Fault tolerant quantum computation with high threshold in two dimensions

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Motivation

Operational requirements

We need experimentally viable methods for fault tolerance

- High threshold
- Threshold should be robust against variations in the error model
- Moderate overhead
- Simple architecture (e.g. no long range interaction)

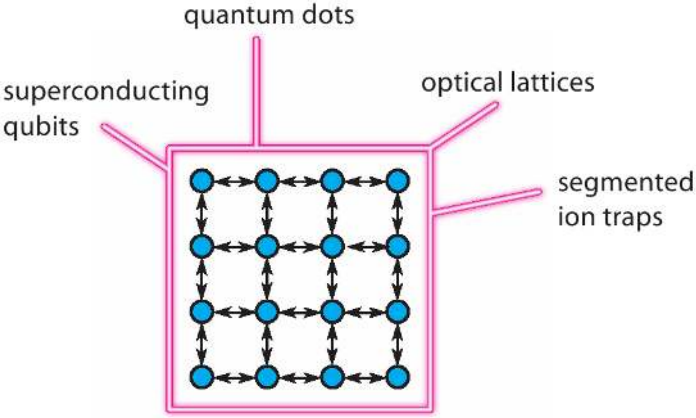
Motivation

Operational requirements

- Preparation of 2D cluster state by translation invariant nearest-neighbor Ising type interactions
- Hadamard gate
- Single qubit measurements in the $X + Y$, X, Y and Z bases
- Classical post-processing of measurement results

Motivation

Operational requirements

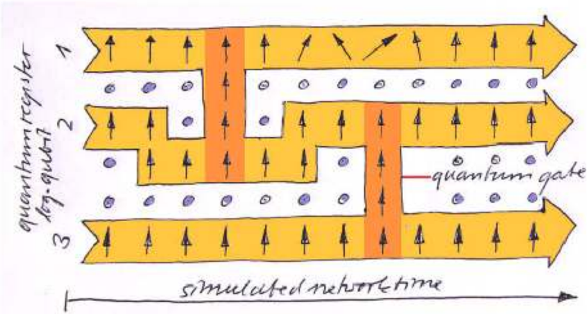


2D, short range translation invariant interactions

Outline

1. Introduction
2. The fault tolerant $QC_{\mathcal{C}}$
3. Threshold and overhead

The one way quantum computer



measurement of Z ($\odot$), X ($\uparrow$), $\cos\alpha X + \sin\alpha Y$ ($\nearrow$)

- Universal computational resource: 2D cluster state.
- Information is written onto the cluster, processed and read out by single qubit measurements only.

The threshold theorem

Theorem

If the noise per elementary operation is below a constant non-zero threshold then an arbitrarily long quantum computation can be performed with arbitrary accuracy and ***small operational overhead****.*[a]

[a] Ahronov & Ben-Or (1996), Kitaev (1997), Knill, Laflamme & Zurek (1998), Aliferis, Gottesman & Preskill (2005)

- What is the threshold value?
- What is the overhead?
- What are the requirements on interaction?

Known thresholds

No constraint

[1] — 0.03, est.

[2] — 10^{-3}, est.

[3] — 10^{-4}, est.

[4] — 10^{-5}, bd.

Geometric constraint

2D

[5] — $7.5 \cdot 10^{-3}$, est.

[6] — $2 \cdot 10^{-5}$, est.

1D

[7] — 10^{-8}, bd.

