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Fault-Tolerant Sliding Mode Observer Synthesis of Markovian Jump Systems Using Quantized Measurements

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This paper investigates the design problem of sliding mode observer using quantized measurements for a class of Markovian jump systems against actuator faults. Such a problem arises in modern networked-based digital systems, where data has to be transmitted and exchanged over a digital communication channel. In this paper, a new descriptor sliding mode observer approach using quantized signals is presented, in which a discontinuous input is synthesized to reject actuator faults by an off-line static compensation of quantization effects. It is revealed that *the lower bound* on the density of a logarithmic quantizer is $1/3$, under which the quantization effects could be compensated completely by using the sliding mode observer approach. Based on the proposed observer method, the asymptotical estimations of sate vector and quantization errors can be obtained simultaneously. Finally, an example of linearized model of an F-404 aircraft engine system is included to show the effectiveness of the presented observer design method.

***Index Terms*— *Markovian jumping parameters, state estimation, quantization, sliding mode observer, actuator fault*.**

I. Introduction

In modern practical industrial devices, system states are generally physically full- or partially unavailable for direct measurement and a full state feedback stabilization scheme is almost impossible to be implemented. In such cases, state estimation via observer is thus of a realistic significance, which has been the subject of extensive research in signal processing domain over the past several decades [12], [2]. Since external disturbances and unknown faults inevitably causes performance degradation for observer synthesis in a variety of industrial processes, disturbances and/or fault-tolerant observer design have thus received a great amount of attention, and a number of design techniques have been reported in literature [19], [23], [20], [21]. Among the existing approaches, sliding mode observer (SMO) has been recognized as one of the most effective approaches to reject disturbances and faults to be an essential basis of a state estimation task. The main characterization of an SMO scheme is that a discontinuous input term is injected into the observer to eliminate faults or disturbances which is synthesized based on the sliding mode control theory. In recent years, SMO has been applied to a wide variety of realistic engineering systems including aircraft, underwater vehicles, spacecraft, flexible space structures and power systems, etc [5], [16].

On another research front, the rapid advances of network technologies has led to a series of successful applications of the so-called networked-based control systems in complex modern industry processes. However, certain limitations induced by the insertion of network devices also arises inevitably including communication delays, intermittent data package losses and signal quantization. In this sense, quantized state estimation has thus attracted considerable research interest as a part of solution of network-based estimation problem, where transmitted information suffers also from transmission delays and packet dropouts. In addition, recently some results on network-based SMO design have been also reported [13], [5], [6].

Markovian jump system (MJS) is an appropriate modeling candidate to describe dynamic systems with random abrupt variation structure. A great number of realistic dynamical systems can be modeled by MJSs, such as chemical processes, communication networks, aerospace industry, and economics systems, etc. Due to MJS's great application potential in a variety of engineering, a great amount of effort has been devoted to address various control and filtering problems of MJS in the past decades [18], [8]. In particular, some researchers have attempted to investigate the aforementioned network-based control and sliding mode control problems for MJS, and some preliminary results have been obtained [17].

Although the applications of network-based control have covered a wide range of variety of realistic industries, those unexpected phenomenon such as random abrupt variation structure of the plant and and components fault, may degrade the control system performances in a network environment. It is therefore essential to improve the application ability of SMO theory and MJS model in a network setting to maintain the robustness performances of networked-based control systems. The main purpose of this paper is to make the first attempt to investigate actuator fault rejection SMO design for MJS with

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network data transmission where signal quantization is taken into account. The achieved combination of SMO, MJS and network-based estimation techniques would clearly possesses both theoretical significance and application potential.

It should be pointed out, however, that there is little work in the literature to consider SMO design for stochastic jump systems with quantization and actuator faults. The main obstacles are in the following two aspects: i) in a network environment, the quantization error of output measurements can refer to a class of additive output disturbances. As pointed out in [4], linear Luenberger observer is not effective to deal with output disturbances and cannot obtain satisfactory results. The reason is that the outputs disturbances will be unavoidably amplified if the observer gain matrices are with a high size [14]; and ii) the piecewise discontinuity of quantized signals will aggravate the difficulty on designing the sliding mode input term of the observer, which is however a crucial role in the whole synthesis procedure since the disturbances/fault rejection of an SMO is totally dependent on the sliding mode input term. As a result, a new observer approach is desirable to fully cope with the aforementioned two difficulties and solve this open research issue, which motivates the current investigation of this paper.

This paper addresses the SMO synthesis problem for a class of continuous-time MJS with quantized measurements and unknown actuator faults. The design approach of this paper is divided into the following several steps: first, the quantization error of measurements is treated as output disturbance and is taken into consideration as an auxiliary state vector of an augmented descriptor system, where the output disturbance (quantization error) is currently converted into an input disturbance; second, a new SMO using quantized measurements is presented to estimate the augmented state including both the state vector of the original system and quantization error vector. During this design, it is proved that, provided the lower bound for the logarithmic quantizer density ρ is larger than $1/3$, by utilizing ρ as a design parameter, the discontinuous input term of the SMO can fully compensate the quantization effects and eliminate the bounded unknown actuator faults effectively. Subsequently, the asymptotic estimates of state vectors of the original plant and output quantization error can be obtained simultaneously. Finally, a simulation example on F-404 aircraft engine system is used to demonstrate the effectiveness of the proposed observer design scheme.

The contribution of the proposed observer approach mainly lies in the following three folds: 1) in this design the output disturbance (quantization error) is converted into input disturbance, which is thus avoided to be amplified when the observer gain is of a high size; 2) different from the traditional observer approaches, in the designed observer a parameter α related to quantizer density is injected into the sliding mode input term, which makes the observer possesses the ability to compensate the quantization effects and guarantee the stability performance of the error dynamic; and 3) the estimates of system states and quantization errors can be obtained simultaneously via the developed observer approach.

Notations: Throughout the paper, $|x|_p$ denotes the $p-norm$ of the vector x, i.e., $|x|_p = (|x_1|^p + |x_2|^p + \cdots + |x_n|^p)^{\frac{1}{p}}$, $p = 1, 2, \cdots$. Given a symmetric matrix A, the notation $A > 0$ (< 0) denotes a positive definite matrix (negative definite, respectively). I_n denotes an identity matrix with dimension n. Given a matrix $X \in \mathbb{R}^{m\times n}$, $|X|_p$ denotes the matrix $p-norm$, that is, $|X|_p = \sup_{x\neq 0} \frac{|Xx|_p}{|x|_p}$, $p = 1, 2, \cdots$.

II. Problem Formulation

Let $\{r_t, t \geq 0\}$ is a right-continuous Markov chain on the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ taking values in a finite state space $\mathbb{S} = \{1, 2, \ldots, s\}$. The mode transition probabilities $\Pi = (\pi_{ij})_{s\times s}$ of the Markov chain is given by

$$p_{ij} = \Pr\{r_{t+\Delta} = j | r_t = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta) & \text{if } i \neq j, \\ 1 + \pi_{ii}\Delta + o(\Delta) & \text{if } i = j, \end{cases}$$

where $\Delta > 0$ and $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0$, π_{ij} is the transition rate from i to j and satisfies: $\pi_{ij} > 0, i \neq j$, and $\pi_{ii} = -\sum_{j\neq i} \pi_{ij} < 0$ for $\forall\ i, j \in \mathbb{S}$.

