta}\frac{N_{A1}}{s}, \tag{A.7}$$

$$t_0 = t^* - \frac{\beta_2}{\alpha_2}\frac{\eta}{1+\eta}\frac{N_{A2}}{s} - \frac{\beta_1}{\alpha_1}\frac{\eta}{1+\eta}\frac{N_{A1}}{s}, \tag{A.8}$$

and the individual cost of group 2

$$C_{A2} = \delta_2\frac{N_{A2}}{s} + \frac{\alpha_2}{\alpha_1}\delta_1\frac{N_{A1}}{s}. \tag{A.9}$$

Eqs. (A.5) and (A.9) are the main results that we use frequently in this paper.

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