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**Published on:** 25 Jun 2008 - Mediterranean Conference on Control and Automation

**Topics:** State observer, Observer (quantum physics), Fuzzy control system, Fault detection and isolation and Robust control

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***Abstract*— This paper addresses fault detection and isolation (FDI) problem using a sliding mode fuzzy observer on the basis of a uncertain Takagi-Sugeno (T-S) fuzzy model. First, a robust fuzzy observer with respect to the uncertainties is designed. The convergence of the fuzzy observer is performed by the search of suitable Lyapunov matrices. It is shown how to synthesis observers using a set of linear matrix inequalities (LMI) conditions. Once the fuzzy observer is designed, FDI problem for nonlinear systems described by T-S fuzzy systems using the fuzzy observer is presented. A bank of fuzzy observer is then designed in order to investigate fault diagnosis problems. The validity of the proposed methodology is illustrated on a dynamic vehicle model.**

## I. Introduction

The objective of fault diagnosis is not only to decide if a fault is present in a system (fault detection), but also to the determination of the kind and the location (fault isolation) or the characterization of the fault by some attributes (fault identification). In general, the task of fault detection and diagnosis is solved in two main steps: symptom generation step and diagnostic step. In the first step, certain quantities called symptoms are generated to indicate the state of the process, and then in the second step, the relation between symptoms and faults is established. Typically, this requires the selection of the most relevant symptoms, which are robust against noise, disturbances and standard changes of the system. Modern approaches are based on a process model and exploit the mathematical relations between different process signals. They enable a fine diagnosis but require deeper insight and understanding of the process and need much effort to develop, particularly for nonlinear and complex processes.

In the conventional model-based FDI schemes, which include parameter estimation, observers and parity space methods are based on the deviation between measured and estimated process states or outputs and nominal ones, leading to analytical symptoms [1][2][3][4]. Obviously, they require accurate mathematical models of the system. However, the task of establishing a mathematical description for complex nonlinear processes is often difficult and time consuming. In general, the nonlinear systems are firstly linearized at an operating point, and then robust techniques are applied to generate residuals, which are robust against limited parameter variations.

However, this assumption of linearity is checked only in a limited vicinity of a particular operating point. The Takagi-Sugeno (T-S) fuzzy model approach can apprehend the nonlinear behavior of a system, while keeping the simplicity of the linear models [5]. Indeed, the real physical systems are often nonlinear. As it is delicate to synthesize an observer for an unspecified nonlinear system, it is preferable to represent this system with a fuzzy model. The idea of the fuzzy model approach is to apprehend the global behavior of a system by a set of local models (linear or affine), each local model characterizing the behavior of the system in a particular zone of operation. The local models are then aggregated by means of an interpolation mechanism.

Recently, several research have exploited the fuzzy modelling approach for fault detection and isolation [4][6][7]. Park et al. [8] have presented the design of a robust adaptive fuzzy observer for uncertain nonlinear dynamic systems. In [9] authors have considered the design of sliding mode fuzzy observer for FDI.

In this paper, we propose a methodology for the diagnosis of dynamic nonlinear processes described by T-S models using fuzzy observers. Typically, the design of a T-S fuzzy observer requires a precise mathematical description of the plant under interest in the form of a T-S dynamic model, which includes both local linear models and activation functions. The local linear models are state space affine models that can be derived directly from first principle or from empirical models.

This paper is organized as follows: section 2 gives the general structure of the considered uncertain T-S fuzzy model. In section 3, the design of sliding mode fuzzy observers is treated. Section 4 gives a sensor and actuator FDI for dynamic vehicle model, which is represented by a T-S fuzzy model. Finally, a conclusion is given in section 5.

**Notation:** Throughout the paper, the following useful notation is used: $X > 0$ means that $X$ is a symmetric positive definite matrix, $\mathbb{I}_M = \{1, 2, ..., M\}$ and $\|.\|$ represents the Euclidean norm for vectors and the spectral norm for matrices.

