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TECHNICAL REPORT

Fermi-Gas Model Parametrization of Nuclear Level Density

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The authors furnish a new parametrization of the Fermi-Gas model description of nuclear level densities at excitation energies corresponding to the neutron binding energy. The model adopted is the standard Fermi-Gas model with pairing and shell-effect corrections. Particular care has been devoted to the inclusion of shell effects and to their parametrization. The procedure for the evaluation of level density parameters has been applied to a data-base of 217 nuclei covering a mass range $41 \leqq A \leqq 253$. A global systematics parametrization has been derived which allows for a derivation of level density parameters for nuclei where experimental information is not available.

***KEYWORDS:* nuclear level density, energy-level density, energy levels, Fermi-Gas model, pairing correlations, shell effects, shell correction energies, level density parameters, global level density parameter systematics**

I. INTRODUCTION

The density of nuclear states is a fundamental ingredient for all the application of the statistical theories of nuclear reactions. Except for the very low excitation energy part of the nuclear spectra, where experimental information is available for a wide class of nuclei, the computation of the density of nuclear states must rely on model calculations. Many nuclear structure models have been employed and tested for the calculation of nuclear state densities at excitation energies corresponding to the neutron binding energy (of the order of 8 MeV for medium and heavy nuclei). This is because at those excitation energies an important experimental quantity can be related to the density of nuclear levels; the average spacings of s-wave neutron resonances $\langle D \rangle_{I=0}$. A large set of information on these quantities has been collected and compiled several times in the past. A relatively recent compilation[1] has been made available which provides a large data-base on which the traditional level density models can be tested. Of course this is not the first time that such an analysis has been made. However, the recent data-base has been used only for particular and specific purposes on limited mass ranges. It is therefore interesting to repeat the calculation using well established techniques in order to furnish the utilizers with the best possible parametrization.

In this work we will apply the Fermi-Gas model with the usual correction for pairing correlations. We will then discuss in detail the influence of the shell effects and their inclusion into the Fermi-Gas description as proposed by a technique which is widely applied in the recent versions of most of the computer codes for nuclear reaction calculations.

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The paper is organized as follows: in the next chapter the basic relations of the Fermi-Gas model will be given. The treatment of the various nuclear structure effects will be given in its sections. In Chap. III we will show the results of our analysis and we will give the parametrization obtained. Finally, the conclusions and some remark on further improvements for the theoretical description will be given in the last chapter.

II. BASIC RELATIONS FOR NUCLEAR LEVEL DENSITY DESCRIPTION

1. Fermi-Gas Model

The density of nuclear states into which a set of A non-interacting nucleons (Fermi-Gas) are arranged from an equally spaced single-particle spectrum with spacing g_f at the fermi energy is given by[2]

$$\rho_A(U)=\frac{1}{\sqrt{48U}}e^{2\sqrt{aU}}, \qquad (1)$$

where $a\equiv(\pi^2/6)g_f$ and U is the excitation energy. This elementary relation can be derived from combinatorial analysis simply counting the ways into which the A nucleons can be arranged into the single-particle spectrum.

The state-density for the equivalent system can be derived in an alternative way, using the equilibrium statistical properties of the Fermi-Gas itself. The result is altogether equivalent to that of Eq. (1). The advantage of the thermodynamical approach is that a number of additional constraints can be readily taken into account. For example, the density of states for a mixture of Z protons and N neutrons can be easily derived and it is given by a relation similar to Eq. (1):

$$\rho_{Z,N}(U)=\frac{\sqrt{\pi}}{12a^{1/4}U^{5/4}}e^{2\sqrt{aU}}. \qquad (2)$$

Other constants of motion can be added to the thermodynamical description. In fact, nuclear states are characterized by good quantum numbers, Π and M, for parity and projection of the total angular momentum. It can be shown that for a scalar constant of motion M, a Gaussian law with dispersion σ holds

$$\rho_{Z,N}(U, M, \Pi)=\rho_{Z,N}(U)\frac{1}{\sqrt{2\pi\sigma^2}}e^{-(M^2/2\sigma^2)}\frac{1}{2}, \qquad (3)$$

where equal probability has been assumed for the two parity values $\Pi=-$ and $\Pi=+$. From Eq. (3), differentiating respect to states with $M=J$ and $M=J+1$, we can derive the expression of the density of nuclear levels with total angular momentum J

$$\rho_{Z,N}(U, J, \Pi)=\frac{\sqrt{\pi}}{12a^{1/4}U^{5/4}}e^{2\sqrt{aU}}\frac{2J+1}{2\sigma^2\sqrt{2\pi\sigma^2}}e^{-(J(J+1)/2\sigma^2)}\frac{1}{2}. \qquad (4)$$

This is the basic relation that we will use below to evaluate the parameter systematics for nuclei throughout the nuclide chart. In addition to the parameter a, the only other quantity to be determined in Eq. (4) is the spin dispersion factor σ. For a gas of fermions constrained into a spherical box of radius R, the dispersion factor σ can be evaluated to give

$$\sigma^2=\frac{2}{5}(M_nR^2/\hbar^2)AT, \qquad (5)$$

where M_n is the nucleon mass and T the nuclear temperature in MeV units. This is equivalent to consider the rotation of a rigid body with moment of inertia

$$I_{rr}=\frac{2}{5}AM_nR^2, \qquad (6)$$

which furnishes

$$\sigma^2=\frac{I_{rr}}{\hbar^2}T=0.01389A^{5/3}T, \qquad (7)$$

when using $R=1.2A^{1/3}$ fm and $M_n=938.926$ MeV.

In the thermodynamical approach, the nuclear temperature is defined by the relation

$$\frac{1}{T}\equiv\frac{1}{\rho}\cdot\frac{d\rho}{dU}. \qquad (8)$$

Using Eq. (1) it is easy to recover the usual relation for the Fermi-Gas

$$U=aT^2. \tag{9}$$

It is also straightforward to derive the value of the temperature from the more elaborate state-density of Eq. (2)

$$\frac{1}{T}=\frac{1}{\sqrt{U/a}}-\frac{5}{4U}. \tag{10}$$

Some care has to be taken when evaluating the spin dispersion factor. In fact, from the relations (7), with the definition of the nuclear temperature in Eq. (9) or (10), a dependence on the parameter a is brought back into the level density expression (4).

The calculation of the level density using Eq. (4) is affected by the assumptions made when deriving the state-density from the thermodynamical equilibrium conditions (saddle-point inversion method, continuous approximation) as well as from the assumptions underlying the Fermi-Gas model itself. Quite generally the assumptions made in deriving Eq. (4) are well justified for excitation energies of a few MeV. In particular those approximations are generally valid for excitation energies corresponding the neutron binding energy. This is an important point because at that energy the density of nuclear states of given J and Π is known from the spacing of neutron resonances (usually s-wave neutron resonances). In these cases it is possible to evaluate the only parameter of Eq. (4), a, from

$$\langle D\rangle_{I=0}=\frac{1}{\rho_{Z,N}(U,\ J=1/2,\ \Pi)}, \tag{11}$$

for nuclei with $I_\alpha=0$, or

$$\langle D\rangle_{I=0}=\frac{1}{\rho_{Z,N}(U,\ J=I_\alpha+1/2,\ \Pi)+\rho_{Z,N}(U,\ J=I_\alpha-1/2,\ \Pi)}, \tag{12}$$

for nuclei with $I_\alpha\neq 0$, where I_α is the spin of the nucleus with Z protons and $N-1$ neutrons (target nucleus in the neutron $+(Z, N\text{-}1)$ process). From the experimental $\langle D\rangle^{\exp}_{I=0}$ it is possible to derive the systematics of the level density parameter a. These kind of analyses have been performed several times in the past, perhaps the most quoted and recognized being those of Erba *et al.*[3], Lynn[4] and Dilig *et al.*[5]. Recently, a much wider compilation of the neutron resonance level spacings has been made available[1]. We have used those experimental values to obtain **Fig. 1**. For clearness, **Fig. 2** shows a portion of Fig. 1 in a restricted region, $100\leqq A\leqq 200$.

