

# Fault Detection and Input Stimulus Determination for the Testing of Analog Integrated Circuits Based on Power-Supply Current Monitoring

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## Abstract

*A new method for the testing and fault detection of analog integrated circuits is presented. Time-domain testing followed by spectral analysis of the power-supply current is used to detect both DC and AC faults. Spectral analysis is applied since the tolerances on the circuit parameters make a direct comparison of waveforms impossible. For the fault detection a probabilistic decision rule is proposed based on a multivariate statistical analysis. Since no extra testing pin is needed and the on-line calculation effort is small, the method can be used for wafer-probe testing as well as final production testing. In addition, a methodology for the selection of the input stimulus is presented that improves the testability. Examples demonstrate the efficiency and the effectiveness of the algorithms.*<sup>\*\*\*</sup>

## 1. Introduction

In recent years a great deal of interest has been shown in the power-supply current monitoring for the testing of CMOS logic circuits [1]. The current passing through the VDD or GND terminal is monitored during the application of an input stimulus or in quiescent condition. The highly successful results achieved with this technique in the digital field have prompted investigation of applying this or a similar technique to the process of analog testing as well. In [2] a DC test input was investigated in detail and the results showed that significant changes in the power-supply current were seen between the fault-free and the faulty circuits. Since only a DC stimulus was used, however, AC faults could not be detected by this method. Another shortcoming is that only the nominal case of the circuit was investigated and no tolerances on the circuit parameters were considered. In the testing of analog circuits, the tolerances on the circuit parameters can result in a significant difference between the power-supply current of a manufactured circuit and its nominal value. So a fault-free circuit may be classified as a faulty one or a faulty circuit as a good one if the effect of tolerances on the circuit parameters are not considered in the method of fault detection. In [3], the spectrum of the power-supply current was used to construct a fault dictionary and to identify a fault in an analog circuit. Good results were presented and AC faults could be detected. However, since the tolerances on the circuit parameters were still not considered in this method, the same shortcoming as in [2] remains. In addition, since different input stimuli may result

in different fault detection results, a methodology to select the most effective input stimulus is also needed.

In this paper, the results of time-domain simulations of the power-supply current are used to construct the signature of the fault-free circuit. So both DC and AC faults can be detected. In order to reduce the complexity of the testing procedure, the spectrum of the power-supply current is used for fault detection, avoiding the storage and direct comparison of the waveforms which is difficult because of the manufacturing tolerances. The method of fault detection by testing the power-supply current is discussed in section 2. The procedure for constructing the signature of the fault-free circuit is presented in section 3. The probabilistic decision criterion is based on the theory of multivariate statistical analysis [4]. In addition, section 4 describes the algorithm for the selection of the most suitable input stimulus. Experimental simulated test results are presented in section 5 and conclusions are given in section 6.

## 2. Analog fault detection by testing the power-supply current

The power-supply current  $I_{PS}$  of an analog circuit depends on the input signal, the state of the circuit (fault-free or faulty) and the value of the circuit parameters. In fact,  $I_{PS}$  is a function of the branch currents of the circuit. For every type of fault in the circuit, the current in some branches must have some degree of change. Those changes in branch currents will cause a more or less significant change in  $I_{PS}$ . To illustrate the faulty behaviour of the power-supply current of an analog circuit, consider the CMOS operational amplifier shown in Fig. 1. The simulated results of  $I_{PS}$  using a periodical pulse input signal are given in Fig. 2 for the fault-free case (A) and for two faulty cases (B and C). According to the waveforms in Fig. 2, it is easy to understand that by careful selection of the input signal, there can exist a significant difference between the  $I_{PS}$  waveform of the fault-free and the faulty circuits.

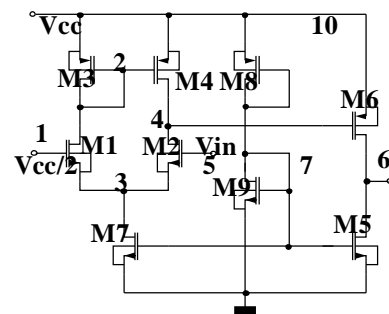
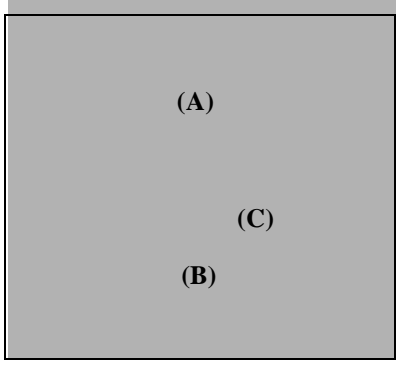


Fig. 1. CMOS operational amplifier.