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## II. TAKAGI-SUGENO FUZZY MODEL REPRESENTATION

The major motivation for the fuzzy modelling methodology is that local modelling is simpler than global modelling because locally there are less relevant phenomena, and interactions are simpler. Typically, this is done by dividing the full range of all possible operating conditions into several regimes where in each regime the system is represented by local linear models [5][10]. The different operating regimes can have either different local model structures (heterogeneous) or same local model structures (homogeneous). Obviously, it is assumed that the whole operating range of the system is completely covered by these regimes. Here, we consider using the following fuzzy uncertain dynamic model to represent a complex nonlinear system with unknown inputs, which includes both local analytic linear models and fuzzy membership functions:

$$\begin{cases} \dot{x} = \sum_{i=1}^{M} \mu_i(\xi) \Big( (A_i + \Delta A_i)x + B_i w + R_i \bar{w} + D_i \Big) \\ y = \sum_{i=1}^{M} \mu_i(\xi) C_i x \end{cases} \tag{1}$$

with: $\sum_{i=1}^{M} \mu_i(\xi) = 1 \quad \text{and} \quad 0 \leq \mu_i(\xi) \leq 1 \quad \forall i \in \mathbb{I}_M$

where $x \in \mathbf{R}^n$ is the state vector, $w \in \mathbf{R}^m$ the input vector, $\bar{w} \in \mathbf{R}^q$, $q < n$, contains the unknown inputs and $y \in \mathbf{R}^p$ the measured outputs. Matrices $A_i \in \mathbf{R}^{n \times n}$ and $B_i \in \mathbf{R}^{n \times m}$ denote the state matrix and the input matrix associated with the $ith$ local model. Matrices $R_i \in \mathbf{R}^{n \times q}$ are the distribution matrices of unknown inputs. $D_i \in \mathbf{R}^n$ is introduced to take into account the operating point of the system. At last, $\xi$ is the so-called decision vector which may depend on some subset of the known inputs and/or measured variables to define the operating regimes.

The matrices $\Delta A_i$ are unknown time-varying matrices with appropriate dimensions, which represent parametric uncertainties in the model. This kind of uncertainties is known as unmatched uncertainties. We also consider that the unknown input $\bar{w}$ are bounded.

$$\|\Delta A_i\| \leq \delta_i \quad \text{and} \quad \|\bar{w}\| \leq \rho \tag{2}$$

The activation functions $\mu_i(\xi)$ are not Boolean ones, then several local models are active at each time and the coefficients $\mu_i(\xi)$ $i \in \{1, ..., M\}$ quantify the relative contribution of each local model to the global model. The choice of the number $M$ of local models for that multiple model may be intuitively done by taking into account a certain number of operating regimes. Matrices $A_i$, $B_i$, $D_i$, $R_i$ and $C_i$ can be obtained by using the direct linearization of an a priori nonlinear model around operating points, or alternatively by using an identification procedure [11], [12], [13]. From a practical point of view, matrices $A_i$, $B_i$, $D_i$, $R_i$ and $C_i$ describe the system's local behaviour around the $ith$ regime.

## III. SLIDING MODE FUZZY OBSERVER

This section proposes sliding mode unknown input fuzzy observer (SMUIFO) based on a nonlinear combination of local unknown input observers. The proposed structure involves sliding terms allowing to compensate the uncertainties and the unknown inputs. The proposed sliding mode fuzzy observer of the T-S model (1) has the following form:

$$\begin{cases} \dot{\hat{x}} = \sum_{i=1}^{M} \mu_i(\xi) \Big( A_i \hat{x} + B_i w + D_i + G_i (y - \hat{y}) + \nu_i + \alpha_i \Big) \\ \hat{y} = \sum_{i=1}^{M} \mu_i(\xi) C_i \hat{x} \end{cases} \tag{3}$$

Let us note that $\nu_i$ and $\alpha_i$ can be considered as variables which compensate respectively the errors due to the unknown inputs and the model uncertainties. Their specific structures will be described further. Our objective is to design gain matrices $G_i$ and variables $\nu_i \in \mathbf{R}^n$ and $\alpha_i \in \mathbf{R}^n$, that guarantee the asymptotic convergence of $\hat{x}$ towards $x$.

