

Fate of gravitational collapse in semiclassical gravity

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While the outcome of gravitational collapse in *classical* general relativity is unquestionably a black hole, up to now no full and complete *semiclassical* description of black hole formation has been thoroughly investigated. Here we revisit the standard scenario for this process. By analyzing how semiclassical collapse proceeds we show that the very formation of a trapping horizon can be seriously questioned for a large set of, possibly realistic, scenarios. We emphasise that in principle the theoretical framework of semiclassical gravity certainly allows the formation of trapping horizons. What we are questioning here is the more subtle point of whether or not the standard black hole picture is appropriate for describing the end point of *realistic* collapse. Indeed if semiclassical physics were in some cases to prevent formation of the trapping horizon, then this suggests the possibility of new collapsed objects which can be much less problematic, making it unnecessary to confront the information paradox or the run-away end point problem.

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I. INTRODUCTION

Although the existence of astrophysical black holes is now commonly accepted, we still lack a detailed understanding of several aspects of these objects. In particular, when dealing with quantum field theory in a spacetime where a classical event horizon forms, one encounters significant conceptual problems, such as the information-loss paradox linked to black hole thermal evaporation [1, 2, 3, 4].

The growing evidence that black hole evaporation may be compatible with unitary evolution in string-inspired scenarios (see, *e.g.*, reference [5])¹ has in recent years led to a revival of interest in, and extensive modification of, early [7] alternative semiclassical scenarios for the late stages of gravitational collapse [8, 9]. (See also [10, 11, 12].) Indeed, while it is by now certain that the outcome of a realistic *classical* collapse is necessarily a standard black hole delimited by an event horizon (that is, a region \mathcal{B} of the total spacetime \mathcal{M} which does not overlap with the causal past of future null infinity: $\mathcal{B} = \mathcal{M} - J^-(\mathcal{I}^+) \neq \emptyset$), it has recently been suggested that only apparent or trapping horizons might actually be allowed in nature, and that somehow *semiclassical* or *quantum gravitational* [9, 13, 14] effects could prevent the formation of a (strict, absolute) event horizon,² and hence possibly evade the necessity of a singular structure

in their interior.

Note that Hawking radiation would still be present, even in the absence of an event horizon [15, 16]. Moreover, the present authors have noticed that, kinematically, a collapsing body could still emit a Hawking-like Planckian flux even if no horizon (of any kind) is ever formed at any finite time [17];³ all that is needed being an exponential approach to apparent/trapping horizon formation in infinite time. Since in this case the evaporation would occur in a spacetime where information by construction *cannot* be lost or trapped, there would be no obstruction in principle to its recovery by suitable measurements of quantum correlations. (The evaporation would be characterized by a Planckian spectrum and not by a truly thermal one.)

Inspired by these investigations we wish here to revisit the basic ideas that led in the past to the standard scenario for semiclassical black hole formation and evaporation. We shall see that, while the formation of the trapping horizon (or indeed most types of horizon) is definitely permitted in semiclassical gravity, nonetheless the actual occurrence or non-occurrence of a horizon will depend delicately on the specific dynamical features of the collapse.

Indeed, we shall argue that in realistic situations one may have alternative end points of semiclassical collapse which are quite different from black holes, and intrinsically semiclassical in nature. Hence, it may well be that the compact objects that astrophysicists currently identify as black holes correspond to a rather different

¹ See, however, a recent article by D. Amati [6] for an alternative point of view on the significance of these results.

² “The way the information gets out seems to be that a true event horizon never forms, just an apparent horizon”. (Stephen Hawking in the abstract to his GR17 talk [13].)

³ Recently, it was brought to our attention that this possibility was also pointed out in a paper by P. Grove [18].

physics. We shall here suggest such an alternative description by proposing a new class of compact objects (that might be called “black stars”) in which no horizons (or ergoregions) are present.⁴ The absence of these features would make such objects free from some of the daunting problems that plague black hole physics.

II. SEMICLASSICAL COLLAPSE: THE STANDARD SCENARIO

Let us begin by revisiting the standard semiclassical scenario for black hole formation. For simplicity, in this paper we shall consider only non-rotating, neutral, Schwarzschild black holes; however, all the discussion can be readily generalized to other black hole solutions.

Consider a star of mass M in hydrostatic equilibrium in empty space. For such a configuration the appropriate quantum state is well known to be the Boulware vacuum state $|0_B\rangle$ [20], which is defined unambiguously as the state with zero particle content for static observers, and is regular everywhere both inside and outside the star (this state is also known as the static, or Schwarzschild, vacuum [21]). If the star is sufficiently dilute (so that the radius is very large compared to $2M$), then the spacetime is nearly Minkowskian and such a state will be virtually indistinguishable from the Minkowski vacuum. Hence, the expectation value of the renormalized stress-energy-momentum tensor (RSET) will be negligible throughout the entire spacetime. This is the reason why, when calculating the spacetime geometry associated with a dilute star, one only needs to care about the classical contribu-

⁴ These “black stars” are nevertheless distinct from the recently introduced “gravastars” [19].

tion to the stress-energy-momentum tensor (SET).

Imagine now that, at some moment, the star begins to collapse. The evolution proceeds as in classical general relativity, but with some extra contributions as spacetime dynamics will also affect the behaviour of any quantum fields that are present, giving place to *both* particle production and additional vacuum polarization effects. Contingent upon the standard scenario being correct, if we work in the Heisenberg picture there is a single globally defined regular quantum state $|C\rangle = |\text{collapse}\rangle$ that describes these phenomena.

For simplicity, consider a massless quantum scalar field and restrict the analysis to spherically symmetric solutions. Every mode of the field can (neglecting back-scattering) be described as a wave coming in from \mathcal{I}^- (*i.e.*, from $r \rightarrow +\infty, t \rightarrow -\infty$), going inwards through the star till bouncing at its center ($r = 0$), and then moving outwards to finally reach \mathcal{I}^+ . As in this paper we are going to work in 1 + 1 dimensions (*i.e.*, we shall ignore any angular dependence), for later notational convenience instead of considering wave reflections at $r = 0$ we will take two mirror-symmetric copies of the spacetime of the collapsing star glued together at $r = 0$ (see Fig. 1). In one copy r will run from $-\infty$ to 0, and in the other from 0 to $+\infty$. Then one can concentrate on how the modes change on their way from $\mathcal{I}_{\text{left}}^-$ (*i.e.*, $r \rightarrow -\infty, t \rightarrow -\infty$) to $\mathcal{I}_{\text{right}}^+$ (*i.e.*, $r \rightarrow +\infty, t \rightarrow +\infty$). Hereafter, we will always implicitly assume this construction and will not explicitly specify “left” and “right” except where it might cause confusion.

