ANL-7907
Mathematics and Computers

ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60439

Fast Fourier Transform Using the PDP-11

by

L. R. Windmiller, J. B. Ketterson, and J. C. Shaw

Solid State Science Division

June 1972

DISCLAIMER

TABLE OF CONTENTS

ABSTRACT

This report describes programs to perform FFT on a real data sequence using a PDP-11 without EAE. The sequence to be transformed can be stored either internally in the memory of the PDP-11, externally in the memory of a Nicolet (Fabri-Tek) Model 1070 signal averager interfaced with the PDP-11, or in some other storage device interfaced with the PDP-11. The calculation is performed in place, using the storage location of the original input data for intermediate results and eventually replacing this data with the Fourier transform. All points in the sequence are considered real and the even odd point method is used in order to avoid the necessity of doubling the data storage area to include the zero imaginary terms. These programs use fixed point arithmetic and can perform 1K FFT in a time typically of order 3 seconds for single precision and 8 seconds for double precision.

Material available includes:

1. Detailed explanation of the algorithm used.
2. Operating instructions.
3. Source or binary tapes.
4. Listings.
5. Executive program for teletype control.
6. Various data handling programs including apodization, magnitude calculation, etc.

Minimum Hardware: 4K PDP-11

Storage Requirements: Program - 900 words
Sine lookup table - dependent on maximum size transform required.
Of order 256 words for 1K or less transform.
Data - Number of elements transformed.

Source Language: PAL-11A

Fast Fourier Transform Using the PDP-11

by

L. R. Windmiller, J. B. Ketterson, and J. C. Shaw

INTRODUCTION

The computational efficiency introduced by the Fast Fourier Transform technique[1] has had a major effect on many areas of research, broadening both the depth and scope of many investigations while opening up other new areas of endeavor. One of the revolutionary new applications using this technique was the use of a minicomputer for both data acquisition and analysis on line within an experimenter's own laboratory. Under such circumstances, the speed at which the minicomputer could perform the Fast Fourier Transform often became the limiting parameter in the design and performance of experiments. With the advent of the Digital Equipment Corporations' PDP-11 minicomputer,[2] it became evident that appreciable reductions in computational time could be realized if programs were written which would take advantage of the unique features of this machine. Such programs have now been written and are the subject of this report. These programs use fixed point arithmetic and can perform 1 K Fast Fourier Transforms in a time typically of order 3 seconds for single precision and 8 seconds for double precision using a computer without hardware multiply/divide. Core storage requirements for the program are modest, the entire program including the routines for calculating the absolute magnitude of the Fourier coefficients reside in about 900 words. In order to achieve high computing speed, the program uses a cosine lookup table. This table adds to the length of the main program by an amount dependent upon the maximum size transform the program is capable of handling. Thus, for a program capable of handling up to $M = 2^m$ point transforms, the cosine lookup table will contain M/4 entries for single precision calculations and M/2 entries for double precision.

The calculation performs the Fast Fourier Transform in place, using the storage locations of the original input data for intermediate storage and eventually replacing this data with the Fourier transform. The data can be stored either internally in the memory of the PDP-11, externally in the memory of a Nicolet Series 1070 signal averager[3] interfaced with the PDP-11, or in some other storage device interfaced with the PDP-11.

Relationship between the Finite Fourier Transform and the Fourier Integral Transform[4,5]

The typical experimental situation in which Fast Fourier Transform techniques are applied approximates the wave form to be transformed by a series of samples of the wave form. (That is, the wave form is sampled at some rate Δt for a finite time T and the $m = T/\Delta t$ samples form a finite discrete series.) It is important to understand the effects that the sampling and the finite observation time have on the transform derived from the fast Fourier transform and how this transform relates to the actual Fourier transform of the complete wave form.

In order to do this, consider the infinite continuous wave form $X(t)$. The Fourier transform is given by

$$a(f) = \int_{-\infty}^{+\infty} X(t) e^{-2\pi i f t} \, dt \qquad (1)$$

The inverse transform is

$$X(t) = \int_{-\infty}^{+\infty} a(f) e^{2\pi i f t} \, dt \qquad (2)$$

When $X(t)$ is sampled at intervals Δt, Eq. (2) is written at the sampling points

$$X(j\Delta t) = \int_{-\infty}^{+\infty} a(f) e^{2\pi i f j \Delta t} \, df$$

$$= \sum_{k=-\infty}^{+\infty} \int_{kF}^{(k+1)F} a(f) e^{2\pi i j f/F} \, df$$

$$= \int_{0}^{F} a_p(f) e^{2\pi i j f/F} \, dF \qquad (3)$$

where $F = 1/\Delta t$ and

$$a_p(f) = \sum_{-\infty}^{+\infty} a(f + kF) \qquad (4)$$

is a continuous function that is periodic with period F. This function is formed by the superposition of the non-periodic function $a(f)$ shifted by all multiples of the fundamental period F. The function $a_p(f)$ is said to be

the "aliased" version of the function a(f) with the aliasing occurring relative to the Nyquist frequency F/2.

Since $a_p(f)$ is a periodic function of f, it has a Fourier series expansion given by

$$a_p(f) = \frac{1}{F} \sum_{j=-\infty}^{+\infty} X(j \cdot \Delta t) e^{-2\pi i f j/F} \qquad (5)$$

Thus, the effect of sampling a continuous function is to produce a periodic continuous Fourier series transform which is the aliased version of the continuous Fourier integral transform.

Eq. (5) still represents an infinite Fourier series. The effect of sampling for only a finite time so that there are only N samples in the series $X(j \cdot \Delta t)$ can be seen by writing Eq. (5) as

$$a_p(f) = \frac{1}{F} \sum_{j=0}^{N-1} \sum_{\ell=-\infty}^{+\infty} X(j \cdot \Delta t + \ell N \Delta t) e^{2\pi i f (j+\ell N)/F}$$

$$= \frac{1}{F} \sum_{j=0}^{N-1} X_p(j \cdot \Delta t) e^{2\pi i f j/F} \qquad (6)$$

where

$$X_p(j \cdot \Delta t) = \sum_{\ell=-\infty}^{+\infty} X(j \cdot \Delta t + \ell N \Delta t) e^{2\pi i f \ell N/F} \qquad (7)$$

In order that $X_p(j \cdot \Delta t) = X(j \cdot \Delta t)$ on the interval $0 \le j < N - 1$, we must have $X(j \cdot \Delta t) = 0$ outside the interval. Thus, the transform (6) is that of a function whose value is zero everywhere but in the interval $0 \le T < (N - 1)\Delta t$ or else it is the transform of a function whose superposition as given by Eq. (7) yields the sampled series.

Before examining the effect of this truncation of the sampled series on the transform, it is worthwhile to point out that Eqs. (6) and (7) simplify if the Fourier series transform $a_p(f)$ is also sampled at intervals $\Delta f = F/N = 1/N\Delta t$.

Then

$$a_p(n\Delta f) = \frac{1}{F} \sum_{j=0}^{N-1} \sum_{\ell=-\infty}^{+\infty} X(j\cdot\Delta t + \ell N\Delta t) e^{2\pi i n(j+\ell N)/N}$$

$$= \frac{1}{F} \cdot \sum_{j=0}^{N-1} X_p(j\cdot\Delta t) e^{2\pi i n j/N} \qquad (6')$$

where

$$X_p(j\cdot\Delta t) = \sum_{\ell=-\infty}^{+\infty} X(j\cdot\Delta t + \ell N\Delta t) \qquad (7')$$

In this case $X_p(j\cdot\Delta t)$ is the aliased version of the function $X(j\cdot\Delta t)$ with aliasing occurring relative to the number of samples N. Here we can relax the condition that the function by zero everywhere outside the interval so long as it is periodic. Thus we have derived the important result that sampling for a finite number of points produces a transform of a function "aliased" as in Eq. (7) or Eq. (7').

Thus, if $X(t)$, $-\infty < t < \infty$ and $a(f)$, $-\infty < f < \infty$ are a Fourier integral transform pair, then

$$T\ X_p(j\Delta t), \qquad j = 0, 1, \ldots N - 1$$

and

$$a_p(n\Delta t) \qquad n = 0,1, \ldots N - 1$$

are a finite Fourier series pair.

Because of this, if we take the finite Fourier series transform of a function $X(j\cdot\Delta t)$, the equivalent Fourier integral function is

$$X'(t) = X(t)\cdot A(t)$$

where $A(t) \neq 0 \quad 0 \leq t < (N - 1)\Delta t$

$= 0$ elsewhere.

Thus

$$a(f) = \int_{-\infty}^{+\infty} X(t)A(t)e^{-2\pi ift}\,dt$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\bar{X}(f')e^{2\pi if't}\,df'\right]\left[\bar{A}(f'')e^{2\pi f''t}\,df''\right]$$

$$e^{-2\pi ift}\,dt$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\bar{X}(f')\bar{A}(f'')e^{2\pi i(f'+f''-f)t}\,dtdf'df''$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\bar{X}(f')\bar{A}(f'')\delta(f' + f'' - f)df'df''$$

$$= \int_{-\infty}^{+\infty}\bar{X}(f')\bar{A}(f - f')df' \qquad (8)$$

where $\bar{X}(f) = \int_{-\infty}^{+\infty} X(t)e^{-2\pi ift}\,dt$

$$\bar{A}(f) = \int_{-\infty}^{+\infty} A(t)e^{-2\pi ift}\,dt \qquad (9)$$

The effect of aliasing on a time sequence is to convolve its transform with the transform of the cutoff function.

This convolution is the source of much of the problems with the fast Fourier transform technique. The transform of a rectangular data window ($A(t) = 1$, $0 \le t < (N - 1)\Delta t$) is the sin f/f function. This function produces a spread in the frequency spectra of the transform and worse produces extra peaks due to the side lobes of the sin f/f function. In the special case that the function to be transformed is exactly periodic over the sampling interval, the central peak of the sin f/f occurs at the frequency sample and all other frequency samples occur at the zeros of this function. Thus the convolution does not affect the sampled frequency spectrum. However, as soon as the function is not exactly periodic, the frequency samples no longer occur at the zeros of the sin f/f function and extra peaks in the frequency spectrum appear.

In order to minimize the distortion of the frequency spectrum due to this cutoff function, function other than a rectangular data window can be used.[6] This procedure, called apodization, consists of multiplying the sampled series with a function whose transform is sharply peaked and which does not have the undesirable side lobes of the sin f/f function.

Examples of such apodization functions are triangular weighting $A(t) = 1 - |t - T/2|$ and Gaussian weighting

$$A(t) = e^{-k(t-T/2)^2} .$$

Description of the Algorithm[7]

The Fast Fourier Transform (FFT) is a method for efficiently computing the discrete Fourier transform of a discrete function. Thus for the discrete series X(t) (t = integer, $0 \le t \le M - 1$), the discrete Fourier transform is

$$A(f) = \sum_{t=0}^{M-1} X(t) \exp(2\pi i\, ft/M) \qquad (10)$$

$$(f = \text{integer},\ 0 \le f \le M - 1).$$

For computing efficiency, we must have the number of terms in the series equal to a power of two; that is, $M = 2^m$ where m is an integer. In the following, the binary representation for the integer t and f will be employed. Thus

$$t = t_{m-1}\, 2^{m-1} + t_{m-2}\, 2^{m-2} + \qquad t_o \equiv (t_{m-1},\ t_{m-2}, \ldots t_o) \qquad (11)$$

$$f = f_{m-1}\, 2^{m-1} + f_{m-2}\, 2^{m-2} + \qquad f_o \equiv (f_{m-1},\ f_{m-2}, \ldots f_o) \qquad (12)$$

where the t_i (or f_i), which have the values either 0 or 1, represent the value of the i^{th} bit in the binary representation of the integer t (or f).

The particular form of the FFT used in this calculation is called decimation in frequency. This form was chosen because of its convenient indexing features. These features will be pointed out in the following.

Decimation in frequency corresponds to writing the argument of the complex exponential of Eq. (10) in the following way:

$$ft/m = (f_{m-1},\ f_{m-2},\ \ldots\ f_o)(t_{m-1},\ t_{m-2},\ \ldots\ t_o)/2^m$$

$$= f_{m-1}\cdot\frac{t}{2} + f_{m-2}\cdot\frac{t}{4} + \ldots\ f_o\cdot\frac{t}{2^m} \qquad (13)$$

Because of the property

$$\exp\,(2\pi i[\text{integer} + \text{fraction}]) = \exp\,(2\pi i[\text{fraction}])$$

we can drop all the integer portions of the fraction $t/2^n$ in Eq. (13) so that Eq. (10) can be rewritten as

$$A(f) = \sum_{t=0}^{m-1} X(t)\left[\exp\left(2\pi i \cdot \frac{f_{m-1}t_o}{2}\right) \times \exp\left(2\pi i \cdot \frac{f_{m-2}(2t_1 + t_o)}{4}\right)\right.$$

$$\ldots \times \exp\left(2\pi i \cdot \frac{f_{m-n}(2^{n-1}t_{n-1} + 2^{n-2}t_{n-2} + \ldots t_o)}{2^n}\right)$$

$$\left. \ldots \times \exp\left(2\pi i \cdot \frac{f_o t}{2^m}\right)\right] \qquad (14)$$

$$= \sum_{t_{m-1}=0}^{1} \sum_{t_{m-2}=0}^{1} \ldots \sum_{t_o=0}^{1} \left[\exp\left(2\pi i \frac{f_{m-1}t_o}{2}\right)\right.$$

$$\left. \times \exp\left(2\pi i \frac{f_{m-2}(2t_1 + t_o)}{4}\right) \times \ldots \exp\left(2\pi i \frac{f_o t}{2^m}\right)\right]$$

The first pass takes all the elements in pairs and performs the calculation

$$X'(f_o, t_{n-2}, \ldots t_o) = \sum_{t_{m-1}=0}^{1} \exp\left(2\pi i \frac{f_o t}{2^m}\right) X(t)$$

Each pair is taken from its place and the calculation performed creating a new pair which is returned to the place just vacated. The next pass sums on the t_{n-2} bit and each subsequent pass sums on the next bit to the right until the entire sum over t is performed. A typical pass performs the calculation

$$X''(f_o, \ldots, f_{m-n}, t_{n-2}, \ldots t_o)$$

$$= \sum_{t_{n-1}} X'(f_o, \ldots f_{m-n-1}, t_{n-1}, \ldots t_o)$$

$$\times \exp\left(2\pi i \frac{f_{m-n}(2^{n-1}t_{n-1} + \ldots + t_o)}{2^n}\right) \qquad (15)$$

This calculation operates on the pairs of elements (where only the underlined argument in Eq. (15) is indicated, the other arguments, of course, determining the pairs and taking on all possible values) in the following way:

$$\text{Re } X''(\text{——},0,\text{——}) = \text{Re } X'(\text{——},0,\text{——}) + \text{Re } X'(\text{——},1,\text{——})$$

$$\text{Imag } X''(\text{——},0,\text{——}) = \text{Imag } X'(\text{——},0,\text{——}) + \text{Imag } X'(\text{——},1,\text{——})$$

$$\text{Re } X''(\text{——},1,\text{——}) = \cos\theta \text{ Re } C - \sin\theta \text{ mag } C$$

$$\text{Imag } X''(\text{——},1,\text{——}) = \cos\theta \text{ Imag } C + \sin\theta \text{ Re } C \qquad (16)$$

where

$$C = X'(\text{——},0,\text{——}) - X'(\text{——},1,\text{——})$$

$$\theta = 2\pi \cdot \left(\frac{2^{n-2} t_{n-2} + \cdots t_0}{2^n} \right) \qquad (17)$$

The advantages of this procedure are now clear since it can be seen that the same bit arrangement used for the address calculation of the elements being operated upon also directly determine the phase θ of the weighting factor.

