Transactions Letters

Fast Initialization of Nyquist Echo Cancelers Using Circular Convolution Technique

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***Abstract*—For full-duplex high-speed data transmission over the two-wire line using the same frequency band, it is required to sufficiently suppress the echo. The use of a conventional adaptation method may take a long time to train the echo canceler. Fast training can be achieved by initializing the coefficients of the echo canceler with an estimate of the impulse response of the echo path. In this letter, we propose a method for fast initialization of the echo canceler by using a circular convolution technique. The proposed method enables the use of real-valued training signals instead of complex-valued ones, resulting in significant reduction of the initialization time as well as the implementation complexity. Finally, the performance of the proposed method is analyzed and verified by computer simulation.**

***Index Terms*—Channel estimation, chirp signals, circular convolution, echo canceler, peak-to-average power ratio (PAR).**

I. Introduction

Hybrid couplers are commonly used for full-duplex data transmission over the two-wire telephone line. It is necessary for high-speed data transmission to sufficiently suppress the echo due to imperfect impedance match in the hybrid circuits. For example, V.34-class voiceband modems may need to suppress the near-end echo as much as 65 dB below [1]. When a conventional adaptation method such as the least mean square (LMS) method is employed, it may require a long time to train the echo canceler [2].

The echo canceler can significantly reduce the training time, if it can start with a set of coefficients initialized by the estimate of the impulse response of the echo channel. There have been proposed a number of estimation methods, including the recursive least square (RLS) method [3], the discrete Fourier transform (DFT) method [4], and the autocorrelation method [5]. The RLS method can quickly provide an estimate, but it requires large computational complexity and may have unstable convergence characteristics under certain conditions [6]. The DFT method requires the use of simple arithmetic, while providing good performance. However, it can have some numerical problems when it is implemented using fixed-point arithmetic. When the autocorrelation method is applied to estimation of the impulse response of the echo path, it requires the use of training signals whose autocorrelation function has a form of Dirac delta function [7]. Moreover, it cannot be applied directly to inband Nyquist echo canceler schemes whose input and output are complex-valued and real-valued signals, respectively. When a polyphase signal is used for training the Nyquist echo canceler, it should be converted into a complex-valued analytic signal using a phase-splitting filter [8]. To avoid the need of such an additional phase-splitting filter, the use of complex-valued periodic signals has been proposed [9]. In order to simultaneously estimate the impulse response of the near-end and the far-end echo channel, the period of the training signals should be at least twice the duration of the impulse response of the echo path. Moreover, the estimation process needs to be done using complex arithmetic, significantly increasing the implementation complexity.

In this letter, we consider the use of a circular convolution technique to estimate the impulse response of an echo path. Unlike the autocorrelation method, the training signals in the transmitter and the receiver need not be the same in the proposed method. This flexibility in designing the training signals makes it possible to use real-valued signals. Moreover, the period of the training signal needs to be equal to or larger than the length of the impulse response of the echo path. This can significantly reduce the initialization time as well as the implementation complexity, while providing performance comparable to that of the autocorrelation method.

In Section II, we briefly review the autocorrelation method for echo canceler initialization. Section III describes the proposed circular convolution method for fast initialization of a Nyquist echo canceler. The design of real training signals is also presented. In Section IV, the performance of the proposed method is compared with that of the previous methods in terms of the initialization time and the implementation complexity. Finally, conclusions are summarized in Section V.

II. Initialization by Autocorrelation Method

Consider a periodic signal $p(n)$ whose autocorrelation is given by

$$p(n) \otimes_L p(n) = L\sigma_p^2 \sum_{j=-\infty}^{\infty} \delta(n+jL) \tag{1}$$

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where $\otimes_L$ denotes circular correlation with period L, $\delta(n)$ is Dirac delta function, and σ_p^2 is the power of the signal $p(n)$. Assume that the impulse response of the echo channel is $h(n)$. When the signal $p(n)$ is transmitted to the channel, the output $y(n)$ of the echo channel can be represented by

$$y(n) = h(n) * p(n) + \nu(n) \tag{2}$$

where $*$ denotes the convolution process, and $\nu(n)$ is additive zero-mean noise introduced in the echo channel.