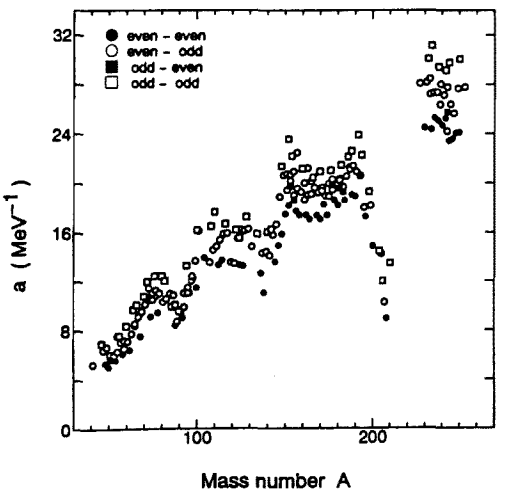


Fig. 1 Level density parameter a calculated from Eq. (4) for $41\leqq A\leqq 235$

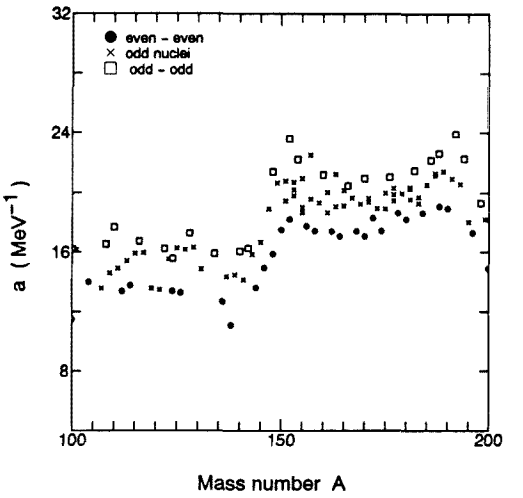


Fig. 2 Level density parameter a calculated from Eq. (4) for $100\leqq A\leqq 200$

The strong fluctuations of the level density parameter a over the entire mass range are due to several nuclear properties whose effects will be discussed below.

2. Pairing Correlations

Of the various effects that must be taken into account to improve the simple description of the nuclear level density based on the Fermi-Gas model, the effect due to pairing correlations in the nuclear motion is clearly revealed in the Figs. 1 and 2. In fact, even-even, odd-mass and odd-odd nuclei are shown to have a different, systematically correlated behavior in the complete mass region. This effect is analogous to the even-odd differences of the nuclear masses and it is easily ascribed to pairing correlations.

A full treatment of the pairing correlation in nuclear motion may be based on statistical as well as on BCS theories[6]. However, a much simpler but effective treatment of this effect can be incorporated into the model description simply redefining the excitation energy of the Fermi-Gas according to

$$U \rightarrow U-\delta, \tag{13}$$

where $\delta=2\Delta$ for even-even nuclei
$=\Delta$ for odd nuclei
$=0$ for odd-odd nuclei.

The pairing correlation parameter Δ may be derived from mass-differences of neighboring nuclei for the entire mass region. However, the values derived from the experimental nuclear masses may be well represented by a simple, smooth function of the mass number. For example the value[7]

$$\Delta=\frac{12}{\sqrt{A}} \tag{14}$$

has shown to be suitable for representing the experimental situation.

The use of this simple prescription may well be questioned from theoretical point of view since the relation (14) has been derived from even-odd difference in nuclear masses, *i.e.* at zero temperature. The situation may be different at finite temperature. However, using this simple procedure we have obtained **Figs.** 3 and 4. It is clear that, overall, the applied correction is effective in condensing the different even-odd behaviors obtained by the straight application of Eq. (4) (Figs. 1 and 2) into a consistent systematics, practically insensitive to even-odd characters.

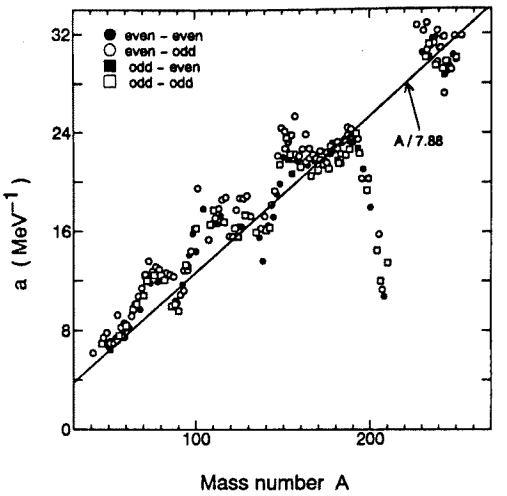


Fig. 3 The same as in Fig. 1, with pairing correction applied (See text for explanations) The least-squares straight line $A/7.88$ is also plotted.

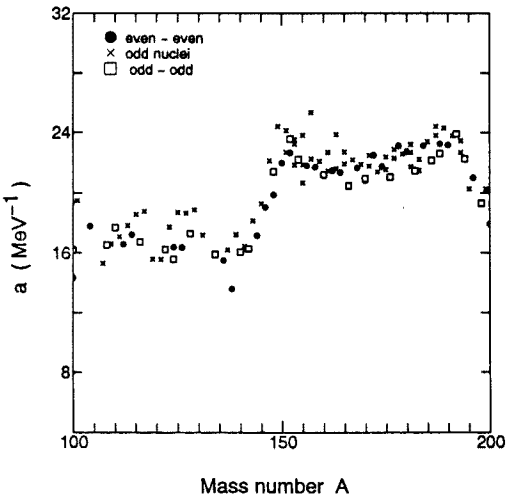


Fig. 4 The same as in Fig. 2, with pairing correction applied (See text for explanations)

3. Shell Effects

From an inspection (Figs. 3 and 4) there still appear to remain strong fluctuations in correspondence of proton and/or neutron shell closures. It is possible to take into account of shell effects in the Fermi-Gas description of the nuclear level density using different approaches. One possibility has been explored by Kataria *et al.*[8]. They have included shell

inhomogeneities into the single-particle spectrum and consequently redefined the thermodynamical quantities necessary to the derivation of Eq. (3) or (4). In this way, the expression for the state-density incorporates the effects of shell closures.

A different approach which produces results altogether equivalent to those obtained by Kataria *et al.* has been proposed by Ignatyuk & coworkers[9]. The advantage of using the Ignatyuk approach is that the Fermi-Gas relations given above remain unchanged except for a.

The Ignatyuk procedure is based on the superfluid model which predicts a phase transition from the superfluid to the normal state at temperatures of the order of $T_{cr}=0.567\Delta$. Above this critical temperature the nucleus behaves as a Fermi-Gas. In this approach a depends on the excitation energy as well as on the shell correction energy E_{sh}

$$a(U)=a(*)\left[1+\frac{E_{sh}}{U}(1-e^{-\gamma U})\right], \qquad (15)$$

where $a(*)$ is the asymptotic level density parameter to which $a(U)$ tends for high excitation energies and γ a dumping parameter.

The shell correction energy E_{sh} is defined as the difference between the experimental nuclear mass and a smooth theoretical mass value M_w, derivable, for example, from a liquid-drop model

$$E_{sh}\equiv M_{\exp}-M_w\,. \qquad (16)$$

The evaluation of this quantity requires some care because different liquid-drop model parametrizations are available from literature and they produce quite different values for E_{sh} (see the next chapter).