This calculation does have the disadvantage that the final result has the Fourier components stored in memory in bit reversed order, that is the elements are stored in memory as

$$X''(f_0, f_1, \cdots f_{m-1})$$

Thus one run through the result is necessary interchanging so that

$$A(f_{m-1}, \cdots f_0) = X''(f_0, \cdots f_{m-1})$$

The subroutine used to perform this rearrangement is called BITINV.

An additional calculation is necessary because of the form of the input data which is, of course, all real. If all the imaginary components of the input series were assumed zero, considerable efficiency in core storage and calculational speed would be lost. In order to avoid this difficulty, the even odd point method is used and only the Fourier components below the Nyquist frequency

are calculated. The even odd point method considers the j^{th} and $j + 1$ data elements as the real and imaginary parts respectively of the $j/2$ element of the series to be transformed. That is, the real series is regrouped so that we actually take the transform of the modified series

$$A'(f) = \sum_{t=0}^{M/2-1} [X(2t) + iX(2t + 1)]e\left(\frac{tf}{M/2}\right) \qquad (18)$$

However, this can be rewritten as

$$A'(f) = \sum_{t=0}^{N-1} [X(t) + iY(t)]e\left(\frac{tf}{N}\right) \qquad (18')$$

Using

$$A'(N - f) = \sum_{t=0}^{N-1} [X(t) + iY(t)]e\left(-\frac{tf}{N}\right) \qquad (18'')$$

it can be shown that

$$\begin{aligned} 2\hat{X}(f) &= \text{Re } C + i \text{ Imag } D \\ 2\hat{Y}(f) &= \text{Imag } C - i \text{ Re } D \end{aligned} \qquad (19)$$

where $\hat{X}(f)$ and $\hat{Y}(f)$ are the transform of $X(t)$ and $Y(t)$ respectively, and

$$\begin{aligned} C &= A'(f) + A'(N - f) \\ D &= A'(f) - A'(N - f) \end{aligned} \qquad (19')$$

However, the Fourier transform actually desired is

$$A(f) = \sum_{t=0}^{M-1} X(t)e\left(\frac{tf}{M}\right)$$

$$= \sum_{t=0}^{N-1} X(2t)e\left(\frac{tf}{N}\right) + \sum_{t=0}^{N-1} X(2t+1)e\left(\frac{tf}{N}\right)e\left(\frac{f}{M}\right)$$

$$= \hat{X}(f) + e\left(\frac{f}{M}\right)\hat{Y}(f) \qquad (20)$$

So after the Fourier transform is computed, it is necessary to make an additional pass through the result modifying the pairs at f and N - f. From above it can be shown that the calculation necessary is

$$\text{Re } A(f) = \frac{1}{2}\left[\text{Re } C + \cos\left(\frac{f}{M}\right)\text{Imag } C + \sin\left(\frac{f}{M}\right)\text{Re } D\right]$$

$$\text{Imag } A(f) = \frac{1}{2}\left[\text{Imag } D + \sin\left(\frac{f}{M}\right)\text{Imag } C - \cos\left(\frac{f}{M}\right)\text{Re } D\right] \qquad (21)$$

and

$$\text{Re } A(N-f) = \frac{1}{2}\left[\text{Re } C - \cos\left(\frac{f}{M}\right)\text{Imag } C - \sin\left(\frac{f}{M}\right)\text{Re } D\right]$$

$$\text{Imag } A(N-f) = \frac{1}{2}\left[-\text{Imag } D + \sin\left(\frac{f}{M}\right)\text{Imag } C - \cos\left(\frac{f}{M}\right)\text{Re } D\right] \qquad (21')$$

The subroutine used to perform this calculation is called COMPLX.

After performing the calculations, the transformed elements are stored in the data area in sequential order with the real and imaginary parts alternating. For many purposes (such as convenient data display), it is desirable to sort the data so that the real elements occupy the first half of the data area in sequential order followed by the corresponding ordering of the imaginary elements in the second half of the data area. Such sorting can be done with a minimum of interchanging by recognizing that the series actually is composed of two interpenetrating series; the series of the real elements and the series of the imaginary elements. Thus, writing the series as

$$R_0,\ I_0,\ R_1,\ I_1,\ R_2,\ I_2,\ R_3,\ I_3,\ R_4,\ I_4 \ldots\ldots$$

it is evident that each element of the real series is twice as far along as it should be; that is, R_1 in position 2 (position numbers start at 0 and go to M-1) should be at position 1, R_2 at position 4 should be at position 2, etc. Less obvious is the fact that the imaginary series also has a similar relationship only instead of with respect to the beginning of the series each element is twice as far as it should be from the end of the series. Thus, sequences of interchanges can be set up based solely on the position of the elements in the original series. The first sequence is then

$$1 \leftarrow 2 \leftarrow 4 \leftarrow 8 \ldots\ldots \leftarrow M/2 \leftarrow I \quad .$$

The recipe of doubling each element to find the next element terminates when the next doubling points to a number larger than the total number of elements. When this occurs, what has actually happened is that the sequence has switched from the real series to the imaginary series. Then, further development of the series is found by doubling the offset from the end of the original series rather than from the beginning. Again, when this doubling leads to an element further from the end than the number of elements in the series, a switch has occurred back to the real series again and the original recipe can be followed. In this way, a sequence of changes can be generated which ends when the first element of the sequence reappears. Since the original series is symmetric with respect to the two interpenetrating series, this sequence can also be used interpreting it as applying to positions from the end rather than the beginning of the series (the case where these two sequences happen to be degenerate in that they operate on the same series elements must be distinguished so that the elements are not shifted twice). This method is applied by generating sequences starting first at element 1, then 3, 5, etc. Sequences can never start on even number elements because of the rule that subsequent members of the sequences are generated by doubling the previous element. Each sequence after the first can be checked to see that it has not been done before by examining its lowest and highest element. The sequence is new if its lowest element is higher than any of the previous sequences lowest elements and its highest element is lower than any of the previous sequences highest elements. Since the first and last elements are in their proper place initially, only M-2 changes are necessary. However, for the case when M/3 = integer, one

of the changes is the interchange of elements M/3 and 2M/3. Thus, in this case the above prescription need be followed for only M-4 changes. The subroutine used to perform this operation is called SORT.

After performing the entire calculation including subroutine SORT, the transformed elements are stored in the data area in sequential order with the real parts in the first half followed by the corresponding imaginary parts in the second half. It is sometimes convenient to have the absolute magnitude of the Fourier transform which entails making the calculation

$$|A(f)| = [(\text{Re } A(f))^2 + (\text{Imag } A(f))^2]^{1/2}$$

for all f. Since no division routine is necessary for the FFT calculation, a square root algorithm was chosen which required no divisions. This algorithm uses the iterative sequence

$$U_o = x$$

$$V_o = x - 1$$

$$U_{k+1} = (U_k(1 - \tfrac{1}{2} V_k) \qquad k = 0, 1, 2, 3 \ldots$$

$$U_{k+1} = \tfrac{1}{4} V_k^2 (V_k - 3)$$

where U_k converges to $x^{1/2}$. For $1/2 \leq x \leq 1$, this requires 3 iterations to converge for single precision and 4 iterations for double precision. The subroutine used to calculate square roots is called SQROOT.

In order to maintain maximum accuracy in the fixed point arithmetic, each pass of the FFT calculation is preceded by a normalization procedure. This procedure looks for the largest data element (in absolute magnitude) and then shifts all the data, either left or right so that $1/8 < |\text{largest element}| \leq 1/4$. The number of shifts performed on the data is maintained at location SCALER. The subroutine used to perform this operation is called either SIZER or NORM depending on the calculational configuration as outlined in the next section.

It should be noted that although the arithmetic is done is fixed point, the operands are always viewed as fraction with the most significant bit used as a sign bit and the following bits as powers of 1/2. Thus 0 111 000 000 000 is + 1/2 + 1/4 + 1/8 = 7/8.

Operating Instructions

Two basic forms of the program are presented here. These programs illustrate both single and double precision fixed point arithmetic with data storage either in the PDP-11 memory or in the memory of a Nicolet Instrument Model 1070 signal averager interfaced with the PDP-11. The Model 1070 is an 18 bit machine in which the most significant 16 bits are accessed by the PDP-11 in the same way as any internal memory word. The least significant 2 bits are accessed rather differently, however, resulting in differences accessing the data between the double precision routine using internal storage and that using storage in the 1070.

The difference between the scaling routines NORM and SIZER is that SIZER performs double precision rotates using the addressing necessary to access the least 2 significant bits in the 1070 while NORM accesses all memory locations in the standard manner. Thus, for double precision arithmetic using the 1070 storage SIZER must be used, while for internal storage NORM must be used. However, for single precision arithmetic SIZER need be used only for the first scaling operating when 1070 storage is used and after that NORM can be used independent of the data storage location.

Since double precision arithmetic using 1070 storage really requires only 18 bits of significance, full double precision multiplication routines and cosine lookup tables are not needed. The integer multiplication routine is therefore modified to operate on 18 bits rather than 32. Seventeen bits of significance are stored in a single word of the double precision cosine lookup table by multiplying each cosine by 2 and using the sign bit (which is always zero).

The main program is completely relocatable and can be loaded anywhere in core.[8] The input needed by the program is the address of the first word of the data storage area which must be put in location LOCATR and the numbers of data words to be transformed (a power of 2) which must be put at location SIZE. The cosine lookup table begins at location SCTABL and extends for M/4 + 1 words for single precision arithmetic with data storage in the 1070 and for M/2 + 2 words for double precision arithmetic with data storage in the PDP-11.

Operation of the program is simple. The FFT program starts at location START and can be invoked by a calling program with the JSR PC, START instruction.

None of the general registers are saved and return to the calling program finds the real part of the transform in the first half of the data area and the imaginary part in the second half. SCALER then contains the factor by which the data has been scaled by the transformation. The absolute magnitude calculation starts at location MAGNTD and also can be entered from a calling program by the JSR PC, MAGNTD instruction. Return to the calling program finds the magnitude replacing the real part of the transform and the imaginary part unchanged. No general registers are saved.

Programs

Two examples of the programs are given here. The first program is a general, single precision computation with data pointed to be location LOCATR. This program can be used to operate on data in the Model 1070 by first using SIZER (SIZER can be found in the second program example) in order not to lose significant bits during the first normalization. After this, the main program using NORM can be used to perform the FFT.

The second program is a double precision routine specialized to operate on 18 bit data in the Model 1070. All the actual computations, however, are performed in a double precision format so that conversion to full 32 bit data is straightforward. All that is necessary is to change the accessing of the data from that peculiar to the Model 1070 to that necessary for full double precision format. In addition the subroutine MULT must be changed to a full 32 bit routine (making sure that it reflects the fact that the numbers are interpreted as fractions so that $40000_8 \times 40000_8 = 20000_8$). An extra iteration in the SQRT routine is necessary. It will also be necessary to access the cosine lookup table in full double precision.

The programs reproduced here can transform a maximum of 1024 points. If larger transforms are desired, the necessary modifications are easy. In addition to the construction of a cosine lookup table having the appropriate number of entries, it is necessary to change the access to the sine in subroutines COMPLX and LOOKUP and the setting up of WGTINC at locations COMPØ and START1.

The double precision program also contains an executive routine in order to control all functions from the teletype keyboard. Thus, if the program is started at location INIT, the program will respond with a CR/LF

on the teletype and wait for an operator response indicating what function to perform. By typing any of the following letters, the functions indicated will be performed:

T - place Model 1070 under computer control, compute FFT, remove Model 1070 from computer control and wait for another operator command.

M - place Model 1070 under computer control, compute absolute magnitude, remove Model 1070 from computer control and wait for another operator command.

O - place Model 1070 under computer control, copy the block of data starting at the address at location SOURCE and extending a number of words equal to the contents of location SIZE into a block starting at the address at location DEST, remove Model 1070 from computer control and wait for another operator command.

I - place Model 1070 under computer control, perform the opposite function of O; that is, copy data block from DEST to SOURCE, remove Model 1070 from computer control and wait for another operator command.

X - place Model 1070 under computer control, interchange the data blocks located at SOURCE and DEST, remove Model 1070 from computer control and wait for another operator command.

S - place Model 1070 under computer control, perform subroutine SIZER on data, remove Model 1070 from computer control and wait for another operator command.

In addition, two more operations can be added to the executive by loading the teletype code for the call character at locations 4034 and 4036 (relative to INIT) and the start address of the subroutines (relative to INIT) at locations 4054 and 4056 (relative to INIT). Because the entire program is relocatable, the executive program uses this information to compute the branch vector in the JSR PC, (subroutine) instructions at locations 4316 and 4320 (relative to INIT). This executive can also be used with the single precision program when used with the Model 1070 or, if the Model 1070 control statements are deleted, with the general program.

