Fault Tolerant Control System Design with Explicit Consideration of Performance Degradation

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A new approach is proposed for active fault tolerant control systems (FTCS), which allows one to explicitly incorporate allowable system performance degradation in the event of partial actuator fault in the design process. The method is based on model-following and command input management techniques. The degradation in dynamic performance is accounted for through a degraded reference model. A novel method for selecting such a model is also presented. The degradation in steady-state performance is dealt with using a command input adjustment technique. When a fault is detected by the fault detection and diagnosis (FDD) scheme, the reconfigurable controller is designed automatically using an eigenstructure assignment algorithm in an explicit model-following framework so that the dynamics of the closed-loop system follow that of the degraded reference model. In the mean time, the command input is also adjusted automatically to prevent the actuators from saturation. The proposed method has been evaluated using the lateral dynamics of an F-8 aircraft against actuator faults subject to constraints on the magnitude of actuator inputs. Very encouraging results have been obtained.

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I. INTRODUCTION

To design fault tolerant control systems (FTCS), one of the important issues to consider is whether to recover the original system performance/functionality completely or to accept some degree of performance degradation after occurrence of a fault. What are the consequences if the performance degradation is not taken into consideration and how to take such performance degradation into account in the design process?

Most of the earlier work on FTCS design is centered around the philosophy to recover the prefault system performance as much as possible [8, 10, 12, 14, 16]. In practice, however, as a result of an actuator fault, the degree of the system redundancy and the available actuator capabilities could be significantly reduced. If the design objective is still to maintain the original system performance, this may force the remaining actuators to work beyond the normal duty to compensate for the handicaps caused by the fault. This situation is highly undesirable in practice due to physical limitations of the actuators. The consequence of the so-designed FTCS may lead to actuator saturation, or worse still, to cause further damage. Therefore, trade-off between achievable performance and available actuator capability should be carefully considered in all FTCS designs. Designing an FTCS against actuator faults to achieve specified degraded performance without violating the actuator limits is therefore the main focus of the work presented here.

In a control system, there are two aspects of performance: dynamic and steady-state. In FTCS, both types should be considered as well. To represent the degradation in dynamic performance, one could use a performance-degraded reference model with a model-following control principle. In general, at least two different models: one for normal and one or more for impaired systems need to be used.

Furthermore, to avoid actuator saturation, adjustment to the system command input levels is often necessary in the event of actuator failure. One way to achieve this is through reference governor/management [1, 4, 5], or command limiting in the context of flight control [2, 9]. The command management can be designed separately from the feedback controller. An adjustment strategy has been proposed here to provide an appropriate command input at both the steady-state and during the initial period of controller reconfiguration.

The paper is organized as follows. Modeling of actuator faults, the concept of performance degradation in FTCS and the overall structure of the proposed FTCS are presented in Section II. A scheme for selecting a degraded reference model and a strategy for managing the command input in the presence of actuator faults are proposed in Section III. Detailed design process and associated algorithms are presented in Section IV. The performance assessment of designed FTCS for an aircraft example is presented in Section V followed by the conclusion in Section VI.

II. MODELING OF ACTUATOR FAULTS AND STRUCTURE OF PROPOSED FTCS

A. Modeling of Actuator Faults

Let's consider a system that is described by the following linear stochastic differential equation under the normal operation:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= A\mathbf{x}(t) + B\mathbf{u}(t) + \mathbf{w}(t) \\ \mathbf{y}(t) &= C_r\mathbf{x}(t) \\ \mathbf{z}(t) &= C\mathbf{x}(t) + \mathbf{v}(t). \end{aligned} \tag{1}$$

The equivalent discrete-time representation can be written as

$$\begin{aligned} \mathbf{x}_{k+1} &= F\mathbf{x}_k + G\mathbf{u}_k + \mathbf{w}_k^{\mathbf{x}} \\ \mathbf{y}_k &= H_r\mathbf{x}_k \\ \mathbf{z}_k &= H\mathbf{x}_k + \mathbf{v}_k \end{aligned} \tag{2}$$

where $F = e^{AT}$, $G = (\int_0^T e^{A\tau}d\tau)B$, $H = C$, $H_r = C_r$, and T stands for the sampling period. $\mathbf{x}_k \in \mathcal{R}^n$ is the system state, $\mathbf{u}_k \in \mathcal{R}^l$ the input, $\mathbf{y}_k \in \mathcal{R}^l$ the output, and $\mathbf{z}_k \in \mathcal{R}^m$ the measured output. $\mathbf{w}_k^{\mathbf{x}} \in \mathcal{R}^n$ is a zero-mean white Gaussian sequence with covariance $Q_k^{\mathbf{x}} \in \mathcal{R}^{n\times n}$ to represent the modeling uncertainties. $\mathbf{v}_k \in \mathcal{R}^m$ is a zero-mean white Gaussian sequence with covariance $R_k \in \mathcal{R}^{m\times m}$ to represent measurement noise. The initial state $\mathbf{x}_0$ is also assumed to be a Gaussian vector with mean $\bar{\mathbf{x}}_0$ and covariance $\bar{P}_0$. $H_r = C_r \in \mathcal{R}^{l\times n}$ is the matrix which relates to those system outputs that track the desired command inputs. $H \in \mathcal{R}^{m\times n}$ is the measurement matrix.

