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Richard E. Blahut
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Fast Algorithms for Signal Processing

Efficient algorithms for signal processing are critical to very large scale future applications such as video processing and four-dimensional medical imaging. Similarly, efficient algorithms are important for embedded and power-limited applications since, by reducing the number of computations, power consumption can be reduced considerably. This unique textbook presents a broad range of computationally-efficient algorithms, describes their structure and implementation, and compares their relative strengths. All the necessary background mathematics is presented, and theorems are rigorously proved. The book is suitable for researchers and practitioners in electrical engineering, applied mathematics, and computer science.

Richard E. Blahut is a Professor of Electrical and Computer Engineering at the University of Illinois, Urbana-Champaign. He is Life Fellow of the IEEE and the recipient of many awards including the IEEE Alexander Graham Bell Medal (1998) and Claude E. Shannon Award (2005), the Tau Beta Pi Daniel C. Drucker Eminent Faculty Award, and the IEEE Millennium Medal. He was named a Fellow of the IBM Corporation in 1980, where he worked for over 30 years, and was elected to the National Academy of Engineering in 1990.

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CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521190497

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First published 2010

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data
Blahut, Richard E.
Fast algorithms for signal processing / Richard E. Blahut.
p. cm.
Includes bibliographical references and index.
ISBN 978-0-521-19049-7 (hardback)
1. Signal processing – Digital techniques. 2. Algorithms. I. Title.
TK5102.9.A378 1995
621.382′2 – dc22 2010007111

ISBN 978-0-521-19049-7 Hardback

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In loving memory of

Jeffrey Paul Blahut

May 2, 1968 – June 13, 2004

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Many small make a great.

— Chaucer

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Frontmatter
More information

Contents

Preface xi
Acknowledgments xiii

1 Introduction 1

1.1 Introduction to fast algorithms 1
1.2 Applications of fast algorithms 6
1.3 Number systems for computation 8
1.4 Digital signal processing 9
1.5 History of fast signal-processing algorithms 17

2 Introduction to abstract algebra 21

2.1 Groups 21
2.2 Rings 26
2.3 Fields 30
2.4 Vector space 34
2.5 Matrix algebra 37
2.6 The integer ring 44
2.7 Polynomial rings 48
2.8 The Chinese remainder theorem 58

3 Fast algorithms for the discrete Fourier transform 68

3.1 The Cooley–Tukey fast Fourier transform 68
3.2 Small-radix Cooley–Tukey algorithms 72
3.3 The Good–Thomas fast Fourier transform 80

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Richard E. Blahut
Frontmatter
More information

3.4 The Goertzel algorithm 83
3.5 The discrete cosine transform 85
3.6 Fourier transforms computed by using convolutions 91
3.7 The Rader–Winograd algorithm 97
3.8 The Winograd small fast Fourier transform 102

4 Fast algorithms based on doubling strategies 115

4.1 Halving and doubling strategies 115
4.2 Data structures 119
4.3 Fast algorithms for sorting 120
4.4 Fast transposition 122
4.5 Matrix multiplication 124
4.6 Computation of trigonometric functions 127
4.7 An accelerated euclidean algorithm for polynomials 130
4.8 A recursive radix-two fast Fourier transform 139

5 Fast algorithms for short convolutions 145

5.1 Cyclic convolution and linear convolution 145
5.2 The Cook–Toom algorithm 148
5.3 Winograd short convolution algorithms 155
5.4 Design of short linear convolution algorithms 164
5.5 Polynomial products modulo a polynomial 168
5.6 Design of short cyclic convolution algorithms 171
5.7 Convolution in general fields and rings 176
5.8 Complexity of convolution algorithms 178

6 Architecture of filters and transforms 194

6.1 Convolution by sections 194
6.2 Algorithms for short filter sections 199
6.3 Iterated filter sections 202
6.4 Symmetric and skew-symmetric filters 207
6.5 Decimating and interpolating filters 213
6.6 Construction of transform computers 216
6.7 Limited-range Fourier transforms 221
6.8 Autocorrelation and crosscorrelation 222

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CAMBRIDGE

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978-0-521-19049-7 - Fast Algorithms for Signal Processing
Richard E. Blahut
Frontmatter
More information

7 Fast algorithms for solving Toeplitz systems 231

7.1 The Levinson and Durbin algorithms 231
7.2 The Trench algorithm 239
7.3 Methods based on the euclidean algorithm 245
7.4 The Berlekamp–Massey algorithm 249
7.5 An accelerated Berlekamp–Massey algorithm 255

8 Fast algorithms for trellis search 262

8.1 Trellis and tree searching 262
8.2 The Viterbi algorithm 267
8.3 Sequential algorithms 270
8.4 The Fano algorithm 274
8.5 The stack algorithm 278
8.6 The Bahl algorithm 280

9 Numbers and fields 286

9.1 Elementary number theory 286
9.2 Fields based on the integer ring 293
9.3 Fields based on polynomial rings 296
9.4 Minimal polynomials and conjugates 299
9.5 Cyclotomic polynomials 300
9.6 Primitive elements 304
9.7 Algebraic integers 306

10 Computation in finite fields and rings 311

10.1 Convolution in surrogate fields 311
10.2 Fermat number transforms 314
10.3 Mersenne number transforms 317
10.4 Arithmetic in a modular integer ring 320
10.5 Convolution algorithms in finite fields 324
10.6 Fourier transform algorithms in finite fields 328
10.7 Complex convolution in surrogate fields 331

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CAMBRIDGE

Cambridge University Press
978-0-521-19049-7 - Fast Algorithms for Signal Processing
Richard E. Blahut
Frontmatter
More information