To establish the conditions for the asymptotic convergence of the fuzzy observer (3), let us define the state and output estimation errors:

$$e = x - \hat{x} \tag{4a}$$

$$r_y = y - \hat{y} = \sum_{i=1}^{M} \mu_i(\xi) C_i e \tag{4b}$$

Using the equations (1) and (3), the dynamic of the state estimation error is:

$$\dot{e} = \sum_{i=1}^{M} \sum_{j=1}^{M} \mu_i(\xi) \mu_j(\xi) \Big( \bar{A}_{ij} e + \Delta A_i x + R_i \bar{w} - \nu_i - \alpha_i \Big) \tag{5}$$

$$\text{with:} \quad \bar{A}_{ij} = A_i - G_i C_j \tag{6}$$

**Theorem 1:** The error of state estimation (10) converges globally asymptotically to zero if there exists a symmetric positive definite matrix $P \in \mathbf{R}^{n \times n}$, matrices $W_i \in \mathbf{R}^{n \times p}$ and positive scalars $\beta_1$, $\beta_2$ and $\beta_3$ satisfying the following conditions for all $i, j \in \mathbb{I}_M$:

$$\begin{bmatrix} A_i^T P + P A_i - C_i^T W_j^T - W_j C_i + (\beta_2 \delta_i^2 + \beta_3) I & P \\ P & -\beta_1 I \end{bmatrix} < 0 \tag{7}$$

The gains $G_i$ and the terms $\nu_i$ and $\alpha_i$ of the fuzzy observer (3) are given by the following equations:

$$\begin{cases} \text{If } r_y \neq 0 & \begin{cases} \nu_i = \rho^2 \beta_3^{-1} \dfrac{\|P R_i\|^2}{2\, r_y^T r_y} P^{-1} \sum_{j=1}^{M} \mu_j(\xi) C_j^T r_y \\ \alpha_i = \beta_1 (1 + \beta_2) \delta_i^2 \dfrac{\hat{x}^T \hat{x}}{2\, r_y^T r_y} P^{-1} \sum_{j=1}^{M} \mu_j(\xi) C_j^T r_y \end{cases} \\ \text{If } r_y = 0 & \begin{cases} \nu_i = 0 \\ \alpha_i = 0 \end{cases} \end{cases} \tag{8}$$

$$G_i = P^{-1} W_i. \tag{9}$$

The proof of the asymptotic convergence of the T-S observer and also relaxed conditions can be found in [14].

In the case of common output matrix ($C_i = C$), we have

$$\dot{e} = \sum_{i=1}^{M} \mu_i(\xi) \left( \bar{A}_{ii} e + \Delta A_i x + R_i \bar{u} - \nu_i - \alpha_i \right) \qquad (10)$$

and it suffices to replace $j$ indices by $i$ in conditions (7).

## IV. Application to automatic steering of vehicle

### A. Vehicle Takagi-Sugeno model representation

Different models related to automatic steering of vehicle have been studied in the literature [15][16][17] [18]. Here, we have chosen to consider the coupling model of longitudinal and lateral motions of a vehicle. This model, already used in [15], is strongly nonlinear and is given by the following equations:

$$\dot{u} = vr - fg + \frac{(fk_1 - k_2)}{M} u^2 + c_f \frac{v + ar}{Mu} \delta + \frac{T}{M} \qquad (11a)$$

$$\dot{v} = -ur - \frac{(c_f + c_r)}{Mu} v + \frac{(bc_r - ac_f)}{Mu} r + \frac{c_f \delta + T\delta}{M} \qquad (11b)$$

$$\dot{r} = \frac{(bc_r - ac_f)}{I_z u} v - \frac{(b^2 c_r + a^2 c_f)}{I_z u} r + \frac{aT\delta + ac_f \delta}{I_z} \qquad (11c)$$

where, $u$, $v$ and $r$ are the longitudinal velocity, the lateral velocity and the yaw rate, respectively, $\delta$ is the steering angle, $T$ is the traction and/or braking force. Table 1 lists the parameters of the above vehicle model.