Considering the following continuous-time Markovian jump system defined in a fixed probability space $(\Omega, \mathcal{F}, \mathcal{P})$

$$\begin{cases} \dot{x}_c(t) &= A_c(r_t)x(t) + B_c(r_t)u(t) + B_{ca}f_a(t) \\ y_c(t) &= C_c x_c(t) \\ q(y_c(t)) &= q(C_c x_c(t)) \end{cases} \tag{1}$$

where $x_c(t) \in \mathbb{R}^n$ denotes the system state vector, $u(t) \in \mathbb{R}^m$ denotes the control input vector, $A_c(r_t) \in \mathbb{R}^{n\times n}$, $B_c(r_t) \in \mathbb{R}^{n\times m}$, $B_{ca} \in \mathbb{R}^{n\times a}$ and $C_c \in \mathbb{R}^{p\times n}$ are known real system matrices. For each possible value $i \in \mathbb{S}$, $A_c(r_t) = A_{ci}$, $B_c(r_t) = B_{ci}$, where A_{ci} and B_{ci} are constant matrices, $q(\cdot) \in \mathbb{R}^p$ is the quantization mapping defined as (8) below. $f_a(t) \in \mathbb{R}^a$ denotes the unknown actuator fault vector which satisfies

$$|f_a(t)|_2 \leqslant r_a \tag{2}$$

where $r_a > 0$ is a known constant.

Throughout this paper, the following assumptions are made

(A1) For each $r_t = i \in \mathbb{S}$, A_{ci} is Hurwitz, and the pair $(A_{ci},\ C_c)$ is a detectable pair;

(A2) Any invariant zero of triple (A_{ci}, B_{ca}, C_c) lies in the left half plane, i.e., for every complex number λ with non-negative real part, rank $\begin{bmatrix} A_{ci} - \lambda I_n & B_{ca} \\ C_c & 0 \end{bmatrix} = n+$ rank(B_{ca}).

(A3) rank$(C_c B_{ca}) =$ rank$(B_{ca}) = a$.

We utilize a coordinate transformation $x(t) = T_1 x_c(t)$ and $y(t) = T_2 y_c(t)$ as in [15] for system (1), such that in the new coordinate system (1) becomes the following form

$$\begin{aligned} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} A_{11i} & A_{12i} \\ A_{21i} & A_{22i} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} B_{1i} \\ B_{2i} \end{bmatrix} u(t) + \begin{bmatrix} B_{a1} \\ 0_{(n-a)\times a} \end{bmatrix} f_a(t) \\ \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} &= \begin{bmatrix} I_a & 0_{a\times(n-a)} \\ 0_{(p-a)\times a} & C_4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \\ \begin{bmatrix} q(y_1) \\ q(y_2) \end{bmatrix} &= \begin{bmatrix} q(x_1(t)) \\ q(C_4 x_2(t)) \end{bmatrix} \end{aligned} \tag{3}$$

with $x_1(t) \in \mathbb{R}^a$, $x_2(t) \in \mathbb{R}^{n-a}$, $y_1(t) \in \mathbb{R}^a$, $y_2(t) \in \mathbb{R}^{p-a}$, $A_{11i} \in \mathbb{R}^{a\times a}$, $A_{12i} \in \mathbb{R}^{a\times(n-a)}$, $A_{21i} \in \mathbb{R}^{(n-a)\times a}$, $A_{22i} \in$

$\mathbb{R}^{(n-a)\times(n-a)}$, $B_{1i} \in \mathbb{R}^{a\times m}$, $B_{2i} \in \mathbb{R}^{(n-a)\times m}$, $B_{a1} \in \mathbb{R}^{a\times a}$, $C_4 \in \mathbb{R}^{(p-a)\times(n-a)}$, and B_{a1} is nonsingular. For symbols simplicity, we denote the vectors and matrices of system (3) as follows

$$\begin{aligned} x(t) &= [x_1^T(t)\ x_2^T(t)]^T,\ y(t) = [y_1^T(t)\ y_2^T(t)]^T, \\ A_i &= \begin{bmatrix} A_{11i} & A_{12i} \\ A_{21i} & A_{22i} \end{bmatrix},\ B_i = \begin{bmatrix} B_{1i} \\ B_{2i} \end{bmatrix}, \\ C_1 &= \begin{bmatrix} I_a & 0_{a\times(n-a)} \end{bmatrix},\ C_2 = \begin{bmatrix} 0_{(p-a)\times a} & C_4 \end{bmatrix}, \\ C &= \left[\frac{C_1}{C_2}\right],\quad B_a = \begin{bmatrix} B_{a1} \\ 0_{(n-a)\times a} \end{bmatrix}. \end{aligned} \tag{4}$$

Considering the quantized output measurement $q(y_2(t))$, we define the quantization error of which as

$$\omega(x,t) = q(y_2(t)) - y_2(t) = q(C_4x_2) - (C_4x_2) \tag{5}$$

then $q(y_2(t))$ can be rewritten as

$$q(y_2(t)) = y_2(t) + \omega(x,t). \tag{6}$$

Based on (3), (4) and (6), we obtain the following quantized fault system

$$\begin{cases} \dot{x}(t) &= A(r_t)x(t) + B(r_t)u(t) + B_a f_a(t) \\ q(y_1(t)) &= q(x_1(t)) \\ q(y_2(t)) &= y_2(t) + \omega(x,t). \end{cases} \tag{7}$$

In this paper, the output measurements $y(t) = [y_1(t)\ \ y_2(t)\ \ \cdots y_p(t)]^T$ are quantized before transmitted over networks with the following logarithmic form

$$q_i(y_i(t)) = \begin{cases} \eta_i^{(j)} & \text{if } \frac{1}{1+\delta_i}\eta_i^{(j)} < y_i(t) \le \frac{1}{1-\delta_i}\eta_i^{(j)}, \\ & \text{and } y_i(t) > 0; \\ 0 & \text{if } y_i(t) = 0; \\ -q_i(-y_i(t)) & \text{if } y_i(t) < 0,\ i = 1,2,\ldots,p, \\ & \ j = 1,2,\ldots \end{cases} \tag{8}$$

where ρ_i denotes the quantizer density of the ith quantizer $q_i(\cdot)$, and $\eta_i^{(0)}$ denotes the initial quantization values for the ith quantizer $q_i(\cdot)$, $\delta_i = (1-\rho_i)/(1+\rho_i)$ is the quantizer parameter. As seen in (8), the real valued signal $y_i(t)$ has been mapped into a piecewise constant signal $q_i(y_i(t))$ taking values in a countable set.

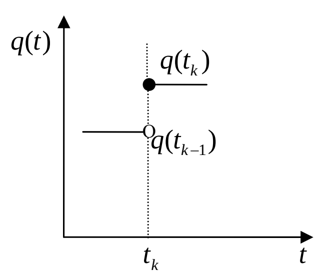

Fig. 1. The updating instant of quantizer: a scalar case

Denote by t_k the updating time instant of the quantizer, and let the set $\mathscr{T} \triangleq \{t_0,\ t_1,\ t_2,\ t_3,\ldots\}$ be a strictly increasing sequence of updating times of the quantizer (8) in $(t_0,\ \infty)$ for some initial time t_0. The quantization mapping $q(\cdot)$ in (8) is indeed right-continuous everywhere, which occurs jumping behavior at each updating time instant $t_k \in \mathscr{T}$ as shown in Figure 1. For simplicity and with some abuse of notation, we use $q(t)$ to denote $q_i(t)$ only within this paragraph. Given $t_k \in \mathscr{T}$, suppose $q(t_k) = q(t_k^+) = \eta_i^{(j)}$ and $q(t_k^-) = \eta_i^{(j-1)}$ for some $j \in \{\pm1, \pm2, \cdots\}$, the following formula (9) describes the discontinuity of $q(\cdot)$ at the updating instant t_k:

$$\begin{aligned} \text{(i) } \dot{q}(t_k^-) &\triangleq \lim_{t\to t_k^-} \frac{q(t)-q(t_k)}{t-t_k} = \lim_{t\to t_k^-} \frac{\eta_i^{(j-1)} - \eta_i^{(j)}}{t-t_k} \\ &= -\infty, \\ \text{(ii) } \dot{q}(t_k^+) &\triangleq \lim_{t\to t_k^+} \frac{q(t)-q(t_k)}{t-t_k} = \lim_{t\to t_k^+} \frac{\eta_i^{(j)} - \eta_i^{(j)}}{t-t_k} \\ &= 0. \end{aligned} \tag{9}$$