Program Listings

```
;               FAST FOURIER TRANSFORM MODULE.
;               PROGRAMMED BY LEE WINDMILLER,
;               ARGONNE NATIONAL LABORATORY,
;               ARGONNE, ILLINOIS  60439.
;
;-----------------------------------------------
;-----------------------------------------------
;----------------SUBROUTINE START---------------
;-----------------------------------------------
;-----------------------------------------------
;
;               CALLING SEQUENCE:
;               JSR PC,START    FOR TRANSFORM
;               JSR PC,MAGNTD   FOR MAGNITUDE
;
```

BITINV	001750	BITIN0	001754	BITIN1	001766	BITIN2	002006
BITIN3	002036	BITIN4	002056	COMPLX	000346	COMPL1	000420
COMPL2	000450	COMP0	000356	COMP0A	000374	COMP2A	000542
COMP2B	000554	COSW	000014	COUNT	001150	FFT	000110
FFT1	000114	FFT2	000314	GETADR	001606	GETAD0	001624
GETAD1	001650	HIADD	001146	IMAGA	000340	IMAGB	000344
INDEX	000006	LOADD	001144	LOCATR	000012	LOOKUP	001670
LOOK1	001716	LOOK2	001746	MAGNTD	002156	MAG1	002162
MAG2	002250	MULT	002060	MULT1	002074	MULT2	002106
MULT3	002114	MULT4	002124	MULT5	002144	MULT6	002146
NORM	000742	NORM1	000764	NORM2	000772	NORM3	001002
NORM4	001020	NORM6	001040	NORM7	001050	NORM7A	001102
NORM7B	001104	NORM7C	001056	NORM8	001122	NORM8A	001124
NORM9	001142	PASS	000002	PC	=%000007	REALA	000336
REALB	000342	R0	=%000000	R1	=%000001	R2	=%000002
R3	=%000003	R4	=%000004	R5	=%000005	SCALER	000010
SCTABL	002474	SINW	000016	SIZE	000000	SORT	001152
SORT0	001172	SORT1	001224	SORT1A	001246	SORT2	001262
SORT2A	001306	SORT2B	001304	SORT3	001314	SORT3A	001346
SORT3B	001344	SORT4	001360	SORT4A	001432	SORT4B	001414
SORT5	001460	SORT5A	001514	SORT6	001524	SORT6A	001522
SORT7	001546	SORT8	001604	SP	=%000006	SQROOT	002252
SQROT1	002266	SQROT2	002332	SQROT3	002344	SQROT4	002346
SQROT5	002352	SQRT	002354	SQRT1	002372	SQRT2	002424
START	000024	START1	000072	SWCNTR	000020	WEIGHT	000004
WGTINC	000022	.	= 003476				

```
        000000          R0=%0
        000001          R1=%1
        000002          R2=%2
        000003          R3=%3
        000004          R4=%4
        000005          R5=%5
        000006          SP=%6
        000007          PC=%7
        000000          .=0
000000 000000  SIZE:    .WORD 0
000002 000000  PASS:    .WORD 0
000004 000000  WEIGHT:  .WORD 0
000006 000000  INDEX:   .WORD 0
000010 000000  SCALER:  .WORD 0
000012 000000  LOCATR:  .WORD 0
000014 000000  COSW:    .WORD 0
000016 000000  SINW:    .WORD 0
000020 000000  SWCNTR:  .WORD 0
000022 000000  WGTINC:  .WORD 0
000024 005067  START:   CLR SCALER
       177760
000030 005067           CLR INDEX
       177752
000034 016700           MOV SIZE,R0
       177740
000040 006200           ASR R0
000042 010067           MOV R0,PASS
       177734
000046 006200           ASR R0
000050 010067           MOV R0,SWCNTR
       177744
000054 005067           CLR WEIGHT
       177724
000060 012767           MOV #1,WGTINC
       000001
       177734
000066 016700           MOV SIZE,R0
       177706
000072 022700  START1:  CMP #2000,R0
       002000
000076 001404           BEQ FFT
000100 006300           ASL R0
000102 006367           ASL WGTINC
       177714
000106 000771           BR START1
000110 004767  FFT:     JSR PC,NORM
       000626
000114 004767  FFT1:    JSR PC,GETADR
       001466
000120 011402           MOV @R4,R2
```

```
000122 061514          ADD @R5,@R4
000124 160215          SUB R2,@R5
000126 005724          TST (R4)+
000130 005725          TST (R5)+
000132 011402          MOV @R4,R2
000134 061514          ADD @R5,@R4
000136 160215          SUB R2,@R5
000140 004767          JSR PC,LOOKUP
       001524
000144 010400          MOV R4,R0
000146 011502          MOV @R5,R2
000150 004767          JSR PC,MULT
       001704
000154 010146          MOV R1,-(SP)
000156 010302          MOV R3,R2
000160 014500          MOV -(R5),R0
000162 004767          JSR PC,MULT
       001672
000166 162601          SUB (SP)+,R1
000170 005401          NEG R1
000172 011502          MOV @R5,R2
000174 010125          MOV R1,(R5)+
000176 010400          MOV R4,R0
000200 004767          JSR PC,MULT
       001654
000204 010104          MOV R1,R4
000206 010300          MOV R3,R0
000210 011502          MOV @R5,R2
000212 004767          JSR PC,MULT
       001642
000216 060401          ADD R4,R1
000220 005401          NEG R1
000222 010115          MOV R1,@R5
000224 062767          ADD #2,INDEX
       000002
       177554
000232 066767          ADD WGTINC,WEIGHT
       177564
       177544
000240 005367          DEC SWCNTR
       177554
000244 001323          BNE FFT1
000246 006367          ASL WGTINC
       177550
000252 006267          ASR PASS
       177524
000256 022767          CMP #1,PASS
       000001
       177516
000264 001413          BEQ FFT2
```

```
000266 005067          CLR WEIGHT
       177512
000272 005067          CLR INDEX
       177510
000276 016700          MOV SIZE,R0
       177476
000302 006200          ASR R0
000304 006200          ASR R0
000306 010067          MOV R0,SWCNTR
       177506
000312 000676          BR FFT
000314 004767  FFT2:   JSR PC,NORM
       000422
000320 004767          JSR PC,BITINV
       001424
000324 004767          JSR PC,COMPLX
       000016
000330 004767          JSR PC,SORT
       000616
000334 000207          RTS PC
                ;-----------------------------------------
                ;-----------------------------------------
                ;-----------SUBROUTINE COMPLX-------------
                ;-----------------------------------------
                ;-----------------------------------------
000336 000000  REALA: .WORD 0
000340 000000  IMAGA: .WORD 0
000342 000000  REALB: .WORD 0
000344 000000  IMAGB: .WORD 0
000346 005067  COMPLX:CLR WGTINC
       177450
000352 016700          MOV SIZE,R0
       177422
000356 022700  COMP0: CMP #2000,R0
       002000
000362 001404          BEQ COMP0A
000364 006300          ASL R0
000366 005267          INC WGTINC
       177430
000372 000771          BR COMP0
000374 016705  COMP0A:MOV LOCATR,R5
       177412
000400 010504          MOV R5,R4
000402 005724          TST (R4)+
000404 061415          ADD @R4,@R5
000406 005014          CLR @R4
000410 012703          MOV #4,R3
       000004
000414 010367          MOV R3,INDEX
       177366
000420 010304  COMPL1:MOV R3,R4
000422 066704          ADD LOCATR,R4
       177364
000426 016705          MOV SIZE,R5
       177346
```

```
000432 006305          ASL R5
000434 160305          SUB R3,R5
000436 066705          ADD LOCATR,R5
       177350
000442 020405          CMP R4,R5
000444 001001          BNE COMPL2
000446 000207          RTS PC
000450 012400  COMPL2:MOV (R4)+,R0
000452 012501          MOV (R5)+,R1
000454 060100          ADD R1,R0
000456 010067          MOV R0,REALA
       177654
000462 006267          ASR REALA
       177650
000466 160100          SUB R1,R0
000470 160100          SUB R1,R0
000472 006200          ASR R0
000474 010067          MOV R0,REALB
       177642
000500 011400          MOV @R4,R0
000502 011501          MOV @R5,R1
000504 060100          ADD R1,R0
000506 010067          MOV R0,IMAGA
       177626
000512 006267          ASR IMAGA
       177622
000516 160100          SUB R1,R0
000520 160100          SUB R1,R0
000522 006200          ASR R0
000524 010067          MOV R0,IMAGB
       177614
000530 016700          MOV INDEX,R0
       177252
000534 006200          ASR R0
000536 016701          MOV WGTINC,R1
       177260
000542 005701  COMP2A:TST R1
000544 003403          BLE COMP2B
000546 006300          ASL R0
000550 005301          DEC R1
000552 000773          BR COMP2A
000554 016067  COMP2B:MOV SCTABL(R0),COSW
       002474
       177232
000562 162700          SUB #1000,R0
       001000
000566 005400          NEG R0
000570 016067          MOV SCTABL(R0),SINW
       002474
       177220
```

```
000576 016700          MOV SINW,R0
       177214
000602 016702          MOV REALB,R2
       177534
000606 004767          JSR PC,MULT
       001246
000612 010103          MOV R1,R3
000614 016700          MOV COSW,R0
       177174
000620 016702          MOV IMAGA,R2
       177514
000624 004767          JSR PC,MULT
       001230
000630 060103          ADD R1,R3
000632 016700          MOV REALA,R0
       177500
000636 060300          ADD R3,R0
000640 010044          MOV R0,-(R4)
000642 160300          SUB R3,R0
000644 160300          SUB R3,R0
000646 010045          MOV R0,-(R5)
000650 005724          TST (R4)+
000652 005725          TST (R5)+
000654 016700          MOV COSW,R0
       177134
000660 016702          MOV REALB,R2
       177456
000664 004767          JSR PC,MULT
       001170
000670 010103          MOV R1,R3
000672 016700          MOV SINW,R0
       177120
000676 016702          MOV IMAGA,R2
       177436
000702 004767          JSR PC,MULT
       001152
000706 160301          SUB R3,R1
000710 016700          MOV IMAGB,R0
       177430
000714 060001          ADD R0,R1
000716 010114          MOV R1,@R4
000720 160001          SUB R0,R1
000722 160001          SUB R0,R1
000724 010115          MOV R1,@R5
000726 062767          ADD #4,INDEX
       000004
       177052
000734 016703          MOV INDEX,R3
       177046
000740 000627          BR COMPL1
```

```
                        ;-------------------------------------------
                        ;-------------------------------------------
                        ;------------------SUBROUTINE NORM-----------
                        ;-------------------------------------------
                        ;-------------------------------------------
000742 016705   NORM:   MOV LOCATR,R5
       177044
000746 016703           MOV SIZE,R3
       177026
000752 005303           DEC R3
000754 010504           MOV R5,R4
000756 012500           MOV (R5)+,R0
000760 100401           BMI NORM1
000762 005400           NEG R0
000764 011501   NORM1:  MOV @R5,R1
000766 100401           BMI NORM2
000770 005401           NEG R1
000772 020001   NORM2:  CMP R0,R1
000774 002402           BLT NORM3
000776 010504           MOV R5,R4
001000 010100           MOV R1,R0
001002 005725   NORM3:  TST (R5)+
001004 005303           DEC R3
001006 001366           BNE NORM1
001010 011400           MOV @R4,R0
001012 001453           BEQ NORM9
001014 100001           BPL NORM4
001016 005400           NEG R0
001020 012704   NORM4:  MOV #2,R4
       000002
001024 022700           CMP #100000,R0
       100000
001030 001407           BEQ NORM7
001032 006300           ASL R0
001034 006300           ASL R0
001036 103404           BCS NORM7
001040 006300   NORM6:  ASL R0
001042 005304           DEC R4
001044 103401           BCS NORM7
001046 000774           BR NORM6
001050 005700   NORM7:  TST R0
001052 001001           BNE NORM7C
001054 005304           DEC R4
001056 005704   NORM7C: TST R4
001060 001430           BEQ NORM9
001062 016705           MOV LOCATR,R5
       176724
001066 016701           MOV SIZE,R1
       176706
001072 060467           ADD R4,SCALER
       176712
001076 005704           TST R4
001100 002410           BLT NORM8
001102 010400   NORM7A: MOV R4,R0
001104 006215   NORM7B: ASR @R5
```

```
001106 005300          DEC R0
001110 003375          BGT NORM7B
001112 005725          TST (R5)+
001114 005301          DEC R1
001116 001411          BEQ NORM9
001120 000770          BR NORM7A
001122 010400  NORM8:  MOV R4,R0
001124 006315  NORM8A: ASL @R5
001126 005200          INC R0
001130 002775          BLT NORM8A
001132 005725          TST (R5)+
001134 005301          DEC R1
001136 001401          BEQ NORM9
001140 000770          BR NORM8
001142 000207  NORM9:  RTS PC
                       ;-----------------------------------------------
                       ;-----------------------------------------------
                       ;-----------------SUBROUTINE SORT---------------
                       ;-----------------------------------------------
                       ;-----------------------------------------------
001144 000000  LOADD:  .WORD 0
001146 000000  HIADD:  .WORD 0
001150 000000  COUNT:  .WORD 0
001152 016767  SORT:   MOV SIZE,COUNT
       176622
       177770
001160 162767          SUB #2,COUNT
       000002
       177762
001166 010700          MOV PC,R0
001170 000415          BR SORT1
001172 000000  SORT0:  .WORD 0,0,0,0,0,0,0,0,0,0,0,0,0
001174 000000
001176 000000
001200 000000
001202 000000
001204 000000
001206 000000
001210 000000
001212 000000
001214 000000
001216 000000
001220 000000
001222 000000
001224 005720  SORT1:  TST (R0)+
001226 005720          TST (R0)+
001230 010067          MOV R0,SORT0
       177736
001234 016701          MOV SIZE,R1
       176540
001240 005301          DEC R1
001242 012702          MOV #1,R2
       000001
001246 005067  SORT1A: CLR HIADD
       177674
```

```
001252 010267          MOV R2,LOADD
       177666
001256 005005          CLR R5
001260 010203          MOV R2,R3
001262 006303  SORT2:  ASL R3
001264 020103          CMP R1,R3
001266 003007          BGT SORT2A
001270 006203          ASR R3
001272 020367          CMP R3,HIADD
       177650
001276 002402          BLT SORT2B
001300 010367          MOV R3,HIADD
       177642
001304 000403  SORT2B:BR SORT3
001306 010320  SORT2A:MOV R3,(R0)+
001310 005205          INC R5
001312 000763          BR SORT2
001314 160103  SORT3:  SUB R1,R3
001316 006303          ASL R3
001320 060103          ADD R1,R3
001322 005703          TST R3
001324 003010          BGT SORT3A
001326 014003          MOV -(R0),R3
001330 005720          TST (R0)+
001332 020367          CMP R3,LOADD
       177606
001336 003002          BGT SORT3B
001340 010367          MOV R3,LOADD
       177600
001344 000746  SORT3B:BR SORT2
001346 010320  SORT3A:MOV R3,(R0)+
001350 005205          INC R5
001352 020302          CMP R3,R2
001354 001401          BEQ SORT4
001356 000756          BR $ORT3
001360 020267  SORT4:  CMP R2,LOADD
       177560
001364 003057          BGT SORT6
001366 160102          SUB R1,R2
001370 005402          NEG R2
001372 020267          CMP R2,HIADD
       177550
001376 002446          BLT SORT5A
001400 010567          MOV R5,LOADD
       177540
001404 160567          SUB R5,COUNT
       177540
001410 160102          SUB R1,R2
001412 005402          NEG R2
001414 010203  SORT4B:MOV R2,R3
```

```
001416 006303          ASL R3
001420 066703          ADD LOCATR,R3
       176366
001424 011346          MOV @R3,-(SP)
001426 016700          MOV SORT0,R0
       177540
001432 011004  SORT4A: MOV @R0,R4
001434 160110          SUB R1,@R0
001436 005420          NEG (R0)+
001440 020204          CMP R2,R4
001442 001406          BEQ SORT5
001444 006304          ASL R4
001446 066704          ADD LOCATR,R4
       176340
001452 011413          MOV @R4,@R3
001454 010403          MOV R4,R3
001456 000765          BR SORT4A
001460 012613  SORT5:  MOV (SP)+,@R3
001462 160102          SUB R1,R2
001464 005402          NEG R2
001466 006302          ASL R2
001470 020201          CMP R2,R1
001472 002413          BLT SORT6A
001474 006202          ASR R2
001476 020267          CMP R2,HIADD
       177444
001502 001404          BEQ SORT5A
001504 166767          SUB LOADD,COUNT
       177434
       177436
001512 000740          BR SORT4B
001514 160102  SORT5A: SUB R1,R2
001516 005402          NEG R2
001520 000401          BR SORT6
001522 006202  SORT6A: ASR R2
001524 022767  SORT6:  CMP #3,COUNT
       000003
       177416
001532 003005          BGT SORT7
001534 062702          ADD #2,R2
       000002
001540 016700          MOV SORT0,R0
       177426
001544 000640          BR SORT1A
001546 005767  SORT7:  TST COUNT
       177376
001552 001414          BEQ SORT8
001554 042701          BIC #125252,R1
       125252
001560 006301          ASL R1
```

```
001562 010102          MOV R1,R2
001564 066701          ADD LOCATR,R1
       176222
001570 011104          MOV @R1,R4
001572 006302          ASL R2
001574 066702          ADD LOCATR,R2
       176212
001600 011214          MOV @R2,@R1