To model actuator faults, control effectiveness factors are used [13, 16]. The dynamic part of the system in the presence of actuator faults can be represented as

$$\mathbf{x}_{k+1} = F\mathbf{x}_k + G^f\mathbf{u}_k + \mathbf{w}_k^{\mathbf{x}} \tag{3}$$

where the post-fault input matrix G^f relates to the nominal input matrix G and the control effectiveness factors γ_k^i, $i = 1,\ldots,l$, in the following manner:

$$G^f = G(I - \Gamma_k), \qquad \Gamma_k = \begin{bmatrix} \gamma_k^1 & 0 & \cdots & 0 \\ 0 & \gamma_k^2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma_k^l \end{bmatrix} \tag{4}$$

where $\gamma_k^i = 0$, $i = 1,\ldots,l$, indicates that the ith actuator is healthy, and $\gamma_k^i = 1$ corresponds to a total failure of the ith actuator, and $0 < \gamma_k^i < 1$ represents partial loss of the control effectiveness in the ith actuator.

To determine the extent of an unknown fault, γ_k^i, $i = 1,\ldots,l$, need to be estimated on-line in real-time. Since Γ_k is a diagonal matrix, for the sake of easy estimation of γ_k^i, the following alternative representation is used

$$\begin{aligned} \mathbf{x}_{k+1} &= F\mathbf{x}_k + G\mathbf{u}_k + G\Gamma_k(-\mathbf{u}_k) + \mathbf{w}_k^{\mathbf{x}} && (5) \\ &= F\mathbf{x}_k + G\mathbf{u}_k + \Pi_k(\mathbf{u}_k)\gamma_k + \mathbf{w}_k^{\mathbf{x}} && (6) \end{aligned}$$

where

$$\Pi_k(\mathbf{u}_k) = GU_k \tag{7}$$

and

$$U_k = \begin{bmatrix} -u_k^1 & 0 & \cdots & 0 \\ 0 & -u_k^2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & -u_k^l \end{bmatrix}, \qquad \gamma_k = \begin{bmatrix} \gamma_k^1 \\ \gamma_k^2 \\ \vdots \\ \gamma_k^l \end{bmatrix}. \tag{8}$$

Due to the random nature of actuator faults and in the absence of the knowledge on their true status, the control effectiveness factors can generally be modeled as a random bias vector:

$$\gamma_{k+1} = \gamma_k + \mathbf{w}_k^{\gamma}. \tag{9}$$

The combined system and the control effectiveness model can then be written as follows:

$$\begin{aligned} \mathbf{x}_{k+1} &= F\mathbf{x}_k + G\mathbf{u}_k + \Pi_k(\mathbf{u}_k)\gamma_k + \mathbf{w}_k^{\mathbf{x}} \\ \gamma_{k+1} &= \gamma_k + \mathbf{w}_k^{\gamma} \\ \mathbf{y}_k &= H_r\mathbf{x}_k \\ \mathbf{z}_k &= H\mathbf{x}_k + \mathbf{v}_k. \end{aligned} \tag{10}$$

This model is used for fault diagnosis and reconfigurable control system design in the rest of this work.

If an actuator fault has occurred at an unknown time instant k_F, the corresponding control effectiveness factor, γ_k^i, $i \in [1,l]$, will become non-zero. The objective of fault detection and diagnosis (FDD) is to determine the extent of the loss in the control effectiveness by estimating γ_k^i on-line in real-time so that an on-line automatic reconfigurable controller can be synthesized.

B. Control Design Objectives in FTCS and Graceful Degradation in Performance

The design objectives for FTCS should include the dynamic and the steady-state performance not only under the normal operation, but also under faults. It is important to point out that the emphasis on system behaviors in these two modes of operation can be significantly different. During the normal operation, one may want to place more emphasis on the quality of the system behavior. In the presence of a fault,

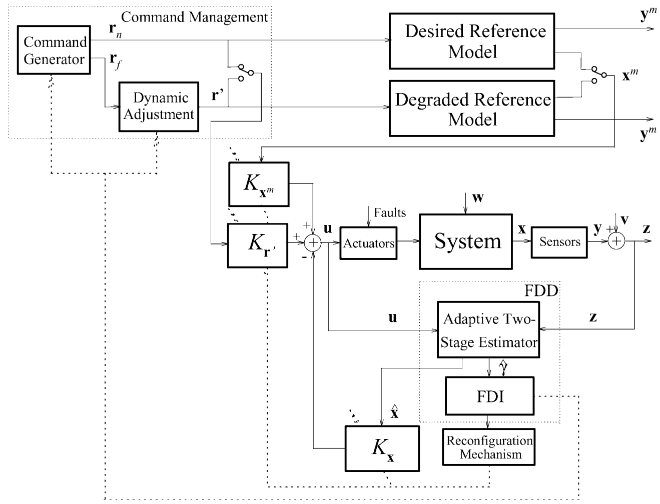

Fig. 1. Overall structure of proposed FTCS.

however, how the system survives with an acceptable performance degradation becomes a predominant issue.

To be more precise, one should assign priorities among the design objectives and rank them accordingly. When a fault occurred it may be necessary to give up some less critical objectives in favor of the more important ones. For example, in a flight control system, the critical objective is to maintain the stability and the integrity of the aircraft. Other objectives may include items such as fuel efficiency, maneuverability, degree of passenger comfort, the optimal flight trajectory, etc. Under a normal flight condition, efforts are made to achieve all these objectives. However, in the event of an emergency, the most important objective would be to maintain the stability of the aircraft and to land the plane safely. The other objectives become far less urgent. This situation is often referred to as graceful degradation in performance. The main contribution of this work is not only to introduce such philosophy in FTCS design, but also to propose a design technique to realize this philosophy.

