Faster Implementation of Scalar Multiplication on Koblitz Curves

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Outline

Contribution

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Key points to achieve this result are:

- Native support of polynomial multiplication over $\mathbb{F}_2$ and use of vectorized instructions.

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- Improvements on point addition implementation.
- Data-dependent precision on computation of $w\tau$-NAF recoding.
- Application of Frobenius endomorphism in a similar way to s-GLV decomposition.

Koblitz Curves

Koblitz curves, E_a, are elliptic curves over $\mathbb{F}_2$ defined by the following equation:

$$E_a\colon\ y^2 + xy = x^3 + ax^2 + 1 \tag{1}$$

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where $a \in \{0, 1\}$.
For cryptographic purposes, we can work in elliptic curves over extension fields, $E_a(\mathbb{F}_{2^m})$ such that,

$$\#E_a(\mathbb{F}_{2^m}) = f \cdot r \qquad (2)$$

where $f = 2^{2-a}$ and r is a large prime.

Koblitz Curves

One important remark on Koblitz curves is the presence of an endomorphic function, say τ. Its evaluation involves application of Frobenius automorphism in $\mathbb{F}_2$ over each coordinate of a given point:

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This endomorphism holds, for every point $P \in E_a$, the following equation:

$$\tau^2(P) + 2P = \mu\tau(P) \qquad \mu = (-1)^{1-a} \quad (4)$$

is also known as the *characteristic equation*.

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- If some extra memory is available, we can think about an $w\tau$-adic representation, which produces a sparse non-zero coefficient expansion of the scalar, enabling the use of precomputed points.
- We can fix w value in such a way that optimizes time/memory to get high speed.

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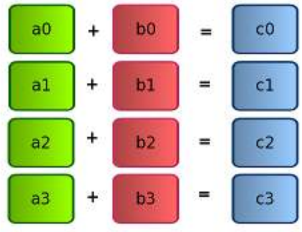

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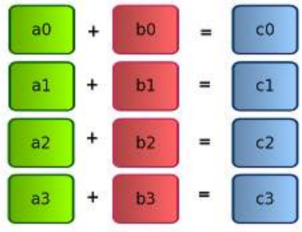

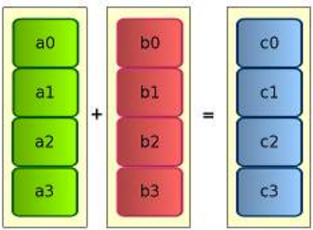

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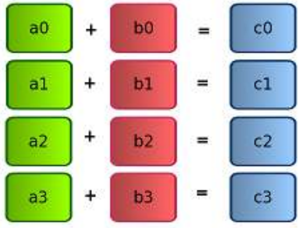

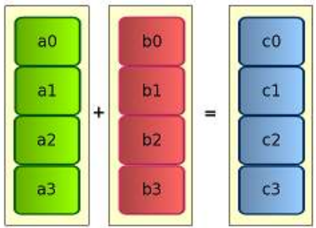

- In this work, we exploit capabilities of AVX and SSE instruction sets on a Sandy Bridge micro-architecture to develop binary field arithmetic.

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- Recent applications of this instruction on binary field arithmetic have shown that increases throughput on high speed implementations such as GCM for authenticated encryption, elliptic curves and η-pairings
- Native support of this operation produces relevant speed-up in the computation of scalar multiplication on Koblitz curves.

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- **Multiplication.** The most performance-critical operation is computed in two stages, first a polynomial multiplication and then the result of that is followed by a modular reduction.
  - **Polynomial multiplication**. Karatsuba approach was applied in 64-bit granularity, thus 13 polynomial multiplications and 32 additions are computed.

    Our algorithm processes all polynomial multiplications independently, so maximizes pipeline occupancy level.

    In order to improve register allocation, operands are stored in an interleaved form.

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    The new optimized reduction algorithm is based on bit and byte-wise shifting and memory alignment instructions.

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- **Multi-squaring.** Given a field element a, evaluating

$$b = a^{2^k}$$

  for a fixed value k, is a time-memory trade-off. In which a look-up table of $16\lceil \frac{m}{4} \rceil$ field elements is stored. Resulting faster than repeatedly squarings when $k > 6$.
  We can increase performance when 256-bit XOR instruction is present with respect to 128-bit XOR instructions.

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  This sequence of powers is done by using multi-squaring operations.

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Due to the choice of irreducible pentanomial, saving one modular reduction represents approximately 15% of a field multiplication,

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- Due to deterministic nature of algorithm, code was completely unrolled to handle only the required precision in current iteration.

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We notice that $\tau^{\lfloor m/s \rfloor}$ function acts as an endomorphism in the context of s-GLV decomposition.

s-GLV method

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The latest equation can be done using an interleaved version of scalar multiplication, saving $m - \lfloor \frac{m}{s} \rfloor$ applications of τ.