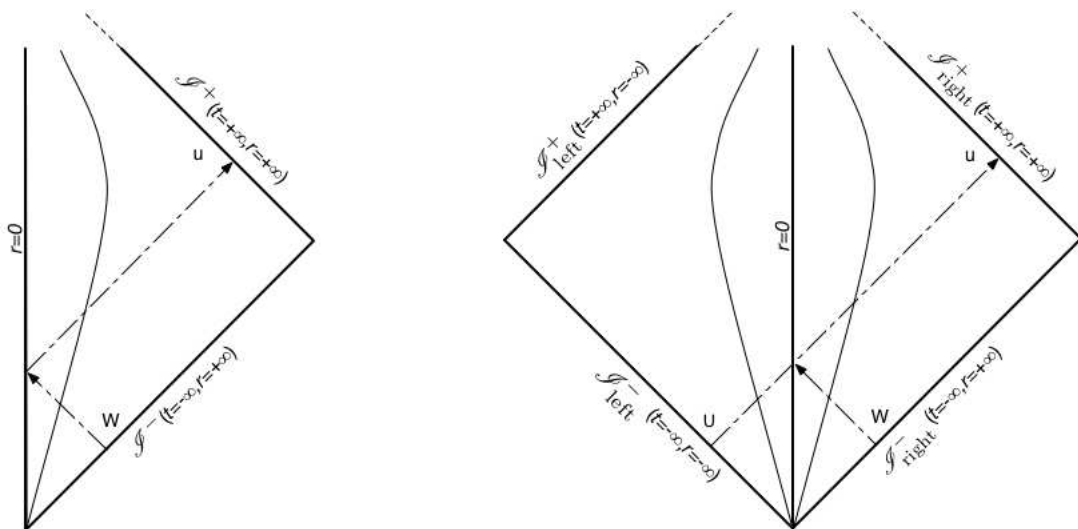


FIG. 1: Standard conformal diagram for a collapsing star, and its mirror-symmetric version.

Now, one can always write the field operator as

$$\hat{\phi}(t, r) = \int d\Omega \left[\hat{a}_\Omega \varphi_\Omega(t, r) + \hat{a}_\Omega^\dagger \varphi_\Omega^*(t, r) \right], \quad (1)$$

where φ_Ω are the modes that near \mathcal{S}^- behave asymptotically as⁵

$$\varphi_\Omega(r, t) \approx \frac{1}{(2\pi)^{3/2} (2\Omega)^{1/2} |r|} e^{-i\Omega U}, \quad (2)$$

with $U = t - r$ and $\Omega > 0$. One can then identify the state $|C\rangle$ as the one that is annihilated by the destruction operators associated with these modes: $\hat{a}_\Omega |C\rangle = 0$. (One could also expand $\hat{\phi}$ using a wave packet basis [2], which is a better choice if one wants to deal with behaviour localized in space and time.) Since the spacetime outside the star is isometric with a corresponding portion of Kruskal spacetime, and is static in the far past, the modes φ_Ω have the same asymptotic expression as the Boulware modes [20] near \mathcal{S}^- (*i.e.*, for $t \rightarrow -\infty$). Hence $|C\rangle$, the quantum state corresponding to the physical collapse, is (near \mathcal{S}^-) indistinguishable from the Boulware vacuum $|0_B\rangle$. (But this will of course no longer be true as one moves significantly away from \mathcal{S}^- .)

Now, the semiclassical collapse problem consists of studying the evolution of the geometry as determined by the semiclassical Einstein equations

$$G_{\mu\nu} = 8\pi \left(T_{\mu\nu}^c + \langle C | \hat{T}_{\mu\nu} | C \rangle \right), \quad (3)$$

where $T_{\mu\nu}^c$ is the classical part of the SET. Significant deviations from the classical collapse scenario can appear only if the RSET in equation (3) becomes comparable with the classical SET. In this analysis there are (at least) two important results from the extant literature that have to be taken into account:

- If a quantum state is such that the singularity structure of the two-point function is initially of the Hadamard form, then Cauchy evolution will preserve this feature [22], at least up to the edge of the spacetime (which might be, for instance, a Cauchy horizon [23]). The state $|C\rangle$ certainly satisfies this Hadamard condition at early times [24], hence it must satisfy it also in the future, even if a trapping/event horizon forms. (A trapping/event horizon is not a Cauchy horizon, and is not an obstruction to maintaining the Hadamard condition.) As a consequence of this fact the RSET cannot become singular anywhere on the collapse geometry,

independently of whether or not a trapping/event horizon is formed.⁶

- For specific semiclassical models of the collapsing star it has been numerically demonstrated (modulo several important technical caveats) that the value of the RSET remains negligibly small throughout the entire collapse process, including the moment of horizon formation [25].⁷ Subsequently, in this scenario quantum effects manifest themselves via the slow evaporation of the black hole.

Thus in this standard scenario nothing prevents the formation of trapped regions (or trapped/apparent/event horizons). Given that quantum-induced violations of the energy conditions [27, 28] are taken to be small enough at this stage of the collapse, one can still use Penrose's singularity theorem to argue that a singularity will then tend to form. Assuming that quantum gravity effects will not conspire to avoid this conclusion, then, in conformity with all extant calculations and the cosmic censorship conjecture, a spacelike singularity and a true event horizon will form. The collapsed star settles down in a quasi-static black hole and then ultimately evaporates.

This last feature can be easily derived by considering an expansion of the field in a basis which contains modes that near \mathcal{S}^+ (*i.e.*, for $r \rightarrow +\infty$, $t \rightarrow +\infty$), behave asymptotically as

$$\psi_\omega(r, t) \approx \frac{1}{(2\pi)^{3/2} (2\omega)^{1/2} r} e^{-i\omega u}, \quad (4)$$

with $u = t - r$ and $\omega > 0$, so defining creation and annihilation operators that differ from those associated with the modes φ_Ω of equation (2). In a static configuration a (spherical) wave coming from \mathcal{S}^- is blue-shifted on its way towards the center of the star, and is then equally red-shifted on its way out to \mathcal{S}^+ , arriving there undistorted. However, in a dynamically collapsing configuration the red-shift exceeds the blue-shift, so that an initial wave at \mathcal{S}^- is distorted once it reaches \mathcal{S}^+ . In this sense the dynamical spacetime acts as a "processing machine" for the normal modes of the field. Expanding the distorted wave in terms of the undistorted basis at \mathcal{S}^+ tells us the amount of particle creation due to the dynamics. In particular one can take a wave packet centered on frequency Ω on \mathcal{S}^- and ask what its typical

⁶ It is important to understand exactly what this theorem does and does not say: If we work in a well-behaved coordinate system (where the matrix of metric coefficients is nonsingular and has finite components), then the coordinate components of the RSET are likewise finite. But note that finite does not necessarily imply small.

⁷ Similar results were, after some discussion, found in (1 + 1)-dimensional models based on dilaton gravity [26].

⁵ We work in natural units.

frequency, say ω , will be when it arrives on \mathcal{I}^+ . The Bogoliubov coefficients that allow us to express the annihilation operators related to the modes (4) in terms of the creation operators pertaining to the modes (2) are related to the number of particles seen by asymptotic observers on \mathcal{I}^+ , which is nothing else than the thermal flux of Hawking radiation [1, 2, 21].

This can be rephrased saying that the physical state $|C\rangle$ corresponding to the collapse behaves like the Unruh vacuum $|0_U\rangle$ [29] of Kruskal spacetime near the event horizon, \mathcal{H}^+ , and near \mathcal{I}^+ (*i.e.*, for $t \rightarrow +\infty$). Indeed, in the Kruskal spacetime the Unruh state $|0_U\rangle$ is a zero-particle state for a freely-falling observer crossing the horizon, and corresponds to a thermal flux of particles at the Hawking temperature for a static observer at infinity [21, 30]. Given that at late times classical black holes generated via classical gravitational collapse are virtually indistinguishable from eternal black holes (see, for instance, the classical theorem in [31]), the Unruh vacuum is the only quantum state on Kruskal spacetime which appropriately (near \mathcal{I}^+ and \mathcal{H}^+) simulates the physical vacuum in a spacetime with an event horizon formed via gravitational collapse.