In a practical control system, failures in actuators may not only result in undesirable transients, but also diminish the capability of the control system to meet the original design specifications. It is generally unreasonable to expect a handicapped system to perform as effectively as when it is healthy, unless a significant amount of redundancy has been built into the system. In some highly reliable systems, one may want to incorporate a large degree of actuator redundancy so that initial failures in some actuators may only lead to reduction of the degree of redundancy, rather than the performance [7]. However, in the majority of practical systems, due to the cost and physical size and weight restrictions associated with redundant control elements, such a situation is not very common. Therefore, upon the occurrence of a failure, one has to scale back on the demand for the performance immediately so that physical constraints of the system are not to be violated. One of these notable constraints is the physical limits (either mechanical or electrical) of the remaining control actuators. If these constraints are not respected, initial failure may lead to potential saturation of the actuators with more serious consequences to follow, which could cause further damage to the rest of the system.

For an aircraft, the degraded performance for dynamic behaviors may include slower climbing rate, wider turns, slower acceleration, and other maneuverability. For steady-state characteristics, the degradation will reflect, for example, in lower cruising speed and altitude. It may even be necessary to drop off some payload or dump extra fuel to achieve a safe landing.

Generally speaking, in comparison with the dynamics of original system, the degraded system will tend to be sluggish with possibly larger overshoots and steady-state errors. In Section III, we will present a technique to select a suitable reference model to account for such behaviors and to propose a strategy for command input adjustment so that the physical constraints of the system will likely not be violated.

C. Overall Structure of Proposed FTCS

The overall structure of the proposed FTCS is depicted in Fig. 1, which includes modules of command management, reference models, FDD, reconfiguration mechanism, and model-following reconfigurable control (MFRC). Note that two reference models are used: one for the system under the normal operation (referred to as the desired reference model) and the other for the system with actuator faults (referred to as the degraded reference model). Note also that to ensure that the closed-loop system follows the degraded reference model, a feedback controller alone is generally not sufficient. Therefore, three reconfigurable controllers, $\{K_{\mathbf{x}}, K_{\mathbf{x}^m}, K_{\mathbf{r}}\}$, need to be synthesized based on the information from FDD to achieve command tracking at the steady-state.

To implement the above fault tolerant control design possible in real-time, the post-fault system model has to be determined on-line and the state variables must be available for feedback. In practice, only part of the state variables may be measurable. To provide required state and fault parameters, simultaneous state and parameter estimation techniques need to be used as shown in Fig. 1. A two-stage adaptive Kalman filter [13, 16] can be used for such a purpose. Furthermore, the fault detection and isolation (FDI) scheme and the reconfiguration mechanism also need to be used. The details of these design have been omitted herein. Interested readers may refer to [15, 16].

III. DEGRADED REFERENCE MODEL AND COMMAND INPUT MANAGEMENT

A. Synthesis of Degraded Reference Model

Assume that the desired reference model of the system with no actuator fault is represented by

$$\begin{aligned} \dot{\mathbf{x}} &= A_d \mathbf{x} + B_d \mathbf{u} \\ \mathbf{y} &= C_d \mathbf{x}. \end{aligned} \tag{11}$$

The corresponding transfer function matrix of the desired reference model is then:

$$T_d(s) = C_d (Is - A_d)^{-1} B_d. \tag{12}$$

Let's assume that the eigenvalues of the closed-loop system are represented as

$$\Lambda_d = \mathrm{diag}[\lambda_1, \lambda_2, \ldots, \lambda_n]. \tag{13}$$

After a fault has occurred, it is expected that the closed-loop system eigenvalues of the degraded reference model will move towards the imaginary axis to reflect the loss of the dynamic performance of the system as well as the reduction in stability margins.

Suppose that the eigenvalues of the degraded reference model are represented as

$$\Lambda_f = \Psi^{-1} \Lambda_d \tag{14}$$

where

$$\Psi = \mathrm{diag}[\alpha_1, \alpha_2, \ldots, \alpha_n], \qquad \alpha_j \geq 1, \qquad \forall \quad j = 1, \ldots, n. \tag{15}$$

This matrix is known as the mode degradation matrix. Each element in this matrix represents the expansion factor of the corresponding mode from the desired reference model.

The transfer function matrix of the reference model for the degraded system then becomes

$$\begin{aligned} T_f(s) &= C_d (Is\Psi - A_d)^{-1} B_d \\ &= C_d (Is - \Psi^{-1} A_d)^{-1} \Psi^{-1} B_d \\ &= C_f (Is - A_f)^{-1} B_f. \end{aligned} \tag{16}$$

Hence, the degraded reference model can be represented as

$$\begin{aligned} \dot{\mathbf{x}} &= A_f \mathbf{x} + B_f \mathbf{u} \\ \mathbf{y} &= C_f \mathbf{x} \end{aligned} \tag{17}$$

where $A_f = \Psi^{-1} A_d$, $B_f = \Psi^{-1} B_d$, $C_f = C_d$.

It is important to note that the desired and the degraded reference models should have unity steady-state gain for the purpose of command input tracking. For this reason, scaling to the reference models in the design may be needed, which is considered in the next section.

B. Command Input Management

The objective of the command input management is to determine appropriate command inputs in the presence of actuator faults for avoiding potential saturation in actuators. Adjustment of command input includes two parts: 1) selection of a new command input to the system at the steady-state, and 2) adjustment of the command input during the initial period of control reconfiguration. Note that only step command inputs have been considered here.