Let $\tilde{h}(n)$ be the circular correlation of $y(n)$ with $p(n)$

$$\tilde{h}(n) = \frac{1}{L\sigma_p^2}\, y(n) \otimes_L p(n). \tag{3}$$

Provided that the period L of $p(n)$ is larger than the duration W of the impulse response $h(n)$ of the echo path, it can be shown that [5]

$$\tilde{h}(n) = h(n) + \frac{1}{L\sigma_p^2} \sum_{m=0}^{L-1} \nu(m) p^*(m+n). \tag{4}$$

Since the noise $\nu(n)$ has zero mean, the impulse response of the echo path can be estimated by taking the expectation of $\tilde{h}(n)$.

Consider a complex-valued polyphase signal whose autocorrelation is a delta function [5]. When such a polyphase signal is applied to initialization of an inband Nyquist echo canceler, the received signal needs to be converted into a complex-valued analytic signal [8]. The analytic signal can be obtained by employing an additional phase-splitting filter whose coefficients are complex valued. Note that the initialization performance directly depends upon the phase splitter.

The need of an additional conversion into the analytic signal can be avoided by using a modified complex chirp signal, the real part of whose autocorrelation is a delta function [9]. We will call this modified chirp signal $\tilde{p}(n)$ Long's signal. Assume that $L \geq W$ and the period of $\tilde{p}(n)$ is $2L$. When $\tilde{p}(n)$ is sent to the echo path, the circular correlation $\tilde{p}(n)$ with the received echo signal yields

$$\begin{aligned} q(n) &= \frac{1}{L\sigma_p^2}\, y(n) \otimes_{2L} \tilde{p}(n) \\ &= \sum_{j=-\infty}^{\infty} [h(n+2jL) + h^*(n+2jL+L)] \\ &\quad + \frac{1}{L\sigma_p^2} \sum_{m=0}^{2L-1} \nu(m) \tilde{p}^*(m+n) \end{aligned} \tag{5}$$

where the superscript $*$ denotes the complex conjugate. $q(n)$ can be decomposed into two parts, each of which has a duration of L and contains the impulse response of the near-end echo path and far-end echo path. Note that the polarity of the impulse response of the far-end echo path is reversed. Thus, the impulse response of the echo path can be estimated by using the first L terms of $q(n)$. It should be noted that this method needs to process $2L$ terms to obtain the first L terms. Moreover, the need of complex-valued training signals results in significant increase of the implementation complexity.

III. Initialization by Circular Convolution Method

Consider a pair of periodic signals $p_1(n)$ and $p_2(n)$ whose circular convolution results in a form of Dirac delta function

$$p_1(n) *_L p_2(n) = L\sigma_{p_1}\sigma_{p_2} \sum_{j=-\infty}^{\infty} \delta(n+jL) \tag{6}$$

where $*_L$ denotes circular convolution with period L, and $\sigma_{p_1}^2$ and $\sigma_{p_2}^2$ are the power of $p_1(n)$ and $p_2(n)$, respectively. Assume that the signal $p_1(n)$ is sent to the echo channel $h(n)$. Taking the circular convolution of the echo channel output $y(n)$ with $p_2(n)$, we have

$$\begin{aligned} \hat{h}(n) &= \frac{1}{L\sigma_{p_1}\sigma_{p_2}}\, y(n) *_L p_2(n) \\ &= \frac{1}{L\sigma_{p_1}\sigma_{p_2}} [h(n) * p_1(n) *_L p_2(n) + \nu(n) *_L p_2(n)] \\ &= \frac{1}{L\sigma_{p_1}\sigma_{p_2}} \Bigg[h(n) * L\sigma_{p_1}\sigma_{p_2} \sum_{j=-\infty}^{\infty} \\ &\qquad \cdot \delta(n+jL) + \nu(n) *_L p_2(n) \Bigg]. \end{aligned} \tag{7}$$