| | Parameters of the vehicle system | |
|---|---|---|
| $M$ | Mass of the full vehicle | 1480 $kg$ |
| $I_z$ | Moment of inertia | 2350 $kg.m^2$ |
| $g$ | Acceleration of gravity force | 9.81 $m/s^2$ |
| $f$ | Rotating friction coefficient | 0.02 |
| $a$ | Distance from front axle to CG[1] | 1.05 $m$ |
| $b$ | Distance from rear axle to CG | 1.63 $m$ |
| $c_f$ | Cornering stiffness of front tyres | 135000 $N/rad$ |
| $c_r$ | Cornering stiffness of rear tyres | 95000 $N/rad$ |
| $k_1$ | Lift parameter from aerodynamics | 0.005 $Ns^2/m^2$ |
| $k_2$ | Drag parameter from aerodynamics | 0.41 $Ns^2/m^2$ |

The nonlinear vehicle dynamics can be written as follows:

$$\dot{x}(t) = F(x(t),\ w(t)) \qquad (12a)$$

$$y(t) = Cx(t) \qquad (12b)$$

with

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (13)$$

where $F$ is a nonlinear function of the state vector $x = [u \;\; v \;\; r]$, $w$ gathers the two inputs $\delta$ and $T$ and $y(t)$ gathers the two inputs $y_1 = u$ and $y_2 = r$. As it is delicate to synthesize an observer for a nonlinear system, we preferred to represent this system with a T-S fuzzy model. Then, we propose to linearize the nonlinear model (12) around some operating points $[x^{(i)} \;\; w^{(i)}]$. Next, we integrate the set of the linear models in a T-S fuzzy model [5]. The proposed T-S model is described as follows [19]:

$$\dot{x} = \sum_{i=1}^{N} \mu_i(y_1)(A_i x + B_i w + D_i) \qquad (14a)$$

$$A_i = \left.\frac{\partial F}{\partial x}\right|_{\substack{x=x^{(i)} \\ w=w^{(i)}}} \quad B_i = \left.\frac{\partial F}{\partial w}\right|_{\substack{x=x^{(i)} \\ w=w^{(i)}}} \qquad (14b)$$

$$D_i = F(x^{(i)},\ w^{(i)}) - A_i x^{(i)} - B_i w^{(i)} \qquad (14c)$$

The previous model (14) has been established on the basis of the nonlinear model (11) considering that the different model parameters are perfectly known. In fact, some parameters are uncertain. It is particularly true for the cornering stiffness coefficients $c_f$ and $c_r$. These uncertainties can be modelled as bounded additive perturbations:

$$c_f = c_{f0} + \Delta c_f \;\text{ and }\; c_r = c_{r0} + \Delta c_r \qquad (15)$$

with $|\Delta c_f| < d_f$ and $|\Delta c_r| < d_r$. Therefore, these uncertainties are taken into account in the considered model which is now written as:

$$\begin{cases} \dot{x} = \sum_{i=1}^{3} \mu_i(y_1) \left( (A_i + \Delta A_i) x + B_i w + D_i \right) \\ y = C x \end{cases} \qquad (16)$$

Three local models were chosen for this application. This number gives a good compromise between the quality of the obtained model and its complexity. The membership functions which are triangular as shown figure 1 only depend on the longitudinal velocity $u$.

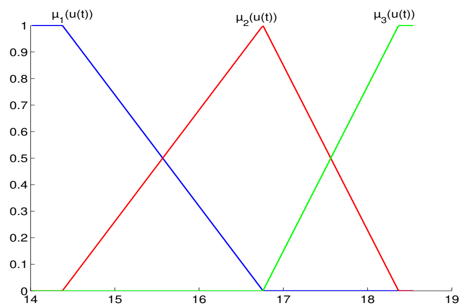


Fig. 1. Membership functions

The model uncertainties are such that:

$$\Delta A_{i,(j,k)} = \theta A_{i,(j,k)} \eta \quad j,k \in \{1,3\} \text{ and } i \in \{1,3\}$$

where $A_{i,(j,k)}$ denotes the $(j,k)$th element of $A_i$ and $\theta = 0.1$. The function $\eta(t)$ is a piece-wise constant function which magnitude is uniformly distributed on the interval $[0 \;\; 1]$. Its time evolution is depicted on figure 2.