The main objective of this paper is to develop an effective observer approach with sliding mode techniques for system (7) in the presences of sensor logarithmic quantization and actuator faults. The key problem is how to cope with the abrupt jumping behavior of quantized signals at each quantizer updating time instant t_k, and compensate the quantization effects such that the discontinuous input term of the SMO should still posses fault-rejection performance despite of signal quantization. At the end of this section, the following definition is adopted from [22].

Definition 1. *[22] The Markovian jump system (7) is said to be stochastically stable if, for $u(t) \equiv 0$, $f_a(t) \equiv 0$ and every initial condition $x_0 \in \mathbb{R}^n$ and $r_0 \in \mathbb{S}$, it follows that* $\mathbb{E}\left\{\int_0^\infty \|x(t)\|^2 \mid x_0, r_0\right\} < \infty$.

III. Observer analysis and design

In this section, we will present an effective method to deal with the aforementioned design problem. We first perform a system augmentation for the original plant (1) to yield a descriptor system, where the output quantization error is transmitted into the input disturbances and is also assembled into the new state vector of the augmented system. Then a new sliding mode observer is constructed for the descriptor augmented system to generate asymptotic estimates for both the original plant state and the quantization error. The remaining part of this section is divided into two sub-sections: III-A *System Augmentation*; and III-B *Descriptor discontinuous Observer*.

A. System Augmentation

To begin with our design approach, we define the following augmented variables and matrices

$$
\begin{aligned}
\bar{n} &\triangleq n+p-a,\\
\bar{x}(t) &\triangleq \begin{bmatrix} x_1(t)\\ x_2(t)\\ \omega(x,t)\end{bmatrix},\ \bar{N}_0 \triangleq \begin{bmatrix} 0_{n\times(p-a)}\\ I_{p-a}\end{bmatrix},\\
\bar{E} &\triangleq \begin{bmatrix} I_a & 0_{a\times(n-a)} & 0_{a\times(p-a)}\\ 0_{(n-a)\times a} & I_{n-a} & 0_{(n-a)\times(p-a)}\\ 0_{(p-a)\times a} & 0_{(p-a)\times(n-a)} & 0_{p-a}\end{bmatrix},\\
\bar{A}_i &\triangleq \begin{bmatrix} A_{11i} & A_{12i} & 0_{a\times(p-a)}\\ A_{21i} & A_{22i} & 0_{(n-a)\times(p-a)}\\ 0_{(p-a)\times a} & 0_{(p-a)\times(n-a)} & -I_{p-a}\end{bmatrix},\\
\bar{B}_i &\triangleq \begin{bmatrix} B_{1i}\\ B_{2i}\\ 0_{(p-a)\times m}\end{bmatrix},\ \bar{B}_a \triangleq \begin{bmatrix} B_{a1}\\ 0_{(n-a)\times a}\\ 0_{(p-a)\times a}\end{bmatrix},\\
\bar{C}_1 &\triangleq \begin{bmatrix} I_a & 0_{a\times(n-a)} & 0_{a\times(p-a)}\end{bmatrix},\\
\bar{C}_2 &\triangleq \begin{bmatrix} 0_{(p-a)\times a} & C_4 & I_{p-a}\end{bmatrix},\\
\bar{F}_2 &\triangleq \begin{bmatrix} 0_{(p-a)\times a} & C_4 & 0_{(p-a)\times(p-a)}\end{bmatrix},\\
\bar{C} &\triangleq \left[\frac{\bar{C}_1}{\bar{C}_2}\right],\ \bar{F}_0 \triangleq \left[\frac{\bar{C}_1}{\bar{F}_2}\right],
\end{aligned}
\tag{10}
$$

and the following augmented descriptor system is constructed

$$
\begin{cases}
\bar{E}\dot{\bar{x}}(t) = \bar{A}_i\bar{x}(t)+\bar{B}_iu(t)+\bar{B}_af_a(t)\\
\qquad\qquad +\bar{N}_0\omega(x,t),\ t\neq t_k,\\
\bar{x}(t_k) = \bar{x}(t_k^+),\quad t=t_k,\ t_k\in\mathscr{T},\\
q(y(t)) = \bar{C}\bar{x}(t).
\end{cases}
\tag{11}
$$

Remark 1. *It is noticed that by a descriptor augmentation strategy, the normal system (7) has been equivalently transmitted into a descriptor hybrid system (11). The main advantage of this handling is that in (11) the output disturbance $\omega(x,t)$ has been transmitted into an input one which facilitates the subsequent design analysis, since output disturbance is indeed more difficult to be dealt with than an input one. However, this augmentation behavior also assembles $\omega(x,t)$ into the new state vector $\bar{x}(t)$, and $\dot{\omega}(x,t)$ is not defined at t_k since $\dot{q}(y_2(t))$ is undefined at $t=t_k$. To overcome this difficulty, in the following discussion, we will propose a new tricky observer technique for the descriptor hybrid system (11), under which the continuity of the resulting error system is ensured strictly in the whole time domain including each quantizer updating time t_k. Based on this property, the stability analysis of the corresponding error dynamic can be performed feasibly.*

We introduce the following lemmas to provide some significant properties for the descriptor augmented system (11), which are useful for the subsequent analysis and design.

Lemma 1. *[14] Define a new matrix $\bar{L}_D\in\mathbb{R}^{(n+p-a)\times(p-a)}$ as $\bar{L}_D=\begin{bmatrix} 0_{n\times(p-a)}\\ L_{(p-a)\times(p-a)}\end{bmatrix}$, where $L=diag(L_1,\ldots,L_{p-a})$, $L_i>0$, $i=1,2,\ldots,p-a$, then the matrix $\bar{S}\triangleq(\bar{E}+\bar{L}_D\bar{C}_2)$ is a nonsingular one which also satisfies the following property*

$$
\bar{C}_2\bar{S}^{-1}\bar{L}_D=I_{p-a},\quad (\bar{A}_i-\bar{N}_0\bar{F}_2)\bar{S}^{-1}\bar{L}_D=-\bar{N}_0. \tag{12}
$$

If we select $\bar{L}_D$ as the aforementioned form, it can be derived that

$$
\bar{S}=\begin{bmatrix} I_n & 0_{n\times(p-a)}\\ LC_2 & L\end{bmatrix},\ \bar{S}^{-1}=\begin{bmatrix} I_n & 0_{n\times(p-a)}\\ -C_2 & L^{-1}\end{bmatrix}. \tag{13}
$$

Lemma 2. *If for each $i\in\mathbb{S}$, A_i is Hurwitz, then $\Big(\bar{S}^{-1}(\bar{A}-\bar{N}_0\bar{F}_2),\ \bar{C}_2\Big)$ is a detectable pair.*