Provided that the period L of $p_1(n)$ is larger than W and the channel is causal, (7) can be rewritten as

$$\begin{aligned} \hat{h}(n) &= h(n) + \sum_{j=1}^{\infty} h(n+jL) + \frac{1}{L\sigma_{p_1}\sigma_{p_2}} \sum_{m=0}^{L-1} \nu(m) p_2^*(n-m) \\ &= h(n) + \frac{1}{L\sigma_{p_1}\sigma_{p_2}} \sum_{m=0}^{L-1} \nu(m) p_2^*(n-m), \\ &\quad 0 \leq n \leq L-1. \end{aligned} \tag{8}$$

Thus, we can estimate the impulse response of the echo channel by taking the expectation of $\hat{h}(n)$.

A periodic signal $p_1(n)$ with period L can be represented by the Fourier series

$$p_1(n) = \sum_{k=0}^{L/2-1} \left[A_k \cos\left(\frac{2\pi kn}{L}\right) + B_k \sin\left(\frac{2\pi kn}{L}\right) \right] \tag{9}$$

where A_k and B_k are the Fourier coefficients given by

$$A_k = \sum_{n=0}^{L-1} p_1(n) \cos\left(\frac{2\pi kn}{L}\right)$$

and

$$B_k = \sum_{n=0}^{L-1} p_1(n) \sin\left(\frac{2\pi kn}{L}\right). \tag{10}$$

The corresponding training signal $p_2(n)$ is simply determined by

$$p_2(n) = \text{IDFT}\left\{ \frac{L\sigma_{p_1}\sigma_{p_2}}{P_1(k)} \right\} \tag{11}$$

where $P_1(k)$ is the discrete Fourier transform (DFT) of $p_1(n)$ and $\text{IDFT}\{A\}$ denotes the inverse DFT (IDFT) of A. Unlike in the autocorrelation method, where $p_1(n)$ and $p_2(n)$ should be the same, the training signals in the transmitter and the receiver do not have to be the same. This flexibility in the design of

