Fault Tolerant Control of a Quadrotor UAV using Sliding Mode Control

Farid Sharifi, Mostafa Mirzaei, Brandon W. Gordon, and Youmin Zhang

***Abstract*—In this paper, the sliding mode approach is used to control of a quadrotor unmanned aerial vehicle (UAV) in the presence of external disturbance and actuator fault. Fault detection unit can detect the actuator fault using a state estimator. Then it reconfigures the structure of controller such that some control performance is achieved. The proposed control structure has the advantage of disturbance rejection in the fault-free condition. Moreover it can recover some of control performances when a fault occurs. Different simulations have been carried out to show the performance and effectiveness of the proposed method.**

I. Introduction

Unmanned aerial vehicles (UAVs) have gained increasing interest in recent years because of a wide area of possible applications such as security, traffic surveillance, management of natural risks, environment exploration, agriculture, and military. For these applications, the ability of helicopters to take off and land vertically, to perform hover flight, as well as their agility, make them ideal vehicles.

One type of aircraft with a strong potential for both indoor and outdoor flight is the rotorcraft and the so-called quadrotor helicopter has been chosen by many researchers as the most promising vehicle [1]-[3]. A quadrotor is an aircraft that is lifted and propelled by four rotors. Control of quadrotor can be achieved by varying the relative speed of each rotor to change the thrust and torque produced by each. Quadrotors are classified as rotorcraft, as opposed to fixed-wing aircraft, because their lift is derived from four rotors. The use of four rotors allows each individual rotor to have a smaller diameter than the equivalent single-rotor helicopter, allowing them to store less kinetic energy during flight and thus reduces the damage caused by the rotors hitting any objects. By enclosing the rotors within a frame, the rotors can be protected during collisions [4].

Different control methods mostly designed for quadrotor UAV are feedback linearization [5]-[7], backstepping [8], and sliding mode control [8], [9]. Some other methods also used for linearized model of quadrotor in literature such as PID [10] and LQR control [11]. In this paper, sliding mode has chosen as controller because of its insensitivity to the model errors, parametric uncertainties and external disturbances. This sliding mode controller is able to make the quadrotor reach a desired height with desired angles.

Fault tolerant control system (FTCS) is a control system with the ability to tolerate faults automatically and continue its operation in the event of a failure in some of its components [12]. FTCS can be classified into two different types which are known as Passive FTCS and Active FTCS. Passive FTCS can tolerate a predefined set of faults by using a specially-designed fixed controller while active FTCS relies on fault detection and diagnosis (FDD) process to monitor system performance and to detect and identify faults in the system and the controller is reconfigured on-line and in real-time [12-14]. Modern UAVs need to be designed to achieve the desired performance under both normal and fault conditions. A relatively small amount of existing research has addressed the FTCS for quadrotor UAVs. Some of few works addressing this issue are multi-observer switching strategy [15], Thau observer for systems with sensor partial failures [16], and gain scheduling based PID controller [17]. In this paper, with the aid of an observer, partial faults in actuator can be detected and then controller reconfigures in such a way to maintain stability and performance of the faulty system.

This paper is organized as follows. In Section II, the nonlinear model of a quadrotor UAV and dynamic of actuators are presented. The sliding mode control for the quadrotor is designed in Section III. The proposed fault detection and fault tolerant control strategy is described in Section IV. Section V is devoted to the presentation of the simulation results obtained for various fault-free situations and fault scenarios when the proposed scheme is applied to the quadrotor UAV. Finally, conclusion is provided in Section VI.

II. Model of the Quadrotor UAV

A. Nonlinear model

In this section, the general dynamic model of a quadrotor UAV has been studied. In lightweight flying systems, the dynamic model ideally includes the gyroscopic effects resulting from both the rigid body rotation in space, and the four propeller's rotation.