However as previously mentioned, this standard scenario leads to several well known problems (or at the very least, disquieting features):

- Modes corresponding to quanta detected at \mathcal{I}^+ have an arbitrarily high frequency on \mathcal{I}^- (this is the so-called trans-Planckian problem [29]).
- The run-away end point of the evaporation process (the Hawking temperature is inversely proportional to the black hole mass) prevents any well-defined semiclassical answer regarding the ultimate fate of a black hole [1].
- If eventually the black hole completely evaporates, leaving just thermal radiation in flat spacetime, then it would seem that nothing would prevent a unitarity-violating evolution of pure states into mixed states, contradicting a basic tenet of (usual) quantum theory (this is one aspect of the so-called information-loss paradox [3, 4]). Such a difficulty for reconciling quantum mechanics with general relativity seems to persist even when imagining many alternative scenarios for the end point of the evaporation, so that one can still continue to talk about an information-loss problem [3, 4].

All in all, it is clear that this semiclassical collapse scenario is evidently plagued by significant difficulties and obscurities that still need to be understood. For this reason we think it is worthwhile to step back to a clean slate, and to revisit the above story uncovering all the hidden assumptions.

III. SEMICLASSICAL COLLAPSE: A CRITIQUE

It is easy to argue that one cannot trust a semiclassical gravity analysis once a collapsing configuration has entered into a high-curvature (Planck-scale) regime; this is expected in the immediate neighborhood of the region in which the classical equations predict the appearance of a curvature singularity. Once the formation of a trapped region is *assumed*, any solution of the problems mentioned above seems (naively) to demand an analysis in a full-fledged theory of quantum gravity. Here, however, we are questioning the very formation of a trapped region in astrophysical collapse. In analyzing this question we will see that semiclassical gravity provides a useful and sensible starting point. Moreover, we will also show that it provides some indications as to how the standard scenario might be modified.

A. The trans-Planckian problem

One potential problem with the semiclassical gravity framework, when used to analyze the onset of horizon formation, is the trans-Planckian problem. While this problem is usually formulated in static spacetimes, for our purposes we wish to look back to its origin in a collapse scenario.

We can, as usual, encode the dynamics of the geometry in the relation $U = p(u)$ between the affine null coordinates U and u , regular on \mathcal{I}^- and \mathcal{I}^+ , respectively. Neglecting back-scattering, a mode of the form (2) near \mathcal{I}^- takes, near \mathcal{I}^+ , the form

$$\varphi_\Omega(r, t) \approx \frac{1}{(2\pi)^{3/2}(2\Omega)^{1/2}r} e^{-i\Omega p(u)}. \quad (5)$$

This can be regarded, approximately, as a mode of the type presented in equation (4), but now with u -dependent frequency $\omega(u, \Omega) = \dot{p}(u) \Omega$, where a dot denotes differentiation with respect to u . (Of course, this formula just expresses the redshift undergone by a signal in travelling from \mathcal{I}^- to \mathcal{I}^+ .)

In general we can expect a mode to be excited if the standard adiabatic condition

$$|\dot{\omega}(u, \Omega)|/\omega^2 \ll 1 \quad (6)$$

does *not* hold. It is not difficult to see that this happens for frequencies smaller than

$$\Omega_0(u) \sim |\ddot{p}(u)|/\dot{p}(u)^2. \quad (7)$$

One can then think of $\Omega_0(u)$ as a frequency marking, at each instant of retarded time u , the separation between the modes that have been excited ($\Omega \ll \Omega_0$) and those that are still unexcited ($\Omega \gg \Omega_0$).

Moreover, Planck-scale modes (as defined on \mathcal{I}^-) are excited in a finite amount of time, even *before* the actual formation of any trapped region. Indeed, they start to be

excited when the surface of the star is above the classical location of the horizon by a proper distance D of about one Planck length, as measured by Schwarzschild static observers. We can see this by observing that the red-shift factor satisfies

$$(1 - 2M/r)^{1/2} \sim \omega/\Omega = \dot{p}(u) \sim \kappa/\Omega_0, \quad (8)$$

where $\kappa = (4M)^{-1}$ is the surface gravity. This then implies $(r - 2M) \sim \kappa/\Omega_0^2$, where we have used $\kappa M \sim 1$. Hence

$$D \sim (r - 2M)(1 - 2M/r)^{-1/2} \sim 1/\Omega_0. \quad (9)$$

Hence, the trans-Planckian problem has its roots at the very onset of the formation of the trapping horizon. Furthermore, any complete description of the semiclassical collapse cannot be achieved without at least *some* assumptions about trans-Planckian physics.

Of course, one can simply assume that there is a natural Planck-scale frequency cutoff for effective field theory in curved spacetimes. Although one cannot completely exclude this possibility, we find that this way of avoiding the trans-Planckian problem is perhaps worse than the problem itself, as it would automatically also imply a shut-down of the Hawking flux in a finite (very small) amount of time. This would eliminate the thermodynamical behaviour of black holes, thus undermining the current explanation for the striking similarity between the laws of black hole mechanics and those of thermodynamics — that they are, in fact, just the same laws [32].

Moreover, such a “hard cutoff” obviously corresponds to a breakdown of Lorentz invariance at the Planck scale. If one is ready to accept such a departure from standard physics, then it seems more plausible (less objectionable?) to conjecture a milder breaking of Lorentz invariance in the form of a modified dispersion relation, a possibility explored in several works on the trans-Planckian problem [33]. While it is seemingly well understood that the Hawking radiation would survive in this case [34], it is however less clear what effect such modified dispersion relations might have on the possibility of forming a (presumably frequency-dependent) trapping horizon, and indeed, on the very definition of such a concept [35].

In what follows we shall adopt a conservative approach and stick, as is usually done, to the standard framework of quantum field theory in curved spacetime, assuming its validity up to arbitrarily high frequencies. Even in the presence of Lorentz violating effects, this would remain a valid framework if, for example, the scale at which Lorentz violations might appear was much higher than the Planck scale [36].

B. Vacuum polarization

The other difficulties of the standard scenario previously listed have been linked by different authors to the presence of horizons and of trapping regions in general.

As we have previously discussed, several departures from semiclassical gravity have often been called for in order to solve these problems. However, the specific question we now want to raise here is rather different: Is the scenario just described guaranteed to be the one actually realized in semiclassical gravity? Or is it possible that semiclassical gravity allows for alternative endpoints of gravitational collapse, in which these problems are not present? In order to answer these questions we look for possible semiclassical effects which could modify the collapse before the very formation of a trapped region.

In any calculation of semiclassical collapse the choice of the properties of the matter involved (which will be encoded in the characteristics of the classical SET) is, obviously, of crucial importance. Normally the initial conditions at early times are chosen so that one has a static star with any quantum field in their “natural” vacuum state. As we have discussed, this will be virtually indistinguishable from the Boulware vacuum state. In this initial configuration we are sure that the RSET is practically zero throughout spacetime, at least before the collapse is initiated. We now want to inquire into the possibility that such a RSET becomes non-negligible during the collapse.

