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Fast-light for astrophysics: super-sensitive gyroscopes and gravitational wave detectors

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We present a theoretical analysis and experimental study of the behaviour of optical cavities filled with slow- and fast-light materials, and show that the fast-light material-filled cavities, which can function as 'white light cavities', have properties useful for astrophysical applications such as enhancing the sensitivity-bandwidth product of gravitational wave detection and terrestrial measurement of Lense–Thirring rotation via precision gyroscopy.

1. Introduction

'Slow light' and 'fast light' in atomic systems have been the subject of a great deal of research over the last decade [1–3]. Researchers have found new ways of controlling the index of refraction of various materials by combining laser beams of different frequencies which interact with them. In particular, we are able to achieve strong dispersion at frequencies where the material is transparent. We also understand how to create materials in which the velocity of pulses, which depends on the slope of the dispersion profile, is much less than the free-space velocity of light, or even significantly greater, without contradicting any laws of physics.

The dispersion profile characterizes the relationship between frequency and wavelength, for continuous waves (CW) as well as for pulses. Since the wavelength of light within a particular material depends on its index of refraction, but the frequency does not, the dispersion is controlled by manipulating the index of refraction. The group velocity of a pulse is given by $\mathrm{d}\omega/\mathrm{d}k$, the local slope of the dispersion profile, and pulse propagation speeds which are either much slower or faster than the free-space velocity of light, c are manifestations of steep dispersion profiles, either normal (positive slope) or anomalous (negative slope). These two types of dispersion profiles have been exploited previously to cause large positive and negative group delays—hence the labels 'slow-light material' and 'fast-light material'. This adjustable delay has many applications [4]. But it is not the behaviour

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of pulses with which we are presently concerned. Instead we are exploring the consequences of these types of dispersion for CW light.

In particular, the wavelength of a CW probe beam in a 'slow-light material' changes dramatically when its frequency is changed slightly. Similarly, the wavelength inside a 'fast-light material' can be made to change very little when the frequency is changed significantly. It is these properties which we are now investigating. Specifically, we are interested in putting these materials into optical cavities. By changing the relationship between frequency and wavelength, we can change the behaviour of the cavity, making it resonate over a larger or smaller range of frequencies, and making the effect of changes in the length of the cavity on its resonance frequency greater or smaller.

We have confirmed experimentally and theoretically [5] that putting a slow-light medium into a cavity narrows its linewidth and causes its resonance frequency to change very little when the length of the cavity is changed. We have also confirmed experimentally [6] and theoretically [7] that putting a fast-light medium into a cavity broadens its linewidth, and that the degree of broadening is controllable, a result which might prove useful in several schemes that have been proposed for gravitational wave detection, for example. Finally, we have shown theoretically [7] that a fast light medium in a cavity will increase the sensitivity of its resonance frequency to length changes, and shown that this result applies even to optical gyroscopes, where the 'length change' is essentially equivalent to the effect of rotation. We are working on using these results to design more sensitive optical gyroscopes for measuring the Lense–Thirring rotation terrestrially, and for enhancing the sensitivity-bandwidth product for gravitational wave detectors.

2. Theory

The principle underlying all of these effects starts with the well-known fact that the wavelength of light inside a medium can be different from its wavelength in free space, even though the frequency is unchanged. The degree to which the wave is compressed or expanded in the material depends on the index of refraction:

$$\lambda = \lambda_{\text{vacuum}}/n = 2\pi c/(n\omega). \tag{1}$$

Since it is possible for different frequencies of input light to experience different indices of refraction, it is possible to make arrangements such that a whole range of frequencies should have approximately the same wavelength inside the material. The condition for this to happen is

$$\left.\frac{\partial(n\omega)}{\partial\omega}\right|_{\omega_o} = 0 \quad \Rightarrow \quad \left.\frac{\partial n}{\partial\omega}\right|_{\omega_o} = -\frac{n_o}{\omega_o}. \tag{2}$$

Consider now the fact that light resonates inside an optical cavity if the length of the cavity is an integer multiple of its wavelength. If we arrange for a whole range of frequencies to have almost exactly the same wavelength, in a particular material, we would expect that all frequencies within that range could resonate in an optical cavity which is filled with this material. Thus, the apparent linewidth of the cavity could be increased. However, the cavity storage time, which is determined primarily by the reflectivity of the cavity mirrors, would remain unaffected by this broadening. This is the principle of the 'white light cavity'. Of course, the reverse is also possible. We can arrange for the wavelengths to be dramatically different for very close frequencies, inside another kind of medium, much more than what would happen in free space. In that case, a much smaller range of frequencies is going to resonate in the medium-filled optical cavity than would resonate in the empty cavity. The apparent linewidth of the cavity would then be highly reduced.

We can describe these effects quantitatively by stating the resonance condition for the cavity in terms of the phase difference between light that is entering the cavity and that which has completed one round trip. If that phase difference is a multiple of 2π, the output of the cavity will be exactly equal to its input, assuming no losses. We will consider the linewidth of the cavity to be the range of frequencies for which the output is at least half the input. The output will drop to half when the phase it has picked up after one round trip is

$$\phi_{1/2\ \max} = 2\pi N + \delta\phi_{1/2\ \max} = \frac{L}{c}\omega_0 + \frac{L}{c}\delta\omega_{1/2\ \max}. \tag{3}$$

Here N is an integer, ω_0 is the resonant frequency of the cavity, L is its length and c is the free-space velocity of light. The quantity $\delta\phi_{1/2\ \max}$ is the amount of excess phase that causes the output light level to drop to half its maximum value, and $\delta\omega_{1/2}$ is the linewidth of the cavity.

