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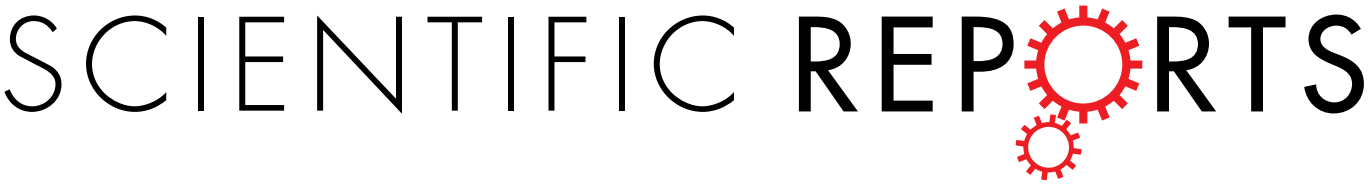

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Fast and simple high-capacity quantum cryptography with error detection

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Quantum cryptography is commonly used to generate fresh secure keys with quantum signal transmission for instant use between two parties. However, research shows that the relatively low key generation rate hinders its practical use where a symmetric cryptography component consumes the shared key. That is, the security of the symmetric cryptography demands frequent rate of key updates, which leads to a higher consumption of the internal one-time-pad communication bandwidth, since it requires the length of the key to be as long as that of the secret. In order to alleviate these issues, we develop a matrix algorithm for fast and simple high-capacity quantum cryptography. Our scheme can achieve secure private communication with fresh keys generated from Fibonacci- and Lucas-valued orbital angular momentum (OAM) states for the seed to construct recursive Fibonacci and Lucas matrices. Moreover, the proposed matrix algorithm for quantum cryptography can ultimately be simplified to matrix multiplication, which is implemented and optimized in modern computers. Most importantly, considerably information capacity can be improved effectively and efficiently by the recursive property of Fibonacci and Lucas matrices, thereby avoiding the restriction of physical conditions, such as the communication bandwidth.

Quantum cryptography provides a feasible solution to the key generation and key management issues for one-time pad (OTP) encryption. Recall that for the classical implementation of OTP, a cryptographic key needs to be generated and distributed to the communicating parties via a secure channel and well ahead of the use of OTP. This constraint is no longer valid in the quantum setting. The randomness necessary to create and distribute a cryptographic key is readily obtained by the parties from observations of quantum signals exchanged between the parties. The quantum key distribution (QKD) provides a straightforward implementation of OTP which preserves its unconditional security[1,2]. However, most QKD protocols suffer from its relatively low rate of key generation, limiting its widespread applications in deployment. This is caused by the nature of quantum computing where it uses polarization to encode only one qubit for each photon. A costly remedy exists with little practical use–qutrit or ququart exploitation can be achieved by adding much more complications to the QKD apparatus.

This paper addresses the problem of efficient generation of cryptographic keys in QKD. This problem has been investigated by many researchers[3–5]. Zhou *et al.*[3] present a four-intensity measurement-device-independent QKD protocol with a decoy state, which significantly improves the rate of key generation. This protocol works well if messages are not too long. Ma *et al.*[4] argue that the rate of key generation for QKD can be increased by using an entanglement parametric down-conversion (PDC) source. Other methods aiming to improve the rate of key generation include information encoding using high-dimensional (HD) photonic degrees of freedom (such

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as position momentum[6] and time energy[7,8]). At a practical level, however, it is difficult to increase the number of dimensions of the states encoded by phase over two, or at most, four.

Therefore, it is desirable to devise a more practical approach of encoding high dimensional states into a photon. On the one hand, spontaneous parametric down conversion (SPDC) is a simple way to produce squeezed and polarization-entangled light. On the other hand, recent advancements of orbital angular momentum (OAM) techniques are able to achieve faster generation of quantum states[9], and to enable better control[10] and easier integration with other systems[11]. First QKD protocols based on OAM have been proposed in refs 12–14. Based on these, Simon *et al.*[15] propose Fibonacci-valued OAM states for high-capacity QKD protocols together with SPDC. Their protocols are easier and simpler to implement than existing SPDC and OAM protocols. However, the rate of key generation can be improved up to 4 times only, which is inadequate for any practical use. Also it is impossible to support long-distance transmission with lower error rate. This means that protocols trade transmission distances with error rates (the further the transmission distance the higher the error rate and vice versa).

It seems that achieving a considerable key rate with a small data size is rather challenging in practical settings. In this paper, we extend and enhance the use of the protocol of Simon *et al.*'s, by enabling each detected Fibonacci number to encode up to a decent number of secret key bits per transmitted entangled photon, while achieving transmission over longer distance with lower error rates. To be exact, we present an approach that uses matrices together with slightly modified QKD protocol[15] to improve the rate of key generation. We observe that the conjugate relation (i.e., $L_{n+2} = F_{n+1} + F_{n-1}$) between Lucas and Fibonacci numbers[16], can be used to reduce side channel information leakage at the key preparation stage and hence to increase the security of QKD protocols[5]. Our observation is valid due to the contribution by Simon *et al.*[15] who have shown that both Fibonacci-valued and Lucas-valued states can also be generated passively by using a beam splitter or by monitoring the idler of a SPDC source.

We propose a quantum cryptography protocol that is based on Fibonacci and Lucas matrix coding. Our new protocol effectively addresses the problem of random key generation for OTP. In particular, our proposed QKD protocol have the following characteristics.

1. It is a slight modification of the QKD protocol based on Simon *et al.*[15]. However, our protocol is free from the restrictions of orbital angular momentum and down-conversion bandwidths.
2. The key generation rate and the key update rate in our protocol are significantly higher than the existing solutions due to the use of the recurrence relations in Fibonacci or Lucas matrices.
3. A signal information leakage can be substantially reduced. This is because we use both Lucas-valued and Fibonacci-valued OAM entangled states, but the values carried by the transmitted entangled photons are all Fibonacci numbers. It is more difficult for the adversary Eve to know the signal information with entangled photons in a spontaneous parametric down conversion (SPDC) source[17,18].
4. The verification of the integrity of encryption/decryption is possible due to the unique mathematical property of a Fibonacci or Lucas matrix. This feature does not exist in any previous QKD protocols.