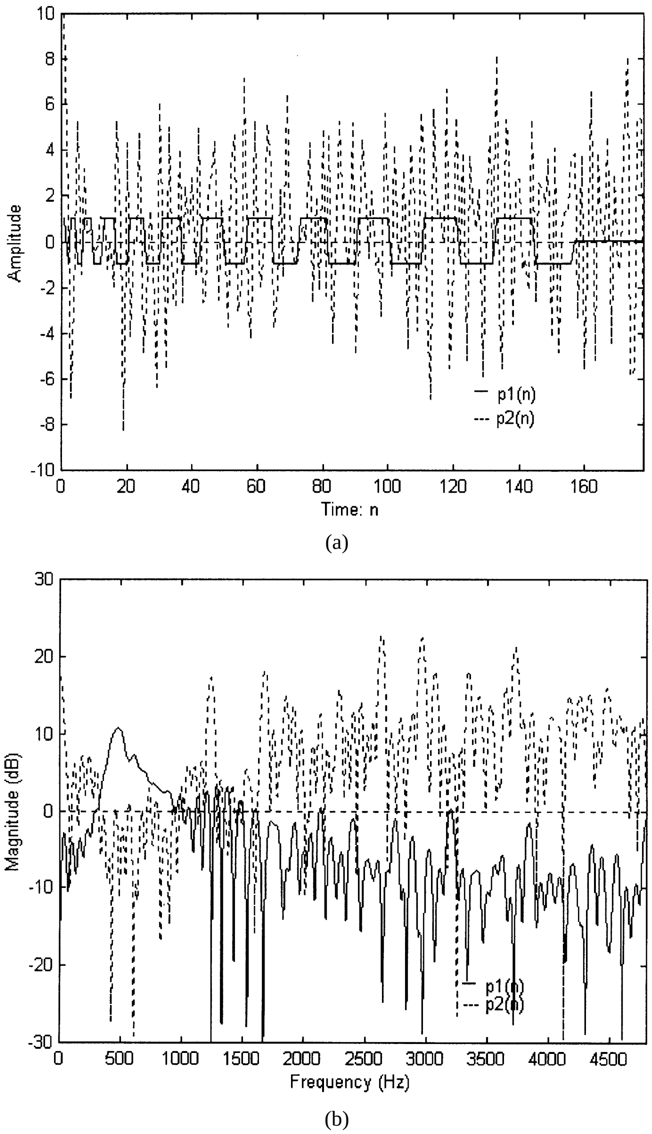

Fig. 1. Composite square-wave signals when $L = 178$. (a) Impulse response of $p_1(n)$ and $p_2(n)$. (b) Frequency response of $p_1(n)$ and $p_2(n)$.

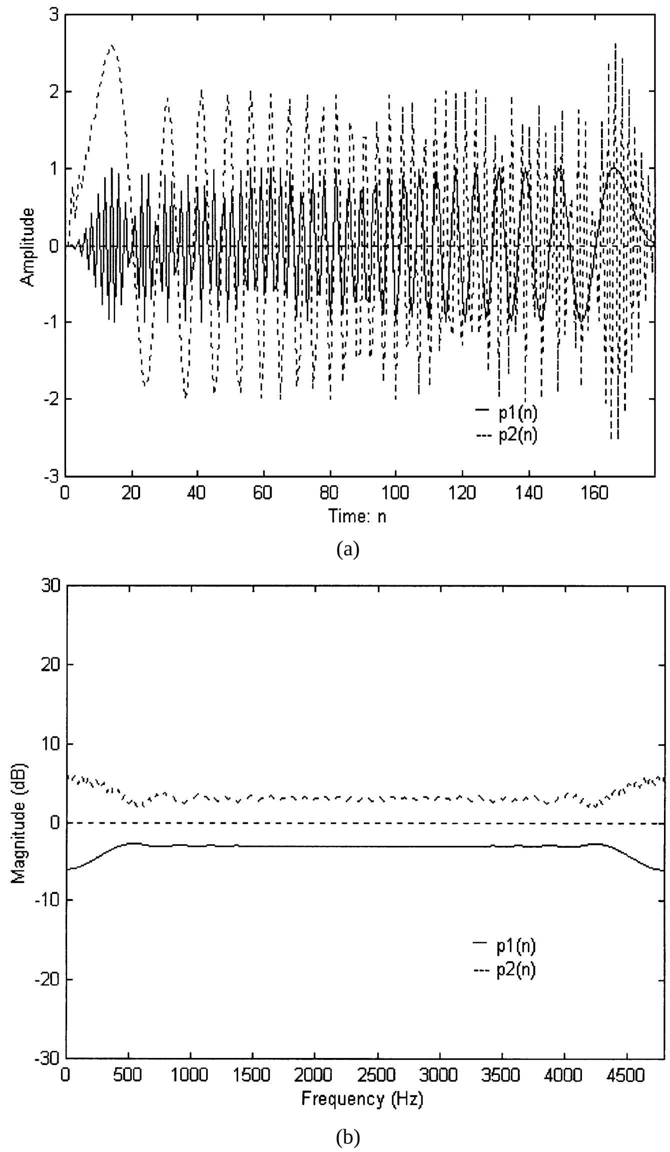

Fig. 2. Real-chirp signals when $L = 178$. (a) Impulse response of $p_1(n)$ and $p_2(n)$. (b) Frequency response of $p_1(n)$ and $p_2(n)$.

$p_1(n)$ and $p_2(n)$ enables us to use real-valued training signals. The use of real-valued training signals significantly reduces the implementation complexity compared to the use of complex-valued ones. Moreover, the training time with the use of training signals with period L becomes almost one half of that with the use of Long's training signal with period $2L$.