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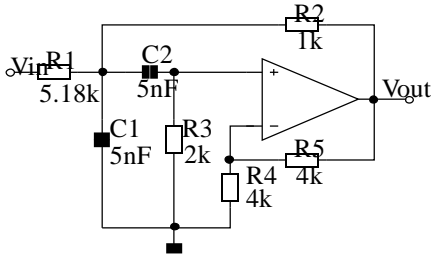
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**Fig. 2. Simulated power-supply current of the CMOS operational amplifier: (A) fault-free, (B) M6 gate-drain short, (C) M4 gate-source short.**

In practice, the manufacturing tolerances on the parameters of an analog circuit also affect the value and waveform of the power-supply current. If the  $I_{PS}$  waveform in the time domain is taken as the measurement for fault detection, the nominal value and the deviation of  $I_{PS}$  at every discrete time point must be simulated and retained during the design stage of the integrated circuit and be compared with the measured value during the testing stage. Thus the testing procedure will be complicated. It is therefore reasonable to avoid direct comparison of the waveforms in the time domain. Instead the spectrum of  $I_{PS}$  is used.

From the waveform in the time domain, the average or RMS value of  $I_{PS}$  can be calculated. The base frequency of the spectrum of the power-supply current  $I_{PS}$  equals the frequency of the input signal, so the higher harmonics of the spectrum can be obtained by the discrete frequency transformation technique. Here again, the tolerances on the circuit parameters make that the RMS value and the harmonics of the  $I_{PS}$  spectrum have a statistical distribution, which makes direct comparisons impossible but necessitates a probabilistic decision criterion for the fault detection.



**Fig. 3. Bandpass filter.**

### Example

Consider the 2nd-order active bandpass filter of Fig. 3 which has a center frequency of 25 kHz. The opamp used in this filter is the one of Fig. 1. Suppose that the W and L of the MOS transistors in the opamp have a tolerance of  $3\sigma = 5\%$  and their model parameters a tolerance of  $3\sigma = 10\%$ . The tolerances of the resistors (R1 to R5) and capacitors (C1, C2) are  $3\sigma = 5\%$ . For a sinusoidal input signal with an amplitude of 50 mV and a frequency of 25 kHz, the mean values and the standard deviations of the RMS value and the first 4 harmon-

ics of  $I_{PS}$  are obtained by Monte-Carlo analysis. The results (with 1000 samples per analysis) for the fault-free case and three faulty cases are given in Table 1.

It can clearly be seen that there exists a significant difference between the mean values and standard deviations of the RMS value and the harmonics of the  $I_{PS}$  spectrum of the fault-free and of the faulty circuits. Additional simulation experiments show that with a careful selection of the input stimulus, this difference exists for most analog circuits. It can therefore be concluded that the power-supply current spectrum can be used to perform effective analog fault detection, a method for which will be described in the next section.

Fault cases		fault-free	C1 short	C2 open	M4 g d sh
RMS value	mean	-6.457e-4	-6.424e-4	-6.488e-4	-5.693e-4
	sigma	1.210e-9	3.962e-9	1.160e-9	1.184e-9
first harmonic	mean	4.021e-5	1.210e-9	6.258e-6	3.256e-6
	sigma	1.996e-11	1.634e-21	1.207e-14	1.985e-14
second harmonic	mean	1.207e-6	2.756e-12	1.321e-9	1.381e-9
	sigma	1.014e-13	5.640e-26	8.958e-20	6.742e-19
third harmonic	mean	5.628e-7	1.544e-12	8.429e-11	1.562e-9
	sigma	2.660e-14	2.423e-26	2.335e-23	7.066e-19
4th harmonic	mean	3.576e-7	1.448e-12	1.509e-10	1.205e-9
	sigma	1.390e-14	1.493e-26	1.599e-23	4.027e-19

**TABLE 1. RMS values and first 4 harmonics of the spectrum of the power-supply current for the bandpass filter.**

### 3. The probabilistic fault detection criterion

Denote the power-supply current spectrum by the vector  $\phi = (\phi_0, \phi_1, \dots, \phi_{m-1})^T \in R^m$ , where  $\phi_0$  is the RMS value and  $\phi_1, \dots, \phi_{m-1}$  are the first  $m-1$  harmonics. If we assume a multivariate normal distribution for the values of  $\phi$  for the fault-free circuit, then the mean vector  $\mu_{(0)}$  and the covariance matrix  $\Sigma_{(0)}$  of  $\phi$  for the fault-free circuit can be estimated by performing a Monte-Carlo analysis with a large number of samples during the design stage of the circuit. During the testing stage of the circuit, a time-domain measurement of the power-supply current is performed for an applied stimulus. Discrete frequency transformation is carried out to obtain the RMS value and the harmonics of the spectrum of the power-supply current. This means that a sample of  $\phi$  is obtained during the testing stage of the analog circuit. If it can be justified that the mean vector of the sample data  $\phi$  is equal to  $\mu_{(0)}$  and the covariance matrix of the sample data  $\phi$  is equal to  $\Sigma_{(0)}$  with a large probability, it is reasonable to believe that the circuit under testing is fault-free.