Results

Elliptic curve. In order to address 128-bit security level and compliance with standards, we chose the Koblitz curve K283 proposed by NIST.

$$E_0(\mathbb{F}_{2^{283}}) : \; y^2 + xy = x^3 + 1$$

$$\mathbb{F}_{2^{283}} \equiv \mathbb{F}_2[z]/(f(z))$$

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Platform. We use a Sandy Bridge architecture present on an Intel processor Core-i7 2600K at 3.4GHz. Some features:

- SSE4.2 and AVX instruction sets.
- PCLMULQDQ instruction included in AES-NI instruction set.
- GCC and ICC compilers were used under Linux environment.

Results

Elliptic curve. In order to address 128-bit security level and compliance with standards, we chose the Koblitz curve K283 proposed by NIST.

$$E_0(\mathbb{F}_{2^{283}}) : \; y^2 + xy = x^3 + 1$$

$$\mathbb{F}_{2^{283}} \equiv \mathbb{F}_2[z]/(f(z))$$

where $f(z) = z^{283} + z^{12} + z^7 + z^5 + 1$.

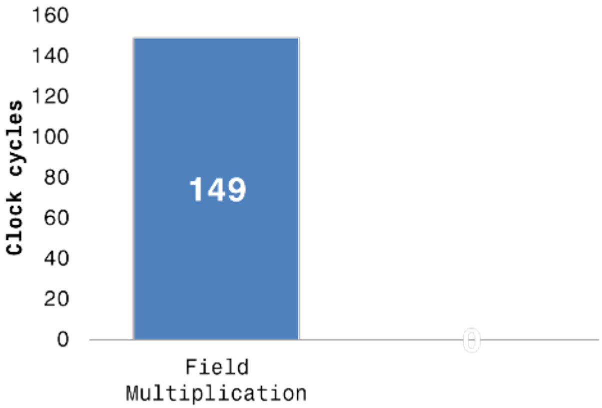
Platform. We use a Sandy Bridge architecture present on an Intel processor Core-i7 2600K at 3.4GHz. Some features:

- SSE4.2 and AVX instruction sets.
- PCLMULQDQ instruction included in AES-NI instruction set.
- GCC and ICC compilers were used under Linux environment.

The benchmarking was conducted using performance guidelines suggested by EBACS website.

Results on binary field operations

Field multiplication $\mathbb{F}_{2^{283}}$.



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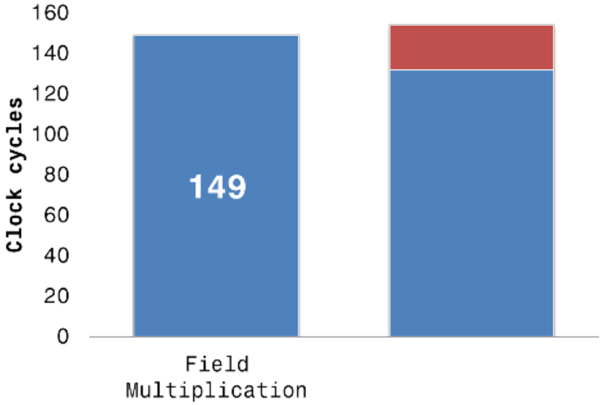

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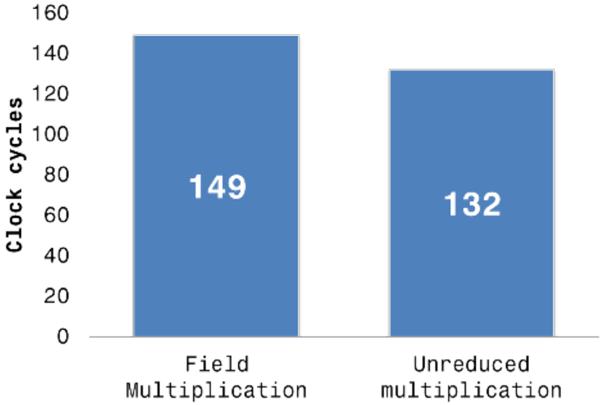
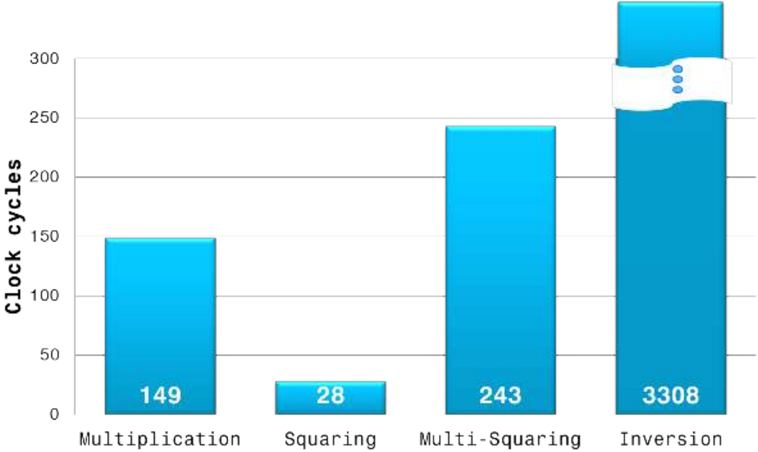


Saving of 15% per unreduced multiplication using lazy reduction technique.