1) *Adjustment of Command Input for Steady-State Command Tracking*: Let the desired command input under no fault condition be $\mathbf{r}_n$. If a fault occurred in the ith actuator which leads to a reduction in the control effectiveness represented by $\gamma_k^i \neq 0$, $i \in [1, l]$, $k \geq k_F$, where k_F is the time instant (unknown) of the fault occurrence. To avoid potential actuator saturation at the steady-state in the reconfigured system, the closed-loop control signals to actuators should all be within the actuator limits. Normally, a smaller control signal in the ith handicapped actuator is expected as a result of reduced command input. Therefore, control redistribution could be carried out by assigning relatively heavier weights on the control signals for the healthy actuators. To implement the above principle, one can find the relationship between the closed-loop control signals $\mathbf{u}_k$ and the associated command inputs $\mathbf{r}_k$ at steady-state and translate the limits of actuator saturation to the desired requirements on the command inputs. From Fig. 1 and the open-loop system model in (1), it can be seen that such a relationship can be represented by

$$\mathbf{r}_f = \mathbf{y}_f = G_\infty W \mathbf{u}_\infty \tag{18}$$

where $\mathbf{u}_\infty$ is the steady-state closed-loop control signal under no fault condition. $\mathbf{y}_f$ is the associated system output. The steady-state open-loop gain of the system G_∞ under the no fault condition can be calculated by

$$G_\infty = \lim_{s\to 0} C_r(Is - A)^{-1}B = C_r(-A)^{-1}B. \tag{19}$$

The weighting matrix W is used to assign proper weights for reducing the magnitude of the closed-loop control signals in each individual control channels and to prioritize control channels for redistributing available control power among the healthy and faulty actuators.

$$W = \begin{bmatrix} \rho^1 & 0 & \cdots & 0 \\ 0 & \rho^2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \rho^l \end{bmatrix} \tag{20}$$

where $0 < \rho^i \leq 1$, $i = 1,\ldots,l$, are the weighting factors. Control prioritization is implemented by selecting different values in ρ^i where larger values are assigned to healthy control channels for allowing relatively larger control signal.

As the estimation of the post-fault system model becomes accurate and the appropriate reconfigurable controller can be synthesized so that the closed-loop control signals will not violate the actuator saturation limits and will eventually settle within a desired range with the above modified command input at the steady-state. However, there are still chances that actuator saturations may occur during the initial period of the reconfiguration due to violent changes in actuator characteristics induced by the fault and unavailability of an accurate post-fault model for feedback control signal synthesis. To deal with the potential actuator saturation in this period, the following dynamic tapering of the command input can be used to reduce the chances of actuator saturation during the transient period.

2) *Dynamic Tapering of Command Input*: Assume that the actuator fault is detected at the time instant k_D, then the following modified command input $\mathbf{r}'_k$ will be generated based on the designed command inputs $\mathbf{r}_k$ as

$$\mathbf{r}'_k = \mathbf{r}'_{k-1} + \kappa_k \cdot [\mathbf{r}_k - \mathbf{r}'_{k-1}] \tag{21}$$

where

$$\mathbf{r}_k = \begin{cases} \mathbf{r}_n, & k < k_D \\ \mathbf{r}_f, & k \geq k_D \end{cases} \quad \text{and} \quad \kappa_k \quad (0 \leq \kappa_k \leq 1)$$

is a weighting parameter chosen to satisfy the actuator constraints during the switching period between the two command inputs: $\mathbf{r}_n$ and $\mathbf{r}_f$. In fact, the command input $\mathbf{r}'_k$ is an interpolation between $\mathbf{r}_k$ and $\mathbf{r}'_{k-1}$. Ideally, $\mathbf{r}'_k = \mathbf{r}'_{k-1}$ if $\kappa_k = 0$, and $\mathbf{r}'_k = \mathbf{r}_k$ when $\kappa_k = 1$. To provide smooth switching between $\mathbf{r}_n$ and $\mathbf{r}_f$, the following variable weighting parameter can be used:

$$\kappa_k = 1 - \sigma e^{-\tau(k-k_D)}, \qquad k \geq k_D \tag{22}$$

where $\tau > 0$ and $\sigma > 0$ are two design parameters. For large k, κ_k will tend to 1, and thus, $\mathbf{r}'_k \to \mathbf{r}_f$.

It is important to note that the degraded performance at the steady-state is achieved by the modified command input $\mathbf{r}_f$. However, performance during the switching period will be affected by the proper selection of the adjustable parameter κ_k so as to avoid actuator saturation. The selection of the parameter κ_k can also be carried out by using optimization-based techniques [1, 5]. However, further discussion is beyond the scope of this paper.

IV. DESIGN OF MODEL-FOLLOWING RECONFIGURABLE CONTROLLER

A. Design Objectives for Model-Following Reconfigurable Control

To better illustrate the reconfigurable control design process, the system model (10) under both the normal and the actuator fault conditions can be written as

$$\begin{aligned} &\begin{cases} \mathbf{x}_{k+1} = F\mathbf{x}_k + G\mathbf{u}_k + \mathbf{w}_k^{\mathbf{x}}, \\ \qquad k < k_F \quad \text{System during normal operation} \\ \mathbf{x}_{k+1} = F\mathbf{x}_k + G^f\mathbf{u}_k + \mathbf{w}_k^{\mathbf{x}}, \\ \qquad k \geq k_F \quad \text{System with actuator faults} \end{cases} \\ &\mathbf{y}_k = H_r\mathbf{x}_k \\ &\mathbf{z}_k = H\mathbf{x}_k + \mathbf{v}_k. \end{aligned} \tag{23}$$

During the normal operation, the system matrices are represented by $\{F, G, H_r\}$. Once an actuator fault occurs, the matrix G becomes G^f at time instant k_F with an unknown change in G.