It is highly desirable for the training signal to be generated by a simple method with a small peak-to-average power ratio (PAR). For simplicity of signal generation, we can generate a training signal $p_1(n)$ by cascading square-wave signals. Design of such a square-wave signal can begin with one period of a bipolar square-wave signal with period $2T$, where T is the duration of one symbol. Then, append another period of a square-wave signal with period $4T$ to the generated signal. In this manner, a composite square-wave signal can be generated by cascading square-wave signals whose period linearly increases. When the remaining interval is less than one period of a square-wave signal to be appended, zero values are simply padded. Fig. 1 depicts the time and frequency response of $p_1(n)$ and $p_2(n)$ signals generated by the proposed method when $L = 178$. It can be seen that the frequency response of $p_1(n)$ and $p_2(n)$ has some magnitude fluctuation because the composite square wave is not an ideal chirp signal [7]. Since the composite square wave is very simple to generate, it may be quite suitable for high-speed processing applications, such as the high-speed digital subscriber loops (HDSL).

It can be possible to obtain the optimum estimate to use a training signal having a flat frequency response. Complex-valued chirp signals and Long's signal can have such a frequency response. We consider the use of real-valued chirp signals generated by taking the real part of a polyphase signal

$$p_1(n) = \sin\left(\frac{M\pi n^2}{L}\right) \tag{12}$$

where M is the parameter that controls the frequency response. It is possible to obtain a flat frequency response with $M = 0.5$. As an example, when $L = 178$, Fig. 2 depicts the time and

TABLE I
SIMULATION CONDITIONS FOR THE V.34 VOICEBAND MODEM

Parameters	
Local (CSA)	Loop 1
Inter-office transmission	C1 trunk
Carrier frequency	1829 Hz
Transmitted signal power	-9 dBm
Near-end echo power	-20 dBm
Far-end echo power	-40 dBm
Received signal power	-30 dBm
Quantization error	-72 dB below the signal level

frequency response of $p_1(n)$ and $p_2(n)$ generated in this way. The PAR of $p_1(n)$ and $p_2(n)$ is 2.11 and 3.12, respectively. Note that there exists a small Gibb's phenomenon effect at both ends in the frequency response, which is mainly due to $p_1(n)$ being generated in discrete time.

IV. INITIALIZATION PERFORMANCE

The initialization performance can be measured in terms of the residual echo power. Let $z(n)$ and $z_o(n)$ be the echo signal and the output of the echo canceler during the transmission of signal $x(n)$, respectively. The power of the residual echo is

$$E\left\{|e(n)|^2\right\} = E\left\{|z(n) - z_o(n)|^2\right\}. \tag{13}$$

By invoking the independence theorem [6], it can be shown that

$$\begin{aligned} E\left\{|e(n)|^2\right\} &= E\left\{\left|x(n) * \left[h(n) - \hat{h}(n)\right] + \nu(n)\right|^2\right\} \\ &= E\left\{\left|x(n) * \sum_{j=1}^{\infty} h(n+jL) + x(n) * \frac{1}{L\sigma_{p_1}\sigma_{p_2}} \right.\right. \\ &\qquad \left.\left. \cdot \sum_{m=0}^{L-1} \nu(m) p_2^*(n-m) + \nu(n)\right|^2\right\} \\ &= \sigma_x^2 \sum_{n=0}^{L-1} \left|\sum_{j=1}^{\infty} h(n+jL)\right|^2 + \frac{1}{L\sigma_{p_1}^2\sigma_{p_2}^2} \\ &\qquad \cdot E\left\{\left|x(n) * \sum_{m=0}^{L-1} \nu(m) p_2^*(n-m)\right|^2\right\} + \sigma_\nu^2 \\ &= \sigma_x^2 \sum_{n=0}^{L-1} \left|\sum_{j=1}^{\infty} h(n+jL)\right|^2 + \frac{W\sigma_\nu^2\sigma_x^2}{L\sigma_{p_1}^2} + \sigma_\nu^2 \\ &= \sigma_x^2 \sum_{n=0}^{L-1} \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} h(n+jL) h^*(n+kL) \\ &\qquad + \left(1 + \frac{W\sigma_x^2}{L\sigma_{p_1}^2}\right) \sigma_v^2 \end{aligned} \tag{14}$$