The dynamic model is derived using Euler-Lagrange formalism. A body-fixed frame B and the earth-fixed frame E are assumed to be at the center of gravity of the quadrotor UAV, where the z-axis is pointing upwards, as seen in Figure 1. The position of the quadrotor UAV in earth frame

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F. Sharifi, M. Mirzaei, B. W. Gordon, and Y. M. Zhang are with the Department of Mechanical and Industrial Engineering, Concordia University, 1515 St. Catherine, Montreal, Quebec, H3G 1M8, Canada (e-mails: fa_shari@encs.concordia.ca, mo_mir@encs.concordia.ca, bwgordon@encs.concordia.ca, ymzhang@encs.concordia.ca).

is given by a vector (x, y, z). The orientation of quadrotor UAV that referred to as roll, pitch, and yaw is given by a vector (ϕ, θ, ψ) which measured with respect to the earth coordinate frame *E*.

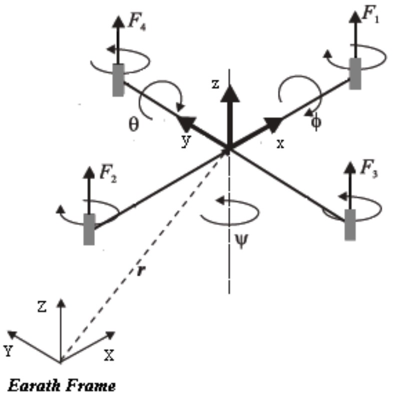

Fig. 1. The structure of quadrotor and its frames

The transformation of vectors from the body-fixed frame to the earth-fixed frame can be obtained based on Euler angles and the rotation matrix R_{EB}.

$$R_{EB} = \begin{bmatrix} C_\psi C_\theta & -S_\psi C_\phi + C_\psi S_\theta S_\phi & S_\psi S_\phi + C_\psi S_\theta C_\phi \\ S_\psi C_\theta & C_\psi C_\phi + S_\psi S_\theta S_\phi & -C_\psi S_\phi + S_\psi S_\theta C_\phi \\ -S_\theta & C_\theta S_\phi & C_\phi C_\theta \end{bmatrix} \quad (1)$$

where the abbreviations $S_{(\cdot)}$ and $C_{(\cdot)}$ have been used for $\sin(\cdot)$ and $\cos(\cdot)$, respectively. It is important to note that $R_{EB} = R_{BE}^T$.

The trust force generated by rotor i, $i = 1, 2, 3, 4$, is $F_i = b\omega_i^2$ where b is the trust factor and ω_i is the speed of rotor i. In the body-fixed frame, the forces are defined as follows:

$$F_B = \begin{bmatrix} F_{xB} \\ F_{yB} \\ F_{zB} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^4 F_i \end{bmatrix}$$

In the earth-fixed frame, the forces can be described as:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = R_{EB}.F_B = (\sum_{i=1}^4 F_i) \begin{bmatrix} S_\psi S_\phi + C_\psi S_\theta C_\phi \\ -C_\psi S_\phi + S_\psi S_\theta C_\phi \\ C_\phi C_\theta \end{bmatrix}$$

Therefore, the equation of motion (x, y, z) in the earth-fixed frame are represented as:

$$m \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} F_x - K_1\dot{x} \\ F_y - K_2\dot{y} \\ F_z - mg - K_3\dot{z} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^4 F_i(C_\psi S_\theta C_\phi + S_\psi S_\phi) - K_1\dot{x} \\ \sum_{i=1}^4 F_i(S_\psi S_\theta C_\phi - C_\psi S_\phi) - K_2\dot{y} \\ \sum_{i=1}^4 F_i C_\phi C_\theta - mg - K_3\dot{z} \end{bmatrix} \quad (2)$$

where K_i is the drag coefficient. Note that these coefficients are negligible at low speed.