The problem of fault detection is therefore structured as to test the null hypothesis that  $E\{\phi\} = \mu_{(0)}$  with a known covariance matrix  $\Sigma_{(0)}$ . According to the theorem 3.3.3 of [4], the vector  $(\phi - \mu_{(0)})^T \Sigma_{(0)}^{-1} (\phi - \mu_{(0)})$  is distributed according to the  $\chi^2$ -distribution with  $m$  degrees of freedom. Let  $\chi^2(\alpha)$  be the number such that

$$\text{Prob}\{\chi^2 \geq \chi^2(\alpha)\} = \alpha \quad (1)$$

Thus

$$\text{Prob} \{ (\varphi - \mu_{(0)})^T \Sigma_{(0)}^{-1} (\varphi - \mu_{(0)}) \geq \chi^2(\alpha) \} = \alpha \quad (2)$$

To test the null hypothesis that  $E\{\varphi\} = \mu_{(0)}$ , the following **decision rule** is used. If

$$(\varphi - \mu_{(0)})^T \Sigma_{(0)}^{-1} (\varphi - \mu_{(0)}) \geq \chi^2(\alpha) \quad (3)$$

then the null hypothesis  $E\{\varphi\} = \mu_{(0)}$  is rejected and the circuit under testing is faulty with probability  $(1 - \alpha)$ . When the result in (3) is less than  $\chi^2(\alpha)$ , the circuit under testing is fault-free with probability  $(1 - \alpha)$ . The probability of falsely rejecting a fault-free circuit is  $\alpha$ . The choice of the value of  $\alpha$  normally depends on the application and the cost of rejecting a good circuit.

To evaluate the result of (3), the inverse of matrix  $\Sigma_{(0)}$  must be calculated. In some cases, the matrix  $\Sigma_{(0)}$  may be singular because of the linear dependence of the components of the  $I_{PS}$  spectrum. For such a singular case, the eigenvalue decomposition of the covariance matrix  $\Sigma_{(0)}$  is used.

The complete fault detection procedure is now summarized in Fig. 4. Fig. 4a describes the procedure for calculating the signature ( $\mu_{(0)}$  and  $\Sigma_{(0)}$ ) of the fault-free circuit during the design phase by means of Monte-Carlo simulations. Fig. 4b describes the procedure of fault detection executed during the testing phase.

free circuit is from the  $I_{PS}$  spectrum distributions of all types of faulty circuits. One technique to improve the fault coverage rate is to select a suitable input stimulus for the given circuit. A method for this is now presented below.

Suppose that there are  $K$  types of faults in the circuit that are likely to occur during the fabrication or field use of the integrated circuit. Denote by  $\mu_{(k)}$  and  $\Sigma_{(k)}$ ,  $k = 1, \dots, K$ , the mean vector and covariance matrix of the RMS value and the harmonics of the  $I_{PS}$  spectrum for the  $k$ th type of faulty circuit. In order to justify if a given input stimulus is suitable for fault detection, a sequence of hypothesis testing must be carried out for this input stimulus :

$$H_0: \mu_{(0)} = \mu_{(k)} \quad \Sigma_{(0)} = \Sigma_{(k)} \quad (4)$$

If the hypothesis  $H_0$  is accepted for any type of faulty circuit, this input stimulus is not suitable. Since the hypothesis testing procedure is repeated for every type of faulty circuit and for all attempts of input signal selection, and a computationally expensive transient circuit simulation is needed for every random sample of the circuit, the sampling size of hypothesis testing is better kept at a reasonably small level, so as to reduce the computational cost of the input stimulus selection procedure.

Let  $\varphi_{(0)}^1, \varphi_{(0)}^2, \dots, \varphi_{(0)}^{N_0}$  be the samples of  $\varphi \in R^m$  (the RMS value and the harmonics of the  $I_{PS}$  spectrum) for the fault-free circuit, and  $\varphi_{(k)}^1, \varphi_{(k)}^2, \dots, \varphi_{(k)}^{N_k}$  be the samples of vector  $\varphi$  for the  $k$ th type of faulty circuit. For the  $k$ th type of faulty circuit, the likelihood ratio criterion for the hypothesis test of  $H_0$  is [4] :

$$\lambda = \left( \frac{|A_0|^{N_0} |A_k|^{N_k} (N_0 + N_k)^{m(N_0 + N_k)}}{N_0^{mN_0} N_k^{mN_k} |B|^{(N_0 + N_k)}} \right)^{1/2} \quad (5)$$