The numerical values of the different matrices $A_i$, $B_i$, $D_i$ and $C$ are:

$$A_1 = \begin{bmatrix} -0.052 & 0.403 & 0.239 \\ -0.366 & -10.82 & -13.743 \\ 0.728 & 0.388 & -11.890 \end{bmatrix} \quad B_1 = \begin{bmatrix} 10.99 & 7 \times 10^{-4} \\ 91.216 & -10^{-4} \\ 60.319 & 0 \end{bmatrix}$$

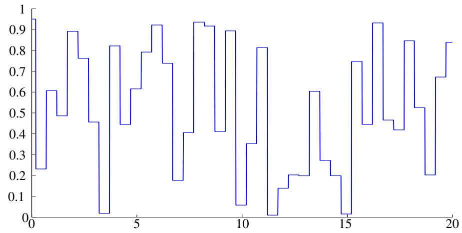


Fig. 2. Piece-wise constant function $\eta$

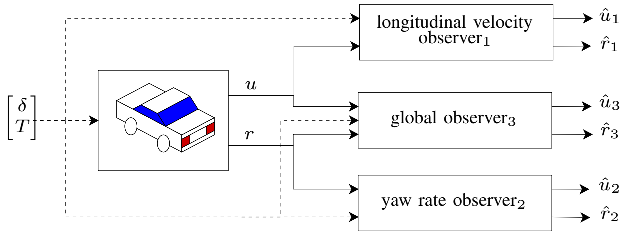


Fig. 3. Block diagram of the banc observer-based FDI

$$A_2 = \begin{bmatrix} -0.085 & 2.895 & 1.925 \\ -0.989 & -9.282 & -16.213 \\ 0.507 & 0.333 & -10.198 \end{bmatrix} \quad B_2 = \begin{bmatrix} 3.359 & 7\times 10^{-4} \\ 91.216 & 3\times 10^{-4} \\ 60.319 & 2\times 10^{-4} \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -0.031 & 2.065 & 0.693 \\ -1.141 & -8.468 & -17.870 \\ 0.441 & 0.303 & -9.303 \end{bmatrix} \quad B_3 = \begin{bmatrix} 1.548 & 7\times 10^{-4} \\ 91.216 & 2\times 10^{-4} \\ 60.319 & 1\times 10^{-4} \end{bmatrix}$$

$$D_1 = \begin{bmatrix} -0.832 \\ 5.259 \\ -10.46 \end{bmatrix} \quad D_2 = \begin{bmatrix} 0.087 \\ 16.562 \\ -8.496 \end{bmatrix} \quad D_3 = \begin{bmatrix} 0.392 \\ 20.951 \\ -8.092 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

## V. Fault Detection and Isolation for Vehicle Model

The objective of this part is to generate residuals that reflect the faults acting on the system (16). An ideal residual signal should remain zero in the fault-free case and non-zero when fault occurs. Once a fault has been detected, it must be estimated. The fault estimation will specify the type of fault, its duration, its amplitude and eventually its probable evolution. In the literature, there are several fault detection techniques. They are generally based on the change detection of the average and the variance. In this FDI study, we will not deal with the detection thresholds of residuals. We will confine ourselves only to the detection and localization of sensor and actuator faults taking into account the uncertainties modelling.

### A. Sensor fault detection and isolation

In order to identify the sensor fault, we consider that the actuators are faultless ($\bar{w} = 0$) while the output vector $y$ is corrupted by the sensor fault $\Delta y$. Then the system (16) becomes:

$$\left\{ \begin{array}{rcl} \dot{x} & = & \sum_{i=1}^{M} \mu_i(y_1) \Big( (A_i + \Delta A_i)x + B_i w + D_i \Big) \\ y & = & Cx + \Delta y \end{array} \right. \tag{17}$$

Three fuzzy observers are designed, one based on the longitudinal velocity observer $y_1 = u$, the second based on the yaw rate $y_2 = r$ and the last is based on the two outputs $u$ and $r$.

Using the numerical values of state matrices $A_i$ and output matrix $C$, we can easily checked that the following observability conditions are satisfied.

$$\forall\, i \in \{1,\,2,\,3\} \text{ and } j \in \{1,\,2\}, \quad \operatorname{rank}(A_i,\, C(j,:)) = 3$$

which implies that it is possible to estimate the state through either the first output $u$ $(y_1)$ or the second one $r$ $(y_2)$.