Proof. For $\forall\ \sigma\in\mathbb{R}^+$ it is derived that

$$
\begin{aligned}
&\text{rank}\begin{bmatrix} \sigma I_{\bar{n}}-\bar{S}^{-1}(\bar{A}_i-\bar{N}_0\bar{F}_2)\\ \bar{C}_2\end{bmatrix}\\
&=\text{rank}\left(\begin{bmatrix} \bar{S}^{-1} & 0\\ 0 & I_p\end{bmatrix}\begin{bmatrix} \sigma(\bar{E}+\bar{L}_D\bar{C})-(\bar{A}_i-\bar{N}_0\bar{F}_2)\\ \bar{C}_2\end{bmatrix}\right)\\
&=\text{rank}\left(\begin{bmatrix} \sigma(\bar{E}+\bar{L}_D\bar{C})-(\bar{A}_i-\bar{N}_0\bar{F}_2)\\ \bar{C}_2\end{bmatrix}\right)\\
&=\text{rank}\left(\begin{bmatrix} I_n & \sigma\bar{L}_D\\ 0 & I_p\end{bmatrix}\begin{bmatrix} \sigma\bar{E}-\bar{A}_i+\bar{N}_0\bar{F}_2\\ \bar{C}_2\end{bmatrix}\right)\\
&=\text{rank}\left(\begin{bmatrix} \sigma\bar{E}-\bar{A}_i+\bar{N}_0\bar{F}_2\\ \bar{C}_2\end{bmatrix}\right)\\
&=\text{rank}\left(\begin{bmatrix} \sigma I_n-A_i & 0_{n\times(p-a)}\\ \begin{bmatrix}0_{(p-a)\times a} & -C_4\end{bmatrix} & I_{p-a}\\ \begin{bmatrix}0_{(p-a)\times a} & C_4\end{bmatrix} & I_{p-a}\end{bmatrix}\right)\\
&=p-a+\text{rank}\left(\begin{bmatrix} \sigma I_n-A_i\\ \begin{bmatrix}0_{(p-a)\times a} & -C_4\end{bmatrix}\end{bmatrix}\right)
\end{aligned}
$$

In fact, if A_i is a stable pair, then $\text{rank}(\sigma I_n-A_i)=n$, and further implies that $\text{rank}\left(\begin{bmatrix} \sigma I_n-A_i\\ \begin{bmatrix}0_{(p-a)\times a} & -C_4\end{bmatrix}\end{bmatrix}\right)=n$, which means that $\text{rank}\begin{bmatrix} \sigma I_{\bar{n}}-\bar{S}^{-1}(\bar{A}_i-\bar{N}_0\bar{F}_2)\\ \bar{C}_2\end{bmatrix}=\bar{n}$. Therefore, $\Big(\bar{S}^{-1}(\bar{A}_i-\bar{N}_0\bar{F}_2),\ \bar{C}_2\Big)$ is a detectable pair. The proof is completed. □

B. Descriptor discontinuous observer

Motivated by the discussion of Section III-A, we construct the following new form sliding mode observer

$$
\begin{cases}
\bar{S}\dot{\bar{z}}(t) = (\bar{A}_i-\bar{N}_0\bar{F}_2-\bar{L}_{pi}\bar{C}_2)\bar{z}(t)+\bar{B}_iu(t)\\
\qquad\qquad +\bar{B}_au_s(t)\\
\hat{\bar{x}}(t) = \bar{z}(t)+\bar{S}^{-1}\bar{L}_Dq(y_2(t)),
\end{cases}
\tag{14}
$$

where $\bar{S}$ is defined as in (12), $\bar{z}(t)\triangleq[z_x^T(t),\ z_\omega^T(t)]^T\in\mathbb{R}^{\bar{n}}$ is the intermediate vector of the dynamic system (14) with $z_x(t)\in\mathbb{R}^n$ and $z_\omega^T(t)\in\mathbb{R}^{p-a}$. $\hat{\bar{x}}(t)\triangleq[\hat{x}^T(t),\ \hat{\omega}^T(t)]^T\in\mathbb{R}^{\bar{n}}$ with $\hat{x}(t)\triangleq[\hat{x}_1^T(t),\ \hat{x}_2^T(t)]^T\in\mathbb{R}^n$ is the estimation of $\bar{x}(t)$. $\bar{L}_D$ and $\bar{L}_{pi}\in\mathbb{R}^{\bar{n}\times(p-a)}$ are the proportional gain and derivative gain matrices, respectively, where $\bar{L}_D$ is designed based on Lemma 1, and $\bar{L}_{pi}$ is to be designed later. As shown in Lemma 2, if A_i is Hurwitz, then $\Big(\bar{S}^{-1}(\bar{A}_i-\bar{N}_0\bar{F}_2),\ \bar{C}_2\Big)$ is detectable and it is feasible to find $\bar{L}_{pi}$ such that $\bar{A}_i-\bar{N}_0\bar{F}_2-\bar{L}_{pi}\bar{C}_2$ is Hurwitz. $u_s(t)\in\mathbb{R}^a$ is the discontinuous input designed as in (28) below to eliminate the effect of actuator fault $f_a(t)$.

Remark 2. *It is noticed that only $q(y_2(t))$ being a part of the quantized output measurements $q(y(t))$ has been injected into the observer (14). Indeed, $q(y_1(t))$ being the remaining part* *of $q(y(t))$ will be also adopted in observer (14) which is to synthesize the discontinuous input $u_s(t)$ as seen in (28) below. That means, a full use of the total output $q(y(t))$ is required in this design even though only $q(y_2(t))$ emerges in observer (14).*

We define the following error variables

$$\begin{aligned}\bar{e}(t) &\triangleq \hat{\bar{x}}(t) - \bar{x}(t), \quad \bar{e}_x^T(t) \triangleq \hat{x}(t) - x(t),\\ \bar{e}_1^T(t) &\triangleq \hat{x}_1(t) - x_1(t), \quad \bar{e}_2^T(t) \triangleq \hat{x}_2(t) - x_2(t),\\ \bar{e}_\omega(x,t) &\triangleq \hat{\omega}(t) - \omega(x,t), \end{aligned} \quad (15)$$

and it is obvious that the following relationship holds

$$\bar{e}(t) = [\bar{e}_x^T(t)\ \bar{e}_\omega(x,t)]^T, \quad \bar{e}_x^T(t) = [\bar{e}_1^T(t)\ \bar{e}_2^T(t)]^T. \quad (16)$$

We should now analyze and establish the error dynamic from plant (11) and observer (14).