The quadrotor dynamic model describing the roll, pitch and yaw angles contains the gyroscopic effect resulting from the rigid body rotation, the gyroscopic effect resulting from the propeller rotation coupled with the body rotation, the actuators action and finally the drag effects. Using the Lagrangian method, quadrotor rotational dynamic model is as follows:

$$\begin{aligned} \ddot{\phi} &= (\dot{\theta}\dot{\psi}(I_y - I_z) - J\dot{\theta}\omega + lU_1)/I_x - K_4\dot{\phi} \\ \ddot{\theta} &= (\dot{\phi}\dot{\psi}(I_z - I_x) + J\dot{\phi}\omega + lU_2)/I_y - K_5\dot{\theta} \\ \ddot{\psi} &= (\dot{\phi}\dot{\theta}(I_x - I_y) + U_3)/I_z - K_6\dot{\psi} \end{aligned} \quad (3)$$

where K_i is the drag coefficient, and system's inputs U_1, U_2, U_3, U_4 are defined as follows:

$$\begin{aligned} U_1 &= b(\omega_4^2 - \omega_3^2) = F_4 - F_3 \\ U_2 &= b(\omega_2^2 - \omega_1^2) = F_2 - F_1 \\ U_3 &= d(\omega_1^2 + \omega_2^2 - \omega_3^2 - \omega_4^2) \\ U_4 &= b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) = \sum_{i=1}^4 F_i \end{aligned} \quad (4)$$

where b and d are the trust and drag factor respectively, and $\omega = \omega_4 + \omega_3 - \omega_2 - \omega_1$ is considered as a disturbance.

The relation between system's inputs and speed of rotors can be shown in the matrix form as follows:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -b & b \\ -b & b & 0 & 0 \\ d & d & -d & -d \\ b & b & b & b \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} = \Omega \cdot \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \quad (5)$$

Since Ω is nonsingular, it is obvious that for each U_i we can find appropriate $\omega_j^2, j = 1, \dots, 4$ while the other inputs, $U_k, k \neq i$, do not change. Finally, this model can be rewritten in the state-space form $\dot{x} = f(x, u)$ as follows:

$$\dot{x} = \begin{bmatrix} x_2 \\ (C_{x_7} S_{x_9} C_{x_{11}} + S_{x_7} S_{x_{11}}).U_4/m - K_1 x_2 \\ x_4 \\ (C_{x_7} S_{x_9} S_{x_{11}} - S_{x_7} C_{x_{11}}).U_4/m - K_2 x_4 \\ x_6 \\ -g + (C_{x_7} C_{x_9}).U_4/m - K_3 x_6 \\ x_8 \\ (x_{12} x_{10}(I_y - I_z) - Jx_{10}\omega + lU_1)/I_x - K_4 x_8 \\ x_{10} \\ (x_{12} x_8(I_z - I_x) - Jx_8\omega + lU_2)/I_y - K_5 x_{10} \\ x_{12} \\ (x_{10} x_8(I_x - I_y) + U_3)/I_z - K_6 x_{12} \end{bmatrix} \quad (6)$$

where $u^T = (U_1, U_2, U_3, U_4)$ is the vector of input variables and $x^T = (x, \dot{x}, y, \dot{y}, z, \dot{z}, \phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi})$, $x \in \Re^{12}$ is the vector of state variables.

B. Actuator dynamics

The thrust generated by each propeller is modeled using the following first-order system

$$F_i = K\frac{\omega_0}{s+\omega_0}u_i \tag{7}$$

where u_i is the input to the *i*-th actuator, ω_0 is the actuator bandwidth and K is a positive gain. A state variable, v_i, will be used to represent the actuator dynamics, which is defined as follows:

$$v_i = \frac{\omega_0}{s+\omega_0}u_i \tag{8}$$

III. Sliding Mode Control

In this section, a sliding mode controller is used to control the height and rotational angles of quadrotor.