001602 010412          MOV R4,@R2
001604 000207  SORT0:  RTS PC
                       ;-------------------------------------------
                       ;-------------------------------------------
                       ;-----------------SUBROUTINE GETADR---------
                       ;-------------------------------------------
                       ;-------------------------------------------
001606 016700  GETADR:MOV INDEX,R0
       176174
001612 016701          MOV PASS,R1
       176164
001616 005700          TST R0
001620 001001          BNE GETAD0
001622 000412          BR GETAD1
001624 010002  GETAD0:MOV R0,R2
001626 010103          MOV R1,R3
001630 005403          NEG R3
001632 040302          BIC R3,R2
001634 001005          BNE GETAD1
001636 060100          ADD R1,R0
001640 010067          MOV R0,INDEX
       176142
001644 005067          CLR WEIGHT
       176134
001650 006300  GETAD1:ASL R0
001652 066700          ADD LOCATR,R0
       176134
001656 010004          MOV R0,R4
001660 006301          ASL R1
001662 060100          ADD R1,R0
001664 010005          MOV R0,R5
001666 000207          RTS PC
                       ;-------------------------------------------
                       ;-------------------------------------------
                       ;-----------------SUBROUTINE LOOKUP---------
                       ;-------------------------------------------
                       ;-------------------------------------------
001670 016700  LOOKUP:MOV WEIGHT,R0
       176110
001674 005002          CLR R2
001676 010001          MOV R0,R1
001700 042701          BIC #177577,R1
       177577
001704 001404          BEQ LOOK1
001706 162700          SUB #400,R0
       000400
001712 005400          NEG R0
001714 005202          INC R2
001716 006300  LOOK1: ASL R0
001720 006300          ASL R0
001722 016003          MOV SCTABL(R0),R3
       002474
```

```
001726 162700          SUB #1000,R0
       001000
001732 005400          NEG R0
001734 016004          MOV SCTABL(R0),R4
       002474
001740 005702          TST R2
001742 001401          BEQ LOOK2
001744 005403          NEG R3
001746 000207  LOOK2: RTS PC
                       ;-------------------------------------------
                       ;-------------------------------------------
                       ;-----------------SUBROUTINE BITINV---------
                       ;-------------------------------------------
                       ;-------------------------------------------
001750 012700  BITINV:MOV #4,R0
       000004
001754 010001  BITIN0:MOV R0,R1
001756 010104          MOV R1,R4
001760 016703          MOV SIZE,R3
       176014
001764 005002          CLR R2
001766 006201  BITIN1:ASR R1
001770 006102          ROL R2
001772 006203          ASR R3
001774 103374          BCC BITIN1
001776 006302          ASL R2
002000 006302          ASL R2
002002 020204          CMP R2,R4
002004 003414          BLE BITIN3
002006 066704  BITIN2:ADD LOCATR,R4
       176000
002012 066702          ADD LOCATR,R2
       175774
002016 011405          MOV @R4,R5
002020 011214          MOV @R2,@R4
002022 010512          MOV R5,@R2
002024 005724          TST (R4)+
002026 005722          TST (R2)+
002030 011405          MOV @R4,R5
002032 011214          MOV @R2,@R4
002034 010512          MOV R5,@R2
002036 062700  BITIN3:ADD #4,R0
       000004
002042 006200          ASR R0
002044 020067          CMP R0,SIZE
       175730
002050 001402          BEQ BITIN4
002052 006300          ASL R0
002054 000737          BR BITIN0
002056 000207  BITIN4:RTS PC
                       ;-------------------------------------------
                       ;-------------------------------------------
                       ;-----------------SUBROUTINE MULT-----------
                       ;-------------------------------------------
                       ;-------------------------------------------
002060 005046  MULT:   CLR -(SP)
002062 005702          TST R2
002064 001430          BEQ MULT6
002066 002002          BGE MULT1
002070 005402          NEG R2
```

```
002072 005216          INC @SP
002074 005700  MULT1:  TST R0
002076 001423          BEQ MULT6
002100 002002          BGE MULT2
002102 005400          NEG R0
002104 005316          DEC @SP
002106 012746  MULT2:  MOV #20,-(SP)
       000020
002112 005001          CLR R1
002114 006001  MULT3:  ROR R1
002116 006002          ROR R2
002120 103001          BCC MULT4
002122 060001          ADD R0,R1
002124 005316  MULT4:  DEC @SP
002126 001372          BNE MULT3
002130 005726          TST (SP)+
002132 005726          TST (SP)+
002134 001403          BEQ MULT5
002136 005401          NEG R1
002140 005402          NEG R2
002142 005601          SBC R1
002144 000207  MULT5:  RTS PC
002146 005001  MULT6:  CLR R1
002150 005002          CLR R2
002152 005726          TST (SP)+
002154 000207          RTS PC
                       ;-------------------------------------------
                       ;-------------------------------------------
                       ;-----------------SUBROUTINE MAGNTO---------
                       ;-------------------------------------------
                       ;-------------------------------------------
002156 016705  MAGNTO:MOV LOCATR,R5
       175630
002162 011500  MAG1:   MOV @R5,R0
002164 010002          MOV R0,R2
002166 004767          JSR PC,MULT
       177666
002172 010504          MOV R5,R4
002174 066704          ADD SIZE,R4
       175600
002200 011400          MOV @R4,R0
002202 010103          MOV R1,R3
002204 010204          MOV R2,R4
002206 010002          MOV R0,R2
002210 004767          JSR PC,MULT
       177644
002214 060402          ADD R4,R2
002216 005501          ADC R1
002220 060301          ADD R3,R1
002222 004767          JSR PC,SQROOT
       000024
002226 010115          MOV R1,@R5
002230 005725          TST (R5)+
002232 010504          MOV R5,R4
002234 166704          SUB LOCATR,R4
       175552
```

```
002240 026704          CMP SIZE,R4
       175534
002244 001401          BEQ MAG2
002246 000745          BR MAG1
002250 000207  MAG2:   RTS PC
                       ;-------------------------------------------
                       ;-------------------------------------------
                       ;-----------------**SUBROUTINE SQROOT-------
                       ;-------------------------------------------
                       ;-------------------------------------------
002252 005004  SQROOT:CLR R4
002254 005701          TST R1
002256 002435          BLT SQROT5
002260 003002          BGT SQROT1
002262 005702          TST R2
002264 001430          BEQ SQROT4
002266 005204  SQROT1:INC R4
002270 006302          ASL R2
002272 006101          ROL R1
002274 100374          BPL SQROT1
002276 000241          CLC
002300 006001          ROR R1
002302 006002          ROR R2
002304 005304          DEC R4
002306 010102          MOV R1,R2
002310 004767          JSR PC,SQRT
       000040
002314 006204          ASR R4
002316 103005          BCC SQROT2
002320 012700          MOV #55202,R0
       055202
002324 010102          MOV R1,R2
002326 004767          JSR PC,MULT
       177526
002332 005704  SQROT2:TST R4
002334 001403          BEQ SQROT3
002336 006201          ASR R1
002340 005304          DEC R4
002342 000773          BR SQROT2
002344 000207  SQROT3:RTS PC
002346 005001  SQROT4:CLR R1
002350 000207          RTS PC
002352 000000  SQROT5:HALT
                       ;-------------------------------------------
                       ;-------------------------------------------
                       ;-----------------**SUBROUTINE SQRT--------
                       ;-------------------------------------------
                       ;-------------------------------------------
002354 012746  SQRT:   MOV #4,-(SP)
       000004
002360 010203          MOV R2,R3
002362 010246          MOV R2,-(SP)
002364 062703          ADD #100000,R3
       100000
002370 010346          MOV R3,-(SP)
002372 006203  SQRT1:  ASR R3
002374 005403          NEG R3
002376 010300          MOV R3,R0
002400 004767          JSR PC,MULT
       177454
```

```
002404 066601          ADD 2(SP),R1
       000002
002410 005366          DEC 4(SP)
       000004
002414 001003          BNE SQRT2
002416 062706          ADD #6,SP
       000006
002422 000207          RTS PC
002424 010166  SQRT2:  MOV R1,2(SP)
       000002
002430 011600          MOV @SP,R0
002432 010002          MOV R0,R2
002434 004767          JSR PC,MULT
       177420
002440 011602          MOV @SP,R2
002442 010146          MOV R1,-(SP)
002444 006202          ASR R2
002446 006202          ASR R2
002450 062702          ADD #120000,R2
       120000
002454 012600          MOV (SP)+,R0
002456 004767          JSR PC,MULT
       177376
002462 010116          MOV R1,@SP
002464 010103          MOV R1,R3
002466 016602          MOV 2(SP),R2
       000002
002472 000737          BR SQRT1
                       ;/////////////////////////////////////////////////////
                       ;/////////////////////////////////////////////////////
                       ;////////////// SINE-COSINE TABLE ////////////////////
                       ;/////////////////////////////////////////////////////
                       ;/////////////////////////////////////////////////////
002474 077777  SCTABL: .WORD   77777,77777,77775,77772
002476 077777
002500 077775
002502 077772
002504 077766          .WORD   77766,77760,77751,77741
002506 077760
002510 077751
002512 077741
002514 077730          .WORD   77730,77716,77702,77665
002516 077716
002520 077702
002522 077665
002524 077647          .WORD   77647,77627,77607,77565
002526 077627
002530 077607
002532 077565
002534 077542          .WORD   77542,77515,77470,77441
002536 077515
002540 077470
002542 077441
002544 077411          .WORD   77411,77360,77325,77272
002546 077360
```

```
002550 077325
002552 077272
002554 077235          .WORD   77235,77177,77137,77077
002556 077177
002560 077137
002562 077077
002564 077035          .WORD   77035,76772,76726,76660
002566 076772
002570 076726
002572 076660
002574 076612          .WORD   76612,76542,76471,76417
002576 076542
002600 076471
002602 076417
002604 076343          .WORD   76343,76267,76211,76132
002606 076267
002610 076211
002612 076132
002614 076051          .WORD   76051,75770,75705,75622
002616 075770
002620 075705
002622 075622
002624 075535          .WORD   75535,75446,75357,75266
002626 075446
002630 075357
002632 075266
002634 075175          .WORD   75175,75102,75005,74710
002636 075102
002640 075005
002642 074710
002644 074612          .WORD   74612,74512,74411,74307
002646 074512
002650 074411
002652 074307
002654 074204          .WORD   74204,74100,73772,73664
002656 074100
002660 073772
002662 073664
002664 073554          .WORD   73554,73443,73331,73216
002666 073443
002670 073331
002672 073216
002674 073101          .WORD   73101,72764,72645,72525
002676 072764
002700 072645
002702 072525
002704 072404          .WORD   72404,72262,72137,72013
002706 072262
002710 072137
002712 072013
```

```
002714 071665          .WORD   71665,71537,71407,71257
002716 071537
002720 071407
002722 071257
002724 071125          .WORD   71125,70772,70636,70501
002726 070772
002730 070636
002732 070501
002734 070342          .WORD   70342,70203,70043,67701
002736 070203
002740 070043
002742 067701
002744 067537          .WORD   67537,67373,67226,67060
002746 067373
002750 067226
002752 067060
002754 066712          .WORD   66712,66542,66371,66217
002756 066542
002760 066371
002762 066217
002764 066044          .WORD   66044,65670,65512,65334
002766 065670
002770 065512
002772 065334
002774 065155          .WORD   65155,64775,64614,64431
002776 064775
003000 064614
003002 064431
003004 064246          .WORD   64246,64062,63675,63506
003006 064062
003010 063675
003012 063506
003014 063317          .WORD   63317,63127,62735,62543
003016 063127
003020 062735
003022 062543
003024 062350          .WORD   62350,62154,61757,61561
003026 062154
003030 061757
003032 061561
003034 061362          .WORD   61362,61161,60761,60557
003036 061161
003040 060761
003042 060557
003044 060354          .WORD   60354,60150,57743,57536
003046 060150
003050 057743
003052 057536
003054 057327          .WORD   57327,57120,56707,56476
003056 057120
```

```
003060 056707
003062 056476
003064 056264          .WORD   56264,56051,55635,55420
003066 056051
003070 055635
003072 055420
003074 055202          .WORD   55202,54763,54544,54324
003076 054763
003100 054544
003102 054324
003104 054102          .WORD   54102,53660,53435,53212
003106 053660
003110 053435
003112 053212
003114 052765          .WORD   52765,52540,52312,52063
003116 052540
003120 052312
003122 052063
003124 051633          .WORD   51633,51402,51151,50716
003126 051402
003130 051151
003132 050716
003134 050463          .WORD   50463,50227,47773,47536
003136 050227
003140 047773
003142 047536
003144 047277          .WORD   47277,47041,46601,46341
003146 047041
003150 046601
003152 046341
003154 046077          .WORD   46077,45636,45373,45130
003156 045636
003160 045373
003162 045130
003164 044664          .WORD   44664,44417,44151,43703
003166 044417
003170 044151
003172 043703
003174 043434          .WORD   43434,43165,42715,42444
003176 043165
003200 042715
003202 042444
003204 042172          .WORD   42172,41720,41445,41172
003206 041720
003210 041445
003212 041172
003214 040716          .WORD   40716,40441,40163,37705
003216 040441
003220 040163
003222 037705
```

```
003224 037427          .WORD   37427,37150,36670,36407
003226 037150
003230 036670
003232 036407
003234 036126          .WORD   36126,35645,35362,35100
003236 035645
003240 035362
003242 035100
003244 034614          .WORD   34614,34330,34044,33557
003246 034330
003250 034044
003252 033557
003254 033272          .WORD   33272,33004,32515,32226
003256 033004
003260 032515
003262 032226
003264 031736          .WORD   31736,31446,31156,30665
003266 031446
003270 031156
003272 030665
003274 030373          .WORD   30373,30101,27607,27314
003276 030101
003300 027607
003302 027314
003304 027021          .WORD   27021,26525,26230,25734
003306 026525
003310 026230
003312 025734
003314 025437          .WORD   25437,25141,24643,24345
003316 025141
003320 024643
003322 024345
003324 024046          .WORD   24046,23547,23250,22750
003326 023547
003330 023250
003332 022750
003334 022450          .WORD   22450,22147,21646,21345
003336 022147
003340 021646
003342 021345
003344 021043          .WORD   21043,20541,20237,17734
003346 020541
003350 020237
003352 017734
003354 017431          .WORD   17431,17126,16623,16317
003356 017126
003360 016623
003362 016317
003364 016013          .WORD   16013,15507,15202,14675
003366 015507
```

```
003370 015202
003372 014675
003374 014370          .WORD   14370,14063,13555,13250
003376 014063
003400 013555
003402 013250
003404 012742          .WORD   12742,12433,12125,11616
003406 012433
003410 012125
003412 011616
003414 011310          .WORD   11310,11001,10471,10162
003416 011001
003420 010471
003422 010162
003424 007653          .WORD   07653,07343,07033,06523
003426 007343
003430 007033
003432 006523
003434 006213          .WORD   06213,05703,05373,05063
003436 005703
003440 005373
003442 005063
003444 004552          .WORD   04552,04242,03731,03420
003446 004242
003450 003731
003452 003420
003454 003107          .WORD   03107,02577,02266,01755
003456 002577
003460 002266
003462 001755
003464 001444          .WORD   01444,01133,00622,00311
003466 001133
003470 000622
003472 000311
003474 000000          .WORD   00000
                ;
                ;
       000001          .END
```

```
;               FAST FOURIER TRANSFORM MODULE.
;               DOUBLE WORD INTEGER FORMAT.
;               NICOLET 1070 STORAGE FORMAT.
;               PROGRAMMED BY LEE WINDMILLER.
;               ARGONNE NATIONAL LABORATORY.
;               ARGONNE,ILLINOIS  60439.
;
;               REGISTER DEFINITIONS
```