[1] Knill, (2005); [2] Zalka (1999); [3] Dawson & Nielsen (2005); [4] Aliferis, Gottesman & Preskill (2005), [5] Raussendorf, Harrington & Goyal quant-ph/0703143; [6] Svore, DiVincenzo & Terhal, quant-ph/0604090, [7] Aharonov & Ben-Or (1999)

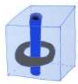
Fault tolerant $QC_{\mathcal{C}}$

Main idea

Replace 2D cluster state with 3D cluster state

- The 3D cluster state is a fault tolerant substrate

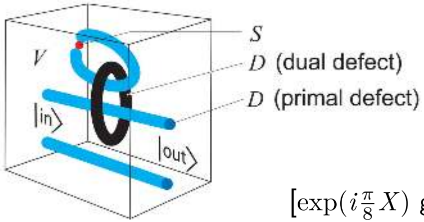
- Topological quantum logic via lattice defects



- Mapping to 2D physical lattice
- Threshold: 7.5×10^{-3}

Macroscopic view

- Example CNOT



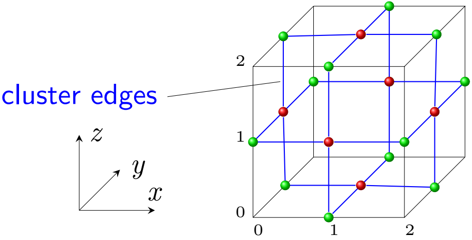
[$\exp(i\frac{\pi}{8}X)$ gate]

- Three cluster regions
  V (Vacuum), D (Defect) and S (Singular)

  V: local X measurements
  D: local Z measurements
  S: local $\frac{X+Y}{2}$, Y measurements

- Defect region D is string like. The quantum circuit is encoded in the topology of D.

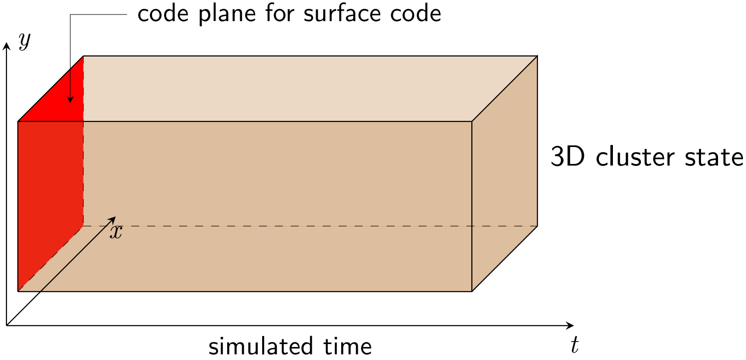
Microscopic view



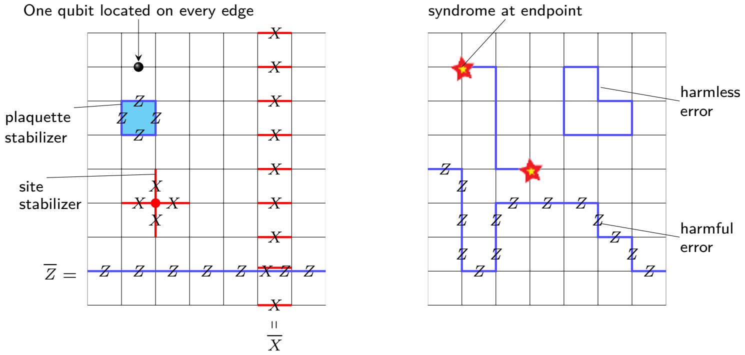
elementary cell of the $\mathcal{L}$attice

qubit location	:	(even, odd, odd)	-	face of $\mathcal{L}$
qubit location	:	(odd, even, even)	-	edge of $\mathcal{L}$
syndrome location	:	(odd, odd, odd)	-	cube of $\mathcal{L}$
syndrome location	:	(even, even, even)	-	site of $\mathcal{L}$

Key to the scheme



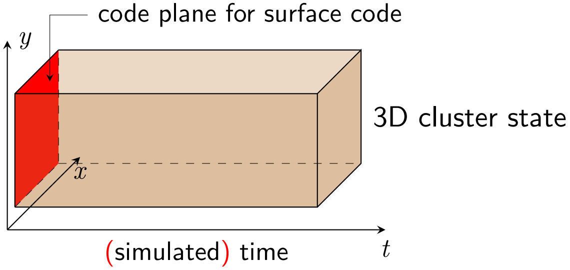
Surface codes



- Surface codes† are CSS codes associated with planar lattices
- Harmful errors stretch across the entire lattice (rare events)