Following this approach, for each isotope, $a(*)$ is determined from the average neutron resonance spacing. In fact, the shell correction energy can be evaluated using Eq. (16) and the corresponding level density parameter a can be adjusted to fit the experimental $\langle D\rangle^{\exp}_{l=0}$ by varying $a(*)$.

Using this technique we have repeated the calculations for the nuclei in our data-base and the result of the $a(*)$-systematics is shown in **Fig. 5**. It clearly appears that the shell effects have, to a great extent, been appropriately taken into account by the procedure described above. In Fig. 5 we also show an evaluated "experimental" error associated to each $a(*)$. This is just the direct effect of the experimental uncertainties on $\langle D\rangle^{\exp}_{l=0}$. The value of the dumping parameter γ adopted in our calculation was[10]

$$\gamma=\frac{0.40}{A^{1/3}} \quad \mathrm{MeV}^{-1}. \qquad (17)$$

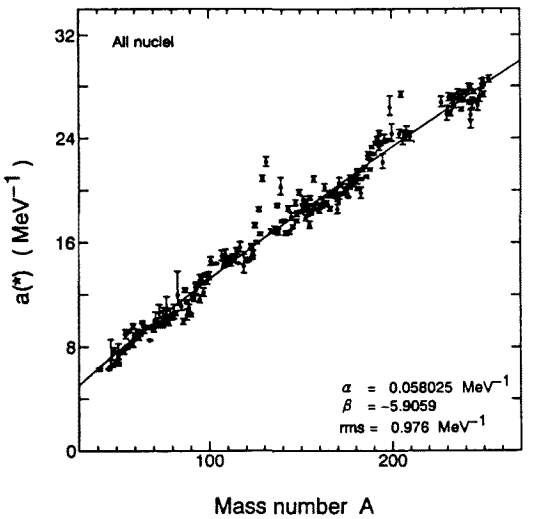


Fig. 5 Level density parameter $a(*)$ systematics Error-bars associated to $a(*)$ correspond to the experimental uncertainties on $\langle D\rangle^{\exp}_{l=0}$. The solid line corresponds to the least-squares fit to Eq. (20). *rms* stands for root mean square deviation.

4. Other Effects

There are a number of other effects that can be incorporated into the Fermi-Gas description of the nuclear level densities. The most important of those are parity and collective effects.

Nuclear energy levels have a clearly unsymmetrical parity distribution in the lower part of the spectrum. As the excitation energy increases, however, the number (or density) of levels with opposite parity tends to be the same. Except for some light nuclei, the parity distribution at excitation energies corresponding to the neutron separation energy

can be assumed to be symmetrical. To treat the cases in which the parity distribution is asymmetrical some technique has been proposed[11]. The possibility of including this effect has been analyzed in several practical applications but a systematic analysis on a large mass range has not yet been done. For this reason the commonly adopted assumption of equal probability for different parity states has been adopted in the present analysis.

The contribution of collective effects is much more important and different techniques have been proposed for its inclusion into the Fermi-Gas description. It is possible to show[12] that the contribution of collective excitations to the state-density can be factorized into the form

$$\rho_{Z,N}(U)=\rho^{i}_{Z,N}(U)Z_{\mathrm{coll}}, \qquad (18)$$

where Z_{coll} is the partition function for the collective degrees of freedom of the system with excitation energies E_c

$$Z_{\mathrm{coll}}=\sum_{c} e^{-(E_c/T)}, \qquad (19)$$

and $\rho^{i}_{N,Z}(U)$ is the state density for the intrinsic (non-collective) excitations.

Various collective models have been employed in the past to evaluate the contribution of collective degrees of freedom to nuclear excitations[7][9]. Also in this case, however, the prescriptions proposed cannot be applied to the complete mass range. In fact, most of the proposed techniques either assume the nucleus to have rotational or vibrational degrees of freedom to be added to the intrinsic excitations. This classification is in many cases not possible. Also, the most important effect due to collective excitations should result on the total nuclear state density. Instead, the information on the neutron resonances concerns only the density of state with fixed spin and parity. In this case the effect of collective excitations at energy close to the neutron binding may be strongly reduced[13].

Finally we would like to mention that our aim here is to furnish a parametrization of the Fermi-Gas description that includes the pairing and shell effect corrections because this will provide the best basis on which further improvements can be constructed.

III. EVALUATION OF LEVEL DENSITY PARAMETERS

All the numerical constants and parameters used in our calculations are given in the APPENDICES. As already mentioned the evaluation of the shell correction energy requires some care. As suggested by Ignatyuk *et al.* we have adopted the droplet model of Myers & Swiatecki[14] and its parametrization is given in the APPENDIX 1.

Several mass formulae can be used to evaluate E_{sh} from Eq. (16). However, the different choices of parametrization lead to different values of the shell correction energy. To give an idea of this problem we have plotted in **Fig. 6**, the results of the calculations made using three diverse prescriptions for the evaluation of the smooth part of the mass formula. As can be seen, even though the overall behavior of E_{sh} is very similar for the three cases shown, the absolute values disagree considerably. In addition, looking at the magnitudes of E_{sh} at the lower and upper part of the mass region considered, a dissimilar mass dependence can be detected. When comparing the results of different

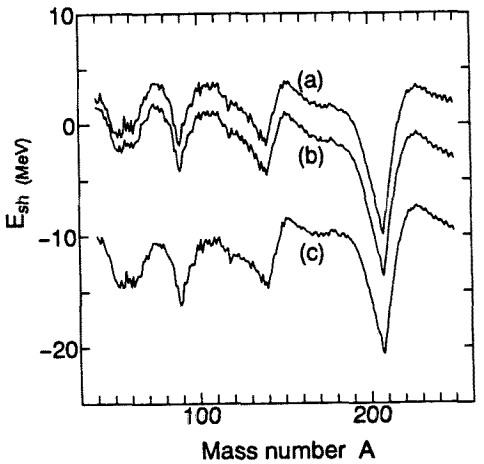


Fig. 6 Shell correction energies as derived from three different mass formulae (a) is from the mass formula of Myers & Swiatecki, adopted in our level density analysis. (b) and (c) show the same quantity as derived from the mass formulae of Refs. (15) and (17) respectively.

authors for the level density parameter systematics, one should be careful to use the same values of the shell correction energies.

Of course, the shell correction energy can be also calculated using some microscopic model as suggested for example by the macroscopic-microscopic model of Möller & Nix[15], but this procedure is rarely necessary since for most of the nuclei of interest the value of the experimental mass is available and E_{sh} can be evaluated using Eq. (16).

As definition of nuclear temperature we have adopted Eq. (10), which is consistent with the state-density relation (4). In the case in which a is energy-dependent (as in Eq. (17)), the nuclear temperature can be calculated by differentiating numerically Eq. (2). However, for the present analysis, the nuclear temperature only enters into the definition of the spin dispersion factor σ. On the other hand, the analytical expression (10) gives results very close to those obtained by numerical differentiation. Weh ave therefore adopted the analytical expression (10) in the analysis.

Our data-base of nuclear information consisted of the experimental average neutron resonances of Mughabghab *et al.*[1] for 217 nuclei. The ground-state spins of the target nuclei have also been taken from the same reference. The nuclear masses were from a CERN compilation[16]. The nuclear information of our data-base and the results of the calculation for the various quantities defined above are given in **APPENDIX 2**.

The systematics of the $a(*)$ parameter is shown in Fig. 5 above and can be well described by a smooth function of the mass number A of type

$$a(*)=\alpha A(1-\beta A^{-1/3}), \qquad (20)$$

where the two parameters α and β can be determined from a least-squares fit of the $a(*)$ values. We have obtained

$$\alpha=0.058025\ \mathrm{MeV}^{-1}, \quad \beta=-5.9059, \qquad (21)$$

with root mean square deviation, rms=0.976 MeV^{-1}.