ADDF	003716	ADDR1I	000556	ADDR1R	000554	ADDR2I	000562
ADDR2R	000560	ADDTBL	000552	BITINV	002620	BITIN0	002630
BITIN1	002642	BITIN2	002666	BITIN3	002724	BITIN4	002744
CHNGE1	000500	COMPLX	002746	COMPL1	003062	COMPL2	003134
COMP0	002756	COMP0A	002774	COMP2A	003324	COMP2B	003336
COMP2C	003412	COMP2D	003424	CONTRL	000006	COSF	002406
COSW	000526	COUNT	001600	CRLF	000366	DATIN	000440
DATIN1	000454	DATOT1	000430	DATOUT	000414	DATTBL	000600
DEST	000412	ERROR	000000	ERROR1	003742	EXEC	000002
EXEC0	000070	EXEC1	000140	EXEC10	000122	EXEC2	000226
EXEC3	000242	EXEC4	000334	FFT	000716	FFT1	000722
FFT2	001302	GETADR	002306	GETAD0	002324	GETAD1	002350
HIADD	001576	IMAG1	000606	IMAG2	000616	INDEX	000522
LOADD	001574	LOOKUP	002472	LOOK1	002520	LOOK2	002616
MAGNTD	004052	MAG1	004056	MAG2	004200	MULT	003746
MULT1	003764	MULT2	004000	MULT3	004018	MULT4	004024
MULT5	004050	PASS	000516	PC	=%000007	REAL1	000602
REAL2	000612	R0	=%000000	R1	=%000001	R2	=%000002
R3	=%000003	R4	=%000004	R5	=%000005	SCALER	000524
SCTABL	004542	SINF	002434	SINH	000532	SIZE	000514
SIZER	001524	SIZER1	001346	SIZER2	001354	SIZER3	001364
SIZER4	001402	SIZER6	001434	SIZER7	001444	SIZER8	001534
SIZER9	001572	SIZE7A	001476	SIZE7B	001506	SIZE7C	001452
SIZE8A	001544	SORT	001602	SORT0	001622	SORT1	001654
SORT1A	001676	SORT2	001712	SORT2A	001736	SORT2B	001734
SORT3	001744	SORT3A	001776	SORT3B	001774	SORT4	002010
SORT4A	002070	SORT4B	002044	SORT5	002126	SORT5A	002174
SORT6	002204	SORT6A	002202	SORT7	002226	SORT8	002304
SOURCE	000410	SP	=%000006	SQROOT	004204	SQROT1	004212
SQROT2	004274	SQROT3	004310	SQROT4	004314	SQROT5	004324
SQRT	004326	SQRT1	004352	SQRT2	004420	START	000546
START1	000564	START2	000622	START3	000700	STOR	000536
SUBF	003738	SWCNTR	000542	TYPE	000356	WEIGHT	000520
WGTINC	000544	XCHNGE	000464	.	= 005544		

```
        000000          R0=%0
        000001          R1=%1
        000002          R2=%2
        000003          R3=%3
        000004          R4=%4
        000005          R5=%5
        000006          SP=%6
        000007          PC=%7
                ;
        000000                  .=0
                ;               ERROR HALT
000000  000000  ERROR:  HALT
                ;-------------------------------------------------
                ;-------------------------------------------------
                ;-----------------SUBROUTINE EXEC-----------------
                ;-------------------------------------------------
                ;-------------------------------------------------
                ;               CALLING SEQUENCE,
                ;               JSR PC,EXEC
                ;               SEE TEXT FOR EXPLANATION OF
                ;               OPTIONS AVAILABLE.
                ;
000002  010701  EXEC:   MOV PC,R1
000004  000431         BR EXEC0
000006  000000 CONTRL:,WORD 0,324,315,317,311,323,330,0,0
000010  000324
000012  000315
000014  000317
000016  000311
000020  000323
000022  000330
000024  000000
000026  000000
000030  000546          .WORD START,MAGNTD,DATOUT,DATIN,SIZER,XCHNGE,0,0
000032  004052
000034  000414
000036  000440
000040  001324
000042  000464
000044  000000
000046  000000
000050  000000          .WORD 0,0,0,0,0,0,0,0
000052  000000
000054  000000
000056  000000
000060  000000
000062  000000
000064  000000
000066  000000
000070  005721  EXEC0:  TST (R1)+
000072  005721          TST (R1)+
000074  010167          MOV R1,CONTRL
        177706
000100  162700          SUB #40,R0
        000040
000104  012702          MOV #10,R2
        000010
000110  010104          MOV R1,R4
000112  062701          ADD #20,R1
        000020
000116  062704          ADD #40,R4
        000040
```

```
000122 011114  EXEC10:MOV @R1,@R4
000124 060014         ADD R0,@R4
000126 005302         DEC R2
000130 001403         BEQ EXEC1
000132 005721         TST (R1)+
000134 005724         TST (R4)+
000136 000771         BR EXEC10
000140 012700  EXEC1: MOV #177560,R0
       177560
000144 012701         MOV #177562,R1
       177562
000150 012702         MOV #177564,R2
       177564
000154 012703         MOV #177566,R3
       177566
000160 105712         TSTB @R2
000162 100376         BPL .-2
000164 004767         JSR PC,CRLF
       000176
000170 005210         INC @R0
000172 105710         TSTB @R0
000174 100376         BPL .-2
000176 111104         MOVB @R1,R4
000200 105712         TSTB @R2
000202 100376         BPL .-2
000204 110405         MOVB R4,R5
000206 004767         JSR PC,TYPE
       000144
000212 005704         TST R4
000214 001447         BEQ EXEC4
000216 012700         MOV #10,R0
       000010
000222 016701         MOV CONTRL,R1
       177560
000226 120411  EXEC2: CMPB R4,@R1
000230 001404         BEQ EXEC3
000232 005721         TST (R1)+
000234 005300         DEC R0
000236 001436         BEQ EXEC4
000240 000772         BR EXEC2
000242 012700  EXEC3: MOV #167776,R0
       167776
000246 012710         MOV #114000,@R0
       114000
000252 013700         MOV @#167774,R0
       167774
000256 042700         BIC #140000,R0
       140000
000262 005700         TST R0
000264 001372         BNE .-12
```

```
000266 016100          MOV 40(R1),R0
       000040
000272 010701          MOV PC,R1
000274 062701          ADD #16,R1
       000016
000300 160100          SUB R1,R0
000302 010067          MOV R0,2(PC)
       000002
000306 004767          .WORD 4767
000310 000000          .WORD 0
000312 005737          TST @#140000
       140000
000316 112737          MOVB #0,@#167774
       000000
       167774
000324 012737          MOV #124000,@#167776
       124000
       167776
000332 000702          BR EXEC1
000334 105712  EXEC4:  TSTB @R2
000336 100376          BPL .-2
000340 004767          JSR PC,CRLF
       000022
000344 012705          MOV #277,R5
       000277
000350 004767          JSR PC,TYPE
       000002
000354 000671          BR EXEC1
000356 010513  TYPE:   MOV R5,@R3
000360 105712          TSTB @R2
000362 100376          BPL .-2
000364 000207          RTS PC
000366 012705  CRLF:   MOV #215,R5
       000215
000372 004767          JSR PC,TYPE
       177760
000376 012705          MOV #212,R5
       000212
000402 004767          JSR PC,TYPE
       177750
000406 000207          RTS PC
000410 000000  SOURCE:.WORD 0
000412 000000  DEST:   .WORD 0
000414 016700  DATOUT:MOV SIZE,R0
       000074
000420 016701          MOV DEST,R1
       177766
000424 016702          MOV SOURCE,R2
       177760
000430 012221  DATOT1:MOV (R2)+,(R1)+
```

```
000432 005300          DEC R0
000434 001375          BNE DATOT1
000436 000207          RTS PC
000440 016700  DATIN: MOV SIZE,R0
       000050
000444 016701          MOV DEST,R1
       177742
000450 016702          MOV SOURCE,R2
       177734
000454 012122  DATIN1:MOV (R1)+,(R2)+
000456 005300          DEC R0
000460 001375          BNE DATIN1
000462 000207          RTS PC
000464 016700  XCHNGE:MOV SIZE,R0
       000024
000470 016701          MOV DEST,R1
       177716
000474 016702          MOV SOURCE,R2
       177710
000500 011103  CHNGE1:MOV @R1,R3
000502 011221          MOV @R2,(R1)+
000504 010322          MOV R3,(R2)+
000506 005300          DEC R0
000510 001373          BNE CHNGE1
000512 000207          RTS PC
                       ;-----------------------------------------------
                       ;-----------------------------------------------
                       ;-----------------SUBROUTINE START--------------
                       ;-----------------------------------------------
                       ;-----------------------------------------------
000514 000000  SIZE:           .WORD 0
000516 000000  PASS:           .WORD 0
000520 000000  HEIGHT:         .WORD 0
000522 000000  INDEX:          .WORD 0
000524 000000  SCALER:         .WORD 0
000526 000000  COSW:           .WORD 0,0
000530 000000
000532 000000  SINW:           .WORD 0,0
000534 000000
000536 000000  STOR:           .WORD 0,0
000540 000000
000542 000000  SWCNTR:         .WORD 0
000544 000000  WGTINC:         .WORD 0
000546 010700  START:          MOV PC,R0
000550 000405                  BR START1
000552 000000  ADDTBL:         .WORD 0
000554 000000  ADDR1R:         .WORD 0
000556 000000  ADDR1I:         .WORD 0
000560 000000  ADDR2R:         .WORD 0
000562 000000  ADDR2I:         .WORD 0
000564 005720  START1:         TST (R0)+
000566 005720                  TST (R0)+
000570 010067                  MOV R0,ADDTBL
       177756
000574 010700                  MOV PC,R0
```

```
000576 000411                  BR START2
000600 000000  DATTBL:         .WORD 0
000602 000000  REAL1:          .WORD 0,0
000604 000000
000606 000000  IMAG1:          .WORD 0,0
000610 000000
000612 000000  REAL2:          .WORD 0,0
000614 000000
000616 000000  IMAG2:          .WORD 0,0
000620 000000
000622 005720  START2:         TST (R0)+
000624 005720                  TST (R0)+
000626 010067                  MOV R0,DATTBL
       177746
000632 005067                  CLR SCALER
       177666
000636 005067                  CLR INDEX
       177660
000642 016700                  MOV SIZE,R0
       177646
000646 006200                  ASR R0
000650 010067                  MOV R0,PASS
       177642
000654 006200                  ASR R0
000656 010067                  MOV R0,SWCNTR
       177660
000662 005067                  CLR WEIGHT
       177632
000666 012767                  MOV #1,WGTINC
       000001
       177650
000674 016700                  MOV SIZE,R0
       177614
000700 022700  START3:         CMP #2000,R0
       002000
000704 001404                  BEQ FFT
000706 006300                  ASL R0
000710 006367                  ASL WGTINC
       177630
000714 000771                  BR START3
000716 004767  FFT:            JSR PC,SIZER
       000402
000722 004767  FFT1:           JSR PC,GETADR
       001360
000726 016705                  MOV ADDTBL,R5
       177620
000732 016702                  MOV DATTBL,R2
       177642
000736 013500                  MOV @(R5)+,R0
000740 013701                  MOV @#167774,R1
       167774
```

```
000744 010022            MOV R0,(R2)+
000746 010122            MOV R1,(R2)+
000750 013522            MOV @(R5)+,(R2)+
000752 013722            MOV @#167774,(R2)+
       167774
000756 013503            MOV @(R5)+,R3
000760 013704            MOV @#167774,R4
       167774
000764 010322            MOV R3,(R2)+
000766 010422            MOV R4,(R2)+
000770 013522            MOV @(R5)+,(R2)+
000772 013712            MOV @#167774,@R2
       167774
000776 004767            JSR PC,ADDF
       002714
001002 010137            MOV R1,@#167774
       167774
001006 010077            MOV R0,@ADDR1R
       177542
001012 004767            JSR PC,SUBF
       002712
001016 004767            JSR PC,SUBF
       002706
001022 010067            MOV R0,REAL1
       177554
001026 010167            MOV R1,REAL1+2
       177552
001032 016700            MOV IMAG1,R0
       177550
001036 016701            MOV IMAG1+2,R1
       177546
001042 016703            MOV IMAG2,R3
       177550
001046 016704            MOV IMAG2+2,R4
       177546
001052 004767            JSR PC,ADDF
       002640
001056 010137            MOV R1,@#167774
       167774
001062 010077            MOV R0,@ADDR1I
       177470
001066 004767            JSR PC,SUBF
       002636
001072 004767            JSR PC,SUBF
       002632
001076 010067            MOV R0,IMAG2
       177514
001102 010167            MOV R1,IMAG2+2
       177512
001106 004767            JSR PC,LOOKUP
       001360
```

```
001112 016702          MOV REAL1,R2
       177464
001116 016703          MOV REAL1+2,R3
       177462
001122 004767          JSR PC,COSF
       001260
001126 016702          MOV IMAG2,R2
       177464
001132 016703          MOV IMAG2+2,R3
       177462
001136 005402          NEG R2
001140 005403          NEG R3
001142 005602          SBC R2
001144 004767          JSR PC,SINF
       001264
001150 010077          MOV R0,@ADDR2R
       177404
001154 016702          MOV IMAG2,R2
       177436
001160 016703          MOV IMAG2+2,R3
       177434
001164 004767          JSR PC,COSF
       001216
001170 016702          MOV REAL1,R2
       177406
001174 016703          MOV REAL1+2,R3
       177404
001200 004767          JSR PC,SINF
       001230
001204 010077          MOV R0,@ADDR2I
       177352
001210 062767          ADD #2,INDEX
       000002
       177304
001216 066767          ADD WGTINC,WEIGHT
       177322
       177274
001224 005367          DEC SWCNTR
       177312
001230 001234          BNE FFT1
001232 006367          ASL WGTINC
       177306
001236 006267          ASR PASS
       177254
001242 022767          CMP #1,PASS
       000001
       177246
001250 001414          BEQ FFT2
001252 005067          CLR WEIGHT
       177242
```

```
001256 005067                   CLR INDEX
       177240
001262 016700                   MOV SIZE,R0
       177226
001266 006200                   ASR R0
001270 006200                   ASR R0
001272 010067                   MOV R0,SWCNTR
       177244
001276 000167                   JMP FFT
       177414
001302 004767   FFT2:           JSR PC,SIZER
       000016
001306 004767                   JSR PC,BITINV
       001306
001312 004767                   JSR PC,COMPLX
       001430
001316 004767                   JSR PC,SORT
       000260
001322 000207                   RTS PC
                ;-----------------------------------------------
                ;-----------------------------------------------
                ;-----------------SUBROUTINE SIZER--------------
                ;-----------------------------------------------
                ;-----------------------------------------------
001324 012705       SIZER:    MOV #140000,R5
       140000
001330 016703                   MOV SIZE,R3
       177160
001334 005303                   DEC R3
001336 010504                   MOV R5,R4
001340 012500                   MOV (R5)+,R0
001342 100401                   BMI SIZER1
001344 005400                   NEG R0
001346 011501       SIZER1:   MOV @R5,R1
001350 100401                   BMI SIZER2
001352 005401                   NEG R1
001354 020001       SIZER2:   CMP R0,R1
001356 002402                   BLT SIZER3
001360 010504                   MOV R5,R4
001362 010100                   MOV R1,R0
001364 005725       SIZER3:   TST (R5)+
001366 005303                   DEC R3
001370 001366                   BNE SIZER1
001372 011400                   MOV @R4,R0
001374 001476                   BEQ SIZER9
001376 100001                   BPL SIZER4
001400 005400                   NEG R0
001402 000241       SIZER4:   CLC
001404 013705                   MOV @#167774,R5
       167774
001410 012704                   MOV #2,R4
       000002
001414 022700                   CMP #100000,R0
       100000
001420 001411                   BEQ SIZER7
```

```
001422 006305                   ASL R5
001424 006100                   ROL R0
001426 006305                   ASL R5
001430 006100                   ROL R0
001432 103404                   BCS SIZER7
001434 006300       SIZER6:   ASL R0
001436 005304                   DEC R4
001440 103401                   BCS SIZER7
001442 000774                   BR SIZER6
001444 005700   SIZER7:         TST R0
001446 001001                   BNE SIZE7C
001450 005304                   DEC R4
001452 005704   SIZE7C:         TST R4
001454 001446                   BEQ SIZER9
001456 012705                     MOV #140000,R5
       140000
001462 016701                   MOV SIZE,R1
       177026
001466 060467                   ADD R4,SCALER
       177032
001472 005704                   TST R4
001474 002417                   BLT SIZER8
001476 010400       SIZE7A:   MOV R4,R0
001500 011503                   MOV @R5,R3
001502 013702                   MOV @#167774,R2
       167774
001506 006203   SIZE7B:         ASR R3
001510 006002                   ROR R2
001512 005300                   DEC R0
001514 003374                   BGT SIZE7B
001516 010237                   MOV R2,@#167774
       167774
001522 010315                   MOV R3,@R5
001524 005725                   TST (R5)+
001526 005301                   DEC R1
001530 001420                   BEQ SIZER9
001532 000761                   BR SIZE7A
001534 010400       SIZER8:   MOV R4,R0
001536 011503                   MOV @R5,R3
001540 013702                   MOV @#167774,R2
       167774
001544 006302   SIZE8A:         ASL R2
001546 006103                   ROL R3
001550 005200                   INC R0
001552 002774                   BLT SIZE8A
001554 010237                   MOV R2,@#167774
       167774
001560 010315                   MOV R3,@R5
001562 005725                   TST(R5)+
001564 005301                   DEC R1
```

```
001566 001401                   BEQ SIZER9
001570 000761                   BR SIZER8
001572 000207   SIZER9:         RTS PC
                ;---------------------------------------
                ;---------------------------------------
                ;-----------------SUBROUTINE SORT-------
                ;---------------------------------------
                ;---------------------------------------
001574 000000   LOADD:          .WORD 0
001576 000000   HIADD:          .WORD 0
001600 000000   COUNT:          .WORD 0
001602 016767   SORT:           MOV SIZE,COUNT
       176706
       177770
001610 162767                   SUB #2,COUNT
       000002
       177762
001616 010700                   MOV PC,R0
001620 000415                   BR SORT1
001622 000000   SORT0:          .WORD 0,0,0,0,0,0,0,0,0,0,0,0,0,0