Results

We illustrate how to use the Fibonacci and Lucas matrix coding to develop a new high-capacity QKD protocol. We also provide a security analysis of the new protocol.

Fibonacci and Lucas Matrix Coding. According to the definitions of Fibonacci and Lucas numbers (for details, see Appendixes I and II), we discuss how they can be used to construct relevant Fibonacci matrices Q_p^n and Lucas matrices R_p^n. Then we explore their basic properties. Finally we investigate the relation between Q_p^n and R_p^n.

The process of creating a sequence of Q_p^n is iterative. We start from $Q_1 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ and then construct subsequent matrices $Q_2, Q_3, \ldots, Q_p$ according to the following relations:

$$Q_2 = \begin{pmatrix} Q_1 & \mathbf{O}_{2\times 1} \\ \mathbf{O}_{1\times 2} & \mathbf{I}_1 \end{pmatrix}, Q_3 = \begin{pmatrix} Q_1 & \mathbf{O}_{2\times 2} \\ \mathbf{O}_{2\times 2} & \mathbf{I}_2 \end{pmatrix}, \ldots, Q_p = \begin{pmatrix} Q_1 & \mathbf{O}_{2\times (p-1)} \\ \mathbf{O}_{(p-1)\times 2} & \mathbf{I}_{p-1} \end{pmatrix}$$

where $\mathbf{O}_{ij}$, $i = j = 1, 2, \ldots, p-1$ is a matrix of the dimension $i \times j$ with zero entries and $\mathbf{I}_i$, $i = 1, 2, \ldots, p-1$ is an identity matrix of the order i. It is easy to show that the matrices $Q_2^n, Q_3^n, \ldots, Q_p^n$ satisfy the following relations:

$$Q_p^n Q_p^m = Q_p^{n+m}; \tag{1}$$

$$Q_p^n = Q_p^{n-1} + Q_p^{n-p-1}; \tag{2}$$

$$Det(Q_p^n) = (-1)^{pn} = (Det(Q_p))^n \tag{3}$$

According to Eq. (3), Q_p^n has its inverse, where $p = 0, 1, 2, 3, \ldots$. As explained in Appendix I, it is easy to find its inverse Q_p^{-n} (see Eqs (26) and (27)). For example, the first four matrices Q_2^n and their inverses are shownin Table 1.

The Lucas matrix (for details, see Appendix II) R_1 can be used to generate matrices of higher dimensions R_2, $R_3, \ldots, R_p$ according to the following recurrence:

n	2	3	4	5
Q_2^n	$\begin{pmatrix}1&1&0\\1&2&0\\0&0&1\end{pmatrix}$	$\begin{pmatrix}1&2&0\\2&3&0\\0&0&1\end{pmatrix}$	$\begin{pmatrix}2&3&0\\3&5&0\\0&0&1\end{pmatrix}$	$\begin{pmatrix}3&5&0\\5&8&0\\0&0&1\end{pmatrix}$
Q_2^{-n}	$\begin{pmatrix}2&-1&0\\-1&1&0\\0&0&1\end{pmatrix}$	$\begin{pmatrix}-3&2&0\\2&-1&0\\0&0&1\end{pmatrix}$	$\begin{pmatrix}5&-3&0\\-3&2&0\\0&0&1\end{pmatrix}$	$\begin{pmatrix}-8&5&0\\5&-3&0\\0&0&1\end{pmatrix}$

Table 1. Q_2^n and Q_2^{-n}, where $n = 2, 3, 4, 5$.

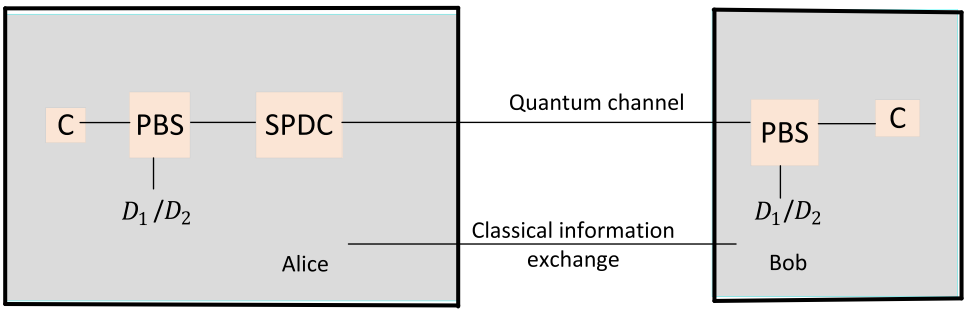

Figure 1. A schematic diagram for the Fibonacci- and Lucas- valued entanglement spontaneous parametric down conversion (SPDC) QKD. Alice and Bob connect to an entangled SPDC source by optical links. There is a C, a D_1 and a D_2 OAM sorter in Alice's and Bob's laboratories respectively. Either of the two entangled photons coming out from the SPDC source goes to Alice's and Bob's laboratories, and then the entangled photon randomly goes through the C, D_1 or D_2 sorter. The C sorter is used for allowing photons to arrive at the arrays of single-photon detectors when they are Fibonacci values. The D_1/D_2 sorter is used for filtering and blocking any non-Fibonacci values against various possible problems, and the D_1 and the D_2 sorters are used for allowing "diagonal" superpositions of the form $\frac{1}{\sqrt{2}}(|L_n\rangle + |F_{n-3}\rangle)$ and $\frac{1}{\sqrt{2}}(|F_n\rangle + |F_{n-2}\rangle)$, respectively. Here, PBS stands for a polarized beam splitter.