The phase light picks up as it travels on each round trip depends on the frequency of the light:

$$\frac{\mathrm{d}\phi}{\mathrm{d}\omega} = \frac{\mathrm{d}}{\mathrm{d}\omega}\left(\omega\frac{n(\omega)L}{c}\right) = \frac{n(\omega)L}{c} + \omega\frac{\mathrm{d}n(\omega)}{\mathrm{d}\omega}\frac{L}{c}. \tag{4}$$

Here L is the round trip length of the cavity, c is the free-space velocity of light, and $n(\omega)$ is the index of refraction of the material as a function of frequency. If we assume that the index depends *linearly* on the frequency over some range centred around a frequency ω_0, and that $n(\omega_0) \approx 1$, we can express the change in frequency necessary to give the change in phase which drops the cavity output to half:

$$\delta\omega_{1/2\ \max} = \frac{c}{L}(1 + \omega_0 n')^{-1}\delta\phi_{1/2\ \max}, \tag{5}$$

where n' indicates the derivative of n with respect to ω, and is constant over the linear region, and ω_0 is the centre frequency. One can see that the linewidth, $\delta\omega_{1/2\ \max}$, will be reduced if $(1 + \omega n')$ is greater than one, and increased if $(1 + \omega n')$ is between 0 and 1. This quantity is defined as the group index of the material. So these simplified calculations suggest that in order to create the broad linewidth cavity, we need a material with a group index of less than one, over some range of frequencies. And in order to create a cavity with a narrow linewidth, we need a material with a group index greater than one. These materials are usually referred to as 'fast-light' and 'slow-light' materials, respectively. We use those names, even though the effect with which we are concerned here has little to do with pulses, or the speed with which they propagate. Fast-light materials can be used to create 'white light' cavities, and slow-light materials to create 'line-narrowed' cavities.

Now we consider the sensitivity of these two kinds of cavities to changes in their path lengths. If we have an optical field resonating in a cavity, and then the mirrors are moved further apart or closer together, how much does the frequency of the input field have to change in order for it to resonate again? In the case of the white light cavity, a large range of frequencies has almost exactly the same wavelength, so the wavelength of the light in the cavity changes very little as the frequency of the input field is changed. So the input frequency will have to be changed by a large amount in order to change the wavelength enough to make the light resonant again. We refer to this large change in resonant frequency, corresponding to a small change in length, as a 'sensitivity enhancement' over the empty cavity. By contrast, in the case of a cavity filled with a medium in which very close frequencies have very different wavelengths, a very small change in the frequency of the input laser will be sufficient to change the wavelength inside the material enough to make it resonant again. We call this small change in resonant frequency for a given change in cavity length a 'sensitivity reduction' over the empty cavity.

The resonance condition that the length of the cavity be an integer multiple of the wavelength can be stated, using the relationship of wavelength to frequency, as

$$L_{\mathrm{res}} = \frac{2\pi Nc}{n(\omega)\omega}. \tag{6}$$

If the length of the cavity changes by an amount $\mathrm{d}L$, the frequency which resonates in it will change by an amount $\mathrm{d}\omega$, so that, assuming $n(\omega)$ is a continuously differentiable function:

$$\frac{\mathrm{d}\omega}{\mathrm{d}L_{\mathrm{res}}} = \left(\frac{\mathrm{d}L_{\mathrm{res}}}{\mathrm{d}\omega}\right)^{-1} = -\frac{n^2(\omega)\omega^2}{2\pi Nc}(n(\omega) + \omega n'(\omega))^{-1} = -\frac{\omega}{L_{\mathrm{res}}}\frac{n(\omega)}{n_{\mathrm{g}}(\omega)}. \tag{7}$$

From this it can be seen, again, that the two cases correspond to materials with a group index greater than one, and those with a group index between zero and one. The slow-light materials, with a group index greater than one, produce line-narrowed cavities and also reduce the sensitivity of a cavity to length changes. The fast-light materials, with a group index between zero and one, can be used to make a white light cavity and enhance the sensitivity of a cavity to length changes.

For the sake of illustration, consider for a moment an unphysical situation where the group index for a material is exactly zero for all frequencies. Suppose further that a cavity filled with such a material is resonant at a particular wavelength. We would find that every other frequency we put into the cavity would also resonate. The linewidth would be infinite. This would be the ideal 'white light cavity'. If we then changed the length of the cavity, we would not be able to make it resonate again no matter how much we changed our laser frequency. This is because we would not be changing the wavelength at all. In this sense, the perfect white light cavity is also infinitely sensitive to length changes. In practice, the group index can vanish only over a limited bandwidth, Γ, which in turn limits the bandwidth of the white light cavity and the reduction in sensitivity to length changes. The details of the actual bandwidth of a white light cavity and the degree of reduction in sensitivity under realistic conditions can be found in [7].

It is also important to note that the broadening with fast-light material does not come at the expense of a decrease in the cavity storage time (and, therefore, the intracavity intensity), which is determined solely by the reflectivities of the cavity mirrors, assuming no loss. In other words, the cavity finesse remains unchanged [7]. Thus, with this technique, it is possible to have high finesse cavities with broad linewidths. Similarly, the narrowing with slow-light material does not entail an increase in the cavity storage time (and, therefore, the intracavity intensity), so that low finesse cavities with narrow linewidths are feasible.