Consider dynamic equation of roll angle

$$\ddot{\phi} = \dot{\theta}\dot{\psi}(\frac{I_y - I_z}{I_x}) - \frac{J}{I_x}\dot{\theta}\omega + \frac{l}{I_x}U_1 - K_4\dot{\phi} + g_1 \tag{9}$$

where U_1 is the control input, ϕ is the output of interest, $f_1 = -\frac{J}{I_x}\dot{\theta}\omega$ is the dynamic associated with disturbance which is unknown but assumed to be bounded by $\hat{f}_1$, and g_1 is the dynamic which is not exactly known but assumed to be bounded by $\hat{g}_1$. In other words, the nominal dynamic equation of roll angle is given as follows:

$$\ddot{\phi} = \dot{\theta}\dot{\psi}(\frac{I_y - I_z}{I_x}) + \frac{l}{I_x}U_1 - K_4\dot{\phi} \tag{10}$$

In order to have the system track $\phi(t) \equiv \phi_d(t)$, we define a sliding surface s as follows:

$$s_\phi = (\frac{d}{dt} + \lambda_\phi)z_\phi = \dot{z}_\phi + \lambda_\phi z_\phi \tag{11}$$

where $z_\phi = \phi_d - \phi = x_{7d} - x_7$. Then we have

$$\begin{aligned} \dot{s}_\phi &= \ddot{\phi} - \ddot{\phi}_d + \lambda_\phi \dot{z}_\phi \\ &= \dot{\theta}\dot{\psi}(\frac{I_y - I_z}{I_x}) + f_1 + \frac{l}{I_x}U_1 - K_4\dot{\phi} + g_1 - \ddot{\phi}_d + \lambda_\phi \dot{z}_\phi \end{aligned} \tag{12}$$

The best approximation $\hat{U}_1$ of a continuous control law that would achieve $\dot{s}_\phi = 0$ is thus

$$\begin{aligned} \hat{U}_1 &= \frac{I_x}{l}(-\dot{\theta}\dot{\psi}(\frac{I_y - I_z}{I_x}) + K_4\dot{\phi} + \ddot{\phi}_d - \lambda_\phi \dot{z}_\phi) \\ &= \frac{I_x}{l}(-x_{10}x_{12}I_1 + K_4 x_8 + \ddot{x}_{7d} - \lambda_\phi(\dot{x}_{7d} - x_8)) \end{aligned}$$

where $I_1 = (\frac{I_y - I_z}{I_x})$. In order to satisfy sliding reachability condition $(s_\phi \dot{s}_\phi < -\eta|s_\phi|)$ despite uncertainty on the dynamics, we add to $\hat{U}_1$ a discontinuous term across the surface $s_\phi = 0$

$$U_1 = \hat{U}_1 - k_1 sign(s_\phi) \tag{13}$$

Then we have

$$s_\phi \dot{s}_\phi = s_\phi(f_1 + g_1 - k_1 sign(s_\phi)) \le (\hat{f}_1 + \hat{g}_1 - k_1)|s_\phi|$$

So by letting

$$k_1 = \hat{f}_1 + \hat{g}_1 + \eta \tag{14}$$

the sliding condition will be satisfied. It means that by choosing Lyapunov function candidate as $V = \frac{1}{2}s_\phi{}^T s_\phi$, its time derivative is negative definite $(\dot{V} = s_\phi \dot{s}_\phi < -\eta|s_\phi|)$. Therefore, under the control (13) the system (9) will reach and thereafter stay on the manifold $s_\phi = 0$ in finite time.

Main drawback of sliding mode control is the chattering effect produced by the discontinuous function *sign*. To eliminate this, the *sign* function is replaced with a saturation function *sat* defined as follows:

$$sat(s) = \begin{cases} sign(s) & if \ |s| > \rho \\ s/\rho & if \ |s| < \rho \end{cases}$$

where ρ is a boundary layer around the sliding surface s.