In the standard semiclassical scenario, it is crucial that the initial Boulware-like structure of the field modes at \mathcal{I}^- is somehow “excited” by the collapse and converted into a Unruh-like structure at both \mathcal{H}^+ and \mathcal{I}^+ — this is necessary for compatibility with the presence of a trapping horizon. In fact, if this excitation and conversion were not to be sufficiently effective so as to get rid of Boulware-like modes in the proximity of the would-be horizon, then a potential obstruction to the very formation of the horizon may arise. We know in fact that in static geometries there is an intrinsic incompatibility between the Boulware vacuum and the existence of a trapping horizon, as the RSET near the horizon (in a simplified calculation in 1+1 dimensions) is found to be [37]

$$\langle 0_B | \widehat{T}_{\hat{\mu}\hat{\nu}}(r) | 0_B \rangle_{\text{ren}} \propto -\frac{1}{M^2} \frac{1}{1 - 2M/r} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad (10)$$

where we work in an orthonormal basis. A similar result remains valid in the more complicated (3+1)-dimensional case [30]. The important point is that the denominator vanishes at the horizon. Hence the RSET acquires a divergent (and energy condition violating [27]) contribution. Note that the divergence *is* present even if the components of the RSET are evaluated in a freely-falling basis [30]. (To see that something *intrinsic* is going on at the horizon it is sufficient to calculate the scalar invariant $T_{\mu\nu} T^{\mu\nu} = T_{\hat{\mu}\hat{\nu}} T^{\hat{\mu}\hat{\nu}}$, and to note that this scalar diverges at the horizon.)

Of course the above result applies to a static spacetime, while we are interested in investigating an intrinsically dynamical scenario, which we moreover know, due to the Fulling–Sweeny–Wald theorem [22], should act in such a way as to avoid the above divergence. We are

hence interested in seeing the precise way in which this happens, and in exploring whether it might leave a route to possibly obtaining large, albeit finite, contributions to the RSET at the onset of horizon formation.

IV. THE RSET

In calculating the RSET in a dynamical collapse several choices must be made. The major assumption is that we shall for the time being restrict attention to 1 + 1 dimensions, since then there is a realistic hope of carrying out a complete analytic calculation. Physically, this is not as bad a truncation as it at first seems, since we can always view it as an s -wave approximation to full (3 + 1)-dimensional problem, with at most a few actors of r^{-2} being inserted at strategic places. (For instance, this analytic approximation underlies the subsequent numerical calculation of Parentani and Piran [25].) A second significant choice we will make is to specifically work in a regular coordinate system, in particular, in Painlevé–Gullstrand coordinates [38, 39]. In regular coordinate systems (where the matrix of metric coefficients is both finite and non-singular), the values of the stress-energy-momentum components are direct and useful diagnostics of the “size” of the stress-energy-momentum tensor.

A. Preliminaries

With reference to the diamond-shaped conformal diagram of Fig. 1, we shall start by considering a set of affine coordinates U and W , defined on $\mathcal{S}_{\text{left}}^-$ and $\mathcal{S}_{\text{right}}^+$ respectively. These coordinates are globally defined over the spacetime and the metric can be written as

$$g = -C(U, W) dU dW . \quad (11)$$

Given that we shall be concerned with events which lie outside of the collapsing star on the right-hand side of our diagram, we can also choose a second double-null coordinate patch (u, W) , where u is taken to be affine on $\mathcal{S}_{\text{right}}^+$, in terms of which the metric is

$$g = -\bar{C}(u, W) du dW . \quad (12)$$

Of course,

$$C(U, W) = \bar{C}(u, W)/\dot{p}(u) , \quad (13)$$

where $U = p(u)$ describes the coordinate transformation. Then

$$\partial_U = \dot{p}^{-1} \partial_u . \quad (14)$$

Furthermore, as long as we are outside the collapsing star it is safe to assume that a Birkhoff-like result holds, and take $\bar{C}(u, W)$ as being that of a static spacetime.

Now for *any* massless quantum field, the RSET (corresponding to a quantum state that is initially Boulware) has components [21, 37]

$$T_{UU} \propto C^{1/2} \partial_U^2 C^{-1/2} , \quad (15)$$

$$T_{WW} \propto C^{1/2} \partial_W^2 C^{-1/2} , \quad (16)$$

$$T_{UW} \propto R . \quad (17)$$

The coefficients arising here are not particularly important, and will in any case depend on the specific type of quantum field under consideration.

The components T_{WW} and T_{UW} will necessarily be well behaved throughout the region of interest; in particular they are the same as in a static spacetime and are known to be regular. On the contrary T_{UU} shows a more complex structure due to the non-trivial relation between U and u . A brief computation yields

$$C^{1/2} \partial_U^2 C^{-1/2} = \frac{1}{\dot{p}^2} \left[\bar{C}^{1/2} \partial_u^2 \bar{C}^{-1/2} - \dot{p}^{1/2} \partial_u^2 \dot{p}^{-1/2} \right] . \quad (18)$$

The key point here is that we have two terms, one ($\bar{C}^{1/2} \partial_u^2 \bar{C}^{-1/2}$) arising purely from the static spacetime outside the collapsing star, and the other ($\dot{p}^{1/2} \partial_u^2 \dot{p}^{-1/2}$) arising purely from the dynamics of the collapse. If, and only if, the horizon is assumed to form at finite time will the leading contributions of these two terms cancel against each other — this is the standard scenario.

Indeed the first term is exactly what one would compute from using standard Boulware vacuum for a static star. As the surface of the star recedes, more and more of the static spacetime is “uncovered”, and one begins to see regions of the spacetime where the Boulware contribution to the RSET is more and more negative, in fact diverging as the surface of the star crosses the horizon.

B. Regular coordinates

To probe the details of the collapse, it is useful to introduce yet a third coordinate chart — a Painlevé–Gullstrand coordinate chart (x, t) in terms of which the metric is [17, 38, 39]

$$g = -c^2(x, t) dt^2 + [dx - v(x, t) dt]^2 . \quad (19)$$

This coordinate chart is particularly useful because it is regular at the horizon, so that the finiteness of the stress-energy-momentum components in this chart has a direct physical meaning in terms of regularity of the stress-energy-momentum *tensor*.⁸ By setting the spacetime interval to zero, it is easy to see that the null rays

⁸ These coordinates are also useful as they allow to straightforwardly apply our calculations to acoustic analogue spacetimes (provided one is in a regime in which one could neglect the existence of modified dispersion relations) [17, 39].

are given by

$$dx = (\pm c + v) dt. \quad (20)$$

Although inside the collapsing star the metric can depend on x and t in a complicated way, the geometry outside the surface of the star is taken to be static, so the functions c and v do not depend on t . Under these conditions we can integrate along the history of an outgoing ray from an event (t, x) just outside the collapsing star to another event (t_f, x_f) at asymptotic future infinity $\mathcal{I}_{\text{right}}^+$:

$$t_f - t = \int_x^{x_f} \frac{dx'}{c(x') + v(x')}. \quad (21)$$

Assuming asymptotic flatness, $c(+\infty) = 1$ and $v(+\infty) = 0$, we find for the u null coordinate in the “out” region,

$$u := \lim_{t_f \rightarrow +\infty} (t_f - x_f) = t - \int^x \frac{dx'}{c(x') + v(x')}. \quad (22)$$