TABLE II
SIMULATION CONDITIONS FOR THE HDSL-2

Parameters	
Channel	CSA-1
Symbol rate	400 Kbaud (16-PAM)
Transmitted signal power	13.5 dBm
Echo signal power	-10.5 dBm
Received signal power	-21.5 dBm
NEXT power	-70 dBm
Quantization error	-72 dB below the signal level

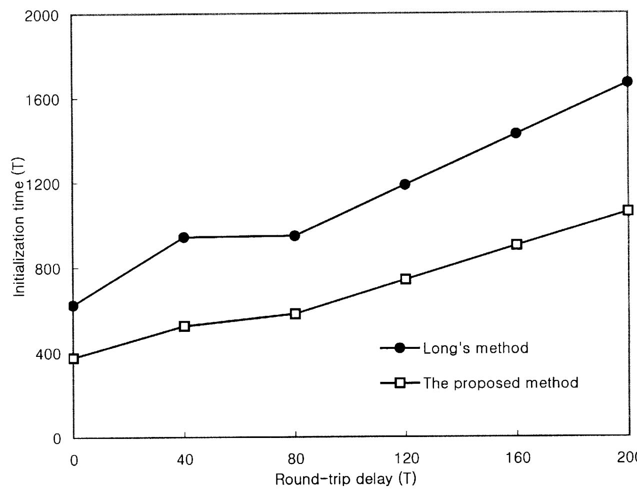

Fig. 3. Initialization time for the echo cancelers.

where σ_x^2 and σ_ν^2 are the power of the data signal and the additive noise term, respectively. Note that the first term in (14) can be zeroed if the length of L is properly chosen. Assuming that the power of $x(n)$ and $p_1(n)$ is the same and $L \geq W$, the power of the residual echo can be represented as

$$E\left\{|e(n)|^2\right\} = \left(1 + \frac{W}{L}\right) \sigma_v^2. \tag{15}$$

To evaluate the initialization performance, the proposed method is applied to initialization of a $T/3$-spaced echo canceler in the V.34-class voiceband modem running at a symbol rate of 3200 (Bd) and a $T/2$-spaced echo canceler in the HDSL-2 modem running at a symbol rate of 400 (kBd). The performance is evaluated in terms of the initialization time and the residual echo power after the initialization. The test conditions for the V.34 and HDSL-2 modems are summarized in Tables I and II, respectively.

The required time for initialization of the echo canceler in the V.34 modem is depicted in Fig. 3 as a function of the round-trip delay time of the far-end echo. Note that the far-end echo canceler cannot start the initialization process until the far-end echo is returned after a certain amount of round-trip delay. Note also that the initialization time is not linearly proportional to the round-trip delay when the far-end echo is overlapped with the

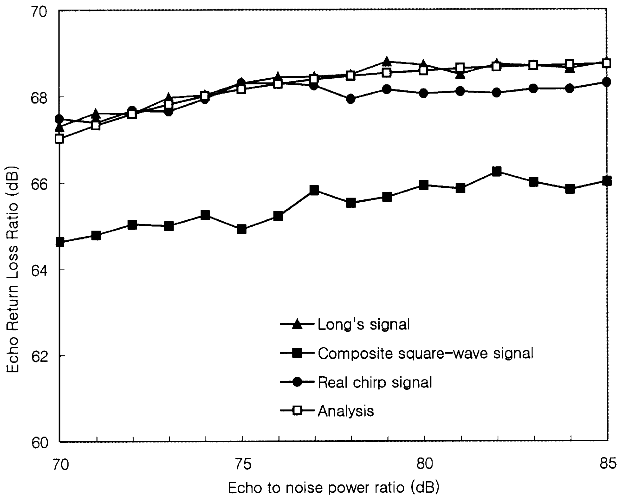

Fig. 4. Power of the residual echo after the initialization.