This parametrization can be used to evaluate the level density parameters in the level density expression Eq. (4), for nuclei whose experimental information on the average level spacings is not available. In fact, from the value of $a(*)$ obtained in Eq. (20), a can be calculated using Eq. (15) and consequently the spin dispersion factor σ.

For a more accurate analysis of our results, we have performed a least-squares fit to Eq. (20), separately for nuclei of given even-odd character. The results are summarized in **Table 1** and the corresponding $a(*)$-systematics are shown in **Figs. 7**(a)~(d). It can be noted that the rms is smaller, compared to the global analysis, in three of the four sub-sets. This fact can allow a more accurate determination of the level density parameters for nuclei of unknown resonance spacings.

Table 1 Parametrization of $a(*)$-systematics according to Eq. (20)

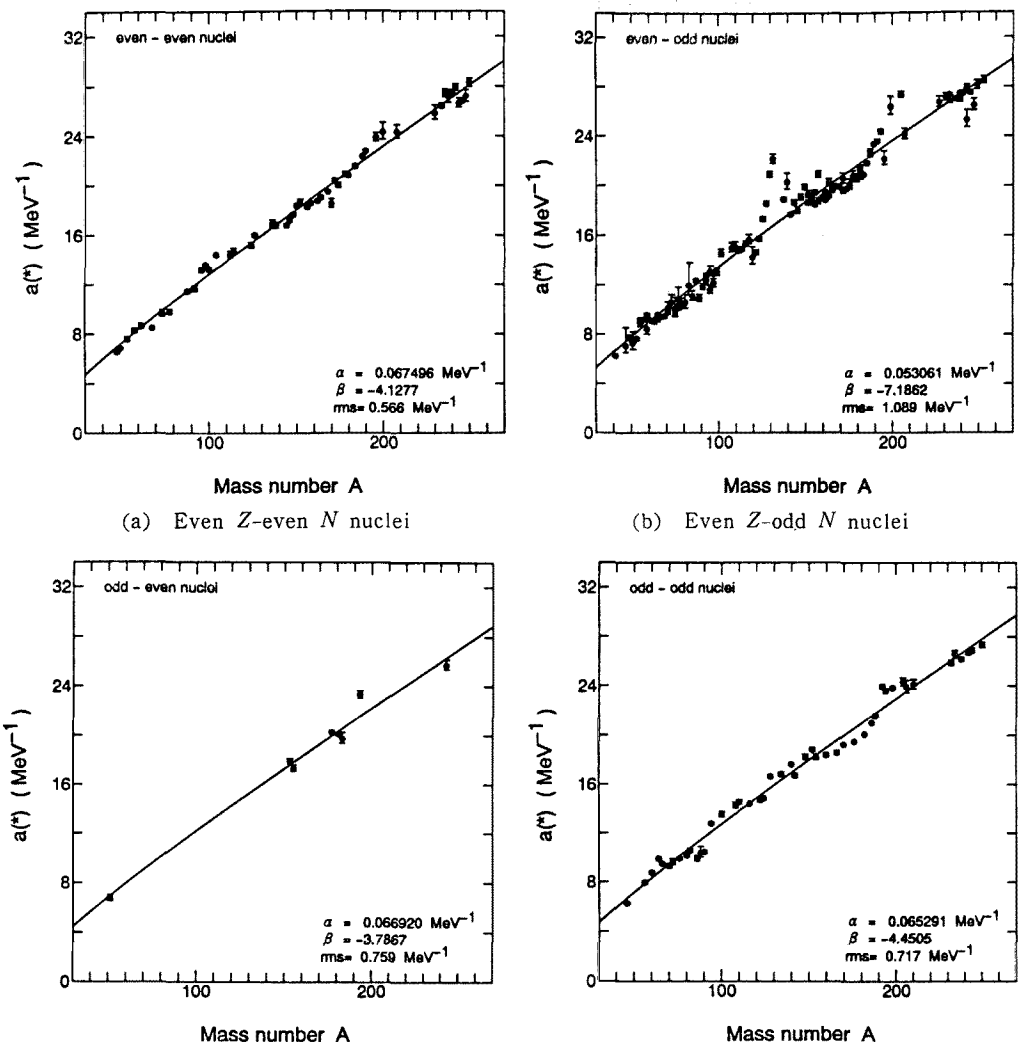
Nuclei	α (MeV^{-1})	β	rms (MeV^{-1})	Number of nuclei
All	0.058025	−5.9059	0.976	217
Even-even	0.067946	−4.1277	0.566	51
Even-odd	0.053061	−7.1862	1.089	112
Odd-even	0.066920	−3.8767	0.759	8
Odd-odd	0.065291	−4.4505	0.717	46

However, to have a better accuracy in the determination of the parameter a, a local systematics of $a(*)$ should be constructed. In fact, the small fluctuation of $a(*)$ in comparison with those of a allows for a much more reliable determination of this parameter and in turn for the level density at the neutron binding.

Finally, we would like also to mention here that we have written a computer program for personal computers that calculates all the quantities described in this paper. It also can construct a level density systematics using different options than those adopted by us. This program is available upon request to the authors.

IV. CONCLUSIONS

We have used a well established technique to evaluate the Fermi-Gas model parametriza-



(a) Even Z-even N nuclei (b) Even Z-odd N nuclei

(c) Odd Z-even N nuclei (d) Odd Z-odd N nuclei

Fig. 7 Level density parameter $a(*)$ systematics for even Z-even N nuclei, even Z-odd N nuclei, odd Z-even N nuclei and odd Z-odd N nuclei

tion of nuclear level density at energies corresponding to the neutron binding energy. Our parametrization can be easily incorporated into those computer codes that use the Fermi-Gas model prescriptions for the calculation of nuclear level densities. Our analysis included pairing correlations as well as shell inhomogeneities effects into the relations based on the Fermi-Gas model.

We would like to remark here that using our global parametrization the evaluation of the nuclear level density parameters can be performed with good accuracy for nuclei where experimental information on the average spacing of neutron resonances is not available. Also, the inclusion of further refinement in the theoretical description can be eased by our analysis. We plan to develop an empirical determination of the collective contribution, in order to widen the physical basis on which the Fermi-Gas model can be employed for the calculation of nuclear level density.

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[APPENDIX]

1. Mass Formula of Myers & Swiatecki[14]

The mass formula that we have used to evaluate M_w is [14]

$$\Delta M_w = M_n N + M_H Z - c_1 A + c_2 A^{2/3} + c_3 \frac{Z^2}{A^{1/3}} - c_4 \frac{Z^2}{A} + p, \qquad \text{(A1)}$$

where $\Delta M_w(\text{amu}) = M_w(\text{amu}) - A$ is the mass-excess,

$$c_i = a_i \left[1 - \kappa \left(\frac{N-Z}{A}\right)^2\right], \qquad i = 1, 2 \qquad \text{(A2)}$$

and

$$p = -\frac{11}{\sqrt{A}} \quad \text{for even-even nuclei}$$
$$= 0 \quad \text{for odd-mass nuclei} \qquad \text{(A3)}$$
$$= +\frac{11}{\sqrt{A}} \quad \text{for odd-odd nuclei.}$$

The numerical constants in Eqs.(A1)~(A3) are

$a_1 = 15.677$ MeV $\quad a_2 = 18.56$ MeV
$\kappa = 1.79$
$c_3 = 0.717$ MeV $\quad c_4 = 1.21129$ MeV
$M_n = 8.07144$ MeV $\quad M_H = 7.28899$ MeV
1 amu = 931.44 MeV.