3. Applications

Two major potential applications for these effects present themselves. First, a white light cavity could be useful in future incarnations of LIGO, for detecting gravitational waves. This prospect is already under study [8]. In fact, a configuration using both the broadened bandwidth and an effect related to the sensitivity enhancement that we are investigating has also been proposed for LIGO [9]. Second, the enhanced length sensitivity could be useful in optical gyroscopes that detect the rotation rate of a cavity as a change in its resonant frequency, due to a change in the effective optical path length. Gyroscopes with enhanced precision would make inertial navigation systems more accurate over long distances, and may also enable measurement of the frame dragging effect predicted by general relativity.

Other applications can also be envisioned. For example, the insensitivity achieved by a very large group index may be useful in forming highly stable cavities. Such cavities may prove useful in laser frequency stabilization, producing squeezed light for gravitational wave interferometers [10], and in other applications that rely on narrow linewidth, low loss optical cavities immune to external perturbations. Here, we limit ourselves to discussions of the enhancement of the sensitivity-bandwidth product of the LIGO system, and enhanced precision of optical gyroscopes.

LIGO detects gravitational waves by their effect on the arms of a Michelson interferometer. As a gravitational wave passes by, travelling normal to the plane in which the interferometer sits, it causes space to periodically contract along one axis of that plane and expand along the other. This induces a phase shift on the light [11] which is different in the two arms. If the interferometer is configured so that, in the absence of gravitational waves, the beams from the two arms interfere destructively at the output, then any signal from the detector at the output would indicate the possible presence of gravitational waves. The oscillating phase shift induced by the gravitational field can also be described in terms of frequency sidebands. In this description, it is the sideband frequencies that travel toward the detector, while the carrier frequency light still interferes destructively at that port.

Meers [12] introduced the idea of 'signal recycling' as a way of enhancing the sensitivity of such a detector. Signal recycling involves putting a partial mirror in front of the detector, so that the sideband light is mostly sent back into the interferometer. This idea relies on the fact that light re-entering the interferometer

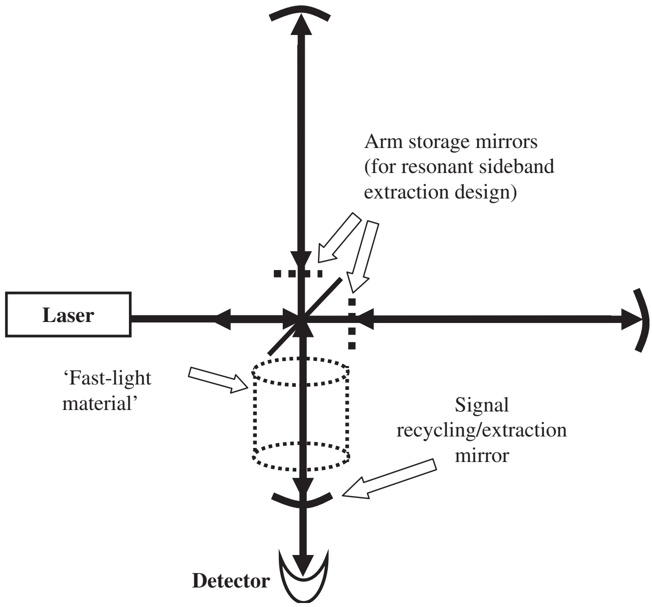

Figure 1. Illustration of interferometric gravitational wave detector incorporating a white light cavity. When arm cavity mirrors (dashed lines) are used, they form, with the signal extraction mirror, a 'signal extraction cavity' for each arm. In the absence of the dashed mirrors, the system is most simply modelled by treating the interferometer itself as one mirror, and the signal recycling mirror as another. In either case, the bandwidth of the detector is increased by the insertion of a fast-light material, so that the relevant cavity is a 'white light cavity'.

from the detector port will be 'reflected' back toward the detector port again. That is, it will traverse the arms and interfere constructively again at the detector port, so that the interferometer can in effect be treated as a mirror. It cannot be a perfect mirror, since a little of the light will be lost to higher order sidebands, but this reduces the magnitude of the first order sidebands by only a negligible amount on each pass. The system can then be described as an optical cavity, with the mirror in front of the detector as one of the mirrors, and the interferometer acting as the other.

The advantage of this is that the sideband-frequency light is in effect created inside the cavity. It never has to pass through an input coupler. Adding the 'signal recycling mirror' in front of the detector (as illustrated in figure 1) can therefore actually increase the amount of light reaching the detector. In all resonating cavities, the output is brighter than the 'first pass' beam inside the cavity. Without the signal recycling mirror, the first pass beam is all the detector sees.

The problem with this scheme is that it severely narrows the bandwidth of the detector. In order to place the signal recycling mirror correctly, one must know in advance what frequency of sideband light one expects to have. That means knowing in advance what frequencies of gravitational wave are present, and choosing only one of them to attempt to detect. Gravitational wave frequencies which induce sideband components outside the cavity bandwidth will not be detected. Variations on this idea which use other mirror geometries to store the sidebands in the interferometer arms face the same problem, trading sensitivity for bandwidth. A white light cavity would eliminate this problem.