where

$$\begin{aligned} A_0 &= \sum_{i=1}^{N_0} (\varphi_{(0)}^i - \overline{\varphi_{(0)}}) (\varphi_{(0)}^i - \overline{\varphi_{(0)}})^T \\ A_k &= \sum_{i=1}^{N_k} (\varphi_{(k)}^i - \overline{\varphi_{(k)}}) (\varphi_{(k)}^i - \overline{\varphi_{(k)}})^T \\ B &= \sum_{j=0, k} \left( \sum_{i=1}^{N_j} (\varphi_{(j)}^i - \overline{\varphi_{(j)}}) (\varphi_{(j)}^i - \overline{\varphi_{(j)}})^T \right) \end{aligned}$$

The critical region for this likelihood ratio test consists in rejecting the null hypothesis if  $\lambda$  is less than a predetermined constant. The resulting procedure for the input stimulus selection for analog fault detection is then summarized in Fig. 5.

## 5. Experimental results

Experimental results have been carried out to test the effectiveness of the fault detection method discussed above. A large number of circuits under testing, including both the fault-free and the derived faulty circuits, were simulated by Monte-Carlo analysis with tolerances on all parameters of the devices in the circuit. Measurements are obtained from the simulated power-supply current followed by a discrete frequency transformation.

**Fig. 4. Overview of the procedure for analog fault detection by monitoring the power-supply current.**

### 4. The method to select the input signal

The effectiveness of the above technique strongly depends on how separable the  $I_{PS}$  spectrum distribution of the fault-

### 5.1. Example 1: CMOS operational amplifier

The schematic of this opamp is shown in Fig. 1. The faults considered are the most frequent faults for MOS devices listed in [5]: gate to drain short, gate to source short, drain contact open and source contact open. This results in 30 possible types of hard faults in this circuit. To construct the signature of the fault-free circuit, a 1000-sample Monte-Carlo analysis is performed with tolerances of  $3\sigma = 10\%$  on the model parameters and 5% on the geometric sizes of the MOS transistors. The selected input stimulus is a periodical pulse signal with a period of 200 ns and a peak-to-peak value of 200 mV. The CPU time for the Monte-Carlo simulations is 15'31" on an HP workstation. Since this procedure is carried out in the design stage and the cost of on-line calculation in the testing stage is very small, this calculation cost is acceptable. In addition, for every type of faulty circuit 500 statistical samples have been considered. The total of 16000 resulting circuits have been taken as the circuits under test (CUT). For a value of  $\alpha = 0.05$ , the results of fault detection are given in Table 2. The error rate of a fault-free circuit being considered as a faulty one (ERR1) is 0.85% and the error rate of a faulty circuit being considered as a good one (ERR2) is 0.04%.

2. The error rate of a fault-free circuit being considered as a faulty one (ERR1) is 0.85% and the error rate of a faulty circuit being considered as a good one (ERR2) is even zero.

These results show that the proposed discrimination method based on monitoring the spectrum of the power-supply current is efficient and very effective for the detection of hard faults in both small and large analog CMOS circuits. Similar results have been obtained for bipolar circuits.

circuit	opamp	bandpass filter
nr of CUT	16000	8000
nr of good circuits in CUT	1000	1000
nr of faulty circuits in CUT	15000	7000
nr of possible types of faults	30	14
fault detection ERR1	0.85%	0.85%
fault detection ERR2	0.04%	0.00%

TABLE 2. The simulated fault detection results.

## 6. Conclusions

By monitoring the spectrum of the power-supply current and by applying a probabilistic decision rule that fully takes into account the effect of the tolerances on the circuit parameters, it has been shown that most hard faults (both DC and AC) can be detected in analog circuits. The on-line calculation is small and no extra testing pin is needed, so that the algorithm can be used for final production testing as well as for wafer-probe testing. A significant difference between the spectrum of the fault-free and the faulty circuits can be obtained by a careful selection of the input stimulus, for which a systematic procedure has been proposed. By using analog-to-digital conversion and digital signal processing techniques, the method can be used for built-in self-testing.

## References

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Fig. 5. Procedure for the selection of the input stimulus.

### 5.2. Example 2: Active bandpass filter

The second example is the 2nd-order active bandpass filter of Fig. 3, of which most data were already presented in section 2. We assume that the opamp itself has already been tested and that we are now testing for possible opens and shorts of all resistors and capacitors in the filter. The selected input signal is a sinusoidal with an amplitude of 50 mV and a frequency of 25 kHz. To construct the signature, a Monte-Carlo simulation procedure with 1000 samples is performed for the fault-free circuit in 20'43". The fault detection procedure has then been applied to these circuits together with 7000 faulty circuit samples. The results are also given in Table