The sensor fault detection and localization is based on the analysis of the residuals $r_{y_{ik}} = y_i - \hat{y}_{i,k}$, with $k \in \{1, 2, 3\}$, generated by three observers and $i \in \{1,\,2\}$ The three observers, diagrammed in figure 3, depend on two inputs $\delta$ and $T$ applied to the system (11). The longitudinal velocity observer$_1$ and the yaw rate observer$_2$ use respectively only one output $u$ and $r$. The global observer$_3$ uses two outputs $u$ and $r$.

It is important to note that the implementation of this sliding mode fuzzy observer induces a practical problem: when the estimation error $r_y$ tends towards zero, the magnitude of $\alpha_i$ may increase without bound. This problem is overcome as follows:

$$\left\{ \begin{array}{lll} \text{If } \|r_y\| \geq \varepsilon & \Rightarrow & \alpha_i = \beta_1 (1+\beta_2)\, \delta_i^2 \dfrac{\hat{x}^T \hat{x}}{2\, r_y^T r_y} P^{-1} C^T r_y \\ \text{If } \|r_y\| < \varepsilon & \Rightarrow & \alpha_i = 0 \end{array} \right.$$

The terms $\alpha_i$ are fixed to zero when the output estimation error is such that $\|r_y\| \leq \varepsilon$, where $\varepsilon$ is a threshold chosen by the user. In this case, the estimation error cannot converge to zero asymptotically but to a small neighborhood of zero depending on the choice of $\varepsilon$. For this example, we fixed $\varepsilon$ at $10^{-3}$.

Figures 4 and 5 show the additive signals that represent sensor failures, the first one has been added to sensor 1 output $y_1$ between 5 and 10s, and the second one has been added to sensor 2 output $y_2$ between 13s and 18s.

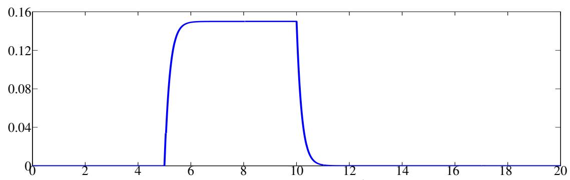


Fig. 4. Sensor failure $\Delta y_1$

*1) FDI using global observer$_3$:* the simulation results of the fault detection and isolation based on the global observer$_3$ are illustrated on the figures 6 and 7. The residuals $(u - \hat{u}_3)$ and $(r - \hat{r}_3)$ (see figures 6 and 7) show only the moment of the appearance and disappearance of sensor faults without being able to locate the fault. So, there is an instantaneous fault detection at time of appearance $5s$ and disappearance

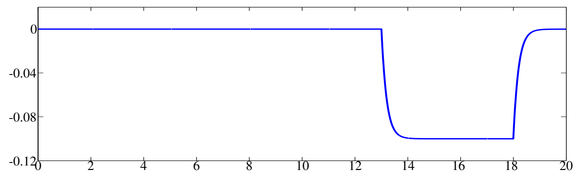

Fig. 5. Sensor failure $\Delta y_2$

$10s$ of fault. Between this two times $]5s \;\; 10s[$ there is a non detection of fault. We can conclude that this are derivator residues because they was not well conceived.

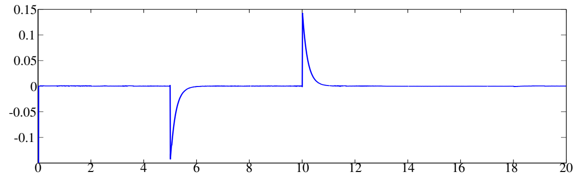

Fig. 6. $u - \hat{u}_3$ using global observer$_3$

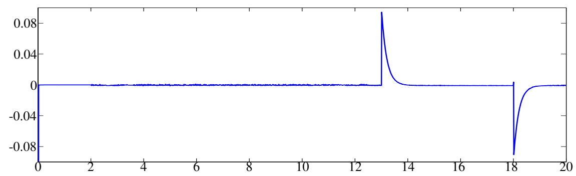

Fig. 7. $r - \hat{r}_3$ using global observer$_3$

*2) FDI using longitudinal velocity observer$_1$:* the simulation results of the fault detection and isolation based on the longitudinal velocity observer$_1$ are illustrated on the figures 8, 9, 10 and 11. The residuals $(u - \hat{u}_1)$ and $(r - \hat{r}_1)$ (see figures 9-11) generated by the observer$_1$ allow to detect and locate the fault sensor on the yaw rate output $r$. The fault detection and localization is possible by this longitudinal velocity observer$_1$, because this observer does not depend on the faulty output .