001624 000000
001626 000000
001630 000000
001632 000000
001634 000000
001636 000000
001640 000000
001642 000000
001644 000000
001646 000000
001650 000000
001652 000000
001654 005720   SORT1:          TST (R0)+
001656 005720                   TST (R0)+
001660 010067                   MOV R0,SORT0
       177736
001664 016701                   MOV SIZE,R1
       176624
001670 005301                   DEC R1
001672 012702                   MOV #1,R2
       000001
001676 005067   SORT1A:         CLR HIADD
       177674
001702 010267                   MOV R2,LOADD
       177666
001706 005005                   CLR R5
001710 010203                   MOV R2,R3
001712 006303   SORT2:          ASL R3
001714 020103                   CMP R1,R3
001716 003007                   BGT SORT2A
001720 006203                   ASR R3
001722 020367                   CMP R3,HIADD
       177650
001726 002402                   BLT SORT2B
001730 010367                   MOV R3,HIADD
       177642
```

```
001734 000403  SORT2B:        BR SORT3
001736 010320  SORT2A:        MOV R3,(R0)+
001740 005205                 INC R5
001742 000763                 BR SORT2
001744 160103  SORT3:         SUB R1,R3
001746 006303                 ASL R3
001750 060103                 ADD R1,R3
001752 005703                 TST R3
001754 003010                 BGT SORT3A
001756 014003                 MOV -(R0),R3
001760 005720                 TST (R0)+
001762 020367                 CMP R3,LOADD
       177606
001766 003002                 BGT SORT3B
001770 010367                 MOV R3,LOADD
       177600
001774 000746  SORT3B:        BR SORT2
001776 010320  SORT3A:        MOV R3,(R0)+
002000 005205                 INC R5
002002 020302                 CMP R3,R2
002004 001401                 BEQ SORT4
002006 000756                 BR SORT3
002010 020267  SORT4:         CMP R2,LOADD
       177560
002014 003003                 BGT SORT6
002016 160102                 SUB R1,R2
002020 005402                 NEG R2
002022 020267                 CMP R2,HIADD
       177550
002026 002462                 BLT SORT5A
002030 010567                 MOV R5,LOADD
       177540
002034 160567                 SUB R5,COUNT
       177540
002040 160102                 SUB R1,R2
002042 005402                 NEG R2
002044 010203  SORT4B:        MOV R2,R3
002046 006303                 ASL R3
002050 016367                 MOV 140000(R3),REAL1
       140000
       176524
002056 013767                 MOV @#167774,REAL1+2
       167774
       176526
002064 016700                 MOV SORT0,R0
       177532
002070 011004  SORT4A:        MOV @R0,R4
002072 160110                 SUB R1,@R0
002074 005420                 NEG (R0)+
002076 020204                 CMP R2,R4
```

```
002100 001412                   BEQ SORT5
002102 006304                   ASL R4
002104 016405                   MOV 140000(R4),R5
       140000
002110 013767                   MOV @#167774,@#167774
       167774
       167774
002116 010563                   MOV R5,140000(R3)
       140000
002122 010403                   MOV R4,R3
002124 000761                   BR SORT4A
002126 016737   SORT5:          MOV REAL1+2,@#167774
       176452
       167774
002134 016763                   MOV REAL1,140000(R3)
       176442
       140000
002142 160102                   SUB R1,R2
002144 005402                   NEG R2
002146 006302                   ASL R2
002150 020201                   CMP R2,R1
002152 002413                   BLT SORT6A
002154 006202                   ASR R2
002156 020267                   CMP R2,HIADD
       177414
002162 001404                   BEQ SORT5A
002164 166767                   SUB LOADD,COUNT
       177404
       177406
002172 000724                   BR SORT4B
002174 160102   SORT5A:         SUB R1,R2
002176 005402                   NEG R2
002200 000401                   BR SORT6
002202 006202   SORT6A:         ASR R2
002204 022767   SORT6:          CMP #3,COUNT
       000003
       177366
002212 003005                   BGT SORT7
002214 062702                   ADD #2,R2
       000002
002220 016700                   MOV SORT0,R0
       177376
002224 000624                   BR SORT1A
002226 005767   SORT7:          TST COUNT
       177346
002232 001424                   BEQ SORT8
002234 042701                   BIC #125252,R1
       125252
002240 006301                   ASL R1
002242 016104                   MOV 140000(R1),R4
       140000
```

```
002246 013705                  MOV @#167774,R5
       167774
002252 010102                  MOV R1,R2
002254 006302                  ASL R2
002256 016203                  MOV 140000(R2),R3
       140000
002262 013737                  MOV @#167774,@#167774
       167774
       167774
002270 010361                  MOV R3,140000(R1)
       140000
002274 010567                  MOV R5,@#167774
       167774
002300 010462                  MOV R4,140000(R2)
       140000
002304 000207   SORT8:         RTS PC
                ;-----------------------------------------------
                ;-----------------------------------------------
                ;----------------SUBROUTINE GETADR--------------
                ;-----------------------------------------------
                ;-----------------------------------------------
002306 016700   GETADR:        MOV INDEX,R0
       176210
002312 016701                  MOV PASS,R1
       176200
002316 005700                  TST R0
002320 001001                  BNE GETAD0
002322 000412                  BR GETAD1
002324 010002   GETAD0:        MOV R0,R2
002326 010103                  MOV R1,R3
002330 005403                  NEG R3
002332 040302                  BIC R3,R2
002334 001005                  BNE GETAD1
002336 060100                  ADD R1,R0
002340 010067                  MOV R0,INDEX
       176156
002344 005067                  CLR WEIGHT
       176150
002350 006300   GETAD1:        ASL R0
002352 062700                  ADD #140000,R0
       140000
002356 016702                  MOV ADDTBL,R2
       176170
002362 010022                  MOV R0,(R2)+
002364 005720                  TST (R0)+
002366 010022                  MOV R0,(R2)+
002370 006301                  ASL R1
002372 060100                  ADD R1,R0
002374 005740                  TST -(R0)
002376 010022                  MOV R0,(R2)+
002400 005720                  TST (R0)+
002402 010012                  MOV R0,@R2
002404 000207                  RTS PC
                ;-----------------------------------------------
                ;-----------------------------------------------
                ;----------SUBROUTINES COSF AND SINF------------
                ;-----------------------------------------------
                ;-----------------------------------------------
002406 016704   COSF:          MOV COSW,R4
       176114
```

```
002412 016705                   MOV COSW+2,R5
       176112
002416 004767                   JSR PC,MULT
       001324
002422 010067                   MOV R0,STOR
       176110
002426 010167                   MOV R1,STOR+2
       176106
002432 000207                   RTS PC
002434 016704   SINF:           MOV SINW,R4
       176072
002440 016705                   MOV SINW+2,R5
       176070
002444 004767                   JSR PC,MULT
       001276
002450 016703                   MOV STOR,R3
       176062
002454 016704                   MOV STOR+2,R4
       176060
002460 004767                   JSR PC,ADDF
       001232
002464 010137                   MOV R1,@#167774
       167774
002470 000207                   RTS PC
                ;---------------------------------------------
                ;---------------------------------------------
                ;----------------SUBROUTINE LOOKUP------------
                ;---------------------------------------------
                ;---------------------------------------------
002472 016700   LOOKUP:         MOV WEIGHT,R0
       176022
002476 005002                   CLR R2
002500 010001                   MOV R0,R1
002502 042701                   BIC #177577,R1
       177577
002506 001404                   BEQ LOOK1
002510 162700                   SUB #400,R0
       000400
002514 005400                   NEG R0
002516 005202                   INC R2
002520 006300   LOOK1:         ASL R0
002522 006300                   ASL R0
002524 016067                   MOV 7000(R0),COSW
       007000
       175774
002532 005067                   CLR COSW+2
       175772
002536 006067                   ROR COSW
       175764
002542 006067                   ROR COSW+2
       175762
002546 162700                   SUB #1000,R0
       001000
002552 005400                   NEG R0
002554 016067                   MOV 7000(R0),SINW
       007000
       175750
```

```
002562 005067                  CLR SINW+2
       175746
002566 006067                  ROR SINW
       175740
002572 006067                  ROR SINW+2
       175736
002576 005702                  TST R2
002600 001406                  BEQ LOOK2
002602 005467                  NEG COSW
       175720
002606 005467                  NEG COSW+2
       175716
002612 005667                  SBC COSW
       175710
002616 000207  LOOK2:          RTS PC
               ;---------------------------------------
               ;---------------------------------------
               ;-----------------SUBROUTINE BITINV-----
               ;---------------------------------------
               ;---------------------------------------
002620 012700  BITINV:         MOV #4,R0
       000004
002624 012705                  MOV #167774,R5
       167774
002630 010001  BITIN0:         MOV R0,R1
002632 010104                  MOV R1,R4
002634 016703                  MOV SIZE,R3
       175654
002640 005002                  CLR R2
002642 006201  BITIN1:         ASR R1
002644 006102                  ROL R2
002646 006203                  ASR R3
002650 103374                  BCC BITIN1
002652 006302                  ASL R2
002654 006302                  ASL R2
002656 020204                  CMP R2,R4
002660 003421                  BLE BITIN3
002662 012701                  MOV #2,R1
       000002
002666 016446  BITIN2:         MOV 140000(R4),-(SP)
       140000
002672 011546                  MOV @R5,-(SP)
002674 016203                  MOV 140000(R2),R3
       140000
002700 011515                  MOV @R5,@R5
002702 010364                  MOV R3,140000(R4)
       140000
002706 012615                  MOV (SP)+,@R5
002710 012662                  MOV (SP)+,140000(R2)
       140000
002714 005722                  TST (R2)+
002716 005724                  TST(R4)+
002720 005301                  DEC R1
002722 001361                  BNE BITIN2
002724 062700  BITIN3:         ADD #4,R0
       000004
```

```
002730 006200                   ASR R0
002732 020067                   CMP R0,SIZE
       175556
002736 001402                   BEQ BITIN4
002740 006300                   ASL R0
002742 000732                   BR BITIN0
002744 000207   BITIN4:         RTS PC
                ;-----------------------------------------------
                ;-----------------------------------------------
                ;-------------------SUBROUTINE COMPLX-----------
                ;-----------------------------------------------
                ;-----------------------------------------------
002746 005067   COMPLX:         CLR WGTINC
       175572
002752 016700                   MOV SIZE,R0
       175536
002756 022700   COMP0:          CMP #2000,R0
       002000
002762 001404                   BEQ COMP0A
002764 006300                   ASL R0
002766 005267                   INC WGTINC
       175552
002772 000771                   BR COMP0
002774 005005   COMP0A:         CLR R5
002776 016503                   MOV 140000(R5),R3
       140000
003002 013704                   MOV @#167774,R4
       167774
003006 005725                   TST (R5)+
003010 016500                   MOV 140000(R5),R0
       140000
003014 013701                   MOV @#167774,R1
       167774
003020 004767                   JSR PC,ADDF
       000672
003024 005745                   TST -(R5)
003026 010137                   MOV R1,@#167774
       167774
003032 010065                   MOV R0,140000(R5)
       140000
003036 005000                   CLR R0
003040 005725                   TST (R5)+
003042 010037                   MOV R0,@#167774
       167774
003046 010065                   MOV R0,140000(R5)
       140000
003052 012700                   MOV #4,R0
       000004
003056 010067                   MOV R0,INDEX
       175440
003062 010001   COMPL1:         MOV R0,R1
003064 062700                   ADD #140000,R0
       140000
003070 010067                   MOV R0,ADDR1A
       175460
```

```
003074 005720                   TST (R0)+
003076 010067                   MOV R0,ADDR11
       175454
003102 016700                   MOV SIZE,R0
       175406
003106 006300                   ASL R0
003110 160100                   SUB R1,R0
003112 062700                   ADD #140000,R0
       140000
003116 010067                   MOV R0,ADDR2R
       175436
003122 026767                   CMP ADDR1R,ADDR2R
       175426
       175430
003130 001001                   BNE COMPL2
003132 000207                   RTS PC
003134 005720   COMPL2:         TST (R0)+
003136 010067                   MOV R0,ADDR21
       175420
003142 017703                   MOV @ADDR2R,R3
       175412
003146 013704                   MOV @#167774,R4
       167774
003152 006203                   ASR R3
003154 006004                   ROR R4
003156 017700                   MOV @ADDR1R,R0
       175372
003162 013701                   MOV @#167774,R1
       167774
003166 006200                   ASR R0
003170 006001                   ROR R1
003172 004767                   JSR PC,ADDF
       000520
003176 010067                   MOV R0,REAL1
       175400
003202 010167                   MOV R1,REAL1+2
       175376
003206 004767                   JSR PC,SUBF
       000516
003212 004767                   JSR PC,SUBF
       000512
003216 010067                   MOV R0,REAL2
       175378
003222 010167                   MOV R1,REAL2+2
       175366
003226 017703                   MOV @ADDR21,R3
       175338
003232 013704                   MOV @#167774,R4
       167774
003236 006203                   ASR R3
```

```
003240 006004                   ROR R4
003242 017700                   MOV @ADDR11,R0
       175310
003246 013701                   MOV @#167774,R1
       167774
003252 006200                   ASR R0
003254 006001                   ROR R1
003256 004767                   JSR PC,ADDF
       000434
003262 010067                   MOV R0,IMAG1
       175320
003266 010167                   MOV R1,IMAG1+2
       175316
003272 004767                   JSR PC,SUBF
       000432
003276 004767                   JSR PC,SUBF
       000426
003302 010067                   MOV R0,IMAG2
       175310
003306 010167                   MOV R1,IMAG2+2
       175306
003312 016700                   MOV INDEX,R0
       175204
003316 006200                   ASR R0
003320 016701                   MOV MGTINC,R1
       175220
003324 005701   COMP2A:         TST R1
003326 003403                   BLE COMP2B
003330 006300                   ASL R0
003332 005301                   DEC R1
003334 000773                   BR COMP2A
003336 016004   COMP2B:         MOV SCTABL(R0),R4
       004542
003342 005005                   CLR R5
003344 006004                   ROR R4
003346 006005                   ROR R5
003350 010467                   MOV R4,COSW
       175152
003354 010567                   MOV R5,COSW+2
       175150
003360 016702                   MOV IMAG1,R2
       175222
003364 016703                   MOV IMAG1+2,R3
       175220
003370 004767                   JSR PC,MULT
       000352
003374 010046                   MOV R0,-(SP)
003376 010146                   MOV R1,-(SP)
003400 016700                   MOV INDEX,R0
       175116
```

```
003404 006200                   ASN R0
003406 016701                   MOV WGTINC,R1
       175132
003412 005701  COMP2C:          TST R1
003414 003403                   BLE COMP2D
003416 006300                   ASL R0
003420 005301                   DEC R1
003422 000773                   BR COMP2C
003424 162700  COMP2D:          SUB #1000,R0
       001000
003430 005400                   NEG R0
003432 016004                   MOV SCTABL(R0),R4
       004542
003436 005005                   CLR R5
003440 006004                   ROR R4
003442 006005                   ROR R5
003444 010467                   MOV R4,SINH
       175062
003450 010567                   MOV R5,SINH+2
       175060
003454 016702                   MOV REAL2,R2
       175132
003460 016703                   MOV REAL2+2,R3
       175130
003464 004767                   JSR PC,MULT
       000256
003470 012604                   MOV (SP)+,R4
003472 012603                   MOV (SP)+,R3
003474 004767                   JSR PC,ADDF
       000216
003500 016703                   MOV REAL1,R3
       175076
003504 016704                   MOV REAL1+2,R4
       175074
003510 004767                   JSR PC,ADDF
       000202
003514 010137                   MOV R1,@#167774
       167774
003520 010077                   MOV R0,@ADDR1R
       175030
003524 004767                   JSR PC,SUBF
       000200
003530 004767                   JSR PC,SUBF
       000174