Note that the even-odd correction term p is slightly different from the corresponding parameter in Eq. (14). We have maintained this difference in our calculations in order to keep the values of the other numerical constants of the mass formula consistent with the original determination by Myers & Swiatecki. For the other mass formulae used to make Fig. 6, see Refs. (15) and (17).

2. Nuclear Information and Level Density Parameter Systematics

Here follows the table containing the nuclear information and the parameter systematics, calculated as described in the text.

Table A1 Data-base containing nuclear information and level density parameter systematics

I_α, Π_α, $\langle D\rangle^{exp}_{l=0}$ and its error are from Mughabghab *et al.*[1]. The neutron binding energy S_n is evaluated from the nuclear mass table[16]. All the other quantities have been defined in the text except for $-a(*)$ and $+a(*)$. These are the uncertainties associated with the level density parameter $a(*)$, evaluated from the experimental uncertainty on $\langle D\rangle^{exp}_{l=0}$.

Z	A	I_α	Π	$\langle D\rangle^{exp}_{l=0}$ KeV	$\Delta\langle D\rangle^{exp}_{l=0}$ KeV	S_n MeV	E_{sh} MeV	T MeV	σ^2	$a(*)$ MeV^{-1}	$-a(*)$ MeV^{-1}	$+a(*)$ MeV^{-1}	a MeV^{-1}
20	41	0.0	1	45.0	6.0	8.3631	-0.150	1.273	8.620	6.285	-0.112	0.129	6.208
21	46	3.5	-1	1.3	0.1	8.7605	1.460	1.336	10.957	6.303	-0.058	0.063	6.959
22	47	0.0	1	10.0	8.0	8.8797	0.727	1.180	10.032	7.064	-0.513	1.516	7.458
22	48	2.5	-1	2.2	0.4	11.6281	0.663	1.304	11.478	6.596	-0.138	0.168	6.914
22	49	0.0	1	13.0	3.0	8.1424	0.244	1.099	10.013	7.696	-0.205	0.263	7.844
22	50	3.5	-1	5.0	0.5	10.9449	-0.468	1.295	12.208	6.876	-0.089	0.099	6.638
22	51	0.0	1	125.0	70.0	6.3724	-0.319	1.039	10.118	7.286	-0.492	0.961	7.089
23	51	6.0	1	2.7	0.7	11.0512	-0.645	1.431	13.940	6.787	-0.205	0.272	6.490
24	51	0.0	1	15.0	2.0	9.2612	-1.089	1.260	12.274	7.573	-0.119	0.138	6.966
24	53	0.0	1	42.0	5.0	7.9398	-1.135	1.170	12.150	7.657	-0.118	0.134	6.983
24	54	1.5	-1	7.1	1.2	9.7190	-0.308	1.137	12.184	7.611	-0.158	0.190	7.431
24	55	0.0	1	26.0	7.0	6.2464	0.424	0.874	9.660	8.935	-0.293	0.394	9.250
25	56	2.5	-1	2.7	0.4	7.2702	-0.643	1.175	13.378	7.983	-0.139	0.163	7.607
26	55	0.0	1	13.0	2.0	9.2986	-2.980	1.242	13.725	9.162	-0.162	0.191	7.193
26	57	0.0	1	17.0	2.0	7.6464	-1.310	1.039	12.182	9.206	-0.132	0.150	8.276
26	58	0.5	-1	6.0	1.0	10.0443	-0.528	1.117	13.487	8.309	-0.159	0.190	7.985
26	59	0.0	1	35.0	15.0	6.5810	0.281	0.942	11.701	8.428	-0.410	0.666	8.618
27	60	3.5	-1	1.1	0.1	7.4916	-0.605	1.121	14.318	8.780	-0.092	0.102	8.400
28	59	0.0	1	13.7	2.0	9.0000	-3.080	1.199	14.895	9.586	-0.161	0.188	7.465
28	61	0.0	1	16.0	2.5	7.8203	-1.252	1.052	13.806	9.171	-0.168	0.199	8.309
28	62	1.5	-1	1.8	0.3	10.5977	-0.767	1.139	15.367	8.691	-0.158	0.189	8.219
28	63	0.0	1	19.1	3.6	6.8387	0.156	0.927	12.849	9.071	-0.204	0.250	9.182
28	65	0.0	1	19.9	3.6	6.0981	1.228	0.824	12.021	9.261	-0.204	0.248	10.169
29	64	1.5	-1	0.32	0.03	7.9159	-0.258	1.052	14.964	9.900	-0.095	0.105	9.724
29	66	1.5	-1	0.51	0.06	7.0666	0.915	0.980	14.671	9.511	-0.117	0.133	10.131
30	65	0.0	1	3.44	0.23	7.9799	0.542	0.955	13.943	9.589	-0.071	0.077	9.969
30	67	0.0	1	4.7	0.4	7.0527	1.846	0.858	13.168	9.463	-0.092	0.101	10.786