The same steps are followed to extract U_2 and U_3 as follows:

$$\begin{aligned} U_2 &= \frac{I_y}{l}(-x_8 x_{12} I_2 + K_5 x_{10} + \ddot{x}_{9d} \\ &\quad - \lambda_\theta(\dot{x}_{9d} - x_{10})) - k_2 sat(s_\theta) \\ U_3 &= I_z(-x_8 x_{10} I_3 + K_6 x_{12} + \ddot{x}_{11d} \\ &\quad - \lambda_\psi(\dot{x}_{11d} - x_{12})) - k_3 sat(s_\psi) \end{aligned} \tag{15}$$

where

$$\begin{aligned} z_\theta &= x_{9d} - x_9 \quad , \quad z_\psi = x_{11d} - x_{11} \\ s_\theta &= \dot{z}_\theta + \lambda_\theta z_\theta \quad , \quad s_\psi = \dot{z}_3 + \lambda_\psi z_\psi \\ k_2 &= \hat{f}_2 + \hat{g}_2 + \eta \quad , \quad k_3 = \hat{g}_3 + \eta \\ I_2 &= (\frac{I_z - I_x}{I_y}) \quad , \quad I_3 = (\frac{I_x - I_y}{I_z}) \end{aligned}$$

The altitude control U_4 is obtained using the same approach described for rotational angles.

$$U_4 = \frac{m}{C_{x_7}C_{x_9}}(g + K_3 x_6 + \ddot{x}_{5d} - \lambda_z(\dot{x}_{5d} - x_6)) - k_4 sat(s_4) \tag{16}$$

where:

$$z_z = x_{5d} - x_5$$
$$s_z = \dot{z}_z + \lambda_z z_z$$
$$k_4 = \hat{g}_4 + \eta$$

After the control signals U_i are calculated, desired output of each actuator can be obtained using (4) and (5) as follows:

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} 0 & -1/2 & b/4d & 1/4 \\ 0 & 1/2 & b/4d & 1/4 \\ -1/2 & 0 & -b/4d & 1/4 \\ 1/2 & 0 & -b/4d & 1/4 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} \tag{17}$$

By knowing the dynamic of actuators (7), the control input of rotors u_i can be easily computed such that the outputs of actuators, F_i, follow the desired trajectory.

IV. Fault Detection and Fault Tolerant Control

The sliding mode control that described in the previous section is able to control a quadrotor in the presence of external disturbances. Although that method is completely effective in the normal mode, it is not able to control the system when a fault occurs. Two main sources of faults in the system are sensor's faults and actuator's faults. In this section we consider the effect of actuator's fault on the system performance.

During a long time mission it is very probable that one of rotors of a quadrotor becomes faulty and partially loses its effectiveness. It means that the rotor cannot produce its desired effectiveness any more but it is still working and is not completely shut down. If one uses a tool to detect this fault then it might be possible to choose a strategy to overcome this faulty condition and at least land the quadrotor safely.

Normally there are no sensors in quadrotor to measure the outputs of rotors. With the help of these sensors, it might be easy to determine whether a fault has been occurred in one of rotors. In the absence of such sensors, a state estimator can be used to estimate the outputs of rotors. We use a Luenberger linear state estimator for this purpose [18]. Although the dynamic model of quadrotor is nonlinear, we can use the linearizred model around the equilibrium point to design the linear state estimator. Since our objective is to estimate the outputs of actuators as the states of system, the linearized model of quadrotor has to include the actuator dynamics as well.

When the observer estimates the outputs of each rotors, the fault detection unit has to compare these amounts with the desired rotors outputs that calculated by system controller. If the difference between these two amounts is more than a threshold for a considerable period of time, it means that the rotor is in a faulty condition.

After fault detection unit realizes that the quadrotor has been faced with a partial loss in one of its rotors, it has to change the control strategy to mitigate the effect of fault. This control strategy has to be able to land the quadrotor safely. In faulty situation, we do not change the control signals U_i, but we modify transform matrix (17) to change the rotors trust forces F_i. For this purpose, we propose the algorithm of controller reconfiguration in the faulty condition which is shown in Figure 2.

