Fast Finite Element Analysis of Motors Using Block Model Order Reduction

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This paper proposes a model order reduction (MOR) based on the proper orthogonal decomposition (POD) to perform fast analysis of-motors. When POD-based MOR is applied to the motor analysis, the number of basis vectors has to be increased to express changes in magnetic fields due to rotational movement. Its computational efficiency is thus greatly deteriorated. To overcome this difficulty, block-MOR is first applied to motor analysis. In this method, a parameter space is subdivided into several blocks, which correspond to angular ranges in motor analysis, in each of which the basis vectors are constructed from snapshotted fields. The computational time of block-MOR is shown to be shorter than that of the conventional MOR while accuracy of both methods is almost identical.

Index Terms—**Finite element analysis, interior permanent magnet motor, model order reduction, proper orthogonal decomposition.**

I. Introduction

In the design of the control and driving systems of the motors, equivalent circuits and behavior models of the motors are widely used rather than finite element (FE) models whose computational time is rather long for dynamic simulations [1]. Accuracy of the former two methods is, however, often unsatisfactory especially for loss evaluation. For this reason, fast and accurate computational methods for the motor analysis have been required.

In order to reduce the computational time in FE analysis, the model order reduction (MOR) based on the proper orthogonal decomposition (POD) has been proposed, in which unknown fields are approximated by a linear combination of small number of basis vectors obtained by principal component analysis of the snapshotted fields. POD-based MOR has been successfully applied to analysis of stationary electromagnetic devices [2-5]. Moreover, it has been applied to the analysis of a surface permanent magnet motor considering rotation movement [6] in which magnetic saturation in the motor core is not taken into account.

When POD-based MOR is applied to moving objects, large number of the basis vectors must be involved to express field changes due to motion. Its computational efficiency is thus greatly deteriorated. Moreover, when we consider magnetic saturation in core materials, a number of matrix-matrix products in the Newton-Raphson iteration [4] must be carried out. This is another factor to limit efficiency of POD-based MOR.

In order to circumvent these difficulties, the block-MOR has been proposed for fast analysis of moving objects [7] considering nonlinearity of magnetic material. In this method, a parameter space is subdivided into several blocks. The basis vectors are generated for each block from snapshotted fields. It will be shown that computational burden of block-MOR becomes smaller as the number of blocks increases, and therefore it is smaller than that of the conventional MOR.

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The block-MOR method has been successfully applied to analysis of a vibration energy harvester [7]. However, its applicability of the motor analysis has remained unclear.

In this study, we apply the block-MOR for the first time to analysis of reluctance and interior permanent magnet (IPM) motors, to evaluate computational efficiency and accuracy.

II. Numerical method

A. *Proper orthogonal decomposition*

It is assumed that the magnetic fields in a motor do not vary along its axial direction parallel to z-axis. The two-dimensional magnetostatic fields on x-y plane are analyzed here by finite element method (FEM), which solves

$$\sum_j A_j \int_\Omega \nu(A) \mathrm{grad} N_j \cdot \mathrm{grad} N_i \mathrm{d}S = \int_\Omega N_i J \mathrm{d}S + \int_\Omega \left(-M_y \frac{\partial N_i}{\partial x} + M_x \frac{\partial N_i}{\partial y} \right) \mathrm{d}S \quad 1 \le i, j \le n \tag{1}$$

where A_j, $\nu(A)$, N_j, J, M_x, M_y and n are magnetic vector potential, magnetic reluctivity, scalar interpolation function, current density and x and y components of the magnetization and the number of nodal points respectively. Eq. (1) can be written as the matrix form that is $\mathrm{A}\boldsymbol{x}=\boldsymbol{b}$. Applying the Newton-Raphson method to (1), we obtain

$$\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{A}} \Delta \boldsymbol{A} = -\boldsymbol{G} \tag{2}$$

where $\boldsymbol{A} \in \mathrm{R}^n$ and $\boldsymbol{G}=\boldsymbol{b}-\mathrm{A}\boldsymbol{x}$ are the unknown and residual vectors. We apply POD-based MOR to (2) in order to reduce DoF. To do so, we solve (2) at s sampling angles θ_l, l = 1, 2,$\cdots$, s, where s is much smaller than n. Then, the data matrix X is constructed as follows:

$$\mathrm{X} = \left[\boldsymbol{A}(\theta_1) \quad \boldsymbol{A}(\theta_2) \quad \cdots \quad \boldsymbol{A}(\theta_s) \right] \tag{3}$$