† A. Kitaev, quant-ph/9707021 (1997)

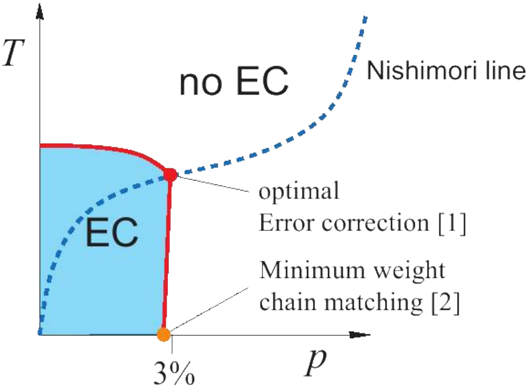
$QC_{\mathcal{C}}$: topological error correction in V



- Fault tolerant quantum memory with planar code ⇔ *Random plaquette* Z_2 *gauge model* (RPGM)†.
- Same error correction applies to the 3D cluster state

† Dennis et al., quant-ph/0110143 (2001).

Phase diagram of the RPGM



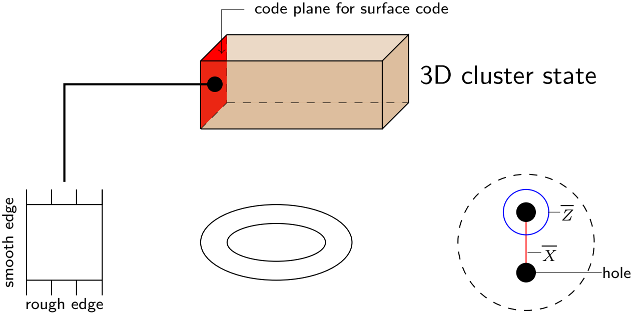
- Have an error budget of 3%

[1] T. Ohno et al., quant-ph/0401101 (2004).

[2] E. Dennis et al., quant-ph/0110143 (2001); J. Edmonds, Canadian J. Math. 17, 449 (1965).

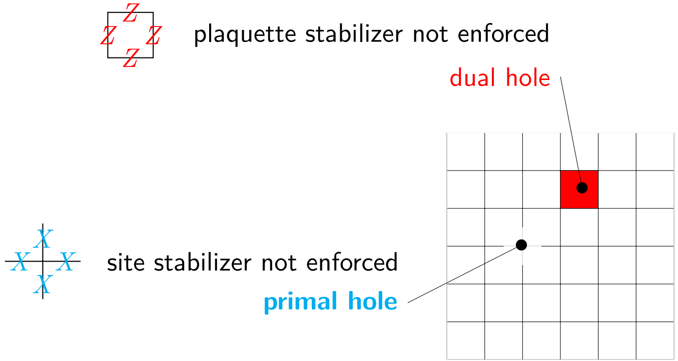
Fault tolerant quantum logic

Encoding capacity of the code depends on the topology of the code surface



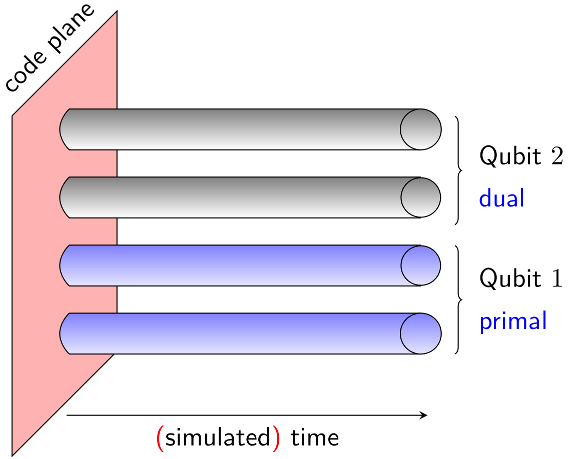
Plane segment	Torus	Plane with two holes
1 Qubit	2 Qubits	1 Qubit