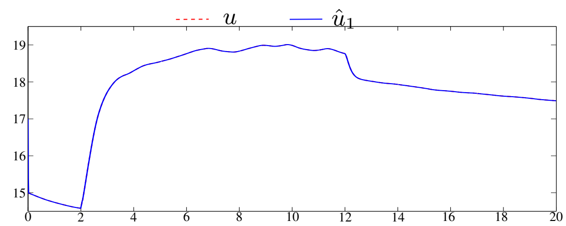

Fig. 8. $u$ and $\hat{u}_1$ using longitudinal velocity observer$_1$

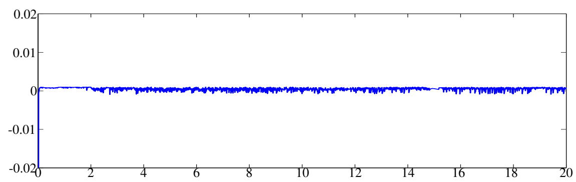

Fig. 9. $u - \hat{u}_1$ using longitudinal velocity observer$_1$

## B. Actuator fault detection and isolation

In this section, an unknown input sliding mode fuzzy observer (3) is proposed as a method for actuator fault detection and isolation (localization). We consider that the sensors are faultless ($\Delta y = 0$). The figure 12 shows the additive signal that represent the actuator failure $\Delta\delta$ in the steering angle $\delta$ between 7 and 15s. The T-S fuzzy model (11) described the vehicle model (14) becomes as follows:

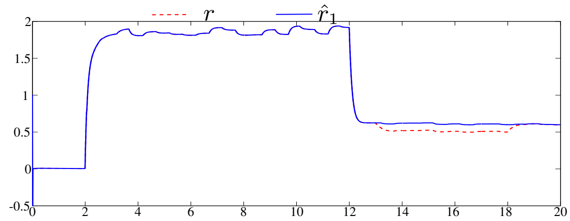

Fig. 10. $r$ and $\hat{r}_1$ using longitudinal velocity observer$_1$

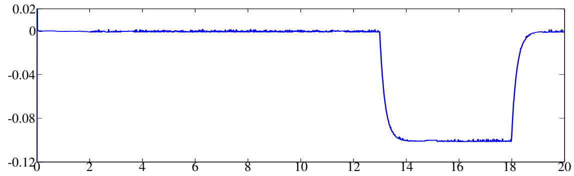

Fig. 11. $r - \hat{r}_1$ using longitudinal velocity observer$_1$

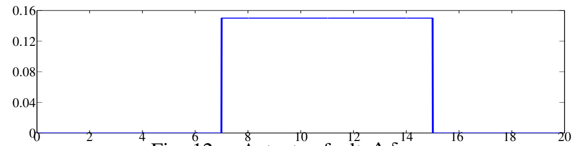

Fig. 12. Actuator fault $\Delta\delta$

$$\dot{x} \;=\; \sum_{i=1}^{N} \mu_i\,(y_1)\left((A_i + \Delta A_i)x + B_i \begin{bmatrix} \delta + \Delta\delta \\ T \end{bmatrix} + D_i\right) \quad (18)$$

In order to develop an actuator fault detection and localization method, a sliding mode unknown input fuzzy observers can be used. The first idea most obvious is to develop an sliding mode observer without inputs $\delta$ and $T$. The structure of the observer is given by the following equations:

$$\begin{cases} \dot{\hat{x}} = \displaystyle\sum_{i=1}^{M} \mu_i\,(y_1)\left(A_i\hat{x} + D_i + G_i\,(y - C\hat{x}) + \nu_i + \alpha_i\right) \\ \hat{y} = C\hat{x} \end{cases} \quad (19)$$

where $\nu_i$, $\alpha_i$ and $G_i$ are given by (8) and (9).