30	68	2.5	-1	0.51	0.02	10.1982	1.953	1.018	16.018	8.535	-0.037	0.039	9.702
30	69	0.0	1	5.77	0.73	6.4821	2.642	0.795	12.824	9.484	-0.137	0.157	11.415
30	71	0.0	1	6.9	1.0	5.8356	3.271	0.719	12.165	9.824	-0.165	0.193	12.352
31	70	1.5	-1	0.181	0.035	7.6551	2.335	0.975	16.102	9.327	-0.170	0.209	10.819
31	72	1.5	-1	0.225	0.048	6.5209	3.406	0.858	14.850	9.660	-0.198	0.249	12.010
32	71	0.0	1	0.93	0.36	7.4159	3.220	0.808	13.668	10.131	-0.354	0.543	12.523
32	73	0.0	1	0.96	0.40	6.7823	3.763	0.736	13.038	10.619	-0.399	0.634	13.608
32	74	4.5	1	0.082	0.020	10.2001	3.219	0.914	16.552	9.692	-0.220	0.286	11.824
32	75	0.0	1	3.0	1.5	6.5054	3.393	0.759	14.054	9.958	-0.462	0.818	12.497
32	77	0.0	1	3.75	2.00	6.0723	2.607	0.715	13.843	10.872	-0.542	1.004	13.026
33	76	1.5	-1	0.075	0.005	7.3282	3.700	0.882	16.708	9.953	-0.064	0.068	12.463
34	75	0.0	1	0.42	0.24	8.0278	3.711	0.834	15.445	10.142	-0.461	0.895	12.790
34	77	0.0	1	0.667	0.270	7.4184	3.756	0.789	15.265	10.365	-0.366	0.573	13.157
34	78	0.5	-1	0.146	0.030	10.4968	3.179	0.929	18.367	9.834	-0.179	0.223	11.913
34	79	0.0	1	1.39	0.50	6.9606	3.241	0.772	15.587	10.477	-0.349	0.518	12.942
34	81	0.0	1	3.5	1.5	6.7011	1.998	0.787	16.578	10.589	-0.430	0.694	12.131
34	83	0.0	1	6.7	4.7	5.8954	0.732	0.719	15.779	12.005	-0.760	1.831	12.663
35	80	1.5	-1	0.047	0.005	7.8924	3.384	0.911	18.785	10.192	-0.098	0.110	12.462
35	82	1.5	-1	0.094	0.015	7.5935	2.091	0.912	19.613	10.599	-0.154	0.183	12.067
37	86	2.5	-1	0.200	0.045	8.6504	0.011	1.078	25.097	9.930	-0.205	0.261	9.937
37	88	1.5	-1	2.64	0.88	6.0778	-0.435	0.923	22.323	10.411	-0.352	0.508	10.097
38	85	0.0	1	0.383	0.130	8.5254	1.866	0.875	19.977	11.118	-0.319	0.461	12.501
38	87	0.0	1	0.500	0.066	8.4282	-0.018	0.877	20.812	12.365	-0.153	0.176	12.350
38	88	4.5	1	0.121	0.013	11.1132	-1.505	1.046	25.296	11.473	-0.120	0.134	10.390
38	89	0.0	1	25.0	5.0	6.3639	-0.949	0.855	21.056	10.953	-0.253	0.314	10.206
39	90	0.5	-1	4.0	0.4	6.8568	-1.285	1.002	25.146	10.454	-0.114	0.127	9.557
40	91	0.0	1	6.4	1.1	7.1993	-1.231	0.875	22.377	11.890	-0.216	0.260	10.879
40	92	2.5	1	0.57	0.10	8.6351	-0.005	0.849	22.117	11.704	-0.210	0.253	11.700
40	93	0.0	1	2.6	0.7	6.7320	0.480	0.768	20.374	12.414	-0.326	0.436	12.832
40	95	0.0	1	3.6	0.8	6.4709	1.504	0.753	20.678	11.637	-0.263	0.334	12.867
40	97	0.0	1	4.5	1.0	5.5810	2.493	0.654	18.601	12.181	-0.284	0.361	14.380
41	94	4.5	1	0.044	0.004	7.2296	0.581	0.845	22.801	12.822	-0.110	0.121	13.307
42	93	0.0	1	2.1	0.3	8.0675	-1.844	0.911	24.143	12.765	-0.181	0.211	11.204
42	95	0.0	1	0.975	0.260	7.3713	0.090	0.792	21.753	13.124	-0.321	0.427	13.204
42	96	2.5	1	0.055	0.008	9.1542	1.019	0.792	22.130	13.200	-0.175	0.204	14.089
42	97	0.0	1	0.850	0.045	6.8211	1.694	0.724	20.599	12.906	-0.069	0.073	14.413
42	98	2.5	1	0.032	0.003	8.6423	2.448	0.717	20.758	13.585	-0.117	0.129	15.815
42	99	0.0	1	0.97	0.20	5.9256	3.372	0.629	18.520	13.085	-0.261	0.325	16.218
42	101	0.0	1	0.40	0.08	5.3984	4.667	0.539	16.394	14.584	-0.274	0.339	19.490
43	100	4.5	1	0.0107	0.0018	6.7641	3.021	0.733	21.928	13.562	-0.195	0.233	16.238
44	100	2.5	1	0.025	0.002	9.6733	1.286	0.810	24.246	13.285	-0.095	0.104	14.378
45	104	0.5	-1	0.016	0.001	6.9984	2.968	0.727	23.212	14.214	-0.075	0.080	16.919
47	108	0.5	-1	0.016	0.003	7.2695	2.519	0.748	25.442	14.291	-0.212	0.258	16.554