The objective of this fault tolerant control strategy is to land the quadrotor horizontally $(roll = 0$ and $pitch = 0)$ when an actuator fault occurs. The zeros in the 3rd column of all matrices in Figure 2 mean that there is no control on the yaw angle in the faulty mode. The other difference between control strategies in the faulty mode and the normal mode is that the faulty rotor does not participate in the control of roll or pitch angles. However it still participates in the control of height. For instance, in the normal mode, signal U_2 that controls the pitch angle is equal to the difference of trust of the motor 1 and 2. But when there is a fault in the motor 1, it is possible that it cannot provide sufficient trust for control signal U_2. In order to resolve this problem, we omit the effect of the motor 1 in control signal U_2 and provide this control signal only with the motor 2. In fact, the relation between the control signals U_1 and U_2 and rotor outputs in the faulty condition is as follows:

$$\begin{aligned}
&\textit{If Motor}\,\#1\ \textit{is faulty} \quad U_2 = +F_2 \quad U_1 = F_4 - F_3 \\
&\textit{If Motor}\,\#2\ \textit{is faulty} \quad U_2 = -F_1 \quad U_1 = F_4 - F_3 \\
&\textit{If Motor}\,\#3\ \textit{is faulty} \quad U_1 = +F_4 \quad U_2 = F_2 - F_1 \\
&\textit{If Motor}\,\#4\ \textit{is faulty} \quad U_1 = -F_3 \quad U_2 = F_2 - F_1
\end{aligned}$$

But the relation between control signal U_4 and rotor outputs is still $U_4 = \sum_{i=1}^{4} F_i$

If Motor #1 is faulty

$$Set\ [F_1 \quad F_2 \quad F_3 \quad F_4] = \begin{bmatrix} 0 & 0 & 0 & .25 \\ 0 & 1 & 0 & .25 \\ -.5 & 0 & 0 & .25 \\ .5 & 0 & 0 & .25 \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

If Motor #2 is faulty

$$Set\ [F_1 \quad F_2 \quad F_3 \quad F_4] = \begin{bmatrix} 0 & -1 & 0 & .25 \\ 0 & 0 & 0 & .25 \\ -.5 & 0 & 0 & .25 \\ .5 & 0 & 0 & .25 \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

If Motor #3 is faulty

$$Set\ [F_1 \quad F_2 \quad F_3 \quad F_4] = \begin{bmatrix} 0 & -.5 & 0 & .25 \\ 0 & .5 & 0 & .25 \\ 0 & 0 & 0 & .25 \\ 1 & 0 & 0 & .25 \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

If Motor #4 is faulty

$$Set\ [F_1 \quad F_2 \quad F_3 \quad F_4] = \begin{bmatrix} 0 & -.5 & 0 & .25 \\ 0 & .5 & 0 & .25 \\ -1 & 0 & 0 & .25 \\ 0 & 0 & 0 & .25 \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

Fig. 2. The algorithm of controller reconfiguration for fault tolerant control

The schematic of fault detection and fault tolerant control is illustrated in Figure 3.

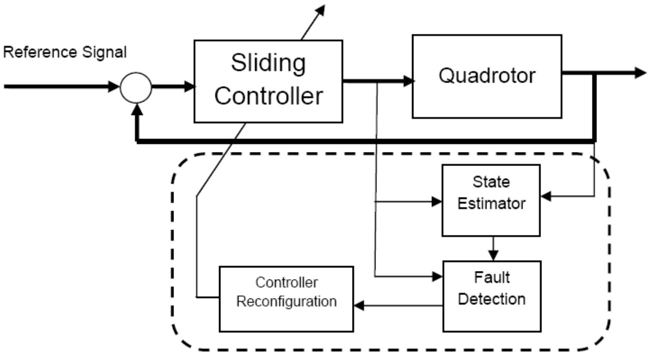

Fig. 3. The schematic of fault detection and fault tolerant control

V. Simulation Results

In order to show the performance of sliding mode control and effectiveness of fault detection and fault tolerant control we simulate a quadrotor UAV in MATLAB/SIMULINK® environment. The desired trajectories for angles and height are shown in Figure 4. In all simulations, a pulse of disturbance is inserted to the system in the interval of [3-6] seconds.