Another way of increasing sensitivity is to increase the laser power. In 'Resonant Sideband Extraction' (RSE) [13], additional mirrors are put into the arms of the interferometer (also illustrated in figure 1, with dashed lines) to form arm end cavities that are resonant for the carrier frequency. This allows the power in the arms to be high, so that the sidebands created have larger amplitudes, while keeping the power on the beamsplitter low, decreasing the distortion and noise due to heating. These arm cavities would ordinarily trap the sidebands also (not being resonant, they would not transmit out of the cavity effectively). But with a mirror placed in front of the detector, the situation changes. This mirror lies on the exit path of the sidebands, but not on the path taken by the carrier frequency. Therefore, from each arm, the sideband light sees two mirrors on its way to the detector instead of one. If these two form a resonant cavity for one of the sidebands, then this two-mirror "compound output coupler" is actually 100% transmitting, and the sideband is not trapped in the arm-end-cavity for the carrier. Since this requires the same basic geometry as signal recycling, hybrids of RSE and signal recycling can be imagined, with the reflectivity of the compound output coupler varied between zero and one and the arm-end-cavities tunable to resonate the sidebands or the carrier. In choosing between modes of operation for such a hybrid system, however, one is always making tradeoffs: resonate the carrier or the sidebands, greater detection bandwidth or greater sensitivity. With a white light cavity somewhere in the system (where it is placed depends on the details of the geometry chosen), these tradeoffs are unnecessary.

Other geometries have been proposed, including some using types of interferometer other than the Michelson. All of these designs benefit from high laser intensity and long storage time for the sidebands, and thus, from the use of cavities. But the use of cavities generally comes at the cost of detection bandwidth, unless a workable white light cavity can be implemented.

As an example, consider a realistic situation where, using signal recycling or resonant sideband extraction or another resonant configuration, the next-generation LIGO, in the narrow-band mode operates with a bandwidth of around 10 Hz. Instead, if it were to operate in a broadband mode with a bandwidth of about 10 kHz (covering the whole frequency range over which gravitational waves are expected to be detectable in practice), the finesse would have to be reduced by a factor of 10^3. This would correspond to a reduction in sensitivity by a factor of $10^{3/2}$ (≈ 32), which is quite significant. With a white light cavity, the system can operate in the broadband mode, covering the whole 10 kHz bandwidth, without any reduction in sensitivity. Therefore, this can be viewed as a net increase in sensitivity by a factor of about 32.

The second application, in optical gyroscopes, makes use of the enhanced length sensitivity. An optical gyroscope is a practical application of the Sagnac effect. This can be visualized most easily by imagining a cavity in which the light travels a circular path. (The analysis as presented below, chosen for simplicity, happens to yield the correct result only for light propagating in vacuum; a more accurate description of the process using the relativistic addition of velocities in a medium is presented later on in this paper.) The light enters the loop, travels all the way around it, exits, bounces off other mirrors, and re-enters going in the same direction as it did the first time. Suppose the light travels around the loop in the counterclockwise direction. If the cavity is also rotating counterclockwise around the centre of the loop, then by the time the light reaches the exit, that exit point has moved. The light has to travel a distance somewhat greater than the circumference of the loop in order to exit. So by the time it completes one circuit of the cavity, it has travelled a length that is longer, by vt (where v is the velocity of the 'exit point'), than if the cavity were stationary. The longer path length means that a longer wavelength, and therefore a lower frequency of light, is resonant in the moving cavity than would be resonant in a stationary one. But a clockwise moving cavity with the light traversing it in the counterclockwise direction would, by the same token, resonate at a higher frequency. One measures the resonant frequency and thereby determines the rate and direction of rotation. A cavity with a fast-light medium inside, whose resonant frequency would change more than that for an empty cavity for the same rotation would thus seem to be well suited for use in optical gyroscopes.

As indicated above, this non-relativistic description of the problem happens to predict the correct behaviour only for a cavity in vacuum. However, in order to understand what will happen when a fast-light medium is introduced, we need to treat the problem more carefully, taking into account rules of relativistic addition of velocities in the presence of a moving medium [7]. Let us consider first waves constrained to propagate in a circular path with a radius R (the effect occurs for paths that are rectilinear as well, but for simplicity we will consider only the circular case). We can treat this problem using special relativity only, assuming that the experiment is done in the relatively flat space–time on Earth. For the clockwise (+) and counter-clockwise (−) directions, the relativistic velocities $V_R^{\pm}$ of the phase fronts, the time $T^{\pm}$ of phase fronts to travel from the loop input to the loop exit and the effective distances $\pm L$ from the input to the exit are related as follows:

$$\begin{aligned}
L^{\pm} &= 2\pi R \pm vT^{\pm}, \\
V_P &= \frac{c}{n_0}, \\
V_R^{\pm} &= \frac{V_P \pm v}{1 \pm V_P v/c^2} = (c/n_0 \pm v)(1 \pm v/n_0 c)^{-1}, \\
T^{\pm} &= \frac{L^{\pm}}{V_R^{\pm}} = \frac{2\pi R}{V_R^{\pm} \mp v}.
\end{aligned} \tag{8}$$

Here V_P is the velocity of each phase front in the absence of rotation, c is the velocity of light in vacuum, $v = \Omega R$ is the tangential velocity of rotation, and Ω is the rotation rate.