Theorem 1. *Considering the descriptor system (11) and observer (14), and recalling $t_k \in \mathscr{T}$, $k = 1, \ldots$ denotes the time sequences of quantizer's updating, the following proportions hold: (i) the error vector $\bar{e}(t)$ defined in (15) is strictly continuous for $t \in [0,\ \infty)$, and $\bar{e}(t_k)$ does not grow along each updating instant $t_k \in \mathscr{T}$; (ii) when $t \in (t_k,\ t_{k+1})$ the error dynamic can be established by*

$$\begin{aligned}\dot{\bar{e}}(t) &= \bar{S}^{-1}(\bar{A}_i - \bar{N}_0\bar{F}_2 - \bar{L}_{pi}\bar{C}_2)\bar{e}(t)\\ &\quad + \bar{B}_a(u_s(t) - f_a(t)), \quad t \in (t_k,\ t_{k+1}). \end{aligned} \quad (17)$$

Proof. (i) First, we prove that the error vector $\bar{e}(t)$ is strictly continuous for $t \in [0, +\infty)$, and the value of which does not increase abruptly along each $t_k \in \mathscr{T}$. It is derived from (12) that $\bar{S}^{-1}\bar{L}_D = \begin{bmatrix} 0_{n\times(p-a)} \\ I_{p-a} \end{bmatrix}$. Accordance with the observer dynamics (14) it is derived that

$$\hat{\bar{x}}(t) = \begin{bmatrix} \hat{x}(t) \\ \hat{\omega}(t) \end{bmatrix} = \begin{bmatrix} z_x(t) \\ z_\omega(t) \end{bmatrix} + \begin{bmatrix} 0_{n\times(p-a)} \\ I_{p-a} \end{bmatrix} q(y_2(t)) \quad (18)$$

which yields that $\hat{x}(t) = z_x(t)$ and $\hat{\omega}(t) = z_\omega(t) + q(y_2(t))$. Therefore $\bar{e}_x(t) = \hat{x}(t) - x(t) = z_x(t) - x(t)$. It is easy to see that $\bar{e}_x(t)$ is strictly continuous for $t \in [0, +\infty)$, since both $z_x(t)$ and $x(t)$ are continuous for $t \in [0, +\infty)$. On the other hand, we consider $\bar{e}_\omega(x,t)$, recalling $\omega(x,t) = q(y_2(t)) - y_2(t)$ it is derived that

$$\begin{aligned}\bar{e}_\omega(x,t) &= \hat{\omega}(t) - \omega(x,t) = z_\omega(t) + y_2(t)\\ &= z_\omega(t) + C_4 x_2(t). \end{aligned} \quad (19)$$

It can be seen intuitively that $\bar{e}_\omega(x,t)$ is strictly continuous for $t \in [0, +\infty)$. We thus conclude that $\bar{e}(t) = [\bar{e}_x(t),\ \bar{e}_\omega(x,t)]$ is strictly continuous for $t \in [0, +\infty)$, and $\dot{\bar{e}}(t)|_{t=t_k}$ is well defined for $k = 0, 1, \ldots, \infty$. Consequently, the value of $\bar{e}(t)$ does not increase abruptly along each $t_k \in \mathscr{T}$.

(ii) We now prove that for $t \in (t_k,\ t_{k+1})$, the error dynamic can be derived as (17). Notice that $\dot{q}(y_2(t)) = 0$ for $t \in (t_k,\ t_{k+1})$, from observer (14), the following can be derived

$$\begin{aligned}&\bar{S}\dot{\hat{\bar{x}}}(t) = \bar{S}\dot{\bar{z}}(t) + \bar{L}_D\dot{q}(y_2(t))\\ =\ &(\bar{A}_i - \bar{N}_0\bar{F}_2 - \bar{L}_{pi}\bar{C}_2)\hat{\bar{x}}(t) + \bar{L}_D\dot{q}(y_2(t))\\ &+\bar{B}_a u_s(t) + \bar{B}_i u(t) - (\bar{A}_i - \bar{N}_0\bar{F}_2)\bar{S}^{-1}\bar{L}_D y_2(t)\\ &+\bar{L}_{pi}\bar{C}_2\bar{S}^{-1}\bar{L}_D y_2(t) \end{aligned} \quad (20)$$

Recall that $(\bar{A}_i - \bar{N}_0\bar{F}_2)\bar{S}^{-1}\bar{L}_D = -\bar{N}_0$ and $\bar{C}_2\bar{S}^{-1}\bar{L}_D = I_{p-a}$, (20) becomes

$$\begin{aligned}\bar{S}\dot{\hat{\bar{x}}}(t) &= (\bar{A}_i - \bar{N}_0\bar{F}_2 - \bar{L}_{pi}\bar{C}_2)\hat{\bar{x}}(t) + \bar{L}_D\dot{q}(y_2)(t)\\ &\quad +\bar{B}_a u_s(t) + \bar{B}_i u(t) + \bar{N}_0 y_2(t) + \bar{L}_{pi} y_2(t) \end{aligned} \quad (21)$$

On the other hand, by adding $\bar{L}_D\dot{q}(y_2(t)) = 0$ in both sides of the plant (11), and noting that $\bar{E}\dot{\bar{x}}(t) + \bar{L}_D\dot{q}(y_2(t)) = \bar{E}\dot{\bar{x}}(t) + \bar{L}_D\bar{C}_2\dot{\bar{x}}(t) = \bar{S}\dot{\bar{x}}(t)$, one can obtain

$$\begin{aligned}\bar{S}\dot{\bar{x}}(t) &= (\bar{A}_i - \bar{L}_{pi}\bar{C}_2)\bar{x}(t) + \bar{L}_{pi}\bar{C}_2\bar{x}(t) + \bar{B}_a f_a(t)\\ &\quad +\bar{N}_0\omega(x,t) + \bar{B}_i u(t) + \bar{L}_D\dot{q}(y_2(t)) \end{aligned} \quad (22)$$

Recalling that $\bar{L}_{pi}\bar{C}_2\bar{x}(t) = \bar{L}_{pi}y_2(t)$ and $\bar{N}_0\omega(x,t) = \bar{N}_0(\bar{C}_2\bar{x}(t) - \bar{F}_2\bar{x}(t))$, it is derived from (22)

$$\begin{aligned}\bar{S}\dot{\bar{x}}(t) &= (\bar{A}_i - \bar{N}_0\bar{F}_2 - \bar{L}_{pi}\bar{C}_2)\bar{x}(t) + \bar{L}_D\dot{q}(y_2(t))\\ &\quad \bar{B}_a f_a(t) + \bar{B}_i u(t) + \bar{N}_0 y_2(t) + \bar{L}_{pi} y_2(t) \end{aligned} \quad (23)$$

Subtracting (21) from (23), one can obtain

$$\begin{aligned}\dot{\bar{e}}(t) &= \bar{S}^{-1}(\bar{A}_i - \bar{N}_0\bar{F}_2 - \bar{L}_{pi}\bar{C}_2)\bar{e}(t)\\ &\quad +\bar{S}^{-1}\bar{B}_a(u_s(t) - f_a(t)), \quad t \in (t_k,\ t_{k+1}) \end{aligned} \quad (24)$$

Notice that in error system (24) it is derived that $\bar{S}^{-1}\bar{B}_a = \bar{B}_a$, and finally we obtain the error dynamic (17). □

IV. Synthesis of error dynamic

As seen in the error dynamics (17) established in Section III-B, the derivative observer gain $\bar{L}_D$ has been designed. In this section, we are focused on designing the discontinuous input $u_s(t)$ and observer gain $\bar{L}_{pi}$ for observer (14) to stabilize the error system (17). Since in the setting of this paper data needs to be transmitted over a digital communication channel, we have to use only quantized error estimation to implement the synthesis work, while an ideal design without quantization is impossible. The remaining part of this section is divided into two subsections: Section IV-A *Quantizer analysis and design*, considering the logarithmic quantizer (8) under investigation, we analyze and establish a relationship between the quantization errors and the quantized values $q(y(t))$ refered to quantizer density; and Section IV-B *Error dynamic analysis*, it is proved that, based on the result of Section IV-A, the input $u_s(t)$ can be designed based on quantized error estimation successfully to ensure the stochastic stability of error dynamic (17).