Let's assume that the reference models for the desired and the degraded conditions be represented as

Desired reference model:

$$\begin{cases} \mathbf{x}_{k+1}^m = F_n^m\mathbf{x}_k^m + G_n^m\mathbf{r}_k \\ \mathbf{y}_k^m = H_n^m\mathbf{x}_k^m \end{cases}, \qquad k < k_F$$

Degraded reference model:

$$\begin{cases} \mathbf{x}_{k+1}^m = F_f^m\mathbf{x}_k^m + G_f^m\mathbf{r}'_k \\ \mathbf{y}_k^m = H_f^m\mathbf{x}_k^m \end{cases}, \qquad k \geq k_F \tag{24}$$

where $\mathbf{x}_k^m \in \mathcal{R}^{n^m}$ is the state, $\mathbf{y}_k^m \in \mathcal{R}^{l^m}$ is the reference model output, and $\mathbf{r}_k = \mathbf{r}_n, \mathbf{r}'_k \in \mathcal{R}^{l^m}$ are the original and modified command inputs, respectively. The constant matrices $\{F_n^m, G_n^m, H_n^m\}$ and $\{F_f^m, G_f^m, H_f^m\}$ are of appropriate dimensions.

Based on the system representation (23), and the desired reference model (24), one needs to synthesize the following control gains $\{K_{\mathbf{x}}^n, K_{\mathbf{x}^m}^n, K_{\mathbf{r}}^n\}$

for generating the desired control signals under the normal system operation:

$$\mathbf{u}_k^n = \underbrace{-K_\mathbf{x}^n \mathbf{x}_k}_{\text{feedback}} + \underbrace{K_{\mathbf{x}^m}^n \mathbf{x}_k^m}_{\text{reference model}} + \underbrace{K_\mathbf{r}^n \mathbf{r}_k}_{\text{feedforward}}. \tag{25}$$

Once a fault is detected, new controller gains $\{K_\mathbf{x}^f, K_{\mathbf{x}^m}^f, K_\mathbf{r}^f\}$ will have to be synthesized based on the degraded reference model in (24) so that the closed-loop system follows the degraded reference model with the new control signal:

$$\mathbf{u}_k^f = -K_\mathbf{x}^f \mathbf{x}_k^f + K_{\mathbf{x}^m}^f \mathbf{x}_k^m + K_\mathbf{r}^f \mathbf{r}'_k, \qquad k \geq k_R \tag{26}$$

where k_R represents the controller reconfiguration time.

B. Design of Feedforward and Feedback Controllers

The feedforward control is mainly to ensure that the selected controlled variables follow the outputs of the desired and the degraded reference models during the normal and fault conditions. Therefore, the objective of the controller design is to find a system input $\mathbf{u}_k$ ($\mathbf{u}_k^n$ or $\mathbf{u}_k^f$) that drives the tracking error $\mathbf{e}_k$ to zero asymptotically. The error $\mathbf{e}_k$ is defined as follows:

$$\mathbf{e}_k = \mathbf{y}_k - \mathbf{y}_k^m = H_r \mathbf{x}_k - H^m \mathbf{x}_k^m. \tag{27}$$

When this condition is satisfied, the following will be true:

$$\mathbf{y}_k^* = H_r \mathbf{x}_k^* = H^m \mathbf{x}_k^m. \tag{28}$$

Under the assumption that the ideal system state $\mathbf{x}_k^*$ and the control trajectories $\mathbf{u}_k^*$ are the linear combinations of the states and the inputs of the reference model, the solutions for $\mathbf{x}_k^*$ and $\mathbf{u}_k^*$ can be determined from [3]:

$$\mathbf{x}_k^* = S_{11} \mathbf{x}_k^m + S_{12} \mathbf{r}'_k + \Delta(\mathbf{r}'_k) \tag{29}$$

$$\mathbf{u}_k^* = S_{21} \mathbf{x}_k^m + S_{22} \mathbf{r}'_k + \Delta(\mathbf{r}'_k) \tag{30}$$

where S_{ij}, $i,j = 1,2$, are constant gain matrices. If we restrict ourselves to step inputs, the higher order terms will vanish, i.e. $\Delta(\mathbf{r}'_k) = 0$. The solution for $\mathbf{x}_k^*$ and $\mathbf{u}_k^*$ to achieve the perfect command tracking can be represented as

$$\mathbf{x}_k^* = S_{11} \mathbf{x}_k^m + S_{12} \mathbf{r}'_k \tag{31}$$

$$\mathbf{u}_k^* = S_{21} \mathbf{x}_k^m + S_{22} \mathbf{r}'_k \tag{32}$$

where S_{ij}, $i,j = 1,2$, can be calculated from

$$S_{11} = \Phi_{11} S_{11} (F^m - I) + \Phi_{12} H^m \tag{33}$$

$$S_{12} = \Phi_{11} S_{11} G^m \tag{34}$$

$$S_{21} = \Phi_{21} S_{11} (F^m - I) + \Phi_{22} H^m \tag{35}$$

$$S_{22} = \Phi_{21} S_{11} G^m \tag{36}$$

and Φ_{ij}, $i,j = 1,2$, are given by

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} = \begin{cases} \begin{bmatrix} F-I & G \\ H_r & 0 \end{bmatrix}^{-1} & \text{System under normal operation} \\ \begin{bmatrix} F-I & \hat{G}_k^f \\ H_r & 0 \end{bmatrix}^{-1} & \text{System with actuator faults} \end{cases} \tag{37}$$

where I is an identity matrix, and $\hat{G}_k^f = G(I - \hat{\Gamma}_k)$ is an estimate of G^f at time k.

It should be noted that Φ_{ij} are functions of the pre- and post-fault system models, whereas S_{ij} depends on both the system and the reference models under the normal and fault conditions.

