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FEEDBACK CONTROL OF A SPACE VEHICLE WITH UNACTUATED FUEL SLOSH DYNAMICS

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ABSTRACT

We develop a mathematical model that describes the accelerating flight of a spacecraft in a fixed plane. The spacecraft is represented as a rigid body and fuel slosh dynamics are included using a common pendulum model. The control inputs are defined by a transverse body fixed force and a pitching moment about the center of mass of the spacecraft; the slosh dynamics are assumed to be unactuated. The model is placed in the form of a nonlinear control system that allows for the study of planar vehicle maneuvers. We develop a nonlinear feedback controller that stabilizes a relative equilibrium corresponding to suppression of the transverse, pitch, and slosh dynamics. This controller is a significant extension of what has been done previously, since it simultaneously controls both the rigid vehicle motion and the fuel slosh dynamics.

INTRODUCTION

The dynamics and control of vehicles with unactuated fuel slosh dynamics has been extensively treated in the literature. It has been demonstrated[5] that lumped spring-mass and pendulum models can approximate complicated fluid and structural dynamics; such pendulum models have formed the basis for many studies on dynamics and control of vehicles with fuel slosh. For accelerating space vehicles, studies have made use of thrust vectoring to suppress the fuel slosh dynamics. This approach has commonly made use of methods of linear control design[2,11,12] and adaptive control.[1] A related paper, following a similar approach, is motivated by a robot arm moving a liquid filled container.[3] A significant deficiency of all of these approaches is that suppression of the slosh dynamics inevitably leads to excitation of the transverse vehicle motion through coupling effects.

In this paper, we assume[1,2,11,12] that a constant axial thrust force acts on the spacecraft but here we assume that additional control effectors, e.g. gas jet thrusters, can be used to independently exert a transverse force and a pitching moment on the vehicle. We investigate the opportunities for obtaining improved vehicle control properties in such cases. In particular, we show that this formulation allows simultaneous control of the transverse dynamics, the pitch dynamics and the slosh dynamics. We compare our results with previous results.[1,2,11,12]

We note that these problems are interesting examples of underactuated control problems; in particular, the objective is to simultaneously control the rigid body vehicle degrees of freedom and the fuel slosh degrees of freedom using only control effectors that act on the rigid vehicle. Control of the unactuated fuel slosh degree of freedom must be achieved through the system coupling. We have previously developed theoretical controllability and stabilizability results for a large class of underactuated mechanical systems using methods from nonlinear control theory;[6-10] we are guided by this previous research but there are important features that are unique to the specific problem that is addressed in this paper.

MODEL FORMULATION

In this paper we study the dynamics and

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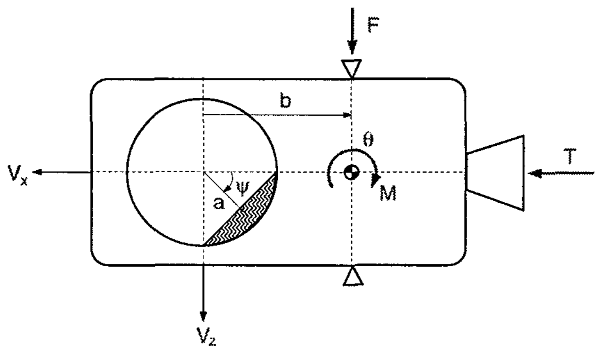

Fig. 1 Model of the spacecraft with slosh dynamics.

control of an accelerating rigid spacecraft that moves in a fixed plane; fuel slosh dynamics are included as represented in Figure 1. The important variables are the axial and transverse components of the vehicle velocity vector v_x, v_z, the attitude angle θ of the vehicle with respect to a fixed reference, and the angle ψ of the pendulum with respect to the vehicle longitudinal axis, representing the fuel slosh. A constant thrust $T > 0$ is assumed to act through the vehicle center of mass along the vehicle's longitudinal axis; a transverse force F and a pitching moment M are available for control purposes. The constants in the problem are the vehicle mass m and moment of inertia I (without fuel), the fuel mass m_f and moment of inertia I_f, the length $a > 0$ of the pendulum, and the distance b between the pendulum point of attachment and the vehicle center of mass location along the longitudinal axis; if the pendulum point of attachment is in front of the vehicle center of mass then $b > 0$. The parameters m_f, I_f and a depend on the shape of the fuel tank, the characteristics of the fuel and the fill ratio of the fuel tank.