Now if we assume $v/n_0 c$ is small, we can use the binomial expansion to move the factor containing that term out of the denominator in the second expression for $V_R^{\pm}$. Then simplifying and dropping terms of order v^2, we arrive at

$$V_R^{\pm} \simeq c/n_0 \mp v(1 - 1/n_0^2) \equiv c/n_0 \pm v\alpha_F, \tag{9}$$

where α_F is the Fresnel drag coefficient.

The resonant frequencies are

$$\omega^{\pm} = \frac{2\pi N c}{n_0 V_{\mathrm{R}}^{\pm} T^{\pm}} = \frac{c}{n_0} \frac{1}{V_{\mathrm{R}}^{\pm}} \frac{2\pi N (V_{\mathrm{R}}^{\pm} \mp v)}{P_0}, \tag{10}$$

where N is an integer, $V_{\mathrm{R}}^{\pm} T^{\pm} = P^{\pm}$ is the effective path length around the circle when it is moving, and P_0 is its stationary length, $2\pi R$.

If we assume that $v\alpha_{\mathrm{F}}$ is small compared to c/n_0, then we can neglect that term where it appears in the denominator. Then the resonant frequencies are

$$\omega^{\pm} \simeq (V_{\mathrm{R}}^{\pm} \mp v) \frac{2\pi N}{P_0} \equiv V_{\mathrm{E}}^{\pm} \frac{2\pi N}{P_0}. \tag{11}$$

The 'effective velocity', $V_{\mathrm{E}}^{\pm}$, we define in this equation depends on the index of refraction, as we can see if we write $V_{\mathrm{R}}^{\pm}$ in terms of n according to equation (9). But the index of refraction is itself a function of frequency. Therefore, in order for ω^{+} or ω^{-} to be resonant, we need

$$V_{\mathrm{E}}^{\pm} = \frac{c}{n(\omega^{\pm})} \mp v/n^2(\omega^{\pm}) = \frac{c}{n(\omega^{\pm})} \left[1 \mp \frac{v}{n(\omega^{\pm}) c} \right]. \tag{12}$$

The full expression for the resonant frequencies of a rotating optical cavity filled with some material with index of refraction $n(\omega)$ is thus:

$$\omega^{\pm} = \frac{c}{n(\omega^{\pm})} \left[1 \mp \frac{v}{n(\omega^{\pm}) c} \right] \frac{2\pi N}{P_0}. \tag{13}$$

We can solve this exactly for the resonant frequencies only if we know the exact functional form of $n(\omega)$. Alternatively, we can approximate it as a power series. If we keep only terms to first order in ω, and assume $v/n_0 c \ll 1$, and $|\Delta\omega/2n_0(\partial n/\partial\omega)|_{\omega_0} \ll 1$ so that terms with a product of those two expression are negligible, then we can express $V_{\mathrm{E}}^{\pm}$ in terms of the first derivative of n with respect to ω, then we can write

$$\begin{aligned} \omega^{-} - \omega^{+} &\approx \frac{2\pi N}{P_0} \left(V_{\mathrm{E}}^{-}(\omega^{-}) - V_{\mathrm{E}}^{+}(\omega^{+}) \right) \\ &= \frac{2\pi N}{P_0} \frac{c}{n_{\mathrm{o}}} \left[\left(1 + \frac{v}{n_{\mathrm{o}} c} - n' \frac{\Delta\omega}{2} \right) - \left(1 - \frac{v}{n_{\mathrm{o}} c} + n' \frac{\Delta\omega}{2} \right) \right], \end{aligned} \tag{14}$$

where

$$\Delta\omega = \omega^{+} - \omega_0 = -(\omega^{-} - \omega_0) = \frac{1}{2}(\omega^{+} - \omega^{-}) \tag{15}$$

and the resonant frequency for the empty cavity is

$$\omega_0 = \frac{2\pi N}{P_0} \frac{c}{n_{\mathrm{o}}}. \tag{16}$$

Writing the left-hand side of the equation in terms of $\Delta\omega$ allows us to actually solve for it, and find what change in resonant frequency a rotation should induce in this cavity. The result is

$$\Delta\omega = \left[\left(\frac{2\pi N}{P_0} \frac{c}{n_{\mathrm{o}}} \right) \frac{2v}{n_{\mathrm{o}} c} \right] \left[1 + n' \left(\frac{2\pi N}{P_0} \frac{c}{n_{\mathrm{o}}} \right) \right]^{-1}. \tag{17}$$

To see the effect of the medium clearly, we first find the splitting for the empty cavity case set by setting $n = 1$, and define this as

$$\frac{2\pi N}{P_0}\frac{2v}{n_0^2} \equiv \Delta\omega_0. \tag{18}$$

So the factor by which the splitting is increased is

$$1 + n'\omega_0 = \frac{\Delta\omega_0}{\Delta\omega} = 1/n_g. \tag{19}$$

Therefore, we can say that $1/n_g$ is the relativistically correct enhancement factor. This expression clearly agrees with the simple sensitivity enhancement expression we arrived at in the non-rotating case. It shows that a fast-light material ($n_g < 1$) could indeed enhance the sensitivity of a gyroscope to rotations, whereas a slow-light material ($n_g > 1$) would decrease that sensitivity.

Of course, this enhancement diverges when n_g vanishes. This is due to the unrealistic assumption made above that the index varies linearly over all frequencies. In practice, the variation in the index is linear only over a limited bandwidth, Γ. When this is taken into account, the divergence is eliminated. The expression for the finite enhancement factor under such a condition is derived in [7].