To follow the model-reference design approach, let's define

$$\tilde{\mathbf{x}}_k = \mathbf{x}_k - \mathbf{x}_k^*, \qquad \tilde{\mathbf{u}}_k = \mathbf{u}_k - \mathbf{u}_k^*, \qquad \tilde{\mathbf{y}}_k = \mathbf{y}_k - \mathbf{y}_k^* \tag{38}$$

then

$$\tilde{\mathbf{x}}_{k+1} = \begin{cases} F\tilde{\mathbf{x}}_k + G\tilde{\mathbf{u}}_k, & \text{System under normal operation} \\ F\tilde{\mathbf{x}}_k + \hat{G}_k^f \tilde{\mathbf{u}}_k, & \text{System with actuator faults} \end{cases} \tag{39}$$

$$\tilde{\mathbf{y}}_k = H_r \tilde{\mathbf{x}}_k \tag{40}$$

and the feedback control signal given by

$$\tilde{\mathbf{u}}_k = -K_\mathbf{x} \tilde{\mathbf{x}}_k = -K_\mathbf{x}(\mathbf{x}_k - \mathbf{x}_k^*) \tag{41}$$

From the definition of $\tilde{\mathbf{u}}_k$ in (38), it is clear that

$$\mathbf{u}_k = \mathbf{u}_k^* + \tilde{\mathbf{u}}_k = \mathbf{u}_k^* - K_\mathbf{x}(\mathbf{x}_k - \mathbf{x}_k^*) \tag{42}$$

Substituting (31) and (32) into (42), the total control signal can be shown as

$$\mathbf{u}_k = \underbrace{-K_\mathbf{x} \mathbf{x}_k}_{\text{feedback}} + \underbrace{(S_{21} + K_\mathbf{x} S_{11}) \mathbf{x}_k^m}_{\text{reference model}} + \underbrace{(S_{22} + K_\mathbf{x} S_{12}) \mathbf{r}'_k}_{\text{feedforward}}. \tag{43}$$

It should be noted that (43) represents actuator inputs for both the normal and the fault conditions. In the presence of an actuator fault, S_{ij} and the three controller gain matrices, $K_\mathbf{x}$, $K_{\mathbf{x}^m} = S_{21} + K_\mathbf{x} S_{11}$, $K_\mathbf{r} = S_{22} + K_\mathbf{x} S_{12}$, all need to be updated accordingly.

The feedback part of the controller is designed using eigenstructure assignment techniques [6, 16]. The design objective is to synthesize a feedback controller so that the eigenstructure of the closed-loop system is as close as possible to that of the desired reference model under the normal operation, and that

TABLE I
System and Reference Models

	A	B	Eigenvalues of A
Open-loop System Model	$\begin{bmatrix} -3.598 & 0.1968 & -35.18 & 0 \\ -0.0377 & -0.3576 & 5.884 & 0 \\ 0.0688 & -0.9957 & -0.2163 & 0.0733 \\ 0.9947 & 0.1027 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 14.65 & 6.538 \\ 0.2179 & -3.087 \\ -0.0054 & 0.0516 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.0258 \\ -3.3547 \\ -0.3957 + j2.7405 \\ -0.3957 - j2.7405 \end{bmatrix}$
Desired Reference Model	$\begin{bmatrix} -10.0 & 0 & -10.0 & 0 \\ 0 & -0.7 & 4.5 & 0 \\ 0 & -0.5 & -0.7 & 0 \\ 1 & 0 & 0 & -0.5 \end{bmatrix}$	$\begin{bmatrix} 10.0 & 5.0 \\ -5.48 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.5 \\ -10.0 \\ -0.7 + j1.5 \\ -0.7 - j1.5 \end{bmatrix}$
Degraded Reference Model	$\begin{bmatrix} -3.3333 & 0 & -3.3333 & 0 \\ 0 & -0.7 & 4.5 & 0 \\ 0 & -0.125 & -0.175 & 0 \\ 0.25 & 0 & 0 & -0.125 \end{bmatrix}$	$\begin{bmatrix} 3.3199 & 1.7089 \\ -5.48 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -0.1250 \\ -3.3333 \\ -0.4375 + j0.7026 \\ -0.4375 - j0.7026 \end{bmatrix}$

of the degraded reference model in the presence of faults. The interested reader is referred to [6, 14, 16] for details about eigenstructure assignment techniques.

V. SIMULATION EXAMPLE AND PERFORMANCE ASSESSMENT

To demonstrate the effectiveness of the proposed approach, a fourth-order lateral F-8 aircraft model [11] with two inputs and two outputs is used in the simulation studies.

A. Aircraft Model

The linearized aircraft model can be described as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= A\mathbf{x}(t) + B\mathbf{u}(t) \\ \mathbf{y}(t) &= C_r\mathbf{x}(t) \end{aligned} \tag{44}$$

where the state and the input vectors are $\mathbf{x} = [p \ \ r \ \ \beta \ \ \phi]^T$ and $\mathbf{u} = [\delta_a \ \ \delta_r]^T$, respectively, with p representing the roll rate, r the yaw rate, β the sideslip angle, ϕ the bank angle, δ_a the aileron deflection, and δ_r the rudder deflection.

To maintain the desired values for the sideslip and the bank angle during both the normal operation and under fault conditions, the output matrix H_r is chosen as

$$H_r = C_r = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Taking into account the presence of noise and representing the system in the discrete domain, the system can be transformed to a discrete form as shown in (2) where $H = C = H_r$ and the sampling period $T = 0.1$ s is used. It should be pointed out that only two of the four state variables, i.e. sideslip and bank angles, are measurable. Such a problem setting will increase the degree of complexity for FDD to provide timely and accurate information on the fault and the post-fault system model, and in turn, for the reconfigurable controller design to achieve good control performance of the overall FTCS.

TABLE II
Command Inputs for Normal and Fault Conditions

Setpoints	Normal	Fault
Sideslip angle	3.0	1.0
Bank angle	8.0	4.0

B. Design of Reference Models and Command Inputs

Following the design consideration outlined in Section IIIA and for the selected weighting matrix

$$\Psi = \mathrm{diag}[3, 1, 4, 4]$$

the parameters of the system, the desired and the degraded reference models, as well as the corresponding eigenvalues are given in Table I.