$$R_2 = \begin{pmatrix} R_1 & \mathbf{O}_{2\times 1} \\ \mathbf{O}_{1\times 2} & \mathbf{I}_1 \end{pmatrix}, R_3 = \begin{pmatrix} R_1 & \mathbf{O}_{2\times 2} \\ \mathbf{O}_{2\times 2} & \mathbf{I}_2 \end{pmatrix}, \ldots, R_p = \begin{pmatrix} R_1 & \mathbf{O}_{2\times (p-1)} \\ \mathbf{O}_{(p-1)\times 2} & \mathbf{I}_{p-1} \end{pmatrix}$$

where $\mathbf{O}_{ij}$, $i = j = 1, 2, \ldots, p-1$ is a zero matrix of the dimension $i \times j$, and $\mathbf{I}_i$, $i = 1, 2, \ldots, p-1$ is an identity matrix of order i. Matrices R_2^n, R_3^n, $\ldots$, R_p^n satisfy the following relations:

$$R_p^n R_p^m = R_p^{n+m}; \tag{4}$$

$$R_p^n = R_p^{n-1} + R_p^{n-p-1}; \tag{5}$$

$$Det(R_p^n) = (-1)^{p(n+1)} 5^p = (Det(R_p))^n \tag{6}$$

For Q_p^n, it is easy to find its inverse Q_p^{-n} (see Eqs (33) and (34) given in Appendix II).

Fibonacci and Lucas matrix encryption and decryption algorithms. Let the plaintext be a sequence of integers: $p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, \cdots$

These integers can be represented in the form of a square matrix M of order $p+1$, $p = 0, 1, 2, \ldots$. Note that the elements of M can be taken as an odd or even number of digits as we want. Therefore, the matrix encryption and decryption algorithms can be defined at a high level as follows[19]:

$$E = M \times K \tag{7}$$

and

$$M = E \times K^{-1} \tag{8}$$

where K can be Q_p^n or R_p^n, and K^{-1} is the inverse matrix of K.

High-capacity quantum cryptographic protocol. To extend and enhance the framework of Simon *et al.*'s[15], we use the Fibonacci and Lucas-valued OAM states detected in a transmission between Alice and Bob.

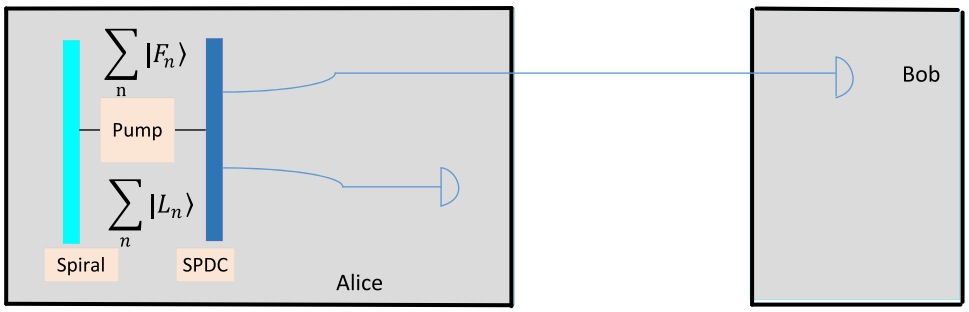

Figure 2. The experimental setup for the QKD protocol based on passive detected-state Fibonacci- and Lucas-valued entangled states. $\sum_n |F_n\rangle = \sum_n (|F_{n-1}\rangle|F_{n-2}\rangle + |F_{n-2}\rangle|F_{n-1}\rangle)_{AB}$; $\sum_n |L_n\rangle = \sum_n (|F_{n-1}\rangle|F_{n-3}\rangle + |F_{n-3}\rangle|F_{n-1}\rangle)_{AB}$. $\sum_n |F_n\rangle$ and $\sum_n |L_n\rangle$ are two-photon output Fibonacci and Lucas entangled states, respectively.

The states are used to construct the Fibonacci matrix Q_p^n and the Lucas matrix R_p^n as the key (see Figs 1 and 2). Note that here the Fibonacci and Lucas values reconstructed from Fibonacci and Lucas-valued OAM states are used as the **seed** for generating recursive matrices Q_p^n and R_p^n in terms of the assumption given below. We take an advantage of the recurrence relation of Fibonacci and Lucas matrices to significantly improve the information capacity of entangled photons to carry more than 4 bits of a cryptographic key.

Assumption. Assume that the order of the key matrix (the Fibonacci matrix Q_p^n or the Lucas matrix R_p^n) is determined by the quantum random number generators of Alice and Bob. The positions of Fibonacci and Lucas numbers in Q_p^n and R_p^n are determined by the outcome of F_n mod 4 or L_n mod 4, respectively. For instance, if $F_n = 13$, then F_n mod $4 = 13$ mod $4 = 1$. For $F_n = 8$, F_n mod $4 = 8$ mod $4 = 0$. Note that the positions of the Fibonacci numbers 13 and 8 in the matrices Q_p^n are illustrated below:

$$\begin{pmatrix} \mathbf{13} & 21 \\ 21 & 34 \end{pmatrix} = Q_1^8, \begin{pmatrix} \mathbf{13} & 21 & 0 \\ 21 & 34 & 0 \\ 0 & 0 & 1 \end{pmatrix} = Q_2^8, \begin{pmatrix} \mathbf{13} & 21 & 0 & 0 \\ 21 & 34 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = Q_3^8;$$

$$\begin{pmatrix} 3 & 5 \\ 5 & \mathbf{8} \end{pmatrix} = Q_1^5, \begin{pmatrix} 3 & 5 & 0 \\ 5 & \mathbf{8} & 0 \\ 0 & 0 & 1 \end{pmatrix} = Q_2^5, \begin{pmatrix} 3 & 5 & 0 & 0 \\ 5 & \mathbf{8} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = Q_3^5;$$

The steps of our protocol are described below.