Under the indicated assumptions, the total kinetic energy is given by

$$\begin{aligned} T &= \frac{1}{2}m((v_x + b\dot{\theta}\sin\theta)^2 + (v_z + b\dot{\theta}\cos\theta)^2) \\ &+ \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}I_f(\dot{\theta}+\dot{\psi})^2 \\ &+ \frac{1}{2}m_f\Big[\left(v_x + a(\dot{\theta}+\dot{\psi})\sin(\theta+\psi)\right)^2 + \\ &\qquad \left(v_z + a(\dot{\theta}+\dot{\psi})\cos(\theta+\psi)\right)^2\Big] . \quad (1) \end{aligned}$$

Since gravitational effects are ignored, there is no potential energy. We include dissipative effects due to fuel slosh, described by a damping coefficient ϵ.

The resulting equations of motion are given by

$$\begin{aligned} &(m+m_f)(\dot{v}_x + \dot{\theta}v_z) + m_f a(\ddot{\theta}+\ddot{\psi})\sin\psi + mb\dot{\theta}^2 \\ &\quad + m_f a(\dot{\theta}+\dot{\psi})^2\cos\psi = T , \\ &(m+m_f)(\dot{v}_z - \dot{\theta}v_x) + m_f a(\ddot{\theta}+\ddot{\psi})\cos\psi \\ &\quad + mb\ddot{\theta} - m_f a(\dot{\theta}+\dot{\psi})^2\sin\psi = F , \\ &(I+mb^2)\ddot{\theta} + mb(\dot{v}_z - \dot{\theta}v_x) = M + Fb , \\ &(I_f + m_f a^2)(\ddot{\theta}+\ddot{\psi}) + m_f a\Big[(\dot{v}_x + \dot{\theta}v_z)\sin\psi + \\ &\quad (\dot{v}_z - \dot{\theta}v_x)\cos\psi\Big] = -\epsilon\dot{\psi} . \end{aligned} \quad (2)$$

For simplicity in our analysis, we do not seek to control the axial vehicle velocity. Hence we ignore the first equation of (2), viewing the axial vehicle velocity as an exogenous variable. The second and third equations of (2) show that the transverse motion and the pitching motion of the vehicle are directly actuated, while the last equation of (2) shows that the fuel slosh dynamics can be controlled only through the coupling. Our previous work[6,10] on dynamics and control of underactuated mechanical systems treat equations that are slight modifications of those given above.

Note that if the axial thrust T is a positive constant and if the transverse force and pitching moment are zero, $F = M = 0$, then the vehicle and fuel slosh dynamics have a relative equilibrium defined by

$$v_z = \bar{v}_z, \theta = \bar{\theta}, \dot{\theta} = 0, \psi = 0, \dot{\psi} = 0 \quad (3)$$

where $\bar{v}_z$, $\bar{\theta}$ are arbitrary constants. Without loss of generality in our subsequent analysis, we consider the relative equilibrium at the origin, i.e. $\bar{v}_z = 0, \bar{\theta} = 0$. Note that the relative equilibrium corresponds to the vehicle axial velocity

$$v_x(t) = \left(\frac{T}{m+m_f}\right)t + v_x(0)$$

where $v_x(0)$ is the initial axial velocity of the vehicle.