The desired reference model is modified to achieve unity steady-state gain from a model in [11] which satisfies all necessary performance requirements under the normal operation. In the selection of the degraded reference model, the following two factors have been taken into consideration: 1) to track the degraded reference model in the presence of actuator faults, and 2) the closed-loop control signals at the steady-state should not violate the amplitude limits of the actuators under all fault conditions considered. The amplitude limits for the two closed-loop control channels are set as $\delta_a^c = \pm 15$ deg and $\delta_r^c = \pm 10$ deg.

For simplicity, the weighting matrix is chosen as $W = \mathrm{diag}[1/3, 1/3]$. The corresponding command inputs for the normal and the fault conditions are given in Table II.

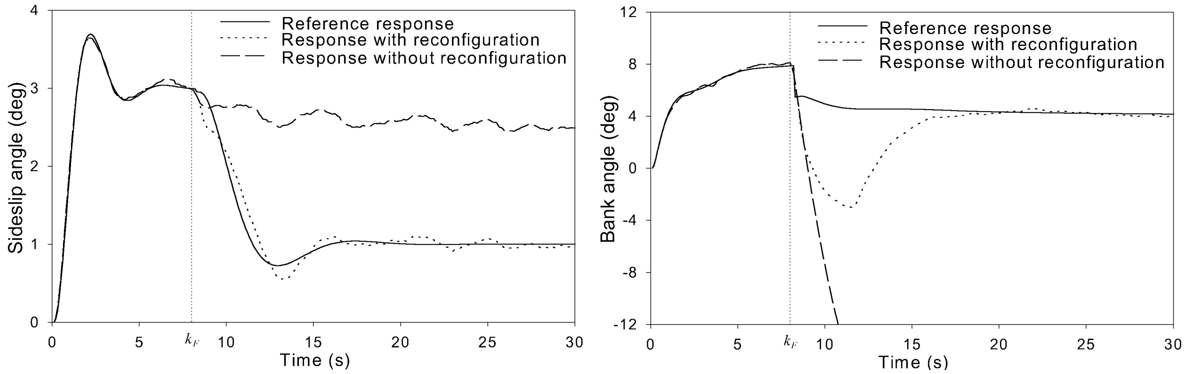

Fig. 2. System outputs with and without reconfiguration.

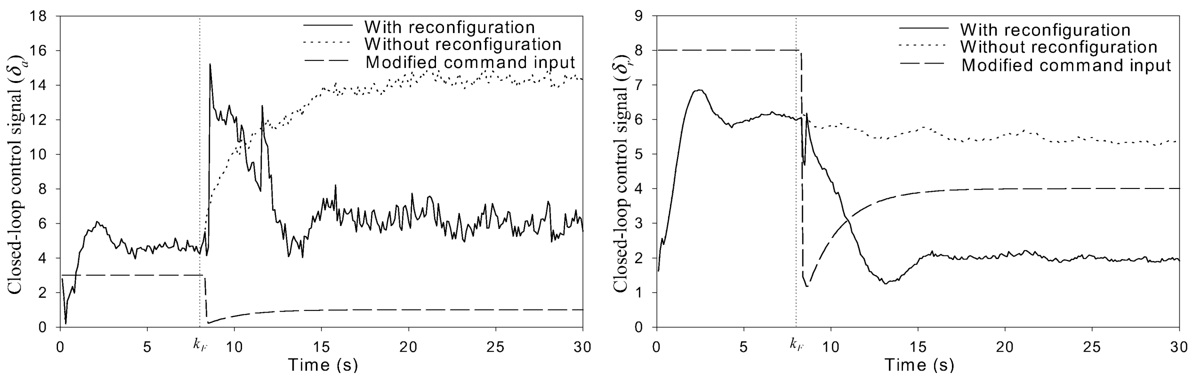

Fig. 3. Closed-loop control signals with and without reconfiguration.

C. Simulation Results and Performance Assessment

To evaluate the performance of the proposed method, a loss of 75% of the control effectiveness in the aileron channel is simulated at time $k_F = 8$ s. Prior to the occurrence of this fault, a constant input vector, $\mathbf{r}_k = [3\ \ 8]^T$, is used as the original command input to represent the desired sideslip and the bank angle. Once the fault has been detected, the new command input becomes $\mathbf{r}_k = [1\ \ 4]^T$ to represent the degraded performance at the steady-state. The parameters in (22) are chosen as $\tau = 0.05$ and $\sigma = 1$.

1) *Performance with and without Reconfiguration*: The responses of the closed-loop system with and without controller reconfiguration following the fault are shown in Fig. 2. The corresponding closed-loop control signals are shown in Fig. 3. To illustrate how the command inputs react to faults, the corresponding command inputs are overlaid on the same graph in Fig. 3.

Since there are two control actuators, two of the system outputs, sideslip and bank angles, can be made to follow the outputs of reference models. With the chosen degraded reference model and the new command input, the original steady-state sideslip 3.0 deg is reduced to 1.0 deg, while the original bank angle 8.0 deg is reduced to 4.0 deg. As shown in Fig. 2, the outputs have successfully tracked those of the desired and the degraded reference models before and after the fault, respectively. The control signals of the reconfigured closed-loop system are also within the saturation limits. However, without reconfiguration the system outputs track neither the outputs of the desired nor the degraded models.

It is interesting to note that after the fault occurrence at $k_F = 8.0$ s and before the reconfiguration is activated at $k_R = 8.6$ s, the outputs of the closed-loop system tend to diverge. After the reconfigurable control law is activated, the system outputs recover back and eventually track those of the degraded reference model with the modified command input.