Z	A	I_α	Π	$\langle D\rangle^{exp}_{J=0}$ KeV	$\Delta\langle D\rangle^{exp}_{J=0}$ KeV	S_n MeV	E_{sh} MeV	T MeV	σ^2	$a(*)$ MeV^{-1}	$-a(*)$ MeV^{-1}	$+a(*)$ MeV^{-1}	a MeV^{-1}
47	110	0.5	-1	0.014	0.002	6.8055	3.379	0.700	24.539	14.571	-0.168	0.195	17.706
48	107	0.0	1	0.135	0.035	7.9274	0.345	0.757	25.366	15.000	-0.319	0.421	15.332
48	109	0.0	1	0.12	0.03	7.3605	1.564	0.697	24.073	15.088	-0.311	0.406	16.629
48	111	0.0	1	0.155	0.020	6.9764	2.380	0.668	23.781	14.779	-0.169	0.193	17.098
48	112	0.5	1	0.020	0.004	9.3953	2.414	0.740	26.743	14.435	-0.229	0.282	16.617
48	113	0.0	1	0.190	0.025	6.5439	2.932	0.631	23.159	14.918	-0.176	0.203	17.835
48	114	0.5	1	0.021	0.004	9.0407	2.741	0.710	26.412	14.703	-0.224	0.274	17.248
48	115	0.0	1	0.235	0.035	6.1448	3.110	0.598	22.571	15.353	-0.207	0.242	18.569
48	117	0.0	1	0.39	0.09	5.7698	2.931	0.575	22.354	15.655	-0.325	0.415	18.775
49	116	4.5	1	0.0094	0.0002	6.7834	2.589	0.721	27.610	14.419	-0.028	0.028	16.767
50	119	0.0	1	0.87	0.39	6.4845	1.467	0.680	27.200	14.228	-0.534	0.880	15.602
50	121	0.0	1	1.40	0.17	6.1714	0.968	0.665	27.323	14.633	-0.176	0.200	15.572
51	122	2.5	1	0.018	0.002	6.8064	1.681	0.734	30.607	14.722	-0.140	0.158	16.257
51	124	3.5	1	0.038	0.004	6.4673	0.766	0.736	31.512	14.871	-0.141	0.158	15.584
52	123	0.0	1	0.132	0.015	6.9329	1.997	0.655	27.656	15.728	-0.157	0.176	17.743
52	124	0.5	1	0.025	0.004	9.4243	1.312	0.752	32.196	15.190	-0.195	0.231	16.402
52	125	0.0	1	0.130	0.015	6.5722	1.250	0.618	26.840	17.306	-0.175	0.198	18.706
52	126	0.5	1	0.038	0.003	9.1184	0.361	0.739	32.518	16.022	-0.108	0.117	16.376
52	127	0.0	1	0.21	0.02	6.2905	0.096	0.606	27.003	18.544	-0.161	0.178	18.660
52	129	0.0	1	0.26	0.03	6.0861	-1.467	0.592	27.075	20.892	-0.220	0.248	18.891
52	131	0.0	1	0.87	0.14	5.9244	-3.416	0.616	28.923	22.166	-0.337	0.400	17.214
53	128	2.5	1	0.0097	0.0008	6.8255	0.640	0.710	32.041	16.654	-0.116	0.126	17.307
55	134	3.5	1	0.02065	0.00230	6.8915	-0.914	0.747	36.398	16.861	-0.162	0.182	15.929
56	136	1.5	1	0.040	0.007	9.1074	-1.422	0.766	38.243	16.959	-0.247	0.297	15.515
56	137	0.0	1	0.430	0.035	6.8985	-2.257	0.690	34.889	18.884	-0.142	0.155	16.227
56	138	1.5	1	0.29	0.04	8.6114	-3.148	0.800	40.967	16.830	-0.216	0.250	13.615
56	139	0.0	1	6.3	1.7	4.7235	-2.242	0.550	28.486	20.276	-0.558	0.747	17.224
57	140	3.5	1	0.208	0.010	5.1605	-1.417	0.657	34.431	17.665	-0.086	0.091	16.074
58	141	0.0	1	3.2	0.2	5.4285	-1.077	0.607	32.224	17.686	-0.117	0.125	16.445
59	142	2.5	1	0.088	0.009	5.8434	-0.437	0.687	36.872	16.745	-0.158	0.176	16.293
60	143	0.0	1	0.440	0.052	6.1225	-0.412	0.610	33.135	18.643	-0.203	0.230	18.157
60	144	3.5	-1	0.045	0.004	7.8174	0.347	0.665	36.556	16.810	-0.136	0.150	17.170
60	145	0.0	1	0.430	0.065	5.7554	1.115	0.571	31.746	18.012	-0.248	0.291	19.295
60	146	3.5	-1	0.022	0.002	7.5645	1.685	0.616	34.639	17.236	-0.139	0.153	19.035
60	147	0.0	1	0.235	0.029	5.2924	2.495	0.505	28.737	19.082	-0.214	0.243	22.161
60	149	0.0	1	0.140	0.015	5.0384	3.525	0.466	27.095	19.883	-0.191	0.214	24.439
60	151	0.0	1	0.174	0.020	5.3345	3.411	0.501	29.780	18.654	-0.192	0.217	22.730
61	148	3.5	1	0.0036	0.0005	5.9014	2.862	0.590	33.952	18.248	-0.202	0.234	21.434
62	148	3.5	-1	0.0057	0.0005	8.1414	2.039	0.628	36.112	17.700	-0.129	0.141	19.882
62	150	3.5	-1	0.0022	0.0002	7.9854	3.252	0.587	34.510	18.401	-0.133	0.147	22.023
62	151	0.0	1	0.055	0.009	5.5965	3.980	0.496	29.494	19.290	-0.261	0.310	24.163
62	152	2.5	-1	0.0012	0.0002	8.2585	3.630	0.589	35.414	18.646	-0.230	0.273	22.688
62	153	0.0	1	0.0518	0.0033	5.8674	3.685	0.519	31.576	18.917	-0.104	0.111	23.282
62	155	0.0	1	0.115	0.012	5.8135	2.941	0.536	33.272	18.495	-0.168	0.187	21.894
63	152	2.5	1	0.00073	0.00007	6.3055	4.223	0.576	34.612	18.862	-0.136	0.150	23.620
63	153	3.0	-1	0.00025	0.00004	8.5505	3.889	0.652	39.617	17.913	-0.198	0.234	21.890
63	154	2.5	1	0.0013	0.0002	6.4344	3.688	0.601	36.904	18.257	-0.209	0.246	22.248
63	155	3.0	-1	0.00092	0.00017	8.1704	3.324	0.657	40.839	17.367	-0.231	0.280	20.693
64	153	0.0	1	0.015	0.002	6.4874	3.748	0.543	33.020	19.179	-0.200	0.230	23.584
64	155	0.0	1	0.0145	0.0015	6.4385	3.709	0.538	33.409	19.449	-0.159	0.177	23.861
64	156	1.5	-1	0.0018	0.0002	8.5364	3.275	0.614	38.573	18.313	-0.153	0.172	21.833
64	157	0.0	1	0.0378	0.0055	6.3605	3.056	0.556	35.261	18.782	-0.220	0.256	22.289
64	158	1.5	-1	0.0049	0.0004	7.9374	2.819	0.591	37.918	18.603	-0.121	0.132	21.734
64	159	0.0	1	0.085	0.003	5.9425	2.635	0.539	34.929	19.039	-0.059	0.062	22.137
64	161	0.0	1	0.202	0.020	5.6354	2.249	0.533	35.294	18.870	-0.167	0.185	21.509
65	160	1.5	1	0.0039	0.0004	6.3755	2.587	0.614	40.210	18.429	-0.147	0.164	21.232
66	157	0.0	1	0.0027	0.0004	6.9695	3.605	0.541	34.350	20.892	-0.223	0.260	25.398
66	161	0.0	1	0.0270	0.0017	6.4534	2.772	0.554	36.685	19.457	-0.101	0.108	22.718
66	162	2.5	1	0.00267	0.00013	8.1965	2.468	0.607	40.561	18.795	-0.073	0.077	21.519
66	163	0.0	1	0.0646	0.0019	6.2724	2.169	0.561	37.926	19.146	-0.049	0.050	21.664
66	164	2.5	-1	0.00685	0.00054	7.6565	2.022	0.586	39.994	19.075	-0.124	0.135	21.374
66	165	0.0	1	0.147	0.009	5.7154	1.914	0.532	36.645	19.638	-0.107	0.115	21.952
67	166	3.5	-1	0.0046	0.0005	6.2425	1.727	0.621	43.217	18.608	-0.162	0.182	20.488
68	163	0.0	1	0.0069	0.0012	6.9045	3.047	0.558	37.678	20.256	-0.258	0.310	23.918
68	165	0.0	1	0.020	0.003	6.6495	2.579	0.563	38.815	19.711	-0.229	0.268	22.744
68	167	0.0	1	0.038	0.003	6.4364	1.928	0.561	39.465	19.935	-0.130	0.141	22.236
68	168	3.5	1	0.0040	0.0002	7.7705	1.856	0.587	41.717	19.548	-0.081	0.085	21.687
68	169	0.0	1	0.094	0.010	6.0034	1.672	0.546	39.169	19.917	-0.180	0.201	21.933
68	171	0.0	1	0.125	0.025	5.6815	1.533	0.523	38.265	20.601	-0.341	0.422	22.527
69	170	0.5	1	0.0073	0.0005	6.5934	1.531	0.627	45.458	19.278	-0.103	0.111	20.974
70	170	3.5	1	0.0022	0.0005	8.4694	2.126	0.631	45.723	18.578	-0.309	0.393	20.844
70	171	0.0	1	0.037	0.006	6.6145	1.895	0.576	42.117	19.615	-0.250	0.297	21.812
70	172	0.5	-1	0.0058	0.0005	8.0194	1.757	0.586	43.313	20.442	-0.136	0.149	22.527
70	173	0.0	1	0.070	0.003	6.3675	1.414	0.571	42.582	19.751	-0.073	0.076	21.410
70	174	2.5	-1	0.0078	0.0009	7.4655	1.468	0.574	43.204	20.061	-0.188	0.212	21.796