The singular value decomposition applied to X results in

$$\mathrm{X} = \mathrm{W}\Sigma\mathrm{V}^{\mathrm{t}} = \sigma_1 \boldsymbol{w}_1 \boldsymbol{v}_1^{\mathrm{t}} + \sigma_2 \boldsymbol{w}_2 \boldsymbol{v}_2^{\mathrm{t}} + \cdots + \sigma_s \boldsymbol{w}_s \boldsymbol{v}_s^{\mathrm{t}} \tag{4}$$

where σ_l is l-th singular value of X and $\boldsymbol{w}_l$, $\boldsymbol{v}_l$ are the eigenvectors of XX^{t}, $\mathrm{X}^{\mathrm{t}}\mathrm{X}$, respectively. The unknown vector is then approximately expressed by $\boldsymbol{A}=\mathrm{W}\boldsymbol{y}$ where $\mathrm{W}\in\mathrm{R}^{n\times s}$, $\boldsymbol{y}\in\mathrm{R}^{s}$. Now (2) reduces to

$$\mathrm{W}^{\mathrm{t}} \frac{\partial \boldsymbol{G}}{\partial \boldsymbol{A}} \mathrm{W}\Delta\boldsymbol{y} = -\mathrm{W}^{\mathrm{t}}\boldsymbol{G} \tag{5}$$

Because $s << n$, (5) can be solved much faster than (2).

B. Block-MOR

In order to reduce the computational time, s should be set as small as possible. However, s must be set sufficiently large to accurately express the magnetic field which significantly changes due to rotational movements shown in Fig. 1. It is, therefore, important to effectively take the snapshots. Fig. 2 shows the procedure of Newton-Raphson method. Increase of s leads to long computational time mainly because of heaby compuational burden in matrix-matrix products in the left hand side of (5). In the conventional POD-based MOR method, the snapshots are taken at equal rotational intervals over the whole angular range of the interest. This results in large s. The computational complexity for the matrix-matrix products is $O(ns^2+n^2s)$.

In the block-MOR, to overcome this difficulty, a parameter space is subdivided into m blocks to generate transformation matrix W_i in each block as shown in Fig. 3. In the motor analysis, the whole the range of mechanical angle is subdivided into $\theta_{i1} < \theta \leq \theta_{iq}$, i=1,2,...,m. The reduced equation is constructed depending on the mechanical angle is solved.

Because changes in the magnetic field in each block, which is here an angular range, are expected to be small, the snapshot number can be reduced to, for example, s/m. Hence, the computational burden for matrix-matrix product in left hand side of (5) can be reduced. Moreover, because the number of unknowns is also reduced, (5) can be solved faster.

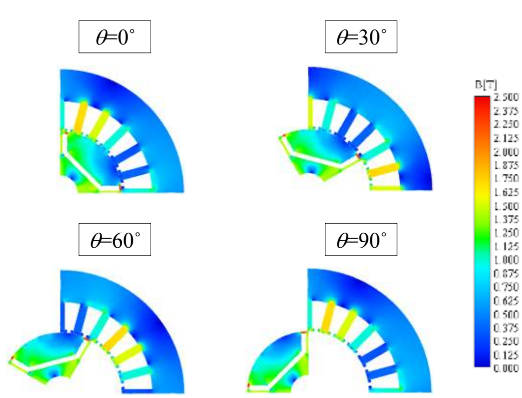

Fig. 1. Distribution of the magnetic fields in the cores.