003534 005400                   NEG R0
003536 005401                   NEG R1
003540 005600                   SBC R0
003542 010137                   MOV R1,@#167774
       167774
003546 010077                   MOV R0,@ADDR2R
       175006
```

```
003552 016704           MOV COSW,R4
       174750
003556 016705           MOV COSW+2,R5
       174746
003562 016702           MOV REAL2,R2
       175024
003566 016703           MOV REAL2+2,R3
       175022
003572 004767           JSR PC,MULT
       000150
003576 010046           MOV R0,-(SP)
003600 010146           MOV R1,-(SP)
003602 016704           MOV SINW,R4
       174724
003606 016705           MOV SINW+2,R5
       174722
003612 016702           MOV IMAG1,R2
       174770
003616 016703           MOV IMAG1+2,R3
       174766
003622 004767           JSR PC,MULT
       000120
003626 012604           MOV (SP)+,R4
003630 012603           MOV (SP)+,R3
003632 004767           JSR PC,SUBF
       000072
003636 016703           MOV IMAG2,R3
       174754
003642 016704           MOV IMAG2+2,R4
       174752
003646 004767           JSR PC,ADDF
       000044
003652 010137           MOV R1,@#167774
       167774
003656 010077           MOV R0,@ADDR1I
       174674
003662 004767           JSR PC,SUBF
       000042
003666 004767           JSR PC,SUBF
       000036
003672 010137           MOV R1,@#167774
       167774
003676 010077           MOV R0,@ADDR2I
       174660
003702 005267           INC INDEX
       174614
003706 016700           MOV INDEX,R0
       174610
003712 000167           JMP COMPL1
       177144
```

```
                        ;-----------------------------------------------
                        ;-----------------------------------------------
                        ;-------------SUBROUTINES ADDF AND SUBF----------
                        ;-----------------------------------------------
                        ;-----------------------------------------------
003716 060401   ADDF:           ADD R4,R1
003720 005500                   ADC R0
003722 060300                   ADD R3,R0
003724 102406                   BVS ERROR1
003726 000207                   RTS PC
003730 160401   SUBF:           SUB R4,R1
003732 005600                   SBC R0
003734 160300                   SUB R3,R0
003736 102401                   BVS ERROR1
003740 000207                   RTS PC
003742 000167   ERROR1:         JMP ERROR
       174032
                        ;-----------------------------------------------
                        ;-----------------------------------------------
                        ;-----------------SUBROUTINE MULT---------------
                        ;-----------------------------------------------
                        ;-----------------------------------------------
003746 005046   MULT:           CLR -(SP)
003750 005702                   TST R2
003752 002004                   BGE MULT1
003754 005402                   NEG R2
003756 005403                   NEG R3
003760 005602                   SBC R2
003762 005216                   INC @SP
003764 005704   MULT1:          TST R4
003766 002004                   BGE MULT2
003770 005404                   NEG R4
003772 005405                   NEG R5
003774 005604                   SBC R4
003776 005316                   DEC @SP
004000 005000   MULT2:          CLR R0
004002 005001                   CLR R1
004004 012746                   MOV #22,-(SP)
       000022
004010 006103   MULT3:          ROL R3
004012 006102                   ROL R2
004014 103003                   BCC MULT4
004016 060501                   ADD R5,R1
004020 005500                   ADC R0
004022 060400                   ADD R4,R0
004024 006004   MULT4:          ROR R4
004026 006005                   ROR R5
004030 005316                   DEC @SP
004032 001366                   BNE MULT3
004034 005726                   TST (SP)+
004036 005726                   TST (SP)+
004040 001403                   BEQ MULT5
004042 005400                   NEG R0
004044 005401                   NEG R1
004046 005600                   SBC R0
004050 000207   MULT5:          RTS PC
                        ;-----------------------------------------------
                        ;-----------------------------------------------
                        ;-----------------SUBROUTINE MAGNTD-------------
                        ;-----------------------------------------------
                        ;-----------------------------------------------
004052 005046   MAGNTD:         CLR -(SP)
004054 005005                   CLR R5
004056 016502   MAG1:           MOV 140000(R5),R2
       140000
```

```
004062 013703                   MOV @#167774,R3
       167774
004066 010204                   MOV R2,R4
004070 010305                   MOV R3,R5
004072 004767                   JSR PC,MULT
       177650
004076 011605                   MOV @SP,R5
004100 010046                   MOV R0,-(SP)
004102 010146                   MOV R1,-(SP)
004104 066705                   ADD SIZE,R5
       174404
004110 016502                   MOV 140000(R5),R2
       140000
004114 013703                   MOV @#167774,R3
       167774
004120 010204                   MOV R2,R4
004122 010305                   MOV R3,R5
004124 004767                   JSR PC,MULT
       177616
004130 012604                   MOV (SP)+,R4
004132 012603                   MOV (SP)+,R3
004134 004767                   JSR PC,ADDF
       177556
004140 004767                   JSR PC,SQROOT
       000040
004144 011605                   MOV @SP,R5
004146 066705                   ADD SIZE,R5
       174342
004152 010137                   MOV R1,@#167774
       167774
004156 010065                   MOV R0,140000(R5)
       140000
004162 011605                   MOV @SP,R5
004164 005725                   TST (R5)+
004166 026705                   CMP SIZE,R5
       174322
004172 001402                   BEQ MAG2
004174 010516                   MOV R5,@SP
004176 000727                   BR MAG1
004200 005726   MAG2:           TST (SP)+
004202 000207                   RTS PC
                ;---------------------------------------------
                ;---------------------------------------------
                ;-----------------SUBROUTINE SQROOT-----------
                ;---------------------------------------------
                ;---------------------------------------------
004204 005700   SQROOT:         TST R0
004206 003442                   BLE SQROT4
004210 005046                   CLR -(SP)
004212 005216   SQROT1:         INC @SP
004214 006301                   ASL R1
004216 006100                   ROL R0
004220 100374                   BPL SQROT1
004222 000241                   CLC
004224 006000                   ROR R0
```

```
004226 006001                   ROR R1
004230 005316                   DEC @SP
004232 010002                   MOV R0,R2
004234 010103                   MOV R1,R3
004236 004767                   JSR PC,SQRT
       000864
004242 011602                   MOV @SP,R2
004244 006202                   ASR R2
004246 103012                   BCC SQROT2
004250 010002                   MOV R0,R2
004252 010103                   MOV R1,R3
004254 012704                   MOV #055202,R4
       055202
004260 012705                   MOV #074631,R5
       074631
004264 004767                   JSR PC,MULT
       177456
004270 011602                   MOV @SP,R2
004272 006202                   ASR R2
004274 005702   SQROT2:         TST R2
004276 001404                   BEQ SQROT3
004300 006200                   ASR R0
004302 006001                   ROR R1
004304 005302                   DEC R2
004306 000772                   BR SQROT2
004310 005726   SQROT3:         TST (SP)+
004312 000207                   RTS PC
004314 002403   SQROT4:         BLT SQROT5
004316 005000                   CLR R0
004320 005001                   CLR R1
004322 000207                   RTS PC
004324 000000   SQROT5:         HALT
                ;---------------------------------------------
                ;---------------------------------------------
                ;-----------------SUBROUTINE SQRT-------------
                ;---------------------------------------------
                ;---------------------------------------------
004326 012746   SQRT:           MOV #4,-(SP)
       000004
004332 010204                   MOV R2,R4
004334 010305                   MOV R3,R5
004336 010246                   MOV R2,-(SP)
004340 010346                   MOV R3,-(SP)
004342 062704                   ADD #100000,R4
       100000
004346 010446                   MOV R4,-(SP)
004350 010546                   MOV R5,-(SP)
004352 006204   SQRT1:          ASR R4
004354 006005                   ROR R5
004356 005404                   NEG R4
004360 005405                   NEG R5
004362 005604                   SBC R4
004364 004767                   JSR PC,MULT
       177356
004370 016603                   MOV 6(SP),R3
       000006
```

```
004374 016604                   MOV 4(SP),R4
       000004
004400 004767                   JSR PC,ADDF
       177312
004404 005366                   DEC 10(SP)
       000010
004410 001003                   BNE SQRT2
004412 062706                   ADD #12,SP
       000012
004416 000207                   RTS PC
004420 010066  SQRT2:           MOV R0,6(SP)
       000006
004424 010166                   MOV R1,4(SP)
       000004
004430 011603                   MOV @SP,R3
004432 016602                   MOV 2(SP),R2
       000002
004436 010204                   MOV R2,R4
004440 010305                   MOV R3,R5
004442 004767                   JSR PC,MULT
       177300
004446 016603                   MOV 2(SP),R3
       000002
004452 011604                   MOV @SP,R4
004454 010046                   MOV R0,-(SP)
004456 010146                   MOV R1,-(SP)
004460 006203                   ASR R3
004462 006004                   ROR R4
004464 006203                   ASR R3
004466 006004                   ROR R4
004470 012700                   MOV #120000,R0
       120000
004474 005001                   CLR R1
004476 004767                   JSR PC,ADDF
       177214
004502 010002                   MOV R0,R2
004504 010103                   MOV R1,R3
004506 012605                   MOV (SP)+,R5
004510 012604                   MOV (SP)+,R4
004512 004767                   JSR PC,MULT
       177230
004516 010066                   MOV R0,2(SP)
       000002
004522 010116                   MOV R1,@SP
004524 010004                   MOV R0,R4
004526 010105                   MOV R1,R5
004530 016602                   MOV 6(SP),R2
       000006
004534 016603                   MOV 4(SP),R3
       000004
```

```
004540 000704                   BR SQRT1
                        ;///////////////////////////////////////////////
                        ;///////////////////////////////////////////////
                        ;///////////////// SINE-COSINE TABLE ////////////
                        ;///////////////////////////////////////////////
                        ;///////////////////////////////////////////////
004542 177777   SCTABL:.WORD 177777,177776,177773,177764
004544 177776
004546 177773
004550 177764
004552 177754           .WORD 177754,177741,177723,177703
004554 177741
004556 177723
004560 177703
004562 177661           .WORD 177661,177634,177604,177552
004564 177634
004566 177604
004570 177552
004572 177516           .WORD 177516,177457,177416,177352
004574 177457
004576 177416
004600 177352
004602 177304           .WORD 177304,177233,177160,177103
004604 177233
004606 177160
004610 177103
004612 177023           .WORD 177023,176740,176653,176564
004614 176740
004616 176653
004620 176564
004622 176472           .WORD 176472,176376,176277,176176
004624 176376
004626 176277
004630 176176
004632 176073           .WORD 176073,175765,175654,175541
004634 175765
004636 175654
004640 175541
004642 175424           .WORD 175424,175305,175163,175036
004644 175305
004646 175163
004650 175036
004652 174707           .WORD 174707,174556,174422,174264
004654 174556
004656 174422
004660 174264
004662 174123           .WORD 174123,173761,173613,173444
004664 173761
004666 173613
004670 173444
004672 173272           .WORD 173272,173115,172736,172555
004674 173115
004676 172736
004700 172555
004702 172372           .WORD 172372,172204,172013,171621
```

```
004704 172204
004706 172013
004710 171621
004712 171424          .WORD 171424,171224,171023,170617
004714 171224
004716 171023
004720 170617
004722 170411          .WORD 170411,170200,167765,167550
004724 170200
004726 167765
004730 167550
004732 167330          .WORD 167330,167106,166662,166434
004734 167106
004736 166662
004740 166434
004742 166203          .WORD 166203,165750,165513,165253
004744 165750
004746 165513
004750 165253
004752 165011          .WORD 165011,164545,164277,164026
004754 164545
004756 164277
004760 164026
004762 163553          .WORD 163553,163276,163017,162536
004764 163276
004766 163017
004770 162536
004772 162252          .WORD 162252,161764,161474,161202
004774 161764
004776 161474
005000 161202
005002 160705          .WORD 160705,160406,160106,157603
005004 160406
005006 160106
005010 157603
005012 157276          .WORD 157276,156766,156455,156141
005014 156766
005016 156455
005020 156141
005022 155624          .WORD 155624,155304,154762,154436
005024 155304
005026 154762
005030 154436
005032 154110          .WORD 154110,153560,153225,152671
005034 153560
005036 153225
005040 152671
005042 152333          .WORD 152333,151772,151430,151063
005044 151772
005046 151430
```

```
005050 151063
005052 150515          .WORD 150515,150144,147572,147215
005054 150144
005056 147572
005060 147215
005062 146637          .WORD 146637,146256,145673,145307
005064 146256
005066 145673
005070 145307
005072 144721          .WORD 144721,144330,143736,143342
005074 144330
005076 143736
005100 143342
005102 142744          .WORD 142744,142343,141742,141336
005104 142343
005106 141742
005110 141336
005112 140730          .WORD 140730,140320,137707,137274
005114 140320
005116 137707
005120 137274
005122 136656          .WORD 136656,136240,135617,135174
005124 136240
005126 135617
005130 135174
005132 134550          .WORD 134550,134122,133472,133040
005134 134122
005136 133472
005140 133040
005142 132404          .WORD 132404,131747,131310,130650
005144 131747
005146 131310
005150 130650
005152 130205          .WORD 130205,127541,127073,126424
005154 127541
005156 127073
005160 126424
005162 125753          .WORD 125753,125300,124624,124146
005164 125300
005166 124624
005170 124146
005172 123466          .WORD 123466,123005,122322,121635
005174 123005
005176 122322
005200 121635
005202 121147          .WORD 121147,120457,117766,117274
005204 120457
005206 117766
005210 117274
005212 116577          .WORD 116577,116102,115402,114702
```

```
005214 116102
005216 115402
005220 114702
005222 114177          .WORD 114177,113474,112766,112260
005224 113474
005226 112766
005230 112260
005232 111550          .WORD 111550,111036,110323,107607
005234 111036
005236 110323
005240 107607
005242 107071          .WORD 107071,106352,105632,105110
005244 106352
005246 105632
005250 105110
005252 104365          .WORD 104365,103641,103113,102364
005254 103641
005256 103113
005260 102364
005262 101634          .WORD 101634,101102,100347,77613
005264 101102
005266 100347
005270 077613
005272 077056          .WORD 77056,76320,75560,75017
005274 076320
005276 075560
005300 075017
005302 074255          .WORD 74255,73512,72745,72200
005304 073512
005306 072745
005310 072200
005312 071431          .WORD 71431,70661,70111,67337
005314 070661
005316 070111
005320 067337
005322 066564          .WORD 66564,66010,65233,64455
005324 066010
005326 065233
005330 064455
005332 063675          .WORD 63675,63115,62334,61552
005334 063115
005336 062334
005340 061552
005342 060767          .WORD 60767,60203,57416,56630
005344 060203
005346 057416
005350 056630
005352 056042          .WORD 56042,55252,54461,53670
005354 055252
005356 054461
```

```
005360 053670
005362 053076          .WORD 53076,52303,51507,50712
005364 052303
005366 051507
005370 050712
005372 050115          .WORD 50115,47317,46520,45720
005374 047317
005376 046520
005400 045720
005402 045120          .WORD 45120,44316,43515,42712
005404 044316
005406 043515
005410 042712
005412 042107          .WORD 42107,41303,40476,37671
005414 041303
005416 040476
005420 037671
005422 037063          .WORD 37063,36255,35446,34637
005424 036255
005426 035446
005430 034637
005432 034027          .WORD 34027,33216,32405,31573
005434 033216
005436 032405
005440 031573
005442 030761          .WORD 30761,30146,27333,26520
005444 030146
005446 027333
005450 026520
005452 025704          .WORD 25704,25067,24252,23435
005454 025067
005456 024252
005460 023435
005462 022620          .WORD 22620,22002,21163,20345
005464 022002
005466 021163
005470 020345
005472 017526          .WORD 17526,16707,16067,15247
005474 016707
005476 016067
005500 015247
005502 014427          .WORD 14427,13607,12766,12146
005504 013607
005506 012766
005510 012146
005512 011325          .WORD 11325,10504,7662,7041
005514 010504
005516 007662
005520 007041
005522 006217          .WORD 6217,5376,4554,3732
```

```
005524 005376
005526 004554
005530 003732
005532 003110          .WORD 3110.2266.1444.622
005534 002266
005536 001444
005540 000622
005542 000000          .WORD 0
       000001                  .END
```