The first equation of (2) allows us to write

$$\begin{aligned} \dot{v}_x + \dot{\theta}v_z = \frac{1}{m+m_f}\Big[&T - m_f a(\ddot{\theta}+\ddot{\psi})\sin\psi \\ &- mb\dot{\theta}^2 - m_f a(\dot{\theta}+\dot{\psi})^2\cos\psi\Big] . \end{aligned} \quad (4)$$

Although the pitch and slosh dynamics have some influence on the axial acceleration, this effect is small so that we substitute the approximation

$$\dot{v}_x + \dot{\theta} v_z = \frac{T}{m + m_f} \tag{5}$$

into the last equation of (2). This leads to

$$\begin{aligned}
&(m + m_f)(\dot{v}_z - \dot{\theta} v_x(t)) + m_f a(\ddot{\theta} + \ddot{\psi}) \cos\psi + mb\ddot{\theta} \\
&\quad - m_f a(\dot{\theta} + \dot{\psi})^2 \sin\psi = F \ , \\
&(I + mb^2)\ddot{\theta} + mb(\dot{v}_z - \dot{\theta} v_x(t)) = M + Fb \ , \\
&(I_f + m_f a^2)(\ddot{\theta} + \ddot{\psi}) + m_f a \Big[(\frac{T}{m + m_f}) \sin\psi \\
&\quad + (\dot{v}_z - \dot{\theta} v_x(t)) \cos\psi \Big] + \epsilon \dot{\psi} = 0 \ ,
\end{aligned} \tag{6}$$

where $v_x(t)$ is considered as an exogenous input. Our subsequent analysis is based on the above equations of motion for the transverse, pitch and slosh dynamics of the vehicle.

STABILIZABILITY ANALYSIS

If we consider only small vehicle motions about the relative equilibrium at the origin, then we can develop a formal linearization of equations (6) which results in:

$$\begin{aligned}
&(m + m_f)(\dot{v}_z - \dot{\theta} v_x(t)) + m_f a(\ddot{\theta} + \ddot{\psi}) + mb\ddot{\theta} = F \ , \\
&(I + mb^2)\ddot{\theta} + mb(\dot{v}_z - \dot{\theta} v_x(t)) = M + Fb \ , \\
&(I_f + m_f a^2)(\ddot{\theta} + \ddot{\psi}) + m_f a(\dot{v}_z - \dot{\theta} v_x(t)) + \epsilon \dot{\psi} = 0 \ .
\end{aligned} \tag{7}$$

It is easy to show that, on the basis of the linearized equations of motion (7), neither of the two control inputs F, M alone can stabilize the relative equilibrium at the origin; if one of the inputs is identically zero, there are necessarily fixed unstable modes. In fact, the case that is commonly treated in the published literature[1,2,11,12] corresponds to $F = 0$ (no transverse force control effector) while the moment control effector is used to stabilize the slosh dynamics; the effect on the transverse dynamics is not normally studied in the published literature.

The case of interest in this paper corresponds to the use of both a transverse force effector and a pitching moment effector as controls. The origin of the linearized time varying equations (7) can be made uniformly asymptotically stable by linear state feedback. In the subsequent sections of this paper, we develop a nonlinear feedback controller that stabilizes the origin of equations (6), that is the relative equilibrium given in (3). Physically, this corresponds to using transverse force and pitching moment controls to stabilize the transverse and pitching dynamics of the vehicle and the fuel slosh dynamics. That is, we simultaneously seek to stabilize all actuated and unactuated degrees of freedom.

One of the difficulties in control design is the time variation representing the non-constant axial velocity of the vehicle that appears in the equations. This time variation formally prohibits the use of transfer function concepts, even for the linearized equations (7). In the subsequent sections, we propose a control design approach that overcomes this difficulty.