Step 1 Preparation of Fibonacci and Lucas-valued OAM states.

We slightly modify the experimental setup of Simon *et al.*'s[15] (see Figs 1 and 2). Alice prepares Fibonacci and Lucas-valued OAM states and makes two-photon output states according to the following encoding

$$\sum_n (|F_{n-1}\rangle|F_{n-2}\rangle + |F_{n-2}\rangle|F_{n-1}\rangle)_{AB}; \tag{9}$$

$$\sum_n (|F_{n-1}\rangle|F_{n-3}\rangle + |F_{n-3}\rangle|F_{n-1}\rangle)_{AB}. \tag{10}$$

One photon of the entangled pair goes to the Alice laboratory and the other to the Bob laboratory. The selection of destination is random. Note that the main difference between our proposed protocol and Simon *et al.*'s[15] is that we use the Fibonacci or Lucas values recovered from photons as a **seed** for constructing the Fibonacci matrix Q_p^n or the Lucas matrix R_p^n. This trick improves the information capacity of photons considerably. We are free to select the **proper** pump values, say, 8, 11, 13, 18, 21, 29, 34, 47.

Step 2 Eavesdropping detection.

As in the Simon *et al.*'s protocol[15], there are six possible cases that need to be considered for entangled photons that arrive at Alice's and Bob's laboratories. They are listed below.

Case I. Both photons go to C.
Case II. One photon goes to C and the other to D_1.
Case III. One photon goes to C and the other to D_2.
Case IV. Both photons go to D_1.
Case V. Both photons go to D_2.
One photon goes to D_1 and the other to D_2.

The possible Fibonacci values obtained by Alice	3	5	8	13	21	34
The classical bits sent by Alice/Bob	00	01	0	10	11	1

Table 2. The possible Fibonacci values obtained by Alice and their corresponding classical representations.

In order to determine the case and detect eavesdropping, Alice and Bob need to exchange classical information over a classical channel (the channel can be an unprotected broadcasting). Let us introduce three events encoded as 0, 01 and 10. The event 0 occurs when the entangled photon goes to C. The second 01 when it goes to D_1 and the third encoded as 10 when it goes to D_2. Assume that there exists an eavesdropper Eve who intercepts an entangled photon, which travels to Alice (or Bob). Clearly, Eve has no information of which type of a detection measurement (C, D_1 or D_2) takes place in Bob's laboratory. So, Eve has to guess. Eve makes a mistake if the photon goes to

- C in Eve's laboratory while Bob's laboratory applies either D_1 or D_2 or
- D_1 in Eve's laboratory while Bob's laboratory applies either C or D_2 or
- D_2 in Eve's laboratory while Bob's laboratory applies either C or D_1

The Alice measurement is going to be erroneous with the probability of $\frac{2}{3}$. Eve's activity is detected by Alice when Alice and Bob compare their transcripts.

Step 3 Reconstruction of the seed for the key matrix.

After eavesdropping detection, if the error rate r_e is larger than the preset threshold r, Alice and Bob discard this communication and return to Steps 1–2. Otherwise, they can exchange classical bits to determine the correct Fibonacci number. Alice and Bob discard the trial if the exchanged classical bits (between Alice and Bob) satisfy one of the following three cases: (I) are both 01 (D_1 sorters), (II) are both 10 (D_2 sorters), (III) are 01 and 10 (D_1 and D_2 sorters), i.e., Cases IV-VI. If the exchanged classical messages are 0, 01 or 0, 10, Alice and Bob need to exchange one more classical message. That is, Alice or Bob need to exchange another classical bit 0 or 1, to let each other know that their trial is either Case II or Case III. This is sufficient for Alice (Bob) to know Bob's (Alice's) state.

If the exchanged classical bits between Alice and Bob are 0, they know the trial is Case I. They know each other's value as the values can be identified from Eqs (9) and (10). In other words, Alice knows that the Bob Fibonacci number is one or two positions before her number (and vice versa, because the angular uncertainty principle links angular position and angular momentum $\Delta\phi\Delta L = \frac{1}{2}|1 - 2\pi P(\pi)|$). However, this case is more complicated than Case II and Case III. To identify the correct OAM value, they need to exchange more classical messages. As we say in Step 1, the pump values of 8, 11, 13, 18, 21, 29, 34, 47 are used, so, the Fibonacci numbers encoded in the entangled photons can be 3, 5, 8, 13, 21, 34. As we can see the Fibonacci number is either even or odd, then, by prior agreement, for Alice, the classical bits 0, 1 are used to denote the first and second even Fibonacci numbers of 3, 5, 8, 13, 21, 34, while the classical bits 00, 01, 10, 11 are used to denote the first, second, third and fourth odd Fibonacci numbers of 3, 5, 8, 13, 21, 34 (see Table 2).

On receiving a classical bit from Alice, Bob can obtain the Alice Fibonacci number, because the number position is adjacent to the position of his Fibonacci number. Likewise, Bob sends the corresponding classical message to Alice in terms of Table 2, and Alice can also confirm the Bob Fibonacci number. That is to say that Alice and Bob are able to confirm to each other's the detected numbers by exchanging classical information. For example, if Alice's detected number is 3, then Alice sends the classical message 00 to Bob over the classical channel and Bob's detected number is 8. So, Bob can obtain the **seed** 11, and Bob sends the classical message 00 to Alice over the classical channel. Alice can also obtain the **seed** 11.

Step 4 Cryptographic key generation with Q_p^n and R_p^n.