2) *Performance with Reconfiguration using the Degraded and the Desired Reference Models*: For comparison purposes, system outputs are illustrated in Fig. 4 for reconfiguring controller designed based on either the degraded or the desired reference models. The corresponding control signals are also shown in Fig. 5.

Results in Fig. 4 have clearly indicated that the outputs of the reconfigured system are able to track those of the degraded reference model satisfactorily. The magnitude of the associated control signal at the steady-state for aileron is almost the same as that in the pre-fault condition. The magnitude of the steady-state signal for the rudder is even significantly smaller than that in the pre-fault case because of the reduced performance demand. However, if one had demanded the post-fault system to follow the desired

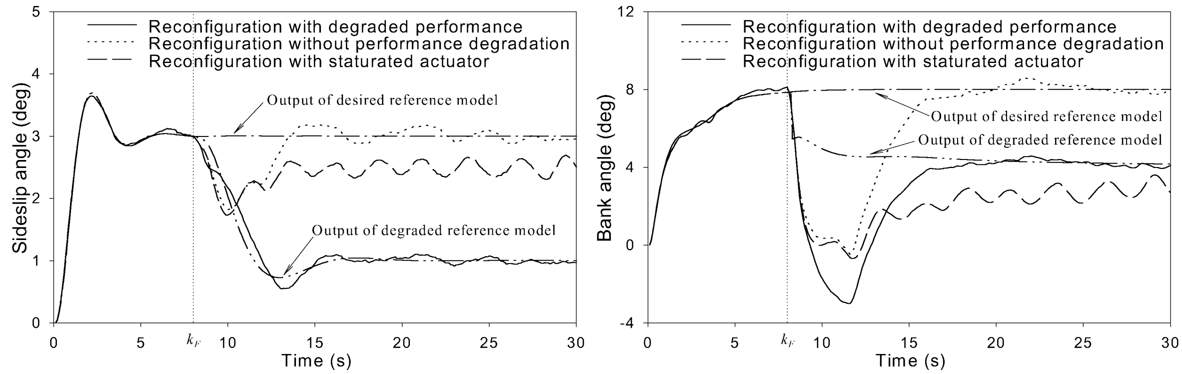

Fig. 4. System outputs using degraded and desired reference models.

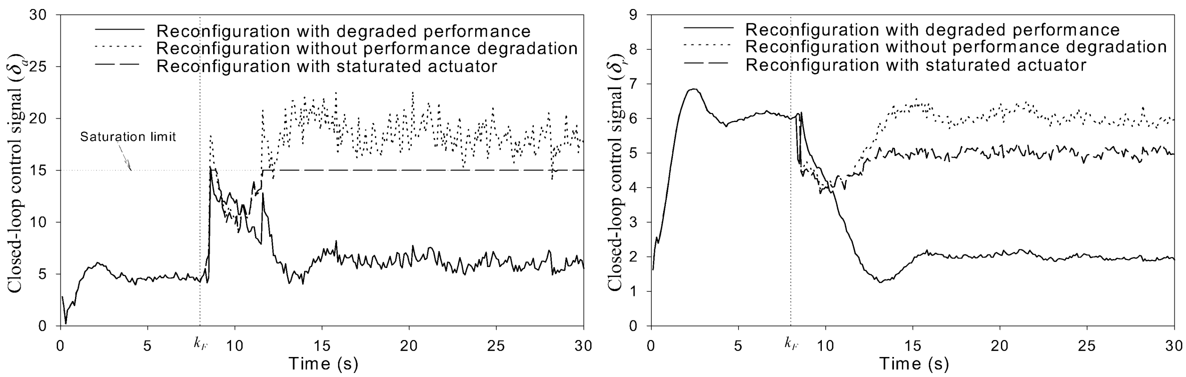

Fig. 5. Control signals using degraded and desired reference models.

reference model, significantly larger control signals (dot lines in Fig. 5) would have been required to track the outputs of desired reference model, particularly in the aileron channel.

In fact, if the performance degradation had not been taken into account, the synthesized closed-loop control signal in the aileron channel would have violated the actuator saturation limit immediately after the fault occurrence (dash line in the left side of Fig. 5). This leads to unacceptable performance deterioration as shown in Fig. 4 (dash lines). However, using the degraded reference model and the command input adjustment techniques, the closed-loop control signals in both channels are well within the limits of the actuators.

3) *Discussion*: The effectiveness and the superiority of the proposed approach have been demonstrated under various conditions in this example. It should be noted that system performances for other type of inputs, including piecewise constant inputs at different levels and different types of fault such as abrupt and incipient, multiple and consecutive faults, have also been investigated. Satisfactory results have been obtained. These results are not included here for the interest of space.

VI. CONCLUSIONS

The design issues for FTCS with explicit consideration of performance degradation in both dynamic and steady-state periods have been addressed in this paper. An integrated approach has been proposed based on the concept of both command input management and MFRC strategy. Two reference models are used, one for normal performance and the other for degraded performance. Novel techniques to synthesize the degraded reference model and to adjust the command input while preventing the actuators from saturation have been proposed. Simulation results have demonstrated the effectiveness of the proposed scheme using an aircraft model.

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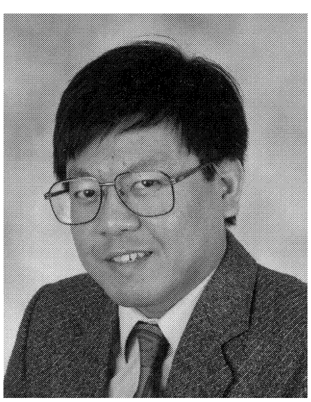

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