NONLINEAR FEEDBACK CONTROLLER

In this section, we design a nonlinear controller to stabilize the relative equilibrium at the origin of the equations (6). Our control design is based on a Lyapunov function approach. By defining control transformations from (F, M) to new control inputs (u_1, u_2):

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \mathbb{M}(\psi)^{-1} \begin{bmatrix} F - m_f a\ddot{\psi} \cos\psi + m_f a(\dot{\theta} + \dot{\psi})^2 \sin\psi \\ M + Fb \end{bmatrix} ,$$

where $\mathbb{M}(\psi) = \begin{bmatrix} m + m_f & m_f a \cos\psi + mb \\ mb & I + mb^2 \end{bmatrix}$, the equations of motion (6) can be written in the following form

$$\begin{aligned}
\dot{v}_z &= u_1 + \dot{\theta} v_x(t) \\
\ddot{\theta} &= u_2 \\
\ddot{\psi} &= -u_2 - c\cos\psi u_1 - d\sin\psi - e\dot{\psi}
\end{aligned} \tag{8}$$

where $c = \dfrac{m_f a}{I_f + m_f a^2}$, $d = \dfrac{m_f a T}{(I_f + m_f a^2)(m + m_f)}$ and $e = \dfrac{\epsilon}{I_f + m_f a^2}$.

The matrix $\mathbb{M}(\psi)$ is invertible for all ψ if

$$(m + m_f)I + mm_f b(b - a) > 0 \ ;$$

this condition is usually satisfied for most spacecraft. Therefore, we assume that the matrix is invertible throughout the paper.

We first make the simplifying assumption that we can directly control $\dot{\theta}$ using u_2. To remove u_2 from the equations, we define a new variable ω as

$$\omega = \dot{\psi} + \dot{\theta} \tag{9}$$

and we obtain the following reduced equations of motion:

$$\begin{aligned} \dot{v}_z &= u_1 + v_x(t)\hat{u}_2 \\ \dot{\theta} &= \hat{u}_2 \\ \dot{\psi} &= -\hat{u}_2 + \omega \\ \dot{\omega} &= -c\cos\psi u_1 + e\hat{u}_2 - d\sin\psi - e\omega \end{aligned} \tag{10}$$

where $\hat{u}_2$ is a new control variable in the reduced equations of motion.

Now, we consider the following candidate Lyapunov function for the reduced system:

$$V = \frac{r_1}{2}v_z^2 + \frac{r_2}{2}\theta^2 + r_3\big[d(1-\cos\psi) + \frac{1}{2}\omega^2\big] \tag{11}$$

where r_1, r_2 and r_3 are positive constants. The function V is positive definite in the domain $D = \{(v_z, \theta, \psi, \omega)| - \pi < \psi < \pi\}$. The time derivative of V along the trajectories of (10) is

$$\begin{aligned} \dot{V} &= r_1 v_z(u_1 + v_x(t)\hat{u}_2) + r_2\theta\hat{u}_2 + r_3 d\sin\psi(-\hat{u}_2 + \omega) \\ &\quad + r_3\omega(-c\cos\psi u_1 + e\hat{u}_2 - d\sin\psi - e\omega) \\ &= [r_1 v_z - r_3 c\omega\cos\psi]u_1 \\ &\quad + [r_1 v_x(t)v_z + r_2\theta - r_3 d\sin\psi + r_3 e\omega]\hat{u}_2 - r_3 e\omega^2 . \end{aligned} \tag{12}$$

To obtain Lyapunov stability for the origin of the reduced system (10), we require $\dot{V} \leq 0$ in D. The obvious choice for the feedback laws that guarantee the condition to be satisfied is :

$$\begin{aligned} u_1 &= -k_1\big[r_1 v_z - r_3 c\omega\cos\psi\big] \\ \hat{u}_2 &= -k_2\big[r_1 v_x(t)v_z + r_2\theta - r_3 d\sin\psi + r_3 e\omega\big] \end{aligned} \tag{13}$$

where k_1 and k_2 are positive constants. To prove asymptotic stability, we consider conditions for $\dot{V} = 0$: $\omega = 0$, $v_z = 0$, and

$$r_2\theta - r_3 d\sin\psi = 0 . \tag{14}$$

By differentiating (14) along the trajectories of (10), it can be easily shown that condition (14) is not satisfied for all time except at the origin. Thus, the origin is the only invariant set in $\{\eta \mid \dot{V}(\eta) = 0\}$ and we conclude that the origin of the reduced system (10) with the feedback law (13) is asymptotically stable.