Surface code on a plane with holes



- There are two types of hole: primal and dual
- A pair of same-type holes form an encoded qubit

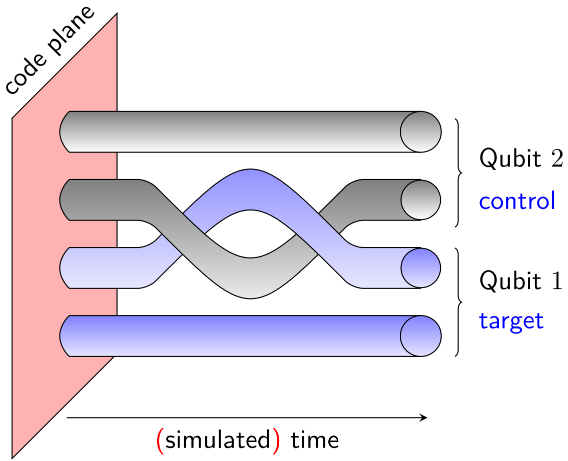
Quantum logic via defect topology



Defects are the extension of holes in the code plane to the third dimension.

Quantum logic via defect topology

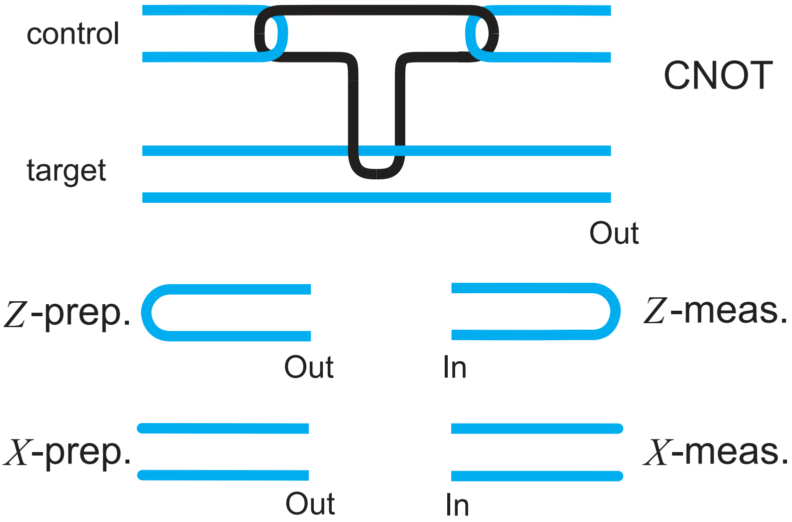
C-NOT gate



Topological quantum gates are encoded in the way primal and dual defects are wound around each other.

Quantum gates

Clifford gates

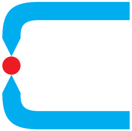


Quantum gates

Non-Clifford gates

- Need one non-Clifford element:

  fault tolerant preparation of $|A\rangle := \frac{X+Y}{\sqrt{2}} |A\rangle$

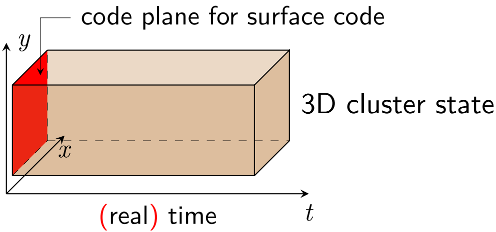


Singular qubit

- FT prep. of $|A\rangle$ is achieved via concatented *magic state distillation*† of logical qubits

† S. Bravyi and A. Kitaev, Phys. Rev. A 71, 022316 (2005).

Mapping to 2D



- Turn simulated time into real time
- Require a single 2D layer

Fault tolerance threshold

Error sources after mapping to 2D

1. $|+\rangle$ preparation: Perfect preparation followed by single qubit partially depolarizing noise with probability p_P.
2. $\Lambda(Z)$ gates (space like edges of $\mathcal{L}$)): Perfect gates followed by two qubit partially depolarizing noise with probability p_2.
3. Hadamard gates (time like edges of $\mathcal{L}$)): Perfect gates followed by single qubit partially depolarizing noise with probability p_1.
4. Measurement: Perfect measurement preceded by single qubit partially depolarizing noise with probability p_M.