Hence, denoting partial derivatives by subscripts:

$$U_x = \dot{p}(u) u_x = -\frac{\dot{p}(u)}{c(x) + v(x)}; \quad (23)$$

$$U_t = \dot{p}(u) u_t = \dot{p}(u). \quad (24)$$

In contrast, along an incoming ray leaving asymptotic past infinity $\mathcal{I}_{\text{right}}^-$ at an event (t_i, x_i) and remaining outside the star,

$$t - t_i = -\int_{x_i}^x \frac{dx'}{c(x') - v(x')}, \quad (25)$$

so we have, for the W null coordinate:

$$W := \lim_{t_i \rightarrow -\infty} (t_i + x_i) = t + \int^x \frac{dx'}{c(x') - v(x')}. \quad (26)$$

Hence

$$W_x = \frac{1}{c(x) - v(x)}; \quad W_t = 1. \quad (27)$$

In addition, by substituting and comparing coefficients of the line element, it is easy to see that the (U, W) and (x, t) coordinates are related by

$$U_t = -(c + v) U_x, \quad (28)$$

$$W_t = (c - v) W_x, \quad (29)$$

and

$$C(x, t) = -\frac{1}{U_x W_x}. \quad (30)$$

Therefore the components of the RSET can be calculated in any of the equivalent forms:

$$T_{tt} = U_t^2 T_{UU} + 2 U_t W_t T_{UW} + W_t^2 T_{WW} \quad (31)$$

$$= (c + v)^2 U_x^2 T_{UU} - 2(c^2 - v^2) U_x W_x T_{UW} + (c - v)^2 W_x^2 T_{WW} \quad (32)$$

$$= \dot{p}^2 T_{UU} - 2\dot{p} T_{UW} + T_{WW}; \quad (33)$$

$$T_{tx} = U_t U_x T_{UU} + (U_t W_x + U_x W_t) T_{UW} + W_t W_x T_{WW} \quad (34)$$

$$= -(c + v) U_x^2 T_{UU} - 2v U_x W_x T_{UW} + (c - v) W_x^2 T_{WW} \quad (35)$$

$$= -\frac{\dot{p}^2}{c + v} T_{UU} + \frac{2\dot{p}v}{c^2 - v^2} T_{UW} + \frac{1}{c - v} T_{WW}; \quad (36)$$

$$T_{xx} = U_x^2 T_{UU} + 2 U_x W_x T_{UW} + W_x^2 T_{WW} \quad (37)$$

$$= \frac{\dot{p}^2}{(c + v)^2} T_{UU} - 2 \frac{\dot{p}}{c^2 - v^2} T_{UW} + \frac{1}{(c - v)^2} T_{WW}. \quad (38)$$

Some of these formulae are more useful for calculating the static Boulware contribution, others are more useful for calculating the dynamical contribution. Since $c + v \rightarrow 0$ at a horizon, while $c - v \rightarrow 2c$ is regular, this is enough to guarantee that the T_{tt} and T_{tx} components of the RSET are always better behaved (less divergent) than the T_{xx} component. Note that no divergence can arise from the terms proportional to T_{WW} .

Equations (22) and (26) also allow us to express the derivative with respect to u in terms of those with respect to the regular coordinates x and t :

$$\partial_u = \frac{c+v}{2c} \partial_t - \frac{c^2-v^2}{2c} \partial_x. \quad (39)$$

C. Calculation assuming normal horizon formation

Hereafter, we shall for simplicity restrict our attention to the case $c(x) \equiv 1$. Placing the horizon at $x = 0$ for convenience, we can write the asymptotic expansion

$$v(x) \approx -1 + \kappa x + \kappa_2 x^2 + \dots, \quad (40)$$

where κ can be identified with the surface gravity [17, 39].

Consider first the static Boulware term in equation (18). We have (placing the horizon at $x = 0$ for convenience)

$$\bar{C} = -\frac{\dot{p}}{U_x W_x} = -\frac{1}{u_x W_x} = 1 - v(x)^2 \approx 2\kappa x. \quad (41)$$

The relevant derivative in ∂_u is then that with respect to x , and we can write

$$\begin{aligned} \bar{C}^{1/2} \partial_u^2 \bar{C}^{-1/2} &\approx (2\kappa x)^{1/2} \kappa x \partial_x \left(\kappa x \partial_x (2\kappa x)^{-1/2} \right) \\ &= \kappa^2/4. \end{aligned} \quad (42)$$

In fact, keeping the subleading terms one finds

$$\bar{C}^{1/2} \partial_u^2 \bar{C}^{-1/2} = \frac{\kappa^2}{4} + \mathcal{O}(x^2). \quad (43)$$

By equations (36) and (38), it is clear that because of the constant term $\kappa^2/4$, the components T_{tx} and T_{xx} of the RSET contain contributions that diverge as x^{-1} and x^{-2} , respectively, as $x \rightarrow 0$. (The sub-leading terms lead to finite contributions of order $\mathcal{O}(x)$ and $\mathcal{O}(1)$ respectively.)

In counterpoint, assuming horizon formation, let us now calculate the dynamical contribution to the RSET ($\dot{p}^{1/2} \partial_u^2 \dot{p}^{-1/2}$). It is well known that any configuration that produces a horizon at a finite time t_H leads to an asymptotic (large u) form

$$p(u) \approx U_H - A_1 e^{-\kappa u}, \quad (44)$$

where U_H and A_1 are suitable constants. Taking into account the asymptotic expression (40) for $v(x)$ near $x = 0$, it is very easy to see that the potential divergence at the horizon due to the static term is exactly cancelled by the dynamical term. In this way we have recovered the standard result that the RSET at the horizon of a collapsing star is regular.

However, the previous relation is an asymptotic one, and for what we are most interested in (the value of the RSET close to horizon formation) it is important to take into account extra terms that will be subdominant at late times. Indeed, we can describe the location of the surface

of a collapsing star that crosses the horizon at time t_H by

$$x = r(t) - 2M = \xi(t) = -\lambda(t - t_H) + \dots, \quad (45)$$

where the expansion makes sense for small values of $|t - t_H|$, and λ represents the velocity with which the surface crosses the gravitational radius. Let t_0 be the time at which a right-moving light ray corresponding to null coordinates u and U crosses the surface of the star. Then on the one hand

$$t_f - t_0 = \int_{\xi(t_0)}^{x_f} \frac{dx'}{1+v(x')}, \quad (46)$$

which for $t_0 \approx t_H$ (implying $r(t_0) \approx 2M$) can be approximated by

$$u \approx (t_0 - t_H) - \frac{1}{\kappa} \ln(-\lambda(t_0 - t_H)) + C_1, \quad (47)$$

so that

$$t_0 - t_H \approx C_2 \frac{e^{-\kappa u}}{\lambda} + \dots \quad (48)$$

On the other hand, since $U(t_0)$ is simply some regular function, we have

$$U(t_0) = U_H + U'_H (t_0 - t_H) + \frac{U''_H}{2} (t_0 - t_H)^2 + \dots \quad (49)$$