After the pump values of Fibonacci and Lucas numbers are determined by Alice and Bob, they can use them as the **seed** for the key matrix, i.e., the Fibonacci matrix Q_p^n or the Lucas matrix R_p^n. The orders of Q_p^n and R_p^n are determined by their quantum random number generators in their laboratories. If the obtained Fibonacci number is $F_4 = 5$ and its matching order is 4, then the inverse of the matrix can be found in Table 1. The matrices can be used to perform basic cryptographic operations such as encryption and decryption. To illustrate them, consider the following example. Assume that a message is a sequence of integers

$$4150\ \ 2313\ \ 6877\ \ 9960,\ \ 132\ \ 214\ \ 054,\ \ \ldots \tag{11}$$

Integers of the message in Eq. (11) can be put as entries of matrices $M_1, M_2, \ldots$, which is

$$M_1 = \begin{pmatrix} 4 & 1 & 5 & 0 \\ 2 & 3 & 1 & 3 \\ 6 & 8 & 7 & 7 \\ 9 & 9 & 6 & 0 \end{pmatrix},$$
$$M_2 = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ 0 & 5 & 4 \end{pmatrix},$$
$$\ldots \tag{12}$$

According to Eq. (7), M can be encrypted as follows

$$E_1 = M_1 \times Q_3^3 = \begin{pmatrix} 4 & 1 & 5 & 0 \\ 2 & 3 & 1 & 3 \\ 6 & 8 & 7 & 7 \\ 9 & 9 & 6 & 0 \end{pmatrix} \times \begin{pmatrix} 5 & 3 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 23 & 14 & 5 & 0 \\ 19 & 12 & 1 & 3 \\ 54 & 34 & 7 & 7 \\ 72 & 45 & 6 & 0 \end{pmatrix} \tag{13}$$

$$E_2 = M_2 \times R_2^5 = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ 0 & 5 & 4 \end{pmatrix} \times \begin{pmatrix} 7 & 11 & 0 \\ 11 & 18 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 40 & 65 & 2 \\ 25 & 40 & 4 \\ 55 & 90 & 4 \end{pmatrix} \tag{14}$$

According to his order and Equation (8), the cryptogram $E = E_1 \| E_2 \| \cdots = 231450191213543477724560406522540455904\cdots$ can be split and decrypted by using the inverse matrix, so Bob obtains

$$M_1 = E \times Q_3^{-3} = \begin{pmatrix} 23 & 14 & 5 & 0 \\ 19 & 12 & 1 & 3 \\ 54 & 34 & 7 & 7 \\ 72 & 45 & 6 & 0 \end{pmatrix} \times \begin{pmatrix} 2 & -3 & 0 & 0 \\ -3 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 1 & 5 & 0 \\ 2 & 3 & 1 & 3 \\ 6 & 8 & 7 & 7 \\ 9 & 9 & 6 & 0 \end{pmatrix} \tag{15}$$

$$M_2 = E_2 \times R_2^{-5} = \begin{pmatrix} 40 & 65 & 2 \\ 25 & 40 & 4 \\ 55 & 90 & 4 \end{pmatrix} \times \begin{pmatrix} \frac{18}{5} & -\frac{11}{5} & 0 \\ -\frac{11}{5} & \frac{7}{5} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ 0 & 5 & 4 \end{pmatrix} \tag{16}$$

$$\vdots \tag{17}$$

Step 5 Integrity verification

According to $E = MQ_p^n$, $E = MR_p^n$, we obtain

$$\begin{aligned} det(E) &= det(MQ_p^n) \\ &= det(M) \times det(Q_p^n) \\ &= (-1)^{np} det(M); \end{aligned} \tag{18}$$

$$\begin{aligned} det(E) &= det(MR_p^n) \\ &= det(M) \times det(R_p^n) \\ &= (-1)^{p(n+1)} 5^p det(M). \end{aligned} \tag{19}$$

If the transmitted and received matrices are E and E' respectively, then E' satisfies Eqs (18) or (19). Hence, the integrity of E is proved.

Security Analysis. The encryption defined by Fibonacci and Lucas matrices is an instance of symmetric-key cryptography. Fibonacci and Lucas matrices have the particular property, i.e., the recurrent property which helps us to know a Fibonacci or Lucas matrix with any one of Fibonacci or Lucas number in the matrix. We call the a Fibonacci or Lucas number the **seed**. As we know, for symmetrical cryptography, the main deficiency is the problem of key distribution. However, in our proposed protocol, the key is generated based on Simon *et al.*'s protocol[15], which is quantum-resistant for enhanced security. That is, Fibonacci or Lucas cryptography can be combined with the quantum one-time-pad for unconditional security. Therefore, in this section, we provide a sufficient security analysis of why seeding the Fibonacci and Lucas matrices used to encrypt the message (and subsequent sending of it) does not increase Eve's information about the secret key as follows.

(1) Firstly, similar entangled states are prepared with the improved experimental setup, by the virtue of the recurrence relations of $F_{n+2}=F_{n+1}+F_n$ and $L_{n+2}=F_{n+1}+F_{n-1}$. Clearly, the entangled photons detected by Alice, Bob and the adversary Eve are Fibonacci-valued. When Eve receives the entangled photon and even she can detect its values, it is more difficult for her to distinguish Fibonacci entangled states from Lucas entangled states than that in Simon *et al.*'s protocol[15]. More precisely, if Eve makes a D_1-type measurement on an entangled photon heading to Bob, which is actually in the eigenstate $|F_i\rangle$. Then she will detect one of the two superposition states $|F_{i-2}\rangle+|F_i\rangle$ or $|F_i\rangle+|F_{i+2}\rangle$, with the probability of $\frac{1}{2}$, respectively. Then Eve transmits either of these two superposition states to Bob. If Bob receives one of these superpositions and makes a *C*-type measurement, he will read out one of the values F_i, F_{i-2}, or F_{i+2}, with the respective probabilities of $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{4}$. However, he should obtain $|F_i\rangle$ with the probability of 1 if there is no eavesdropper. If Eve makes a D_2-type measurement on an entangled photon heading to Bob, which is actually in the eigenstate $|F_i\rangle$, then she will detect one of the two superposition states $|L_{i-2}\rangle+|F_i\rangle$ or $|F_i\rangle+|L_{i+2}\rangle$, with the probability of $\frac{1}{2}$, respectively. Then Eve transmits either of these two superposition states to Bob. If Bob receives one of these superpositions and makes a *C*-type measurement, he will read out one of the values F_i, L_{i-2}, or L_{i+2}, with the respective probabilities of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$. However, he should obtain $|F_i\rangle$ with the probability of 1 if there is no eavesdropper. In these situations, these superposition states do not help Eve to know the **seed**, but expose her eavesdropping action. Consequently, Alice and Bob discard such entangled photons.