A. Quantizer analysis and design

In this section, considering the output measurement $y(t)$, we devote to establish a relationship between the quantized value $q(y(t)$ and the corresponding quantization error. We denote $y(t)$ as y_t for simplicity, and y_t is quantized accordance with the logarithmic scheme (8). We define the following quantization error [3]

$$e_q(t) = q(y_t) - y_t, \quad q(y_t) = (I_p + \Lambda_t)y_t, \tag{25}$$

where $\Lambda_t = \text{diag}(\Lambda_1(t), \ldots, \Lambda_p(t)) \in [-\Delta,\ \Delta]$ is a unknown time-varying matrix, and

$$\Delta \triangleq \text{diag}(\delta_1, \ldots, \delta_p) \tag{26}$$

is defined for the norm bound of Λ_t. Notice that $e_q(t)$ in (25) is different from $\omega(x,t)$ defined in (5). Without loss of generality we suppose that $\rho_1 = \rho_2 \cdots = \rho_p = \rho$ in quantizer (8) with $0 < \rho < 1$ being a given one, which implies $\delta_1 = \delta_2 \cdots = \delta_p = \delta$ and $0 < \delta < 1$. We provide the following Lemma which establishes a relationship between $e_q(t)$ and $q(y_t)$ in view of quantization density ρ.

Lemma 3. *Considering the logarithmic quantizer (8), if its density satisfies $\rho > \frac{1}{3}$, then the quantization error $e_q(t)$ and quantized value $q(y_t)$ satisfy the following relationship*

$$|e_q(t)|_2 \leqslant \frac{1}{\alpha}|q(y_t)|_2 \leqslant |q(y_t)|_2, \tag{27}$$

where the parameter $\alpha = \frac{1-\delta}{\delta}$ satisfies $\alpha > 1$.

The detail proof of Lemma 3 is omitted here for space reason.

B. Error dynamic analysis

In this section, the design tasks of observer gain $\bar{L}_{pi}$ and the discontinuous input $u_s(t)$ are carried out to ensure the stochastic stability of error system (17). On basis of the result of Section IV-A, the term $u_s(t)$ is constructed as the following form

$$u_s(t) = -\left(\frac{(\alpha+1)}{\alpha-1}|P_{1i}B_{a1}|_2 \times \gamma_a + \frac{\alpha\varepsilon}{\alpha-1}\right) \times (P_{1i}B_{a1})^{-1}\,\text{sgn}\,(q(\bar{e}_1)), \tag{28}$$

where α is introduced to adjust and compensate quantization effects, $\gamma_a > 0$ is the norm upper bound of the actuator fault $f_a(t)$ defined in (A4), and $\varepsilon > 0$ is a small parameter which is selected artificially.

We present the stability analysis result of the error dynamics (17) under the quantized controller (28).

Theorem 2. *Considering the sliding mode controller $u_s(t)$ (28) in error dynamics (17), if the density of the logarithmic quantizer (8) satisfies $\rho > \frac{1}{3}$, and for each $i \in \mathbb{S}$ there exist positive and definite matrices $P_{1i} \in \mathbb{R}^{a\times a}$, $P_{2i} \in \mathbb{R}^{(n+p-2a)\times(n+p-2a)}$, $P_i = diag\{P_{1i},\ P_{2i}\}$ and matrices $Y_i \in \mathbb{R}^{(n+p-a)\times(p-a)}$ such that the following matrices inequality holds*

$$\bar{P}_i\bar{S}^{-1}(\bar{A}_i - \bar{N}_0\bar{F}_2) + (\bar{A}_i - \bar{N}_0\bar{F}_2)^T\bar{S}^{-T}\bar{P}_i - Y_i\bar{C}_2 - \bar{C}_2^TY_i^T + \sum_{j=1}^{s}\pi_{ij}\bar{P}_j < 0 \tag{29}$$

then the error dynamics (17) is stochastically stable. Furthermore, the observer gain $\bar{L}_{pi}$ is designed as $\bar{L}_{pi} = \bar{S}\bar{P}_i^{-1}Y_i$.

Proof. Considering the error dynamics (17) with the quantizer updating instant t_k being taken into account, we choose the following Lyapunov function

$$V(t, r_t = i) = \bar{e}^T(t)\bar{P}_i\bar{e}(t), \quad t \in (t_k,\ t_{k+1}), \tag{30}$$

for each $i \in \mathbb{S}$, where $\bar{P}_i > 0$ is as previously defined. Taking the *Weak infinitesimal operator* along the state trajectories of system (17) for $t \in (t_k,\ t_{k+1})$, it can be calculated that

$$\begin{aligned}\mathscr{L}V(t,i) \;=\; & \bar{e}^T(t)\Big[\bar{P}_i\bar{S}^{-1}(\bar{A}_i - \bar{N}_0\bar{F}_2 - \bar{L}_{pi}\bar{C}_2) \\ & +(\bar{A}_i - \bar{N}_0\bar{F}_2 - \bar{L}_{pi}\bar{C}_2)^T\bar{S}^{-T}\bar{P}_i + \\ & \sum_{j=1}^{s}\pi_{ij}\bar{P}_j\Big]\bar{e}(t) \\ & +2\bar{e}^T(t)\bar{P}_i\bar{B}_a(u_s(t) - f_a(t)).\end{aligned} \tag{31}$$

We now consider the term $\bar{e}^T(t)\bar{P}_i\bar{B}_a$ in (31). In fact, it can been seen that

$$\begin{aligned}& \bar{e}^T(t)\bar{P}_i\bar{B}_a \\ =\; & [\bar{e}_1^T(t)\ \underbrace{\bar{e}_2^T(t)\ \bar{e}_\omega^T(x,t)}_{n+p-2a}] \times \left[\begin{array}{c|c} P_{1i} & 0_{a\times(\bar{n}-a)} \\ \hline 0_{(\bar{n}-a)\times a} & P_{2i}\end{array}\right] \\ & \times \left[\begin{array}{cc} B_{a1}^T & 0_{(n+p-2a)\times a}^T \end{array}\right]^T \\ =\; & \bar{e}_1^T(t)P_{1i}B_{a1}\end{aligned} \tag{32}$$

one can thus obtain

$$\bar{e}^T(t)\bar{P}_i\bar{B}_a(u_s(t) - f_a(t)) = \bar{e}_1^T(t)P_{1i}B_{a1}(u_s(t) - f_a(t)) \tag{33}$$

which can be rewritten as

$$\begin{aligned}& \bar{e}^T(t)P_{1i}B_{a1}\Big(u_s(t) - f_a(t)\Big) \\ =\; & [q^T(\bar{e}_1(t)) - e_q^T(t)]P_{1i}B_{a1}(u_s(t) - f_a(t)) \\ =\; & q^T(\bar{e}_1(t))P_{1i}B_{a1}u_s(t) - q^T(\bar{e}_1(t))P_{1i}B_{a1}f_a(t) \\ & -e_q^T(t)P_{1i}B_{a1}u_s(t) + e_q^T(t)P_{1i}B_{a1}f_a(t)\end{aligned} \tag{34}$$

Given vectors x, y with appropriate dimension, using the property $|x^Ty| \leqslant |x|_2|y|_2$, (34) can be enlarged as

$$\begin{aligned}& \bar{e}_1^T(t)P_{1i}B_{a1}\Big(u_s(t) - f_a(t)\Big) \\ \leqslant\; & q^T(\bar{e}_1(t))P_{1i}B_{a1}u_s(t) + |q(\bar{e}_1(t))|_2 \times |P_{1i}B_{a1}f_a(t)|_2 \\ & +|e_q(t)|_2 \times |P_{1i}B_{a1}u_s(t)|_2 \\ & +|e_q(t)|_2 \times |P_{1i}B_{a1}f_a(t)|_2\end{aligned} \tag{35}$$