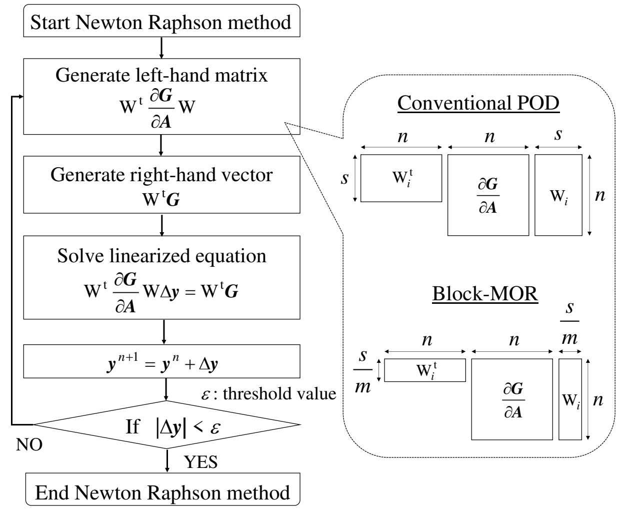

Fig. 2. Procedure of Newton-Raphson.

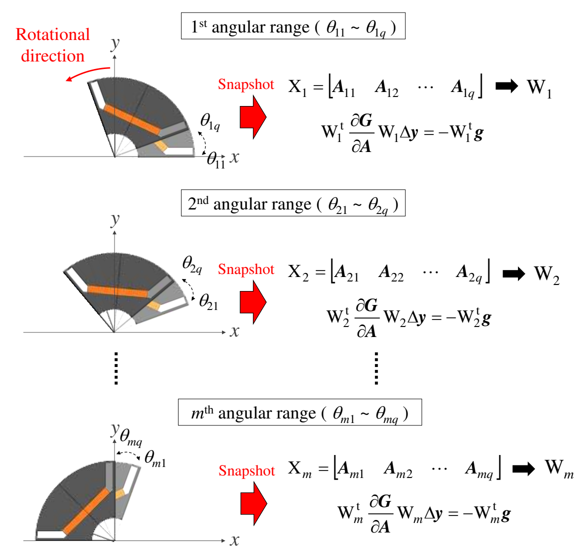

Fig. 3. Transformation matrices for m blocks

C. Loss evaluation using 1-D FE method

After the magnetic field analysis, the iron loss is evaluated by so called 1-D FE method assuming that the motor is sufficiently long in the axial direction and the steel sheet is sufficiently wide [8]. In this method, we analyze one-dimensional quasi-static electromagnetic fields in the thickness direction of a steel sheet under the condition that the computed magnetic flux is imposed in the cross-section of the steel sheet. The hysteresis loss W_h and eddy current loss W_e are obtained from this post-processing. The detailed theory is explained in [8].

III. Numerical Results

A. Reluctance motor

We apply the conventional MOR and block-MOR to analysis of the motor shown in Fig. 4 [9]. We first consider the reluctance motor for which the permanent magnet is assumed to be, for simplicity, non-magnetized, that is, M_x=M_y=0. The analysis conditions are summarized in Table I. The air gap and the other domains are discretized with rectangular and triangular FEs. The rotor angle θ ranges from 0˚ to 90˚. In this case, the cores are scarcely saturated because the permanent magnet is not magnetized.

In the conventional MOR, s snapshots are taken at equal intervals. The snapshot number s is set to 16, 31 and 46, that is, the angular intervals are 6, 3 and 2 degrees, respectively. In the block-MOR, the number of blocks m is set to 2 and 3, and the snapshot number is set to s/m. The total snapshot number is identical to that for the conventional MOR.

The error in torque calculated by the nodal force method is defined by

$$error(T) = \sqrt{\frac{\sum_{k=1}^{N_t}\left(T_k^{\mathrm{orig}} - T_k^{\mathrm{red}}\right)^2}{\sum_{k=1}^{N_t}\left(T_k^{\mathrm{orig}}\right)^2}} \tag{6}$$

where N_t, T_k^{orig} and T_k^{red} are the number of time steps, and the torque obtained by original FE analysis without MOR and with MOR, respectively.

The numerical errors in torque and speed up ratio are plotted in Fig. 5 (a) in which the speed up ratio is defined by $\tau_{\mathrm{FEM}}/\tau_{\mathrm{MOR}}$ where τ_{FEM} and τ_{MOR} are computational times of FEM and MOR. The results marked by m=1 are obtained by the conventional MOR-based FEM. We can find that, although the accuracy becomes better as s increases, the speed up ratio greatly decreases when we use the conventional MOR. This tendency is mainly due to matrix-matrix products performed in the Newton-Raphson iterations. On the other hand, the speed up ratio increases with m because complexity of matrix-matrix products is reduced to $O(n(s/m)^2+n^2(s/m))$ and, in addition, unknowns in (5) are reduced to s/m. Accuracy of block-MOR is almost the same as that of conventional MOR. We conclude from these results for the reluctance motor that block-MOR is superior to conventional MOR with respect to computational efficiency.