To develop a control law for u_2, we use the integrator backstepping idea.[4] The complete system (8) can be reformulated into the following form :

$$\begin{aligned} \dot{\eta} &= f(\eta) + g(\eta)\dot{\theta} \\ \ddot{\theta} &= u_2 \end{aligned} \tag{15}$$

where $\eta = \begin{bmatrix} v_z & \theta & \psi & \omega \end{bmatrix}^T$ and

$$f(\eta) = \begin{bmatrix} u_1 \\ 0 \\ \omega \\ -c\cos\psi u_1 - d\sin\psi - e\omega \end{bmatrix}, \quad g(\eta) = \begin{bmatrix} v_x(t) \\ 1 \\ -1 \\ e \end{bmatrix} .$$

A feedback law for u_2, expressed in terms of $\hat{u}_2$, can be selected as

$$u_2 = -k_3(\dot{\theta} - \hat{u}_2) - k_4\frac{\partial V}{\partial \eta}g(\eta) + \frac{d\hat{u}_2}{dt} \tag{16}$$

to guarantee asymptotic stability of the origin of the complete system. Stability can be checked using the Lyapunov function

$$\begin{aligned} V_a &= \frac{r_1}{2}v_z^2 + \frac{r_2}{2}\theta^2 + r_3\big[d(1-\cos\psi) + \frac{1}{2}\omega^2\big] \\ &\quad + \frac{1}{2k_4}(\dot{\theta} - \hat{u}_2)^2 . \end{aligned} \tag{17}$$

Using LaSalle's principle, we can prove asymptotic stability of the origin of the closed loop defined by (8) and the feedback control laws

$$\begin{aligned} u_1 &= -k_1\big[r_1 v_z - r_3 c\omega\cos\psi\big] \\ u_2 &= -k_3\dot{\theta} - (k_2 k_3 + k_4)\big[r_1 v_x(t)v_z + r_2\theta \\ &\quad - r_3 d\sin\psi + r_3 e\omega\big] - k_2\big[r_1(\frac{d}{c} - \dot{\theta}v_z)v_z \\ &\quad + r_1 v_x(t)(u_1 + \dot{\theta}v_x(t)) + r_2\dot{\theta} \\ &\quad - r_3(d\cos\psi\dot{\psi} - e\dot{\omega})\big] . \end{aligned} \tag{18}$$

Here the gains are $r_i > 0$, $i = 1, 2, 3$ and $k_i > 0$, $i = 1, 2, 3, 4$, but otherwise they can be chosen arbitrarily to achieve good closed loop responses.

SIMULATION

In this section, we demonstrate the effectiveness of the controller by performing simulations of the closed loop defined by the nonlinear equation (2) and the controller (18). The physical parameters used in the simulation are $m = 600$ (kg), $I = 720$ (kg $\cdot$ m^2), $m_f = 100$ (kg), $I_f = 90$ (kg $\cdot$ m^2), $a = 0.32$ (m), $b = 0.25$ (m), $T = 500$ (N) and $\epsilon = 0.19$ (kg $\cdot$ m^2/s). We consider stabilization of a spacecraft in orbital transfer, suppressing the