Notice that, if the quantizer density parameter satisfies $\rho > \frac{1}{3}$, then from Lemma 3 the inequality (27) holds. By replacing y_t with $\bar{e}_1(t)$ in the inequality (27) one can obtain $|e_q(t)|_2 \leqslant \frac{1}{\alpha}q(\bar{e}_1(t))$, and thus the third and fourth terms in the right side of inequality (35) follow that

$$\begin{aligned}& |e_q(t)|_2 \times |P_{1i}B_{a1}u_s(t)|_2 + |e_q(t)|_2 \times |P_{1i}B_{a1}f_a(t)|_2 \\ \leqslant\; & \frac{1}{\alpha}q(\bar{e}_1(t)) \times \Big[|P_{1i}B_{a1}u_s(t)|_2 + |P_{1i}B_{a1}f_a(t)|_2\Big]\end{aligned} \tag{36}$$

On the other hand, we decompose $q^T(\bar{e}_1(t))P_iB_{a1}u_s(t)$ as

$$q^T(\bar{e}_1(t))P_{1i}B_{a1}u_s(t) = \frac{\alpha-1}{\alpha}q^T(\bar{e}_1(t))P_{1i}B_{a1}u_s(t) + \frac{1}{\alpha}q^T(\bar{e}_1(t))P_{1i}B_{a1}u_s(t) \quad (37)$$

then by substituting (36)-(37) into (35) we have

$$\begin{aligned} &\bar{e}_1^T(t)P_{1i}B_{a1}\Big(u_s(t)-f_a(t)\Big) \\ \leqslant\ & \frac{\alpha-1}{\alpha}q^T(\bar{e}_1(t))P_{1i}B_{a1}u_s(t) \\ &+\left(1+\frac{1}{\alpha}\right)|q^T(\bar{e}_1(t))|_2\times|P_{1i}B_{a1}f_a(t)|_2 \\ &+\frac{1}{\alpha}q^T(\bar{e}_1(t))P_{1i}B_{a1}u_s(t) \\ &+\frac{1}{\alpha}|q(\bar{e}_1(t))|_2\times|P_{1i}B_{a1}u_s(t)|_2 \end{aligned} \quad (38)$$

For simplicity we denote the controller gain of (28) as

$$\beta = \frac{(\alpha+1)}{\alpha-1}\gamma_a|P_{1i}B_{a1}|_2 + \frac{\alpha\varepsilon}{\alpha-1},$$

by substituting the controller (28) into (38), for the first and second term in the right side of inequality of (38) we have

$$\begin{aligned} &\frac{(\alpha-1)}{\alpha}q^T(\bar{e}_1(t))P_{1i}B_{a1}u_s(t) \\ &+\left(1+\frac{1}{\alpha}\right)|q^T(\bar{e}_1(t))|_2\times|P_{1i}B_{a1}f_a(t)|_2 \\ =\ & -\frac{(\alpha-1)}{\alpha}q^T(\bar{e}_1(t))(P_{1i}B_{a1})(P_{1i}B_{a1})^{-1}\beta\mathrm{sgn}(q(\bar{e}_1)) \\ &+\frac{(\alpha+1)}{\alpha}|q^T(\bar{e}_1(t))|_2\times|P_{1i}B_{a1}f_a(t)|_2 \\ \leqslant\ & -\frac{(\alpha-1)}{\alpha}\left(\frac{\alpha+1}{\alpha-1}|P_{1i}B_{a1}|_2\gamma_a+\frac{\alpha\varepsilon}{\alpha-1}\right)|q(\bar{e}_1(t))|_2 \\ &+\frac{(\alpha+1)}{\alpha}|q(\bar{e}_1(t))|_2\times|P_{1i}B_{a1}|_2\gamma_a \\ \leqslant\ & -\frac{(\alpha+1)}{\alpha}|q(\bar{e}_1(t))|_2\times|P_{1i}B_{a1}|_2\gamma_a \\ &+\frac{(\alpha+1)}{\alpha}|q(\bar{e}_1(t))|_2\times|P_{1i}B_{a1}|_2\gamma_a-\varepsilon|q(\bar{e}_1(t))|_2 \\ =\ & -\varepsilon|q(\bar{e}_1(t))|_2 \end{aligned} \quad (39)$$

On the other hand, under controller (28) the third and fourth terms in the right side of the inequality (38) follows

$$\begin{aligned} &\frac{1}{\alpha}q^T(\bar{e}_1(t))P_{1i}B_{a1}u_s(t)+\frac{1}{\alpha}|q(\bar{e}_1(t))|_2|P_{1i}B_{a1}u_s(t)|_2 \\ \leqslant\ & -\frac{\beta}{\alpha}|q(\bar{e}_1(t))|_2+\frac{\beta}{\alpha}|q(\bar{e}_1(t))|_2\times|\mathrm{sgn}\left(q(\bar{e}_1)|_2\right). \end{aligned} \quad (40)$$

Notice that $|\mathrm{sgn}\left(q(\bar{e}_1)\right)|_2 = 1$ and thus we have

$$-\frac{\beta}{\alpha}|q(\bar{e}_1(t))|_2+\frac{\beta}{\alpha}|q(\bar{e}_1(t))|_2\times|\mathrm{sgn}\left(q(\bar{e}_1)\right)|_2 = 0, \quad (41)$$

which further implies

$$\frac{1}{\alpha}q^T(\bar{e}_1)P_{1i}B_{a1}u_s(t)+\frac{1}{\alpha}|q(\bar{e}_1)|_2|P_{1i}B_{a1}u_s(t)|_2 = 0. \quad (42)$$

Substituting (39) and (42) into (38) yields

$$\bar{e}_1^T(t)P_{1i}B_{a1}(u_s(t)-f_a(t)) \leqslant -\varepsilon|q(\bar{e}_1(t))|_2 \leqslant 0. \quad (43)$$

It follows from (43) and (31) that

$$\begin{aligned} \mathscr{L}V(t,i) \leqslant\ & \bar{e}^T(t)\Big[\bar{P}_i\bar{S}^{-1}(\bar{A}_i-\bar{N}_0\bar{F}_2-\bar{L}_{pi}\bar{C}_2) \\ &+(\bar{A}_i-\bar{N}_0\bar{F}_2-\bar{L}_{pi}\bar{C}_2)^T\bar{S}^{-T}\bar{P}_i \\ &+\sum_{j=1}^{s}\pi_{ij}\bar{P}_j\Big]\bar{e}(t) \end{aligned} \quad (44)$$

In the light of (29) with letting $\bar{L}_{pi} = \bar{S}\bar{P}_i^{-1}Y_i$, it can be derived that $\bar{P}_i\bar{S}^{-1}(\bar{A}_i-\bar{N}_0\bar{F}_2-\bar{L}_{pi}\bar{C}_2)+(\bar{A}_i-\bar{N}_0\bar{F}_2-\bar{L}_{pi}\bar{C}_2)^T\bar{S}^{-T}\bar{P}_i+\sum_{j=1}^{s}\pi_{ij}\bar{P}_j < 0$. Therefore, it follows from (44) that $\mathscr{L}V(t,i) < 0$ for $\forall\ \bar{e}(t) \neq 0$. Following a similar line in the proof of Theorem 4 in Section 2.2 in [1], it can be proved that $\mathbb{E}\left\{\int_0^\infty \|\bar{e}(t)\|^2 \mid \bar{e}_0, r_0\right\} < \infty$, which means that error system (17) is stochastically stable. This completes the proof. □

We now summarize the design procedure of the proposed robust observer strategy in this paper as follows.