To decide whether this effect is useful in practice, we must put in some realistic numbers. Let us consider a small gyroscope, with a value of (path length)/(area enclosed) = $2\,\text{m}^{-1}$ (e.g. a circle with radius 1 m). Assume the operating frequency to be about 5×10^{14} Hz, a cavity finesse of 1000, and an output power of 1 mW. Then, for the empty cavity case, the minimum measurable rotation rate for an observation time of 1 s is about $1.5 \times 10^{-5}\,\Omega_\oplus$, where $\Omega_\oplus$ is the Earth rotation rate. If a fast-light medium is inserted into the cavity, with a realistic dispersion linewidth of $\Gamma = 2\pi \times 10^6\,\text{s}^{-1}$, the enhancement factor is about 1.8×10^6 [7]. For $\Delta\omega_0 = 1.5 \times 10^{-5}\,\Omega_\oplus$, we can calculate $\Delta\omega$, and find that the minimum measurable rotation rate becomes about $10^{-11}\,\Omega_\oplus$, for an observation time of 1 s. This number can be improved further by increasing the observation time, or by increasing the size of the gyroscope, but it is already smaller than the Lense–Thirring rotation rate of $5.6 \times 10^{-10}\,\Omega_\oplus$ predicted by general relativity for an Earth-bound experiment.

4. Experiment

With these applications in mind we have been working on realizing some of these cavity designs experimentally. The idea is to put slow-light and fast-light media into cavities, and measure the resulting linewidth and sensitivity to length changes. Initially, we experimented with electromagnetically induced transparency (EIT) in sodium vapour as a means of creating a slow-light material. This work is summarized in our 2006 paper [14]. Although we did see the kind of dispersion that would have been appropriate for creating the reduced sensitivity, narrowed linewidth cavity, or alternatively for a slow-light-based relative-rotation sensor, for technical reasons we moved to rubidium vapour for our next set of experiments. The details of these experiments in rubidium can be found in [5, 6]. Here, we present a brief summary of this work.

In rubidium, we have experimented with both EIT and Raman gain to create various dispersion profiles. We found that the dispersion profile associated with a single EIT transmission peak worked best for narrowing the cavity linewidth and reducing its sensitivity [5, 15]. To create the white light cavity, we used an approach akin to that of Wang *et al.* [16] for demonstrating fast-light. This method involves co-propagating two pump beams, at two different frequencies, with the probe beam through the rubidium cell. When the probe (cavity beam) matches either pump frequency, it experiences Raman gain. When it is in-between the two pump frequencies, it experiences a frequency dependent index of refraction, with $n'(\omega) < 0$. This is the condition required for generating fast-light. We found that the linewidth of the cavity did indeed broaden, and moreover, that we could control it by tuning the frequency separation of our pumps and the level of gain [6].

To create either the white light cavity or the line-narrowed cavity that behaves exactly as described in the theory above, it would be ideal to fill the entire cavity with the appropriate dispersive material. This is not, however, necessary; filling the cavity partially with the material gives the same effect as long as the value of the group index is adjusted accordingly [5, 6]. Using of this method, we built a cavity with a one metre path length and placed within it a cell approximately 10 cm long. For this case, the linewidth is given by

$$\delta\omega_{1/2\ \max} \simeq \delta\phi_{1/2\ \max} \frac{c_0}{(L-l)+n_g l}, \tag{20}$$

where L is the cavity lengh and l is the cell length. The change in resonant frequency is given by

$$\frac{\delta\omega}{\delta L} = \frac{-\omega^2}{2\pi Nc}\left(\frac{1}{1+n'(\omega)\omega(l\omega/2\pi Nc)}\right). \tag{21}$$

The experimental set-up is as depicted in figure 2.

Note that the pump frequency light necessary for EIT or Raman gain is introduced to the cavity by a polarizing beamsplitter and then ejected by another, so that it co-propagates with the cavity beam (probe) only for a short distance, inside the cell. An uncorrelated optical pump is used to create the population inversion for Raman gain. The energy level diagrams for the two cases are given in figure 3, with the dashed lines representing the Raman gain case, and the solid lines for the EIT case. These experiments are described individually in [6] and [5], respectively.

All of the beams except the optical pumping beam for the Raman gain were derived from the same laser: a CW Ti:sapphire with a linewidth of approximately 1 MHz. In the EIT case, the laser was locked to the $F=2$ to $F'=3$ saturated absorption resonance in a separate rubidium vapour cell, and the pump was derived directly from the laser. In the Raman gain case, the pumps had to be frequency shifted so that we could control the separation between them, and therefore the saturated absorption locking beam was also frequency shifted. In both cases, the probe was then downshifted from the pump(s) using a 1.5 GHz acousto-optic modulator in double pass mode, to match the frequency difference between the