(2) Secondly, if Eve makes a *C*-type measurement on an entangled photon heading to Bob, which is actually in the eigenstate $|F_i\rangle$, then she will detect $|F_i\rangle$ with the probability of 1. Then Eve transmits $|F_i\rangle$ to Bob. If Bob receives it and makes a *C*-type measurement, he will also read out one of the values F_i, with the probability of 1. Though Eve's eavesdropping action cannot be detected, it is still impossible for her to know the definite OAM value. This is because in Step 3, which classical bit representation of which Fibonacci number is agreed with between Alice and Bob in advance, and the classical channel is the broadcast channel. In other words, when Alice/Bob sends classical messages to Bob/Alice through the classical channel, Bob/Alice can determine the **seed**, but Eve cannot without knowing Alice and Bob's prior agreement, she just knows 0 and 1 that are used to denote even Fibonacci numbers, and 00, 01, 10, 11 to denote odd Fibonacci numbers. So, for Eve, it is at random. Eve has no choice but to guess the obtained Fibonacci and Lucas numbers by Alice and Bob with a probability of $\frac{1}{8}$. Moreover, to prevent Eve from capture two small pulses for the two entangled photons at the same time, the time interval for sending entangled photons is random.

(3) Thirdly, the positions of obtained Fibonacci and Lucas numbers in matrices Q_p^n and R_p^n are not fixed. Instead, the positions are determined by F_n/L_n mod 4. It suggests that our method can be further against Eve's attack. If Eve guesses the wrong obtained Fibonacci or Lucas number by Alice and Bob, she places the position of the wrong obtained Fibonacci or Lucas number in Q_1^n and R_1^n with a probability of $\frac{3}{4}$.

(4) Lastly, the construction of the final key matrices including the Fibonacci and Lucas matrices is determined by these matrices' orders *p*, which are determined by the quantum random number generators in Alice's and Bob's laboratories. Therefore, Eve cannot know the value *p*. Furthermore, the order determines how to split the encrypted message when decrypting them. Even if Eve guesses the correct seed, it is very difficult for her to guess all the right orders for all seeds during the construction of the final key matrices. As long as Eve does not guess all the orders for all corresponding matrices, she also splits the encrypted message in a wrong way. Moreover, in our protocol, we use matrix multiplication to encrypt the message. In matrix multiplication, if the orders of two square matrices are not equal, they cannot be performed multiplication operation. In other words, Eve cannot know any information about the messages if she guess a wrong order matching the matrix. Most importantly, there are no relations among these established Fibonacci or Lucas matrices. So, for Eve, *p* is at random, the probability for Eve to know the right Q_p^n or R_p^n is $\frac{1}{p}$, and the probability for Eve to know a right Q_p^n or R_p^n is $\frac{1}{8}\times\frac{1}{4}\times\frac{1}{p}$, i.e., $\frac{1}{32p}$.

Therefore, by seeding the Fibonacci and Lucas matrices used to encrypt the message to achieve fast and simple high-capacity quantum cryptography, our proposed protocol does not increase Eve's information about the secret key.

Discussion

In this section, we discuss the possibility to improve Simon *et al.*'s[15] experimental setup for quantum cryptography based on the Fibonacci matrix Q_p^n and the Lucas matrix R_p^n, which have more additional features to be obtained, including the higher transmission rates, no limit of communication bandwidths, the considerable information capacity, the selection property of Fibonacci or Lucas numbers, and the powerful detection and correction ability.

Entangled Fibonacci- and Lucas- sequence spiral source. In 2013, Simon *et al.*[15] proposed a high-capacity QKD by Fibonacci coding. In particular, with a Vogel spiral[15] (which refers to the "Fibonacci angle"), they use a source of entangled Fibonacci-valued OAM states to prepare Fibonacci-valued entangled pairs, which then leave the spiral and enter the down-conversion crystal. Moreover, due to the conjugation relation between Lucas numbers and Fibonacci numbers, Simon *et al.*[15] showed that Lucas-valued states can also be generated passively by using a beam splitter or by monitoring the idler of an SPDC source. In addition, the phases of signal photons are totally random due to the spontaneous feature of the SPDC process. This intrinsic phase randomization improves the security of the QKD system by making it immune to source attacks. Therefore, we improve the experimental setup in Simon *et al.*'s protocol to generate both Fibonacci- and Lucas-valued OAM

states (see Figs 1 and 2, note that Simon *et al.*[15] state that in the chosen operating range, it is easy to adjust the fraction of the values that fall on the Fibonacci or Lucas sequence), so that these nonorthogonal states naturally appear and randomly change with Fibonacci- and Lucas-valued entangled pairs.

The overall transmission rate. Assume that the detection probabilities of the entangled photons in the state of Eqs (9) and (10) are independent. Let ξ_A, ξ_A' and ξ_B, ξ_B' be the detection efficiencies for a Fibonacci and Lucas entangled photon for Alice and Bob, respectively. Both ξ_A, ξ_A' and ξ_B, ξ_B' take into account the channel losses, detector efficiencies, coupling efficiencies, and losses inside the detector box. For a $2n$-photon pair, the overall transmission rate is

$$\xi_{2n} = [(1-(1-\xi_A)^n][(1-(1-\xi_B)^n] + [(1-(1-\xi_A')^n][(1-(1-\xi_B')^n] \tag{20}$$

Given that the channel loss is included in ξ_A, ξ_A' and ξ_B, ξ_B', our method can be used to the SPDC source on either Alice's (or Bob's) side or between Alice and Bob.