Design procedure:

(i) Perform a coordinate transformation for system (1) to obtain the standard form (3);

(ii) Select the derivative gain $\bar{L}_D$ according to Lemma 1 such that the matrix $\bar{S} = (\bar{E}+\bar{L}_D\bar{C})$ is nonsingular; solve the condition (29) to obtain the Lyapunov matrix $\bar{P}_i$ and thus obtain the observer gain $\bar{L}_{pi}$.

(iii) Select the logarithmic quantizer density as $\rho > 1/3$ such that the design parameter α satisfies $\frac{1}{\alpha} < 1$, design $u_s(t)$ as the form of (28).

V. SIMULATION EXAMPLE

In this section, we will present a practical example to demonstrate the effectiveness of the proposed observer approach. We consider the nominal system matrix $A(r_t)$ of (3) which is taken from the following linearized model of an F-404 aircraft engine system in [7] $A(t) = \begin{bmatrix} -1.46 & 0 & 2.428 \\ 0.1643+0.5\psi(t) & -0.4+\psi(t) & -0.3788 \\ 0.3107 & 0 & -2.23 \end{bmatrix}$ with $\psi(t)$ being an uncertain model parameter. Let $\psi(t)$ be subject to a Markov process $r(t)$ with $s=2$, and the transition rate be given as $\pi_{11}=-3$, $\pi_{12}=3$, $\pi_{21}=4$, $\pi_{22}=-4$. The uncertainty $\psi(t)$ is assumed to be -0.5 when $r(t)=1$ and -2 when $r(t)=2$, respectively. Under this setting , we have

$$A_1 = \begin{bmatrix} -1.4600 & 0 & 2.428 \\ -0.0857 & -0.9 & -0.3788 \\ 0.3107 & 0 & -2.23 \end{bmatrix}, A_2 = \begin{bmatrix} -1.46 & 0 & 2.428 \\ -0.8357 & -2.4 & -0.3788 \\ 0.3107 & 0 & -2.23 \end{bmatrix}$$

Other coefficient matrices of (3) are set as follows:

$$\begin{aligned} B_1 &= \begin{bmatrix} 0.15 & 0.12 \\ 0.15 & -1.5 \\ 0.2 & -0.2 \end{bmatrix}, B_2 = \begin{bmatrix} 0.15 & 0.2 \\ 0.11 & -1.5 \\ 0.50 & -0.5 \end{bmatrix}, \\ B_a &= \begin{bmatrix} 0.5 & 1 \\ 0.1 & 1 \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1.3 \end{bmatrix} \end{aligned}$$

It is obvious that the system matrices satisfies the standard form (3). It is assumed that the actuator faults $f_a(t) = [f_{a1}^T(t)\ f_{a2}^T(t)]^T$ has the following forms:

$$f_{a1}(t) = \begin{cases} 0.1t, & 0 \leqslant t \leqslant 2, \\ 0.2 & 2 < t < 3, \\ 0.5\sin(10t)+0.2\cos(10t) & 3 \leqslant t \leqslant 5 \end{cases}$$

$$f_{a2}(t) = \begin{cases} 0.3\cos(5t), & 0 \leqslant t \leqslant 1, \\ 2\sin(10t)+0.1 & 1 < t \leqslant 5 \end{cases}$$

and it is calculated that $\gamma_a = 2.1$. Furthermore, the control input is set as $u(t) = \begin{bmatrix} -\cos(t) & -\cos(t) \end{bmatrix}^T$. In this case the system dimensions are $n = 3$, $p = 3$, $a = 2$ and $\bar{n} = 4$. According to the proposed design procedure, we select the derivative gain as $\bar{L}_D = \begin{bmatrix} 0 & 0 & 0 & 0.5 \end{bmatrix}^T$, and it can be calculated $\bar{S}$ is nonsingular. Solve the condition (29), The observer gains $\bar{L}_{pi}$ is thus calculated as

$$\bar{L}_{p1} = \begin{bmatrix} -0.4211 & 0.0255 & -0.3237 & -0.3239 \end{bmatrix}^T,$$
$$\bar{L}_{p2} = \begin{bmatrix} -0.3863 & 0.0441 & -0.2909 & -0.2998 \end{bmatrix}^T.$$

We select the quantizer density $\rho = [\rho_1 \ \rho_2 \ \rho_3]^T$ as $\rho_1 = \rho_2 = \rho_3 = 0.8$, and it is calculated $\alpha = \frac{2\rho}{(1-\rho)} = 8$. Choose $\varepsilon = 10^{-4}$, we design $u_s(t)$ as $u_s(t) = -(2.7 \times |P_{1i}B_{a1}|_2 + 1.1429 \times \varepsilon) \times (P_{1i}B_{a1})^{-1}\text{sgn}(q(\bar{e}_1))$. The simulation results are provided in the following Figures 2-3. The trajectories of state vector $x(t)$ and its estimations are illustrated in Figure 2, and the comparisons between output measurements $y(t)$ and its quantized values are exhibited in Figures 3. It can be seen that the tracking performance of system states $x(t)$ has achieved ideal performances.

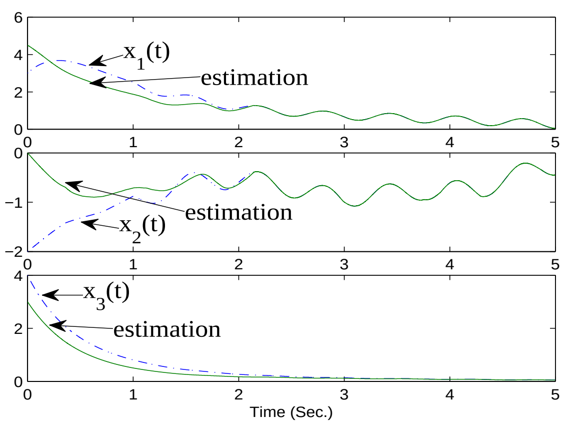

Fig. 2. $x(t)$ and its estimation

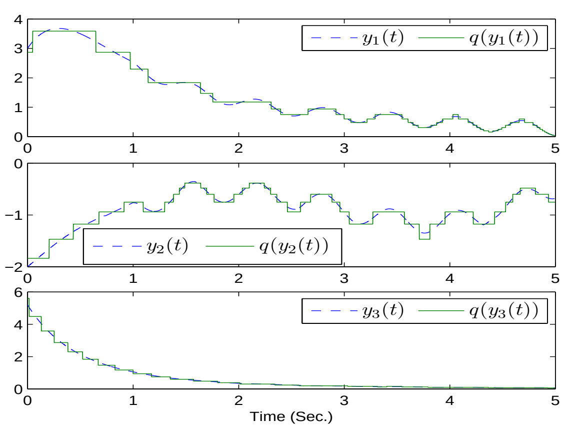

Fig. 3. $y(t)$ and its quantized value $q(y(t))$

VI. Conclusion

In this paper, the robust state observer design problem has been investigated for a class of continuous-time MJSs with quantization and actuator faults. A descriptor augmentation observer approach has been proposed to solve this design problem. A simulation of F-404 aircraft engine system has been provided to illustrate the validness of the developed robust observer approach. Our future work will consider extending the proposed observer approach to nonlinear systems with interval type-2 fuzzy model [11], [9], [10].

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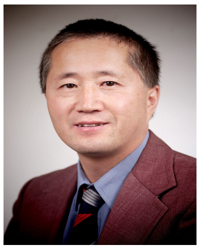

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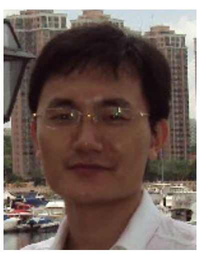

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