Fence patrolling by mobile agents with distinct speeds

Akitoshi Kawamura
University of Tokyo

Yusuke Kobayashi
University of Tokyo

This is based on a paper with the same title [2] to be presented at ISAAC 2012.

Fence patrolling and the partition-based strategy

Suppose we want to patrol a fence (line segment) using k mobile agents with given speeds $v_1, \ldots, v_k$ so that every point on the fence is visited by an agent at least once in every unit time period, indefinitely.

Czyzowicz et al. [1] conjectured that the maximum length of the fence that can be patrolled is $(v_1 + \cdots + v_k)/2$, which is achieved by the simple *partition-based strategy* where each agent i keeps moving back and forth in a sub-segment of length $v_i/2$.

The partition-based strategy is not always optimal

We [2] disproved the conjecture by the counterexample in the figure below, where six agents with speeds 1, 1, 1, 1, 7/3, 1/2 patrol a fence of length 7/2, beating the partition-based strategy attaining 41/12. In the figure, time flows upwards and the agents (four in the left diagram, one in the middle, one in the right) move along the solid lines. The regions that have been visited in the past unit time are shaded. Observe that the regions together cover the whole strip. The dotted lines delimit the regions already covered in previous diagram(s).

Theorem 1 *There is a setting of six agents' speeds for which the partition-based strategy does not patrol the longest possible fence.*

We also showed (see the full version) that the conjecture is true for three agents.

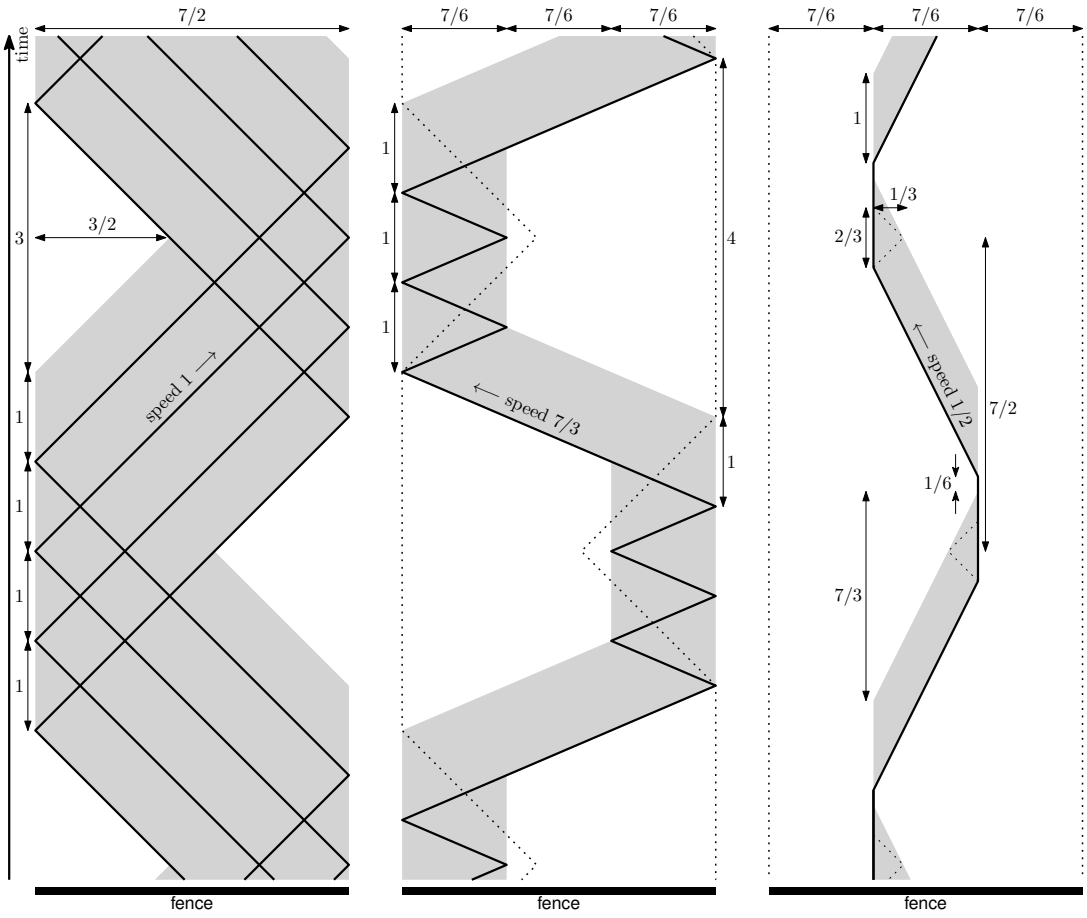
Theorem 4 *For three agents, no strategy patrols a longer fence than the partition-based strategy.*

We do not know whether the conjecture is true for four or five agents.

A revised conjecture

An easy argument about the area of the shaded regions in the diagram shows that no strategy can patrol a fence longer than $v_1 + \cdots + v_k$. This upper bound is twice as big as what the partition-based strategy achieves. Our example in the figure outperforms the partition-based strategy, but barely. Thus we ask if we can improve the upper bound, so that a weakened version of Czyzowicz et al.'s conjecture holds:

Open problem. Is there a constant $c < 1$ such that for any k and any $v_1, \ldots, v_k$, the agents cannot patrol a fence longer than $c(v_1 + \cdots + v_k)$?



[1] J. Czyzowicz, L. Gásieniec, A. Kosowski and E. Kranakis. Boundary patrolling by mobile agents with distinct maximal speeds. In *Proceedings of the 19th Annual European Symposium on Algorithms* (ESA 2011), LNCS 6942, pp. 701–712.

[2] A. Kawamura and Y. Kobayashi. Fence patrolling by mobile agents with distinct speeds. To be presented at the *23rd International Symposium on Algorithms and Computation* (ISAAC 2012).