10.8 Integer ring transforms 336
10.9 Chevillat number transforms 339
10.10 The Preparata–Sarwate algorithm 339

11 Fast algorithms and multidimensional convolutions 345

11.1 Nested convolution algorithms 345
11.2 The Agarwal–Cooley convolution algorithm 350
11.3 Splitting algorithms 357
11.4 Iterated algorithms 362
11.5 Polynomial representation of extension fields 368
11.6 Convolution with polynomial transforms 371
11.7 The Nussbaumer polynomial transforms 372
11.8 Fast convolution of polynomials 376

12 Fast algorithms and multidimensional transforms 384

12.1 Small-radix Cooley–Tukey algorithms 384
12.2 The two-dimensional discrete cosine transform 389
12.3 Nested transform algorithms 391
12.4 The Winograd large fast Fourier transform 395
12.5 The Johnson–Burrus fast Fourier transform 399
12.6 Splitting algorithms 403
12.7 An improved Winograd fast Fourier transform 410
12.8 The Nussbaumer–Quandalle permutation algorithm 411

A A collection of cyclic convolution algorithms 427

B A collection of Winograd small FFT algorithms 435

Bibliography 442
Index 449

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CAMBRIDGE

Cambridge University Press
978-0-521-19049-7 - Fast Algorithms for Signal Processing
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More information

Preface

A quarter of a century has passed since the previous version[1] of this book was published, and signal processing continues to be a very important part of electrical engineering. It forms an essential part of systems for telecommunications, radar and sonar, image formation systems such as medical imaging, and other large computational problems, such as in electromagnetics or fluid dynamics, geophysical exploration, and so on. Fast computational algorithms are necessary in large problems of signal processing, and the study of such algorithms is the subject of this book. Over those several decades, however, the nature of the need for fast algorithms has shifted both to much larger systems on the one hand and to embedded power-limited applications on the other.

Because many processors and many problems are much larger now than they were when the original version of this book was written, and the relative cost of addition and multiplication now may appear to be less dramatic, some of the topics of twenty years ago may be seen by some to be of less importance today. I take exactly the opposite point of view for several reasons. Very large three-dimensional or four-dimensional problems now under consideration require massive amounts of computation and this computation can be reduced by orders of magnitude in many cases by the choice of algorithm. Indeed, these very large problems can be especially suitable for the benefits of fast algorithms. At the same time, smaller signal processing problems now appear frequently in handheld or remote applications where power may be scarce or nonrenewable. The designer's care in treating an embedded application, such as a digital television, can repay itself many times by significantly reducing the power expenditure. Moreover, the unfamiliar algorithms of this book now can often be handled automatically by computerized design tools, and in embedded applications where power dissipation must be minimized, a search for the algorithm with the fewest operations may be essential.

Because the book has changed in its details and the title has been slightly modernized, it is more than a second edition, although most of the topics of the original book have been retained in nearly the same form, but usually with the presentation rewritten. Possibly, in time, some of these topics will re-emerge in a new form, but that time

[1] *Fast Algorithms for Digital Signal Processing*, Addison-Wesley, Reading, MA, 1985.

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is not now. A newly written book might look different in its choice of topics and its balance between topics than does this one. To accommodate this consideration here, the chapters have been rearranged and revised, even those whose content has not changed substantially. Some new sections have been added, and all of the book has been polished, revised, and re-edited. Most of the touch and feel of the original book is still evident in this new version.

The heart of the book is in the Fourier transform algorithms of Chapters 3 and 12 and the convolution algorithms of Chapters 5 and 11. Chapters 12 and 11 are the multidimensional continuations of Chapters 3 and 4, respectively, and can be partially read immediately thereafter if desired. The study of one-dimensional convolution algorithms and Fourier transform algorithms is only completed in the context of the multidimensional problems. Chapters 2 and 9 are mathematical interludes; some readers may prefer to treat them as appendices, consulting them only as needed. The remainder, Chapters 4, 7, and 8, are in large part independent of the rest of the book. Each can be read independently with little difficulty.

This book uses branches of mathematics that the typical reader with an engineering education will not know. Therefore these topics are developed in Chapters 2 and 9, and all theorems are rigorously proved. I believe that if the subject is to continue to mature and stand on its own, the necessary mathematics must be a part of such a book; appeal to a distant authority will not do. Engineers cannot confidently advance through the subject if they are frequently asked to accept an assertion or to visit their mathematics library.

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Acknowledgments

My major debt in writing this book is to Shmuel Winograd. Without his many contributions to the subject, the book would be shapeless and much shorter. He was also generous with his time in clarifying many points to me, and in reviewing early drafts of the original book. The papers of Winograd and also the book of Nussbaumer were a source for much of the material discussed in this book.

The original version of this book could not have reached maturity without being tested, critiqued, and rewritten repeatedly. I remain indebted to Professor B. W. Dickinson, Professor Toby Berger, Professor C. S. Burrus, Professor J. Gibson, Professor J. G. Proakis, Professor T. W. Parks, Dr B. Rice, Professor Y. Sugiyama, Dr W. Vanderkulk, and Professor G. Verghese for their gracious criticisms of the original 1985 manuscript. That book could not have been written without the support that was given by the International Business Machines Corporation. I am deeply grateful to IBM for this support and also to Cornell University for giving me the opportunity to teach several times from the preliminary manuscript of the earlier book. The revised book was written in the wonderful collaborative environment of the Department of Electrical and Computer Engineering and the Coordinated Science Laboratory of the University of Illinois. The quality of the book has much to with the composition skills of Mrs Francie Bridges and the editing skills of Mrs Helen Metzinger. And, as always, Barbara made it possible.

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