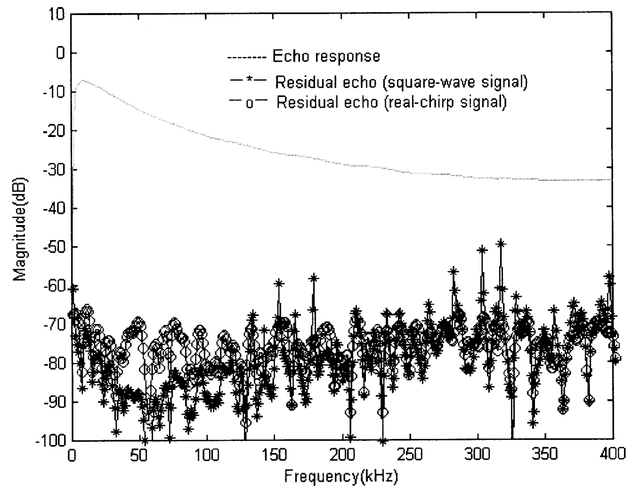

Fig. 6. Initialization performance when applied to $T/2$-spaced HDSL-2 echo canceler.

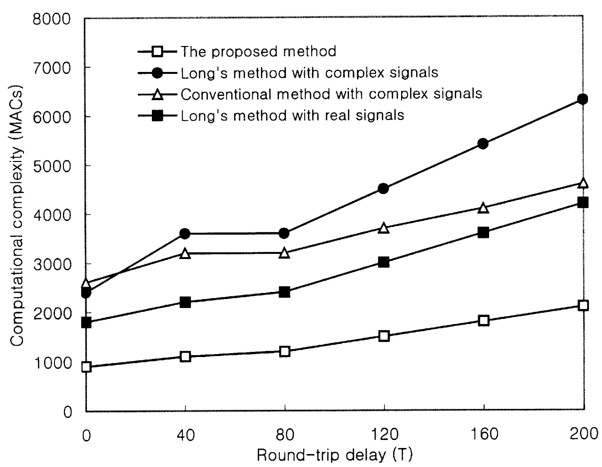

Fig. 5. Computational complexity of the initialization methods.

near-end echo in the time domain. It can be seen that the proposed method can reduce the initialization time about 40∼50% compared to Long's method. This is mainly due to the use of the training signal having a period shorter than that of Long's signal.

Fig. 4 depicts the residual echo power after the initialization as a function of the background noise level. The analytic performance given by (15) is also depicted for reference. It can be seen that the initialization performance by the proposed method with the use of real-valued chirp signals is quite comparable to that by Long's method. The use of composite square-wave signals results in performance about 2∼3 dB worse than the optimum performance. However, it can still provide acceptable initialization performance. After the initialization process, the echo canceler can rapidly reach to the steady state by using a conventional training method such as the LMS adaptation method. Fig. 5 compares the computational complexity per each sample of the estimated echo response in terms of the far-end echo round-trip delay. Note that Long's method can also use real-valued training signals to reduce the implementation complexity, but it may not be applicable when the phase roll exists in the echo path as in the voiceband modem. It can be seen that the proposed method can reduce the implementation complexity by more than one half of the previous methods.

When the symbol rate is very high and low PAR is required, as in the case of HDSL-2 applications, the use of composite square-wave signals is quite practical for the initialization. Fig. 6 illustrates the initialization performance when the proposed method is applied to the HDSL-2 modem. It can be seen that the proposed method with the use of composite square-wave signals can properly initialize the echo canceler under high-speed and interference-dominant channel conditions. The use of real-chirp signals can provide better performance than the use of square-wave signals due to fewer noise enhancement problems in the estimation process.

V. Conclusions

In this letter, we have proposed a method for fast initialization of the echo canceler by employing a circular convolution technique. The proposed method enables the use of real-valued signals, having a shorter period than those of the previous ones, which results in significant reduction in the initialization time. Moreover, the use of real-valued training signals substantially reduces the implementation complexity compared to the use of complex-valued training signals. To illustrate easy application of the proposed method, we have designed two kinds of real-valued training signals, the real-chirp signal and the composite square-wave signal. It has been shown that the use of real-chirp signals can provide initialization performance comparable to the optimum performance, and that the use of composite square-wave signals can be effectively applied to wideband signal transmission such as the HDSL-2.

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