In the first scenario, all rotors works properly and there is no fault in the actuators. The outputs of system using sliding mode controller are shown in the Figure 5. In order to show the performance of sliding mode control in disturbance rejection, the output of system using a backstepping method [8] is also shown in Figure 5.

In the next scenario, after 7 seconds the rotor 1 faces with faulty condition. In this situation, it partially losses its trust and it can only produce half of its normal maximum trust. The outputs of system are shown in Figure 6. The outputs of fault detection unit are depicted in Figure 7. As we can see in Figure 7, the fault detection unit detects the faulty condition in rotor 1 almost immediately. After detection of the fault, the fault tolerant control unit changes the control strategy which enables the quadrotor to follow the desired trajectory for a safe landing. It is worth to mention that the fault detection unit is able to distinguish between external

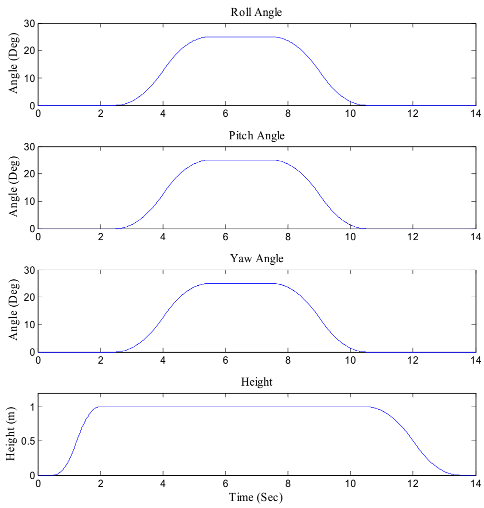

Fig. 4. The desired trajectories of angles and height of the quadrotor UAV

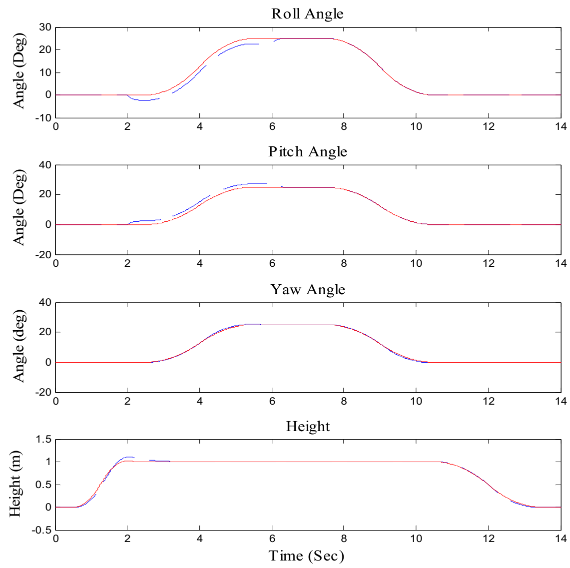

Fig. 5. The outputs of the quadrotor: solid line is sliding mode control, and dashed line is backstepping control