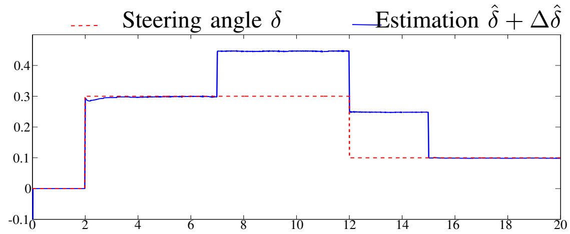

Fig. 13. $\hat{\delta} + \Delta\hat{\delta}$ using unknown input fuzzy observer (19)

It is easy to notice on the figures 13-15 that the actuator of the steering angle is faulty. Indeed, the difference between

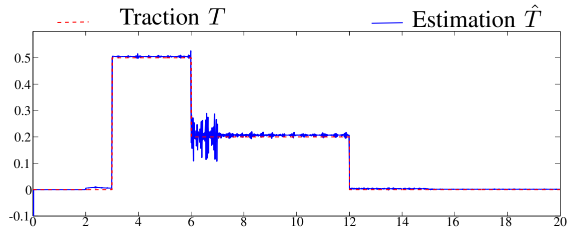


Fig. 14. $\hat{T}$ using unknown input fuzzy observer (19)

the two plots in Figure 13 shows the effect of actuator fault $\Delta\delta$.

The actuator residual generation is obtained if the matrix $\sum_{i=1}^{M} \mu_i(y_1)B_i$ is of full column rank and if the input number is less than the output number of the system ($q \leq p$).

As the output estimation is based on the compensation of the two inputs $\delta + \Delta\delta$ and $T$ (considered unknowns) and model uncertainties $\Delta A_i$ by the sliding mode terms $\alpha_i$ and $\nu_i$, the unknown input estimation is given by considering the T-S model (18) and the sliding mode observer (19) as follows

$$\begin{bmatrix} \hat{\delta} + \Delta\hat{\delta} \\ \hat{T} \end{bmatrix} \approx (K^T K)^{-1} K^T \sum_{i=1}^{M} \mu_i(u) \left( G_i (y - \hat{y}) + \nu_i + \alpha_i \right) \quad (20)$$

with: $K = \left( \sum_{i=1}^{M} \mu_i(y_1) B_i \right)$.

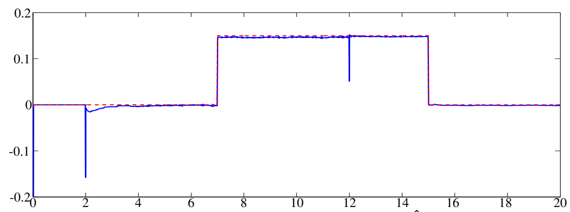

Fig. 15. $\Delta\delta$ and its estimate $\Delta\hat{\delta}$

Figure 16 shows the actuator fault estimation error $\Delta\delta - \Delta\hat{\delta}$. This error is mainly due to the coupling between the terms $\nu_i$ (compensation of model uncertainties) and $\alpha_i$ (compensation of the unknown input) through the estimation error $r_y$.

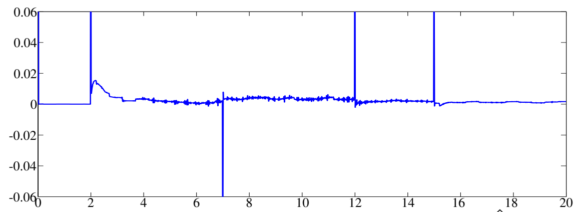

Fig. 16. Actuator fault estimation error: $\Delta\delta - \Delta\hat{\delta}$

## VI. Conclusion

In this work, we are concerned with the fault detection and isolation problem of an uncertain nonlinear system represented by a Takagi-Sugeno fuzzy model. The strategy used is based on the sliding mode unknown input fuzzy observer designed by the resolution of a set LMI conditions. Then detection and isolation of sensor and actuator faults are considered based on the synthesized sliding mode unknown input fuzzy observers.

The validity of the proposed FDI approach has been carried out on a vehicle dynamic model represented by a Takagi-Sugeno fuzzy model taking account parametric uncertainties.

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