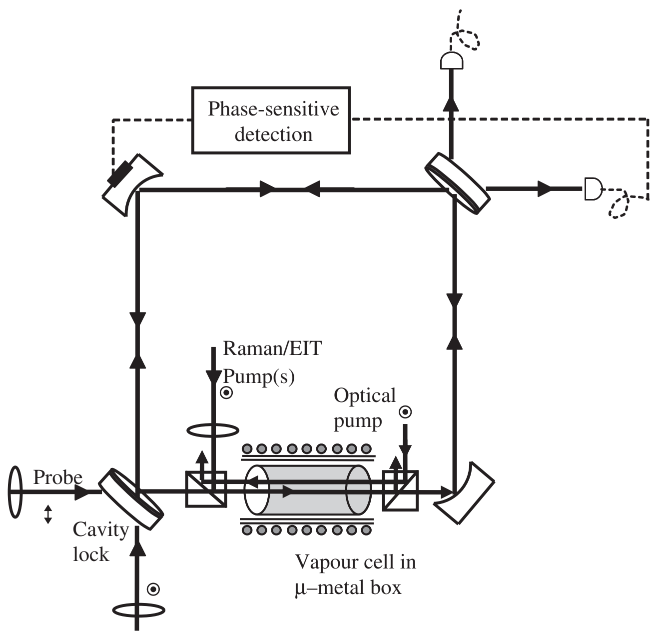

Figure 2. Schematic of the experimental set-up for the white light cavity (using an incoherent optical pump and co-propagating Raman pumps two different frequencies) or line-narrowed cavity (with no optical pump and only one Raman pump frequency).

$F = 2$ and $F = 3$ ground states. The optical pumping beam came from a separate diode laser, also locked by saturated absorption to the appropriate rubidium transition. For both the EIT and the gain experiments, the optical power of the probe field outside the cavity was set at ~100 μW. The Raman/EIT pumps were then made nearly 10 times as strong as the intra-cavity probe intensity. The incoherent optical pump beam was stronger still, >10 mW, to ensure optical pumping over a large velocity group of atoms.

The density of the rubidium vapour in the cell was on the order of 10^{12} atoms per cm^3, at a temperature of approximately 60°C. It was heated by resistance wire which had first been folded in half and twisted into a two strand wire with current going in opposite directions through the two strands, which could be wrapped around the cell without inducing magnetic fields. The cell was also shield by a μ-metal box from external magnetic fields.

We measured the gain coefficient and linewidth for the Raman gain peaks by introducing a flipper mirror in the cavity path and sending the probe directly to a detector. For optimum gain, the probe and the pumps were detuned below resonance, so that the probe was not significantly absorbed in the cell even in the absence of the pump beams. In the EIT case, however, the average detuning was zero.

The cavity was made resonant at the selected probe frequency ω_0 by adjusting the position of one mirror using a piezoelectric actuator. The voltage to the piezo was controlled by an active lock system. The cavity was locked to a transmission peak of

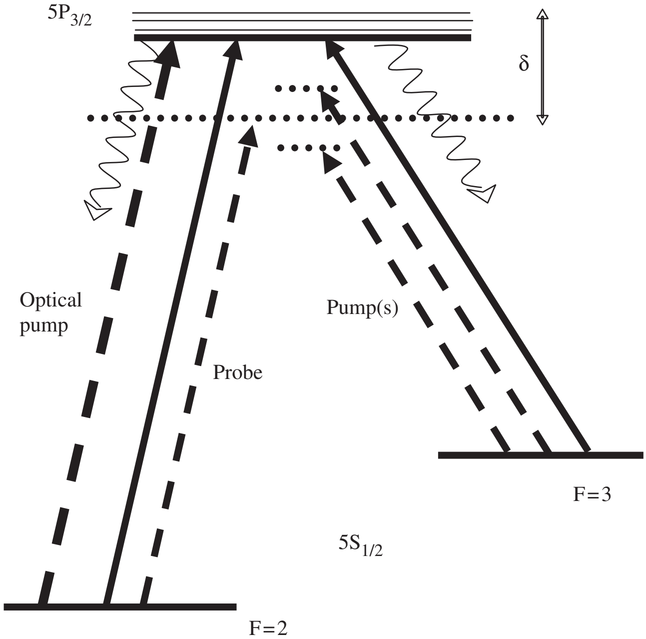

Figure 3. Energy diagram showing of two-photon resonant excitation in D2 line ^{85}Rb used to produce EIT for slow-light (solid lines) or detuned Raman excitation to produce bi-frequency gain for fast-light (dashed lines).

a counter-propagating lock beam, shifted in frequency by many free spectral ranges of the cavity away from the probe beam, and from the rubidium resonances. The lock system used a phase sensitive detection for active feedback and servo-control to keep the cavity length fixed. The cavity linewidth with the various pump beams turned off was measured by scanning the probe frequency around the centre frequency ω_0, detuned from rubidium resonance. Even with the pumps off, the cell and beamsplitter apparatus affected the linewidth of the cavity, introducing extra losses through scattering and residual absorption. The linewidth was found to be approximately 8 MHz, which is nearly three times wider than it was without the vapour cell, and the finesse was measured at about 60.

The gain could be adjusted by changing the pump intensities, which also affected the slope of the negative dispersion. The ideal performance in the white light cavity/enhanced-sensitivity case comes when $n_g = 0$ for $L = l$ only; if $l \ll L$, then we need a value of n_g that is negative: $n_g = 1 - L/l$, and we can adjust the pump intensities, the frequency separation between them, and even the temperature of the cell in order to get to this condition.