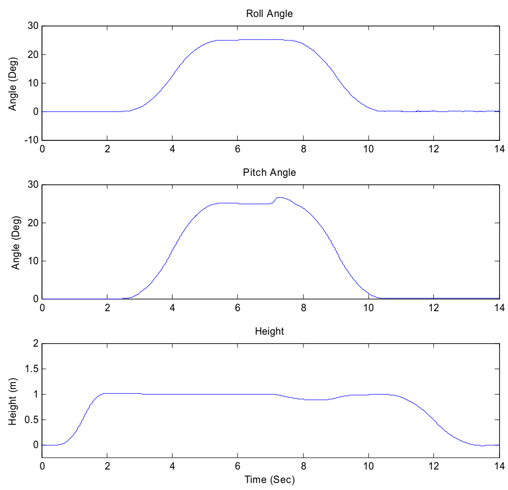

Fig. 6. The outputs of the quadrotor UAV in faulty situation

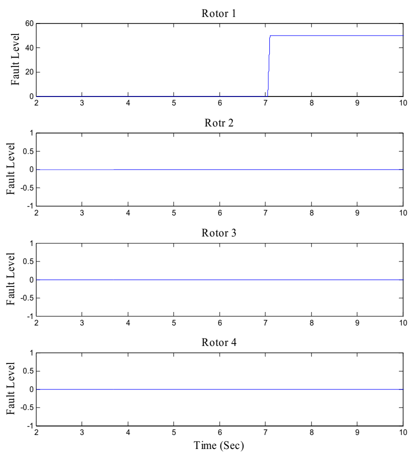

Fig. 7. The outputs of fault detection unit

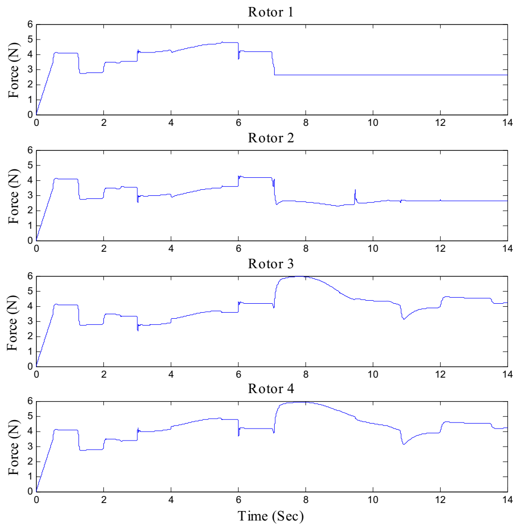

Fig. 8. The output of the actuators in faulty situation

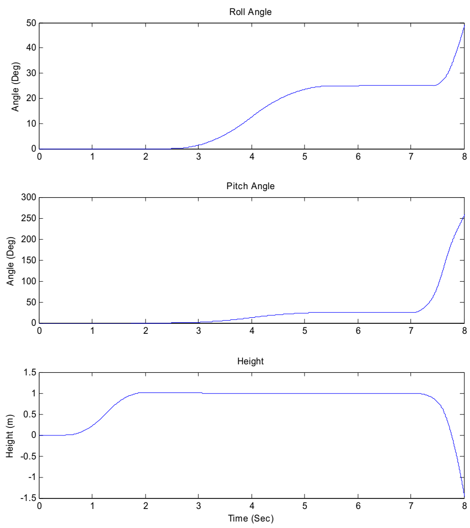

Fig. 9. The outputs of the quadrotor in faulty situation without using fault detection and fault tolerant control strategy

disturbance and actuator fault. It means that we can use the disturbance rejection ability of sliding mode control while using the advantage of fault detection and fault tolerant control strategy. The output of actuators in the faulty situation is depicted in Figure 8. As shown in this figure, the output of first motor has been faced a partial loss at time 7 second. Outputs of system in the faulty situation without using the proposed fault detection and fault tolerant control is also shown in Figure 9 for comparison.

VI. CONCLUSION

The sliding mode approach is able to control the quadrotor in the presence of external disturbance. When an actuator fault occurs, the fault detection unit can detect it using the state estimator. Then the control configuration unit that is a residual-based method using a linear observer modifies the structure of controller such that some of control performances are achieved. It has been shown that the fault detection unit can distinguish between actuator fault and external disturbance. So it is possible to use the advantage of disturbance rejection and fault detection and fault tolerant control at the same time. The simulation results verify the effectiveness of the proposed method.

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