The information capacity. Due to the recursive property of the Fibonacci matrix Q_p^n and the Lucas matrix R_p^n, just one entangled photon can be used as the seed to distribute the entire key, i.e., the information capacity I_c can be even equal to the length of the key ℓ_{key}, i.e., $I_c = \ell_{key}$. This is because the secret can be used to construct a matrix of any order.

No limit of communication bandwidths. Simon *et al.*'s[15] protocol needs more Fibonacci or Lucas numbers to improve key capacity, however, the method is under the limit of orbital angular momentum and down-conversion bandwidths. Therefore, they cannot choose more proper Fibonacci or Lucas numbers to achieve longer distances and lower error rates simultaneously. Moreover, every entangled photon can only be used to encode at most four bits. So, a large number of entangled states should be prepared to establish the key. As a result, when the key is updated frequently with the purpose of security, one-time-pad communication bandwidth increases in a proportional way. However, our protocol improves the key capacity greatly by taking advantages of the recursive property of Fibonacci and Lucas matrices and the method of preparing entangled states by Simon *et al.*'s[15] setup. Therefore, only a few entangled photons are needed to establish the key in practical settings. Therefore, our protocol is free from the limitation of orbital angular momentum and down-conversion bandwidths. Given this, in our protocol, the key can be established in a short time, and the frequent key update is free from the limitation of communication bandwidths.

The selection property of Fibonacci or Lucas numbers. Simon *et al.* used their experiments to come to the conclusion that if the pump values from smaller Fibonacci numbers are used, the photons can travel longer distances but at the expense of higher error rates. If, however, the pump values from larger Fibonacci numbers are used, then error rates reduced but maximal distances over which photons can travel are shorter. That is, to meet the requirements of available orbital angular momentum and down-conversion bandwidths and the longer distances and lower error rates, the **proper** pump values can be selected in our protocol, for example, Fibonacci numbers 8, 13, 21, 34 and Lucas numbers 11, 18, 29, 47. Therefore, the values carried by entangled photons are 3, 5, 8, 13, 21, 34.

Error detection and correction abilities. An additional feature of our protocol is the error detection for the cipertext compared with the existing quantum cryptography, which can keep the integrity of the secret. Stakhov[19] has proposed an error correction algorithm for Fibonacci coding. We present a brief description of this algorithm. For $E = MQ_p^n$, $E = MR_p^n$, we can verify their integrity in terms of Eqs (15) or (16), i.e., $det(E) = det(MQ_p^n) = det(M) \times det(Q_p^n) = (-1)^{np} det(M)$; $det(E) = det(MR_p^n) = det(M) \times det(R_p^n) = (-1)^{p(n+1)} 5^p det(M)$.

If the transmitted matrix is E and the received matrix is E', and E' satisfies Eqs (15) or (16), then there are no errors. Otherwise, there exist errors. In this case, correction is needed, and Stakhov[19] has shown that the correction ability of Fibonacci Q_1^n and Q_2^n matrix coding method is 93.33% and 99.80%, and when p is larger, the correction ability is higher than 99.80%, which exceeds all the other well-known correcting codes.

Because of the considerable information capacity, no limit of communication bandwidths, and the powerful detection and correction abilities, our protocol provides a practical secure way to share more private information with high photon-information efficiency in a short time. In realistic conditions, it is more applicable and feasible in a practical implementation with a slight modification of Simon *et al.*'s protocol. Table 3 compares the features of our proposed protocol with those of the most relevant quantum key distribution protocols in refs 1, 2 and 15. The comparison suggests that our protocol is more suitable for real-world applications.

Conclusions

We have developed a new quantum cryptosystem, i.e., quantum cryptography based on Fibonacci matrix Q_p^n and Lucas matrix R_p^n, which employs technologies similar to Simon *et al.*'s protocol to overcome the previous limitations on communication bandwidths and demonstrate that the number of secret key bits per transmitted entangled photon can be increased up to the length of the key, which is well-over previous demonstrations that were limited to up to 4. Under realistic conditions, the proposed protocol also provides a practical secure way to share more private information with considerably high photon-information efficiency in a short time. Moreover, it can be fast and simple to implement technical realization.

Protocols	Ref. 1	Ref. 2	Ref. 15	Our protocol
The maximal information capacity	1	2	4	ℓ_{key}
The correction ability	n/a	n/a	n/a	Higher than 93.33%
The ability to verify the integrity of ciphertext	No	No	No	Yes
The limitation to bandwidths	Yes	Yes	Yes	No
Achieving longer distances and lower error rates	No	No	No	Yes

Table 3. Performance comparison of our QKD with the most relevant previous QKDs. ℓ_{key} denotes the length of the key, "n/a" which means not applicable.

Methods

Here, we introduce the method of Fibonacci matrix Q_p^n or Lucas matrix R_p^n to quantum cryptography.