B. *IPM motor*

We next consider the IPM motor shown in Fig. 4. There exist saturated region near the magnet in this case. The numerical errors in torque and speed up ratio are plotted in Fig. 5(b). We observe that tendency in (b) is essentially the same as that in (a). It is found from these figures that the errors for IPM motor are larger than those for the reluctance motor. The relatively large errors in IPM motor are attributed to the magneic saturation in the cores. The torques calculated by the conventional FE analysis and FEM with block-MOR at each angular step are shown in Fig. 6, where *error*(T) is also plotted. We find no significant differences in both torques. The error tends to become larger near the upper peak of the torque curves.

Figs. 7(a) and (b) show the distribution of flux density computed by the conventional FEM and FEM with block-MOR under the condition that m=3 and s=31. Fig. 7(c) shows the difference between the flux densities. The differences are found to be less than 0.10 T. There are relatively large errors around the edge of the flux barrier. This error is due to the fact that the flux density strongly saturates in these portions.

Fig. 8 shows the speed up ratios and numerical error in torque when s=45. It is found that speed up ratio gradually improves with m, while it goes down when m=45. When m=15, speed up ratio is about four times as high as that of the conventional method (m=1). On the other hand, numerical error scarcely depends on m expect when m=45. It is concluded that m should be set as great as possible provided that s/m is greater than 2.

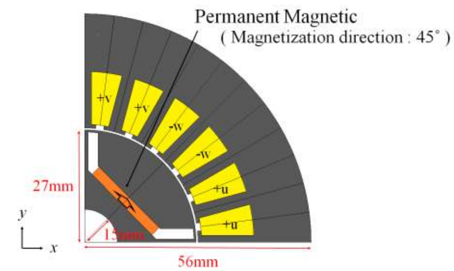

Fig. 4. Motor model

TABLE I
ANALYSIS CONDITION

Current value [AT]	3.0×35
Phase angle of current [degree]	30
Magnetization [T]	1.25
Lamination thickness [mm] / number	0.5 / 130
Initial angle of rotor [degree]	15
Number of the elements / nodes	29498 / 15267
Material of the core	50A470

C. *Iron losses*

As mentioned in II. C, in the post-processing, iron losses W_e and W_h are computed from the magnetic fields obtained from the conventional MOR and block-MOR. The results are summarized in Table II. There are no significant differences in the results obtained by both methods. The computational time for block-MOR in this post-processing is shorter than that for conventional MOR as can be seen in the above results.

IV. CONCLUSION

In this paper, we have described the performance of the block-MOR applied to analysis of reluctance and IPM motors. Accuracy of the present method for torque and loss analysis is almost the same as that of the conventional MOR. On the other hand, the former MOR has larger speed up ratio with respect to the conventional FEM. The error in the flux density computed by block-MOR applied to the IPM motor is less than 0.1 T. Relatively large errors exist in the highly saturated region. Suppress of these errors remains for our future work. Moreover, application of the present method to 3D analysis of motors will be our future work.