Z	A	I_α	Π	$\langle D\rangle^{exp}_{I=0}$ KeV	$\Delta\langle D\rangle^{exp}_{I=0}$ KeV	S_n MeV	E_{sh} MeV	T MeV	σ^2	$a(*)$ MeV^{-1}	$-a(*)$ MeV^{-1}	$+a(*)$ MeV^{-1}	a MeV^{-1}
70	175	0.0	1	0.162	0.018	5.8224	1.322	0.543	41.299	19.992	-0.193	0.217	21.586
70	177	0.0	1	0.18	0.02	5.5674	1.392	0.521	40.386	20.576	-0.200	0.225	22.312
71	176	3.5	1	0.00345	0.00015	6.2935	1.390	0.613	47.056	19.519	-0.069	0.072	21.078
71	177	7.0	-1	0.00174	0.00017	7.0724	1.754	0.588	45.591	20.282	-0.157	0.174	22.333
72	175	0.0	1	0.021	0.005	6.7894	1.964	0.575	43.719	20.105	-0.354	0.456	22.411
72	177	0.0	1	0.032	0.007	6.3834	1.772	0.550	42.646	20.761	-0.346	0.437	22.930
72	178	3.5	-1	0.0024	0.0003	7.6265	1.846	0.562	43.971	20.914	-0.203	0.231	23.162
72	179	0.0	1	0.062	0.011	6.0994	1.625	0.542	42.798	20.633	-0.293	0.354	22.623
72	180	3.5	1	0.0044	0.0002	7.3884	1.589	0.557	44.430	20.845	-0.079	0.082	22.782
72	181	0.0	1	0.094	0.011	5.6954	1.439	0.515	41.456	21.414	-0.212	0.239	23.262
73	181	8.0	1	0.0011	0.0001	7.5825	1.443	0.618	49.727	20.147	-0.143	0.157	21.785
73	182	3.5	1	0.00417	0.00004	6.0634	1.192	0.596	48.397	20.134	-0.016	0.016	21.512
73	183	3.0	-1	0.0047	0.0012	6.9334	1.534	0.595	48.747	19.796	-0.375	0.492	21.538
74	181	0.0	1	0.0225	0.0065	6.6844	1.437	0.567	45.590	20.966	-0.439	0.597	22.714
74	183	0.0	1	0.066	0.004	6.1904	1.134	0.551	45.175	20.879	-0.107	0.114	22.271
74	184	0.5	-1	0.012	0.001	7.4114	1.254	0.554	45.813	21.591	-0.144	0.157	23.163
74	185	0.0	1	0.081	0.005	5.7544	1.243	0.516	43.086	21.814	-0.116	0.123	23.427
74	187	0.0	1	0.087	0.007	5.4664	1.198	0.491	41.721	22.812	-0.158	0.171	24.447
75	186	2.5	1	0.0031	0.0002	6.1794	0.951	0.591	49.722	21.057	-0.107	0.115	22.197
75	188	2.5	1	0.0041	0.0003	5.8724	0.848	0.571	48.944	21.595	-0.126	0.136	22.644
76	187	0.0	1	0.026	0.002	6.2924	0.999	0.536	45.488	22.524	-0.139	0.151	23.834
76	188	0.5	-1	0.0044	0.0002	7.9884	0.715	0.577	49.476	22.386	-0.079	0.083	23.293
76	189	0.0	1	0.040	0.002	5.9245	0.727	0.514	44.400	23.332	-0.097	0.103	24.329
76	190	1.5	-1	0.0033	0.0002	7.7924	0.309	0.571	49.783	22.811	-0.108	0.115	23.211
76	191	0.0	1	0.070	0.005	5.7604	0.167	0.513	45.093	23.589	-0.143	0.154	23.821
76	193	0.0	1	0.115	0.010	5.5834	-0.613	0.509	45.530	24.376	-0.182	0.200	23.493
77	192	1.5	1	0.00180	0.00014	6.1994	-0.025	0.567	50.332	23.969	-0.141	0.153	23.936
77	193	4.0	-1	0.0006	0.0001	7.7644	-0.488	0.612	54.807	23.348	-0.276	0.329	22.721
77	194	1.5	1	0.00554	0.00053	6.0665	-0.993	0.584	52.766	23.626	-0.177	0.195	22.301
78	195	0.0	1	0.240	0.065	6.1084	-1.462	0.579	52.711	22.163	-0.484	0.646	20.287
78	196	0.5	-1	0.018	0.003	7.9214	-2.157	0.610	56.028	23.925	-0.306	0.365	21.033
79	198	1.5	1	0.0165	0.0009	6.5124	-3.432	0.653	61.033	23.856	-0.107	0.114	19.325
80	199	0.0	1	0.105	0.033	6.6644	-4.100	0.605	57.022	26.381	-0.626	0.877	20.268
80	200	0.5	-1	0.10	0.03	8.0284	-4.731	0.673	63.923	24.339	-0.564	0.778	17.946
81	204	1.5	1	0.36	0.05	6.6555	-7.405	0.777	76.270	24.332	-0.303	0.351	14.484
81	206	0.5	1	5.5	1.1	6.5034	-9.095	0.857	85.520	23.943	-0.459	0.569	12.014
82	205	0.0	1	1.52	0.13	6.7314	-7.570	0.702	69.500	27.366	-0.219	0.239	15.782
82	207	0.0	1	37.1	5.5	6.7394	-9.565	0.854	85.923	24.184	-0.378	0.443	11.288
82	208	0.5	-1	35.7	6.5	7.3675	-9.972	0.867	87.923	24.354	-0.479	0.582	10.746
83	210	4.5	-1	4.5	0.6	4.6045	-7.620	0.694	71.523	24.146	-0.357	0.412	13.499
88	227	0.0	1	0.0303	0.0054	4.5521	3.824	0.382	44.809	26.761	-0.375	0.452	32.709
90	230	2.5	1	0.00053	0.00015	6.7910	3.238	0.459	55.035	25.845	-0.488	0.659	30.477
90	231	0.0	1	0.0096	0.0013	5.1206	3.315	0.410	49.494	27.146	-0.277	0.319	32.257
90	233	0.0	1	0.0168	0.0010	4.7863	3.555	0.391	47.965	27.325	-0.131	0.139	32.884
91	232	1.5	-1	0.00045	0.00005	5.5605	2.965	0.476	57.884	25.904	-0.202	0.226	30.100
91	234	1.5	-1	0.00059	0.00009	5.2096	3.122	0.453	55.953	26.587	-0.283	0.332	31.159
92	233	0.0	1	0.0046	0.0007	5.7538	2.465	0.448	54.869	26.979	-0.295	0.346	30.673
92	234	2.5	1	0.00055	0.00005	6.8435	2.555	0.464	57.275	26.458	-0.175	0.192	30.173
92	235	0.0	1	0.0106	0.0005	5.2976	2.623	0.426	52.980	27.071	-0.100	0.105	31.061
92	236	3.5	-1	0.00044	0.00006	6.5459	2.752	0.441	55.164	27.466	-0.268	0.309	31.648
92	237	0.0	1	0.0147	0.0008	5.1247	2.743	0.418	52.665	27.057	-0.117	0.124	31.240
92	238	0.5	1	0.0035	0.0008	6.1531	3.011	0.425	53.892	27.171	-0.440	0.560	31.740
92	239	0.0	1	0.0209	0.0011	4.8063	3.077	0.397	50.738	27.473	-0.117	0.124	32.272
93	238	2.5	1	0.00052	0.00004	5.4881	2.272	0.479	60.808	26.191	-0.146	0.159	29.424
94	239	0.0	1	0.0090	0.0007	5.6472	1.847	0.452	57.762	26.959	-0.160	0.173	29.713
94	240	0.5	1	0.0023	0.0001	6.5337	2.141	0.448	57.621	27.533	-0.090	0.094	30.779
94	241	0.0	1	0.0136	0.0007	5.2412	2.183	0.426	55.210	27.498	-0.112	0.118	30.852
94	242	2.5	1	0.0009	0.0001	6.3094	2.450	0.431	56.304	27.953	-0.229	0.257	31.740
94	243	0.0	1	0.0155	0.0017	5.0339	2.456	0.411	53.974	27.894	-0.237	0.266	31.736
95	242	2.5	-1	0.00055	0.00005	5.5409	1.679	0.483	63.105	26.735	-0.175	0.193	29.159
95	243	5.0	-1	0.00040	0.00008	6.3640	2.094	0.490	64.395	25.777	-0.357	0.441	28.684
95	244	2.5	-1	0.00060	0.00006	5.3629	1.976	0.471	62.304	26.913	-0.194	0.216	29.794
96	243	0.0	1	0.025	0.008	5.6955	1.353	0.477	62.688	25.326	-0.574	0.809	27.209
96	244	2.5	1	0.0011	0.0002	6.7993	1.532	0.475	62.864	26.659	-0.345	0.417	28.879
96	245	0.0	1	0.012	0.001	5.5197	1.458	0.447	59.497	27.547	-0.176	0.192	29.762
96	246	3.5	1	0.0014	0.0001	6.4567	1.730	0.457	61.276	26.855	-0.147	0.159	29.400
96	247	0.0	1	0.034	0.007	5.1573	1.777	0.437	58.960	26.515	-0.415	0.516	29.135
96	248	4.5	-1	0.0014	0.0003	6.2124	2.049	0.440	59.762	27.219	-0.425	0.533	30.289
96	249	0.0	1	0.033	0.005	4.7124	2.305	0.397	54.298	28.163	-0.336	0.393	31.812
97	250	3.5	1	0.0010	0.0001	4.9694	1.823	0.453	62.400	27.353	-0.204	0.227	30.069
98	250	4.5	-1	0.0007	0.0001	6.6233	1.021	0.460	63.364	28.320	-0.294	0.341	29.888
98	253	0.0	1	0.027	0.003	4.8034	2.065	0.401	56.320	28.556	-0.251	0.282	31.847