Inserting (48) into (49) we obtain an asymptotic expansion

$$p(u) = U_H - A_1 e^{-\kappa u} + \frac{A_2}{2} e^{-2\kappa u} + \frac{A_3}{3!} e^{-3\kappa u} + \dots \quad (50)$$

which it is useful to write as

$$p(u) = U_H - F(e^{-\kappa u}), \quad (51)$$

where F is a regular function such that $F(0) = 0$. Then

$$\begin{aligned} \dot{p}^{1/2} \partial_u^2 \dot{p}^{-1/2} &= -\frac{1}{2} \frac{\ddot{p}}{\dot{p}} + \frac{3}{4} \left(\frac{\ddot{p}}{\dot{p}} \right)^2 \\ &= \frac{\kappa^2}{4} + \left[-\frac{1}{2} \frac{F'''}{F'} + \frac{3}{4} \left(\frac{F''}{F'} \right)^2 \right] \kappa^2 e^{-2\kappa u} \\ &= \frac{\kappa^2}{4} + \left[-\frac{1}{2} \frac{A_3}{A_1} + \frac{3}{4} \left(\frac{A_2}{A_1} \right)^2 \right] \kappa^2 e^{-2\kappa u} \\ &\quad + \mathcal{O}(e^{-3\kappa u}). \end{aligned} \quad (52)$$

The point is that this has a universal contribution coming from the surface gravity, plus messy subdominant terms that depend on the details of the collapse. It is important to note, however, that the corresponding additional contributions to the RSET are finite, in contrast to the one associated with the first term. Indeed, for small values of x ,

$$u \approx t - \frac{1}{\kappa} \ln x + \text{const}, \quad (53)$$

so

$$e^{-\kappa u} \propto x e^{-\kappa t}, \quad (54)$$

and so the second term in the right-hand side of equation (52) is $\mathcal{O}(x^2)$, and by equation (38) gives an $\mathcal{O}(1)$ contribution to T_{xx} that does not depend on x , but depends on time as $e^{-2\kappa t}$. In addition, from a comparison of equations (48)–(50) we see that

$$\frac{A_2}{A_1} \propto \frac{1}{\lambda}, \quad \frac{A_3}{A_1} \propto \frac{1}{\lambda^2}, \quad (55)$$

so the leading subdominant term in the RSET is inversely proportional to the square of the speed with which the surface of the star crosses its gravitational radius. In particular, at horizon crossing, that is at $t = t_H$, the value of the RSET can be as large as one wants provided one makes λ very small. This would correspond to a very slow collapse in the proximity of the trapping horizon formation. Thus, there is a concrete possibility that (energy condition violating) quantum contributions to the stress-energy-momentum tensor could lead to significant deviations from classical collapse when a trapping horizon is just about to form.

D. Calculation assuming asymptotic horizon formation

Another interesting case one may want to consider is one in which the horizon is never formed at finite time, but just approached asymptotically as time runs to infinity. In particular, in reference [17] it was shown that collapses characterized by an exponential approach to the horizon,

$$r(t) = 2M + B e^{-\kappa_D t}, \quad (56)$$

lead to a function $p(u)$ of the form

$$p(u) = U_H - A_1 e^{-\kappa_{\text{eff}} u}, \quad (57)$$

where κ_{eff} is half the harmonic mean between κ and the rapidity of the exponential approach κ_D ,

$$\kappa_{\text{eff}} = \frac{\kappa \kappa_D}{\kappa + \kappa_D}, \quad (58)$$

so that one always has $\kappa_{\text{eff}} < \kappa$. In this case, the calculation of the dynamical part of the RSET leads to exactly the same result that when using expression (44), modulo the substitution of κ by κ_{eff} . However, the non-dynamical part of the RSET remains unchanged. This implies that now, at leading order

$$\begin{aligned} \text{RSET}(x \approx 0) &\approx \frac{1}{\kappa^2 x^2} (\kappa_{\text{eff}}^2 - \kappa^2) \\ &= -\frac{\kappa (2\kappa_D + \kappa)}{(\kappa_D + \kappa)^2 x^2}, \end{aligned} \quad (59)$$

which obviously diverges in the limit $x \rightarrow 0$. We stress that this result does not contradict the Fulling–Sweeny–Wald theorem [22], as the calculation applies only outside the surface of the star (*i.e.*, for $x \geq \xi(t)$), and so the divergence appears only at the boundary of spacetime. Nevertheless, particularizing to $x = \xi(t)$, this again indicates that there is a concrete possibility that energy condition violating quantum contributions to the stress-energy-momentum tensor could lead to significant deviations from classical collapse when a trapping horizon is on the verge of being formed.

E. Physical insight

The key bits of physical insight we have garnered from this calculation are:

- In the standard collapse scenario the regularity of the RSET at horizon formation is due to a subtle cancelation between the dynamical and the static contributions.
- Contributions that can be neglected at late times can indeed be very large at the onset of horizon formation. The actual value of these contributions depends on the rapidity with which the configuration approaches its trapping horizon.
- Once the horizon forms, the above contributions will be exponentially damped with time. However, the analysis of the configuration that approaches horizon formation asymptotically tells us that, while horizon formation is delayed, there are contributions that will keep growing with time.

Hence apparently the RSET can acquire large (and energy condition violating [27]) contributions when a collapsing object approaches its Schwarzschild radius, depending on the details of the dynamics. The final lesson to draw from this part of our investigation is that not all the classical matter configurations compatible with the formation of a trapping horizon in classical general relativity necessarily lead to the same final state when semiclassical effects are taken into account. In particular, for classical collapses that exhibit a slow approach to horizon formation, our calculation indicates that there will be a large (albeit always finite in compliance with [22]) contribution from the RSET, a contribution which can potentially lead the semiclassical collapse to classically unforeseeable end points. For these reasons we wish next to further explore the alternative situation in which the horizon is only formed asymptotically.

V. A QUASI-BLACK HOLE SCENARIO

The history of the confrontation between general relativity and quantum physics has already shown several

times that the quantum mechanical effects in matter can prevent the formation of black holes in situations in which classically such formation would seem unavoidable. Without quantum mechanics, objects such as white dwarfs and neutrons stars would have never been predicted in the first place. Similarly, in this paper we have seen that if for any reason the collapse of the matter forces it into some (metastable) state in which horizon formation is approached sufficiently slowly, then large quantum vacuum effects could prevent the very formation of a trapping horizon. The resulting object could then be considered the most compact and quantum mechanical kind of star. These objects, which we shall tentatively call “black stars”,⁹ would be supported by a form of quantum pressure of universal nature, being characterized only by their closeness to the formation of a trapping horizon.

Lacking an understanding of the physics of matter at densities well beyond that characterizing neutron stars, we cannot reliably assert anything about the stability of black stars. However, the first motivation for our investigation was to see whether semiclassical physics can allow for compact objects closely mimicking black hole features, including Hawking radiation, without incurring in the same problems plaguing the standard scenario. In this sense, static configurations do not seem viable candidates as the absence of a trapping horizon together with the staticity prevents any possibility of emission of a Hawking flux.¹⁰ On the other side, evolving configurations that continue to asymptotically approach their would-be horizon¹¹ would produce quantum radiation at late times.

In order for such a scenario to be realized in nature one can speculate that in some cases, once matter has slowed down the collapse so allowing for the piling up of a sizeable RSET, the latter would not be able to completely stop the collapse, but would instead lead to an evolving configuration where every layer of the collapsing star would lie very close to where the classical horizon of the matter inside it would be located, continually and asymptotically approaching it. We can call this object a “quasi-black hole”.