We independently measured the dispersion as seen by the probe field under the EIT and double gain conditions. The measurement was done using a heterodyne technique: an auxiliary reference wave was produced by frequency shifting a fraction of the probe beam outside the cavity using a 40 MHz AOM, and this reference wave, which had not passed through the dispersive media, was recombined with the probe,

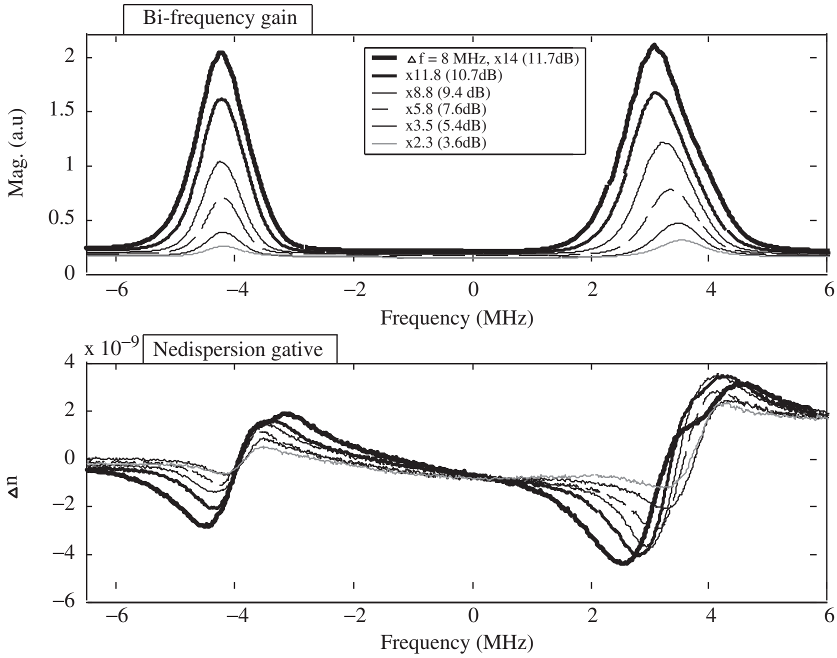

Figure 4. Measured gain and corresponding dispersion with bi-frequency pumped Raman excitation.

and sent to a detector. A low noise rf mixer and a low-pass frequency filter were used to demodulate the rf signal from the detectors. The dispersion showed up as a frequency dependent phase shift between the two beams, and thus a frequency dependent intensity level on the demodulated detector signal. In the EIT case, the dispersion was limited mainly by the small size of the cell and by the Doppler broadening of the medium, which resulted in less than complete transparency, and a broader EIT linewidth. Buffer gases and a larger cell might allow larger, positive slopes of dispersion. In the Raman gain case, the goal was dispersion profiles with a negative slope. We experimented with making n_g close to zero. The smallest negative dispersion slopes were observed with gain as low as 3 dB or less. Such a low gain also avoids other undesirable effects such as self-oscillation and bi-stable behaviour which might be observed in the presence of a stronger gain medium. We display the dispersion measurements for the double pumped Raman gain in figure 4, to show the tunability of the group index.

Finally, to observe the effect of these media on the cavity, we folded down the flipper mirror, closing the cavity. The results, in the EIT case, were almost exactly what the theory predicted. The sensitivity to length changes was reduced by the correct factor, and the linewidth correspondingly reduced, as described in detail in [5]. For the white light cavity case, we observed the line broadening, as expected. We were able to show that the broadening is significantly over and above what would be expected for increased losses alone. Thus, this experiment represents a demonstration of the white light cavity effect. The details of this experiment can be found in [6]. The measured linewidths in both of these cases were in good agreement with theory.

One should expect that the experimental apparatus we used to observe the white light cavity effect, as describe above and in [6], could easily be used to demonstrate the enhancement in sensitivity to cavity length change as well. However, there are some significant difficulties in doing so, for the following reasons. As shown in [7], the enhancement factor is nonlinear: it decreases with increasing values of the empty-cavity frequency shift, $\Delta\omega_o$ (corresponding to ΔL, or equivalently, a rotation rate). Furthermore, in order for the enhancement to be evident, the value of the loaded-cavity frequency shift, $\Delta\omega'_o$ (i.e. the enhanced shift) must be less than the dispersion bandwidth. Thus, for the limited dispersion bandwidth we were able to realize in [6], one must use a very small value of ΔL in order to observe an enhancement. This in turn requires the use of a resonator that has a much higher finesse than the one used in [6], and a more precise voltage supply for the PZT. The required modifications are non-trivial, and efforts are underway in our laboratory to implement these changes to the apparatus.

Finally, we point out that for a passive resonator, the enhancement in sensitivity does not lead to an actual enhancement in the capability of a rotation sensor, which is characterized by the minimum measurable rotation rate. This results from the fact that while the signal level is enhanced, the linewidth broadening compensates for it, so that there is no net improvement in the sensitivity, as shown in detail in [7]. However, this is not the case if an active resonator (i.e. a ring laser gyroscope: RLG) is used. To see why, note that for an RLG, the linewidth depends only on the cavity decay time, which is unaffected by the dispersion [7, 17]. Thus, in order to demonstrate fully that the enhanced sensitivity can be used for ultra-precise rotation sensing, it is necessary to realize an RLG with a built-in fast-light medium. This is a difficult challenge, and efforts are underway in our laboratory to realize such a device as well.

5. Conclusion

Our initial results, showing linewidth narrowing and reduction in sensitivity to length changes in cavities filled with slow-light media, and showing linewidth broadening in cavities filled with fast-light media, are encouraging. The methods we used to produce the slow- and fast-light media are flexible enough to allow the dispersion to be easily measured and tuned. Further research into these phenomena could lead to improved rotation sensors, gravitational wave detectors, or other applications yet to be explored.

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