ACKNOWLEDGMENTS

We would like to acknowledge several stimulating conversations with Professor R. J. Higgens. These conversations helped considerably in clarifying our understanding of the relationship between the infinite Fourier integral transform and the finite Fourier series transform. The help of the staff of the Nicolet Instrument Corporation who related their experiences in programming FFT on minicomputers and helped with the software problems relating to the Model 1070 interface is also gratefully acknowledged. R. L. Aurbach, in using a preliminary version of this program, has helped in finding several coding errors and has made several valuable suggestions for modifying our codes for the relocatable assembler, PAL-11S.

REFERENCES

1. J. W. Cooley and J. W. Tukey, Math. of Computation 19, 297 (1965).

2. Digital Equipment Corporation, Maynard, Massachusetts.

3. Nicolet Instruments Corporation (Formerly Fabri-Tek Instruments Corp.), Madison, Wisconsin.

4. J. W. Cooley, P. A. W. Lewis, and P. D. Welch, IEEE Trans. on Audio and Electroacoustics, Vol. AU-15, No. 2, 79 (1967).

5. R. J. Higgens, private communication.

6. C. H. Costain, Decuscope Vol. 11, No. 2, 25 (1972).

7. W. T. Cochran, et al., IEEE Trans. on Audio and Electroacoustics, Vol. AU-15, No. 2, 45 (1967).

8. During the time this manuscript was being prepared, a relocating assembler PAL-11S was released by Digital Equipment Corporation. Versions of the programs described here are now being prepared using this assembler and these will be supplied upon request.