In order to know exactly how the star asymptotically approaches the horizon in this scenario, one should solve

Einstein’s semiclassical field equations with back-reaction — obviously a very difficult task. Without the result of such an explicit calculation, it is nevertheless reasonable to conjecture that the approach can either follow a power law, or be exponential with a timescale $1/\kappa_D$, say. The case of a power law seems, however, uninteresting for our purposes, because it would not lead to a Planckian emission [17]. On the contrary, an exponential approach is associated with the emission of radiation at a modified temperature $T = (2\pi/\kappa + 2\pi/\kappa_D)^{-1}$ [17]. At least for astrophysical black holes, it is also reasonable to think that $\kappa_D \gg \kappa$ at the beginning of the evaporation process, so that $T \approx \kappa/2\pi$, indistinguishable from the standard Hawking temperature. During evaporation κ increases so, in the long run, T is determined by κ_D and tends to zero. Hence we could in principle have a “graceful exit” from the evaporation process; that is, one could avoid the standard run-away endpoint. Meanwhile, the evaporation could be visualized as a continuous chasing between the surface of the star and its (receding) Schwarzschild radius.

Indeed, possibilities for such a never-ending collapse were already envisaged in 1976, soon after the discovery of Hawking radiation [44, 45] and have been recently proposed again [46] (although via different back-reaction mechanisms). It is important, however, to understand that in the quasi-black hole scenario we discuss here the Hawking flux only affects late-time evolution, and is *not* the agent that prevents horizon formation in the first place. The initial slow-down of the collapse is in this case due to matter-related high energy physics. This provides the time necessary for the vacuum polarization to grow and finally modify the evolution of the collapse toward an asymptotic regime.

Of course, the state at $T = 0$ is reached only after a very long time (for typical estimates of the evaporation timescale, see reference [21]), so according to this scenario a collapsing star forms an object that, for a long period, is indistinguishable from a standard black hole, further justifying our nomenclature of “quasi-black hole”. This object would still evaporate with a Planckian spectrum [17], but (since there is no event horizon) it would not be truly “thermal” (the quantum state is indeed a squeezed state [47]), hence there would be no information-loss problem. The partners of the particles emitted towards infinity, instead of being accumulated inside the trapping horizon as in the standard scenario, would now simply be emitted with a (significant) temporal delay. The radiation received at one instant of time would be correlated with that arriving some time later, so all the information would be recovered in the resulting radiation.

How does back-reaction work in this scenario? During the late time asymptotic collapse, two processes unfold at the same time: (1) the energy associated with vacuum polarization becomes more and more negative; (2) radiation is emitted towards infinity. During a time interval Δu as measured on \mathcal{I}^+ , an arrival of energy

⁹ Newtonian versions of “black stars”, more often called “dark stars”, have a very long history in astrophysics, dating back to Michell [40] and Laplace [41]. For recent commentary on the historical connections between Michell, Cavendish, and Laplace, see [42].

¹⁰ It is perhaps worthwhile to stress here that such static black stars do not belong to the class of objects known in the literature as gravastars, (at least not without the addition of considerable extra assumptions), given that the former are compact agglomerates of matter while the latter have a de Sitter-like interior [19].

¹¹ This approach could be completely monotonic or have oscillating components. These oscillations can also produce burst of radiation at the Hawking temperature [43].

$\Delta E_{\text{rad}} > 0$ is recorded by observers at infinity. Correspondingly, vacuum polarization leads to an extra energy $\Delta E_{\text{vac}} < 0$ (due to the fact that the star becomes more compact), so the Bondi mass of the object decreases by an amount $|\Delta E_{\text{vac}}|$. By energy conservation, one expects that $\Delta E_{\text{vac}} = -\Delta E_{\text{rad}}$, so the emission of radiation is balanced by the increase of vacuum polarization nearby the central object. This balance makes the Bondi mass of the object decrease *as if* it were taken away by radiation, eventually reducing to zero as $T \rightarrow 0$. Note that the expression (10) for the RSET can be rewritten in such a way as to exhibit the fact that vacuum polarization corresponds to the *absence* of black-body radiation at the temperature $T = (8\pi M)^{-1}$ [30]. Although this does not constitute a proof, it is a strong plausibility argument in favour of the energy balance between radiation and vacuum polarization. Also, it strongly suggests that the asymptotic approach to the would-be horizon must be of the exponential type, rather than a power law. Indeed, since a power law would not lead to a Planckian emission, it would be hard to reconcile it with the result presented in reference [30].

Thus, provided that trapping horizons do not form, we have described a plausible scenario for the progressive collapse and evaporation of quasi-black holes. However, the end point of this process seems to still share a problem with the standard scenario: The apparent accumulation of baryon number within the collapsing object [44]. The least massive baryon one can find is the proton. Baryon number is conserved in all experiments realized up to now, and in particular, the proton has been found to be stable (nevertheless, Grand Unification Theories predict it should eventually disintegrate into leptons). In the standard paradigm for the evaporation of a black hole, the trapping horizon and its surroundings is an empty region of spacetime. Therefore, there is only one physical quantity characterizing the quantum emission: The value of its Hawking temperature. For a standard evaporating black hole to be able to nucleate a proton-antiproton pair, it seems necessary that it reaches a temperature larger than $\sim 10^{13}$ K, or equivalently, a tiny mass of less than $\sim 10^{38} m_p$, where m_p is the mass of one proton. However, for example, a black hole having initially one solar-mass would contain a baryon number of around $\sim 10^{57}$. During the evaporation it would conserve this baryon number till it reaches a Bondi mass of $\sim 10^{38} m_p$. But then, even emitting all its remaining energy in the form of baryons (with emission in the form of protons being the most efficient way of removing baryon number), it would end up either: (1) leaving an almost massless relic having a baryon number of $\sim 10^{57} - 10^{38} \sim 10^{57}$ (a rather peculiar state); or (2) completely evaporating producing an enormous violation of baryon-number conservation.

The quasi-black hole scenario, however, adds one extra ingredient to the previous discussion: The would-be horizon and its surroundings is now not an empty region of spacetime. In the vicinity of the would-be horizon

there is always matter progressively being compressed. This fact could significantly affect the way the quasi-black hole radiates its energy. For example, an upper bound for the average density of a solar mass quasi-black hole is given by that of the corresponding black hole $\sim 1/M^2 \sim 10^{19} \text{ kg/m}^3$ (a few times bigger than that of a typical neutron star). At these densities and higher, it is quite plausible that new particle physics effects could come into play and deplete the baryon number much more efficiently than the evaporation process.¹²

Up to this point we have only considered spherically symmetric configurations. However, current observations tell us that most of the observed black hole candidates have a high rate of rotation, sometimes very close to extremality [48]. Hence, for a quasi-black hole scenario to be a feasible description of these objects, it would be necessary to generalize our proposal to rotating configurations. Given the complexity of the vacuum structure around rotating black holes [49] it is very difficult to have a precise proposal in this sense. However, we know that any rotating object possessing an ergoregion but not a horizon would be highly unstable [50]. Hence we expect that any viable model of a rotating quasi-black hole should be characterized by a matter distribution extending up to the outer boundary of the ergoregion.

The fact that most of the progenitors of the observed black hole candidates are characterized by supercritical rotations ($J > M^2$, where J is the angular momentum of the progenitor) is often used as evidence of the validity of the cosmic censorship conjecture. It is interesting to note that if such conjecture holds for standard general relativity it would also be effective in preventing supercritical quasi-black holes. In order to understand this point it is enough to realize that a generalization of the calculation of this article to more general metrics allowing for extremality (*e.g.*, Reissner–Nordström, Kerr, ...) would still imply a pile up of the RSET in proximity of the “would-be horizon” *if and only if* such a horizon can form in the first place. That is, a large quantum-induced RSET can arise only if the collapsing object has already shed the extra charges (*e.g.*, electric charge or angular momentum) so as to be subcritical in proximity of the horizon crossing. So supercritical configurations are likely to be unaffected by the vacuum polarization and behave as in classical general relativity. On the contrary sub-critical configurations will develop (or not develop) trapping horizons according to the details of the dynamics.