Results on binary field operations

Comparison between binary field operations $\mathbb{F}_{2^{283}}$.



Results on Scalar Multiplication on Koblitz curves

Timings are reported in three different scenarios:

- **Unknown point**

$$[k]P$$

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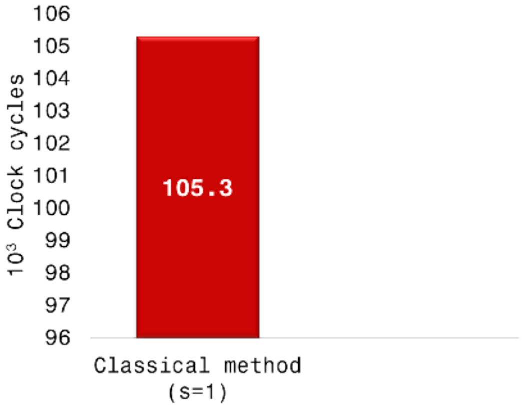
- **Multiple point multiplication.**

$$[k]P + [k']Q$$

P is unknown, Q is known in advance and k and k' are generated online at random, e.g. ECDSA verification.

Results on Scalar Multiplication on Koblitz curves

Unknown point scenario.



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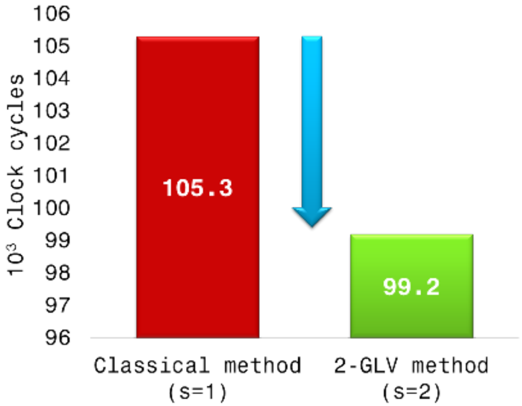
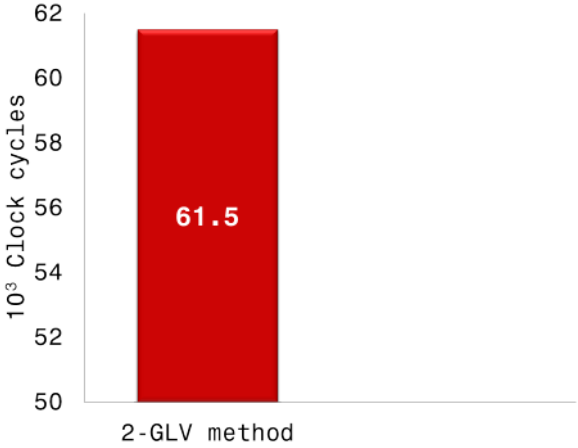


Breaking the barrier of 10^5 clock cycles to compute a scalar multiplication of a random point.

Results on Scalar Multiplication on Koblitz curves

Fixed point scenario. Using a table of 64 precomputed points ($w = 8$).



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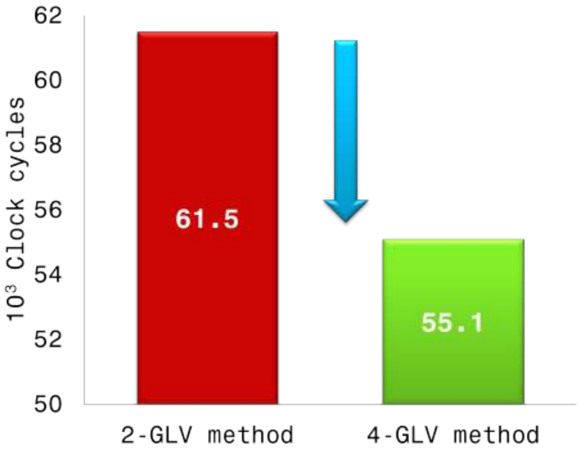
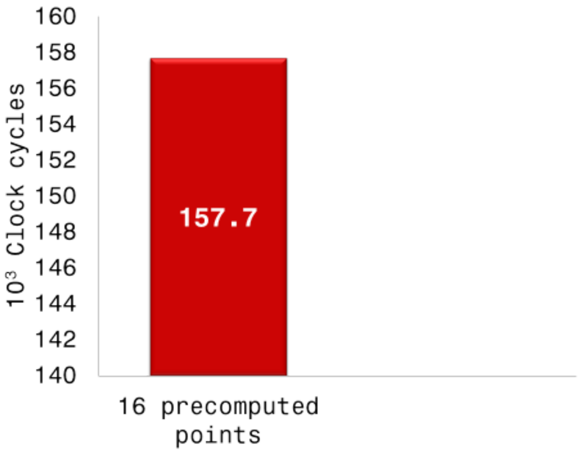