No qubit is idle between preparation and measurement – no memory error

Fault tolerance threshold

Threshold estimate ($p := p_1 = p_2 = p_P = p_M$)

- Topological threshold in cluster region V:

$$p_c = 7.5 \times 10^{-3}$$

- Threshold for magic state distillation:

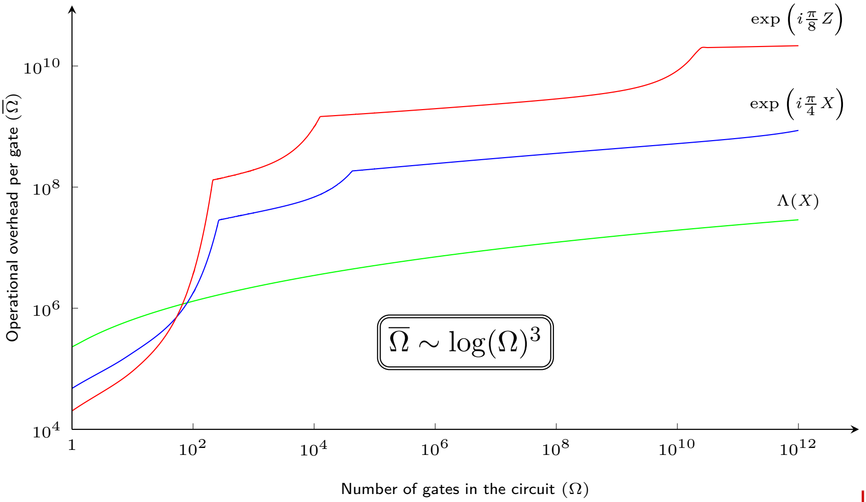
$$p_c = 2.8 \times 10^{-2}$$

- The threshold is robust against variations in the error model such as higher weight elementary errors or decaying long distance errors.

Topological EC sets the overall threshold

Operational overhead

(At $p = \frac{1}{3} p_c$)



Summary

[quant-ph/0703143]

Scenario

- Local and nearest neighbor gates on a 2D lattice

Performance

- Threshold: 7.5×10^{-3}
- Overhead: $\overline{\Omega} \sim \log(\Omega)^3$

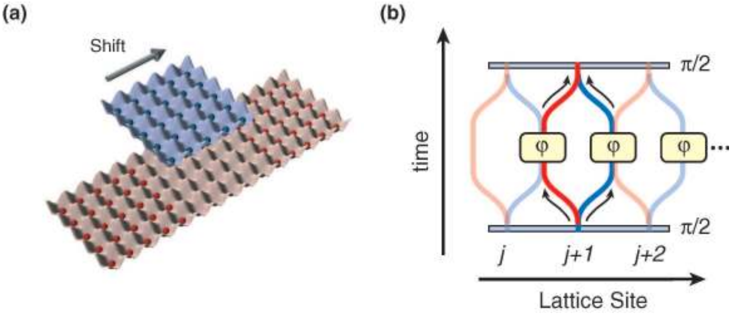
Method

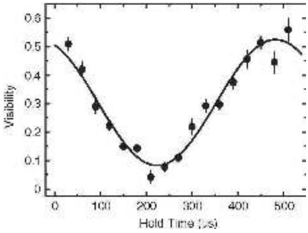
- 3D cluster states provide intrinsic topological error correction and topologically protected quantum gates

Suitable systems for implementation

- Cold atoms in optical lattices, segmented ion traps, superconducting qubits, quantum dots...

Cold atoms in an optical lattice





- Translation invariant 2D C-PHASE†
- Individual atom readout

† Greiner et. al., Nature (2002)