Let the initial message be a "digital signal" of integers: $p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, \cdots$

Assume that we choose the first nine readings and form a 3×3 matrix P_1, which is regarded as a plain text matrix.

$$P_1 = \begin{pmatrix} p_1 & p_2 & p_3 \\ p_4 & p_5 & p_6 \\ p_7 & p_8 & p_9 \end{pmatrix} \tag{21}$$

And the key K, i.e., Q_p^n or R_p^n is obtained by Steps 1–3 in the proposed protocol. In general, the cryptosystem consists of the plain text matrix P, the cipher text matrix C, and the key K

$$\{P, \quad C, \quad K\} \tag{22}$$

Then encryption and decryption algorithm is
If $K = Q_p^n$
then $C \leftarrow P \times Q_p^n$;
$P \leftarrow C \times Q_p^{-n}$
Endif
If $K = R_p^n$
then $C \leftarrow P \times R_p^n$;
$P \leftarrow C \times R_p^{-n}$
Endif

Appendix I

Definition 1. (**Fibonacci numbers**)[20] The sequence $\{F_n\}_{n=1}^{+\infty}$ of Fibonacci numbers is defined by the recurrence relation

$$F_{n+2} = F_{n+1} + F_n \tag{23}$$

Clearly, the integers 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, … are members of Fibonacci sequences. Using the definition of Fibonacci numbers, one can prove that

$$F_{-n} = (-1)^{n+1} F_n \tag{24}$$

Now we are ready to introduce a Fibonacci Q-matrix[21,22] as

$$\overline{Q_1} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}. \tag{25}$$

According to Stakhov's work[19], we can write a Fibonacci Q-matrix of dimension 2 as follows:

$$Q_1 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \tag{26}$$

Now, we can derive a relevant recurrence relation in the form:

$$\begin{aligned} Q_1^n &= \begin{pmatrix} F_{n-1} & F_n \\ F_n & F_{n+1} \end{pmatrix} \\ &= \begin{pmatrix} F_{n-2} + F_{n-3} & F_{n-1} + F_{n-2} \\ F_{n-1} + F_{n-2} & F_n + F_{n-1} \end{pmatrix} \\ &= \begin{pmatrix} F_{n-2} & F_{n-1} \\ F_{n-1} & F_n \end{pmatrix} + \begin{pmatrix} F_{n-3} & F_{n-2} \\ F_{n-2} & F_{n-1} \end{pmatrix} \end{aligned} \tag{27}$$

n	4	5	6	7
Q_1^n	$\begin{pmatrix}2 & 3\\3 & 5\end{pmatrix}$	$\begin{pmatrix}3 & 5\\5 & 8\end{pmatrix}$	$\begin{pmatrix}5 & 8\\8 & 13\end{pmatrix}$	$\begin{pmatrix}8 & 13\\13 & 21\end{pmatrix}$
Q_1^{-n}	$\begin{pmatrix}5 & -3\\-3 & 2\end{pmatrix}$	$\begin{pmatrix}-8 & 5\\5 & -3\end{pmatrix}$	$\begin{pmatrix}13 & -8\\-8 & 5\end{pmatrix}$	$\begin{pmatrix}-21 & 13\\13 & -8\end{pmatrix}$

Table 4. Q_1^n and Q_1^{-n}, where $n = 4, 5, 6, 7$.

$$= Q_1^{n-1} + Q_1^{n-2} \tag{28}$$

Note that Q_1^n satisfies the following two properties:

- $Q_1^n Q_1^m = Q_1^{n+m}$ and
- $Det(Q_1^n) = (-1)^n = (Det(Q_1))^n$.

The inverse matrix Q_1^{-n} of Q_1^n is obtained as follows

$$Q_1^{-2k} = \begin{pmatrix}F_{2k+1} & -F_{2k}\\-F_{2k} & F_{2k-1}\end{pmatrix}, \quad \text{where } n = 2k \tag{29}$$

$$Q_1^{-(2k+1)} = \begin{pmatrix}-F_{2k+2} & F_{2k+1}\\F_{2k+1} & -F_{2k}\end{pmatrix}, \quad \text{where } n = 2k+1 \tag{30}$$

For example, according to Eqs (26) and (27), we can obtain Q_1^{-n} of Q_1^n when $n = 4, 5, 6, 7$ (see Table 4).

Appendix II

Definition 2. (**Lucas numbers**)[16] The Lucas numbers are defined as follows:

$$L_n = \begin{cases}2, & \text{when } n = 0,\\1, & \text{when } n = 1,\\L_{n-1} + L_{n-2}, & \text{when } n \geq 2.\end{cases} \tag{31}$$

In particular, $L_0 = 2$, $L_1 = 1$, $L_2 = 3$, $L_3 = 4$, $L_4 = 7$ … Moreover, Lucas numbers and Fibonacci numbers have a conjugate relation[16] of the following form:

$$L_{n+2} = F_{n+1} + F_{n-1} \tag{32}$$

Let us define a 2×2 matrix R as

$$R_1 = \begin{pmatrix}2 & 1\\1 & 3\end{pmatrix} \tag{33}$$

Therefore, according to Eqs (20), (29) and (30), we have

$$R_1^n = \begin{pmatrix}L_{n-1} & L_n\\L_n & L_{n+1}\end{pmatrix} = Q_1^n \times \begin{pmatrix}-1 & 2\\2 & 1\end{pmatrix} \tag{34}$$

$$\begin{aligned} Det(R_1^n) &= Det\left(Q_1^n \times \begin{pmatrix}1 & 2\\2 & -1\end{pmatrix}\right)\\ &= (-1)^n \times (-5)\\ &= 5 \times (-1)^{n+1} \end{aligned} \tag{35}$$

The inverse matrix R_1^{-n} of R_1^n can also be derived in terms of Eqs (26), (27), (31) and (32), which is as follows

$$R_1^{-2k} = \begin{pmatrix}\frac{L_{2k+1}}{5} & -\frac{L_{2k}}{5}\\-\frac{L_{2k}}{5} & \frac{L_{2k-1}}{5}\end{pmatrix}, \quad \text{where } n = 2k \tag{36}$$

$$R_1^{-(2k+1)} = \begin{pmatrix}-\frac{L_{2k+2}}{5} & \frac{L_{2k+1}}{5}\\\frac{L_{2k+1}}{5} & -\frac{L_{2k}}{5}\end{pmatrix}, \quad \text{where } n = 2k+1 \tag{37}$$

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Author Contributions

L.H. proposed the theoretical method. L.H., L.M.X., P.J., Z.J. and P.L. wrote the manuscript text. O.M. and L.S.D. reviewed the manuscript.

Additional Information

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