Results on Scalar Multiplication on Koblitz curves

Multiple point multiplication scenario. Using 2-GLV method.



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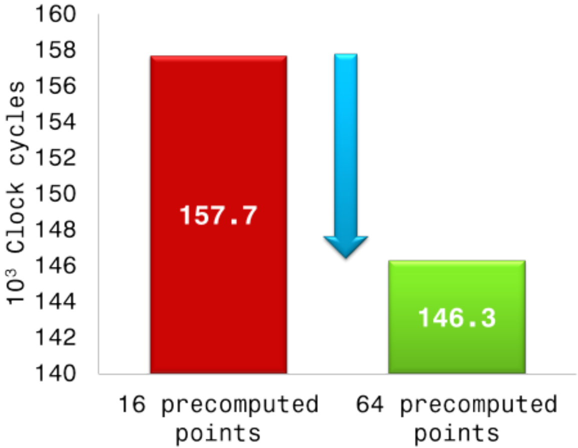

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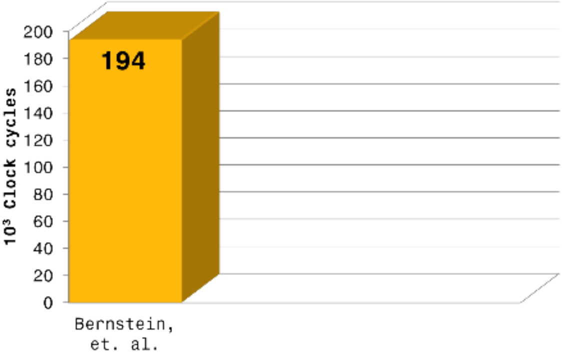

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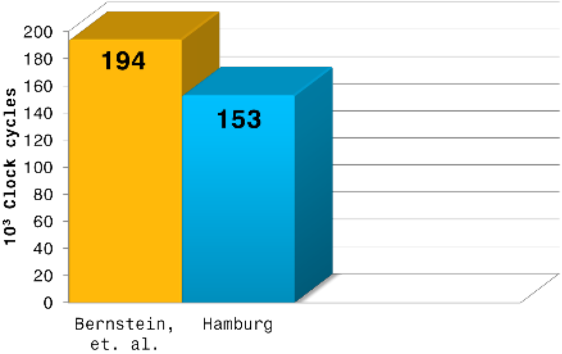

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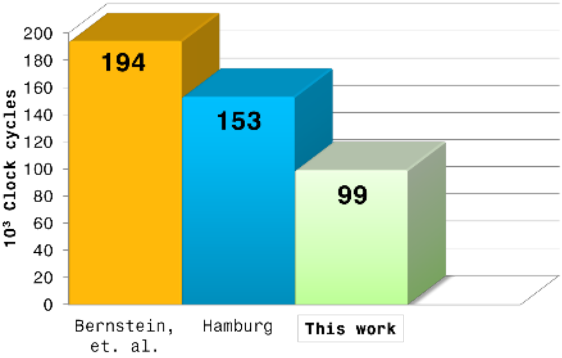

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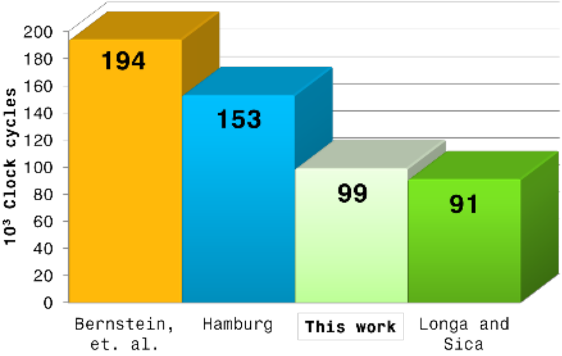

Performance estimates

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This estimate shows that performing scalar multiplication on a Koblitz curve should be considerated faster than a prime curve equipped with endomorphisms, if sufficient support to binary field multiplication is present.

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  - Provide side-channel resistant against timing and memory cache attacks.
  - Submit source code to EBACS website.

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Thanks!, Muchas Gracias!, Obrigado!

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