¹² Of course, for very massive quasi-black holes such effects will be negligible for a very long time, but will eventually become important as the Bondi mass is decreased by the combined effect of Hawking radiation and vacuum polarization.

VI. CONCLUSIONS

Quantum physics imposed upon the description of the collapse of astrophysical objects in situations that would classically lead to black hole formation could unexpectedly lead to observable effects at early times, when the trapping horizon is about to form. In particular we have shown that before forming a trapping horizon, trans-Planckian modes are excited. Hence, whether the trapping horizon forms or not depends critically on assumptions concerning the net effect of any trans-Planckian physics that might be at work.

Assuming that quantum field theory holds unmodified up to arbitrarily high energy (as is commonly done in most of the extant literature) we have shown that there can be large deviations from classical collapse scenarios, if the latter do allow in the first place a piling up of vacuum energy. Most of the classical collapse scenarios so far considered do not allow for such a piling up, due to their intrinsic rapidity. In this sense the prediction of horizon formation in many of these models [25, 51] seems completely correct.

We have argued however, that alternative classical collapse scenarios in which horizon formation is approached in a slow manner are not only foreseeable, but possibly natural in more realistic situations. If this is indeed the case one then would have to add a new class of compact, horizonless, objects (possibly the most compact objects apart from black holes themselves) to the astrophysical bestiary: the black stars.

In the final part of this work we have then considered a particular subclass of these objects, the quasi-black holes, which could closely mimic all the most relevant features of black hole physics, while avoiding at the same time most of its intrinsic problems (such as singularities, the information paradox, and the question of the end point of Hawking evaporation).

Summarizing, the quasi-black hole scenario for collapse and evaporation is the following one (see Fig. 2): As a star of mass M implodes we conjecture that its matter will try to adjust in new, possibly unstable, configurations so to reach a new equilibrium against gravity. If there is ever a significant slowing down of the collapse, for any reason whatsoever, then this allows the vacuum polarization to progressively grow, and further slow down the approach to trapping horizon formation. Provided such an approach is asymptotic with an exponential law controlled by a timescale $1/\kappa_D$, then the quantum radiation produced during this process is still Planckian, with a temperature $T = (2\pi/\kappa + 2\pi/\kappa_D)^{-1}$, where κ is inversely proportional to the total Bondi mass $M + E_{\text{vac}}$ of the star [17]. For a long time, $|E_{\text{vac}}| \ll M$ and $\kappa \ll \kappa_D$, so $T \approx \kappa/2\pi$ and (from the point of view of an external observer) the object is essentially indistinguishable from a standard evaporating black hole. The emission of radiation is accompanied by an increase in vacuum polarization, that progressively diminishes the Bondi mass of the star, so the would-be horizon shrinks and is never crossed

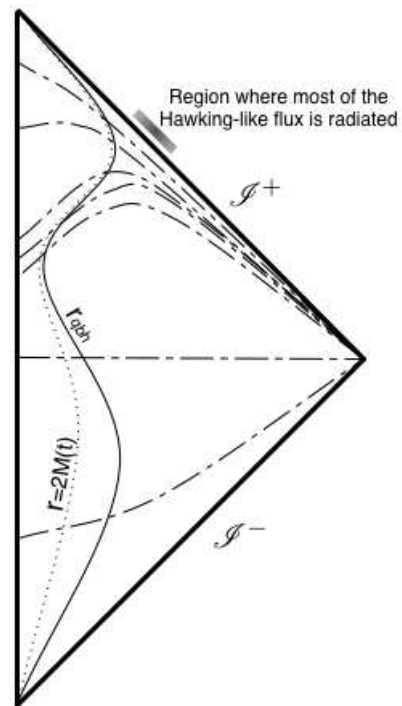


FIG. 2: Conformal diagram of the spacetime for a quasi-black hole. The solid line represents the surface of the collapsing object; the dotted line is at $r = 2M(t)$, where $M(t)$ is the instantaneous mass of the object as measured from \mathcal{S}^+ ; dashed lines correspond to (Schwarzschild) $t = \text{const}$ hypersurfaces. The period of evaporation appears short because of a distortion induced by the representation, but actually corresponds to a very long lapse of time, as one can see from the fact that the lines at $t = \text{const}$ crowd around it. This diagram is compatible with current astrophysical observations of gravitationally active collapse products.

by the matter configuration. When the Bondi mass has become sufficiently small, $1/\kappa$ is negligible and the temperature is approximately equal to $\kappa_D/2\pi$. This quantity is also decreasing, because back-reaction is in fact slowing down collapse, so the temperature, after reaching a maximum value, decreases and approaches zero.

We do not yet have a definitive proposal as to the end-point of the evaporation process. This could only be achieved by understanding the physics of baryon nucleation in the presence of high-density states of matter. The end state of the evaporation could correspond to a zero-temperature relic¹³ with vanishing Bondi mass (hence would at large distances be gravitationally in-

¹³ Note that the nature of such a relic would be quite different from that of a standard black hole remnant, because the relic could be regarded just as a peculiar case of a very compact star. For this reason, the usual issues related to remnants (like the compatibility with CPT invariance or their capacity for storing information) are not present in this scenario.

ert), with an inner structure formed by a core with mass $\sim M$ and a non-vanishing baryon number, immersed into a cloud of polarized vacuum with negative energy $E_{\text{vac}} \sim -M$. Alternatively, it might correspond to plain vacuum.

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- [40] Reverend John Michell, FRS, “On the Means of discovering the Distance, Magnitude, etc. of the Fixed Stars, in consequence of the Diminution of the Velocity of their Light, in case such a Diminution should be found to take place in any of them, and such other Data should be procured from Observations, as would be farther necessary for that Purpose”, Philosophical Transactions of the Royal Society of London **74**, 35–57 (1784); reprinted in *Black Holes*, edited by S. Detweiler (Stony Brook, AAPT, 1982), pp. 8–18. (Warning: The correct spelling is Michell, not Mitchell, though usage is somewhat inconsistent.) The original reference is not easy to obtain and we provide a brief quotation:
- [...] if the semi-diameter of a sphere of the same density with the sun were to exceed that of the sun in the proportion of 500 to 1, a body falling from an infinite height towards it, would have acquired at its surface a greater velocity than that of light, and consequently, supposing light to be attracted by the same force in proportion to its vis inertiae [inertial mass], with other bodies, all light emitted from such a body would be made to return towards it, by its own proper gravity.
- [41] Pierre Simon Marquis de Laplace, *Exposition du Système du Monde* (Paris, Imprimerie du Cercle Social, 1796). The intimate connection between the work of Michell and Laplace can clearly be seen from the following quotation:
- A luminous star, of the same density as the earth, and whose diameter should be two hundred and fifty times larger than that of the Sun, would not, in consequence of its attraction, allow any of its [light] rays to arrive at us; it is therefore possible that the largest luminous bodies in the universe may, through this cause, be invisible.
- An easy-to-find English translation of an essay giving the technical justification for this statement is available as Appendix A of S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge, Cambridge University Press, 1973).
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