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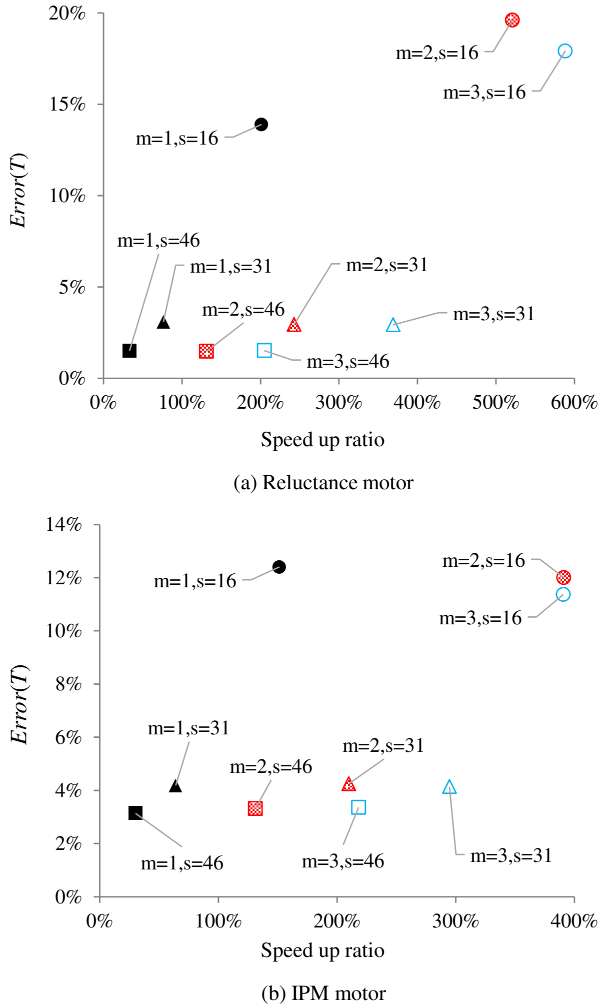

Fig. 5. Diagram of numerical error and computational time. Results marked by m=1 and m=2, 3 represent conventional and block-MOR.

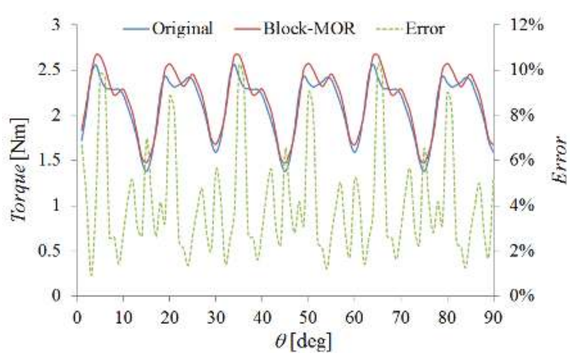

Fig. 6. Comparison of torque calculated by original FE analysis and block-MOR

TABLE II
NUMERICAL ERRORS IN IRON LOSS IN ROTOR

	IPM		Reluctance	
	W_e	W_h	W_e	W_h
Conventional POD (m=1,s=31)	2.62%	3.32%	0.889%	0.774%
block-MOR (m=3,s=31)	1.81%	3.59%	0.669%	1.12%

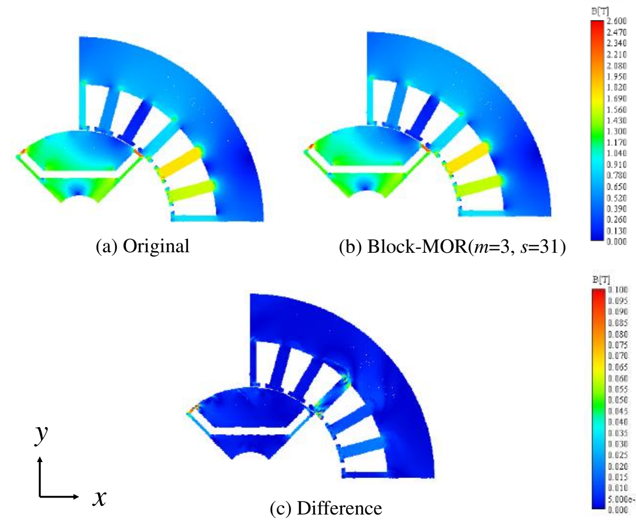

Fig. 7. Distribution of flux density when θ=45˚.

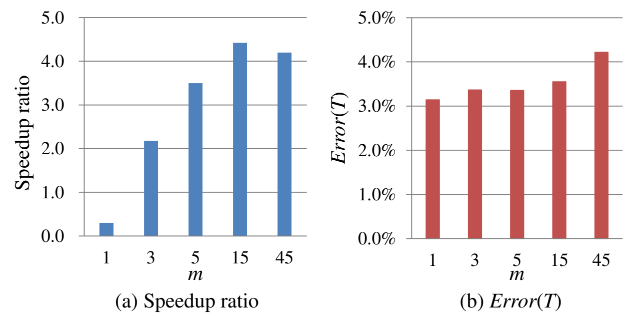

Fig. 8. Results of each number of m (s=45)

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