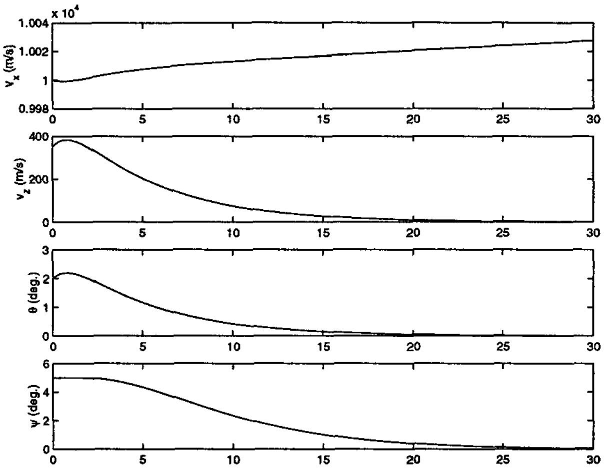

Fig. 2 State Variables v_x, v_z, θ and ψ.

transverse and pitching motion of the spacecraft and sloshing of fuel while the spacecraft is accelerating. This control objective is to stabilize the relative equilibrium corresponding to a constant axial spacecraft acceleration of 0.714 (m/sec^2) and $\theta = 0,\ \dot{\theta} = 0,\ v_z = 0,\ \psi = 0,\ \dot{\psi} = 0$.

Responses are given in Figure 2 corresponding to the initial condition $(v_{x0}, v_{z0}, \theta_0, \dot{\theta}_0, \psi_0, \dot{\psi}_0) =$ $(10000\ \text{(m/s)}, 350\ \text{(m/s)}, 2°, 0.57\ \text{(deg/s)}, 5°, 0)$.

The transverse velocity, attitude angle and the slosh angle converge to the relative equilibrium at zero while the axial velocity v_x increases and $\dot{v}_x$ tends asymptotically to 0.714 (m/sec^2). The responses reflect the fact that control of the slosh dynamics occurs through the system coupling. Our experience indicates that there is trade-off between good responses for the directly actuated degrees of freedom (the transverse and pitch dynamics) and good responses for the unactuated degree of freedom (the slosh dynamics); the controller presented in (18) with gain values $r_1 = 1$, $r_2 = 10^8$, $r_3 = 10^5$, $k_1 = 10^{-4}$, $k_2 = 10^{-9}$, $k_3 = 1$, $k_4 = 10^{-2}$ represent one example of this balance.

Figure 3 shows the time history of the transverse control force F and the pitching moment M. We can see from the figure that the designed controller does not require excessive control force or moment.

CONCLUSION

In this paper, we have developed a new model for the planar dynamics of a spacecraft; the model incorporates both the axial, transverse, and pitch dynamics of the vehicle plus fuel slosh dynamics.

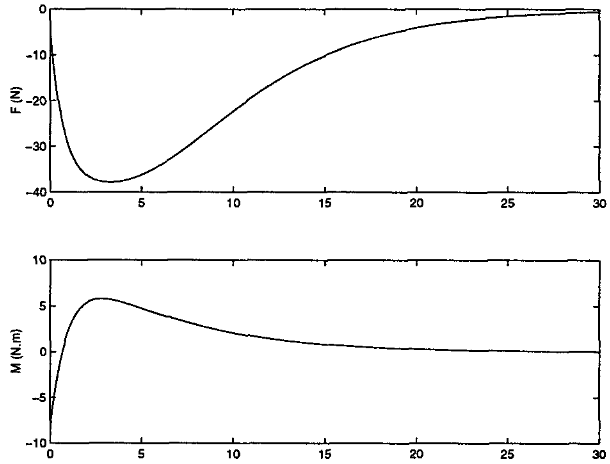

Fig. 3 Transverse Control Force F and Pitching Moment M.

This model is represented in a special nonlinear control form. We have substantially extended the treatment in the published literature by developing a nonlinear controller that suppresses the vehicle transverse and pitch dynamics and the fuel slosh dynamics. The general model that we have developed can also be used to control the axial velocity; the model for the slosh dynamics can be extended and used for control design purposes in a number of ways.

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