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Fermi Liquid Theory of Dilute Submonolayer ^{3}He on Thin ^{4}He II Film

—Dimer Bound State and Cooper Pairs—

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A degenerate dilute Fermi system, with a shallow dimer bound state, is discussed with attention to the possibility of Fermi superfluidity. (Necessary information on interaction is expressed through the bound state energy of the dimer.) A relevant system may be dilute submonolayer ^{3}He on a thin ^{4}He II film just above the onset of the apparent gas-liquid phase separation of ^{3}He.

The experiments of Bhattacharyya and Gasparini,[1] on dilute submonolayer ^{3}He on a thin ^{4}He II film, indicate that both the ^{3}He-^{3}He interaction and effective mass m of a single ^{3}He in a surface state can be varied over a wide range, and that the ^{4}He II film offers an ideally smooth substrate.[2] Another advantage of this system is that the number density n of ^{3}He can be changed continuously from the extremely dilute to the moderately dilute case, e.g., 0.0006 A$^{-2}< n <$ 0.018 A^{-2} (this means the interparticle distance $r_0 \equiv (2/\sqrt{3}n)^{1/2}$ ranges over 45 A$> r_0 >$8 A),[1] which might make theoretical treatments more tractable. Indeed, for dilute solutions of ^{3}He in bulk ^{4}He II, the Fermi liquid properties of ^{3}He are known to be expressed well in terms of the quantities characterizing two-particle scattering.[3),4)]

We examine the properties of a degenerate dilute Fermi system with interactions and effective mass just above the onset of many-particle self-binding (MPSB). (If MPSB occurred, the system would no longer be dilute.) Such a situation is expected for the above ^{3}He-^{4}He II film system at a thickness of ^{4}He II film between 12.2 A and 18.8 A.[1] According to the arguments by Miller and Nosanow (MN)[5] using the quantum theorem of corresponding states, the existence of a dimer bound state is a necessary condition for MPSB in two-dimensional (2D) Fermi systems at $T=0$, and there exists a very shallow dimer S-wave bound state for values of the quantum parameter greater than that corresponding to the onset of MPSB. In the treatment by MN, the strength of interaction was parametrized by the quantum parameter $\eta \equiv \hbar^2/m\varepsilon\sigma^2$ on the basis of 12-6 Lennard-Jones (LJ) potential (notations are usual ones), and the critical value of η for the onset of MPSB was given in the variational approximation by $\eta_{CF}=0.19$, the value of which corresponds to the dimer bound state energy $E_0 \sim -10^{-2}$ K.[6] On the other hand, η for the onset of dimer binding is $\eta_{DB}=0.27$.[5),6)] In general, the variational approach gives a lower bound to the true η_{CF} and, moreover, the actual interaction between ^{3}He particles on ^{4}He II film may be considerably modified from LJ by the processes of exchange involving the various modes in ^{4}He II film.[2] So we cannot rely on the numerical values given by MN. Nevertheless, it might be reasonable to assume that two ^{3}He particles, with interactions and effective mass, can form a shallow

S-wave bound state.*) While a calculation of E_0 at MPSB-onset from microscopic theory is difficult, it is possible to express the Fermi liquid properties in terms of E_0 regarding it as a phenomenological parameter.

A basis of such an approach is the fact that both the interparticle distance r_0 and the size b of dimer ($E_0 \equiv -\hbar^2/mb^2$) are much larger than the force range l_0 of interaction. In this paper, we discuss the Fermi liquid properties with attention to the possibility of Fermi superfluidity.

To determine the Landau interaction $f_{p\sigma,p'\sigma'}$ in the dilute limit, we use a simple approach in which $f_{p\sigma,p'\sigma'}$ is given by the phase-shift for two-particle scattering:[8),9)]

$$f_{p\sigma,p'\sigma'} = -\frac{2\hbar^2}{m}\delta_0(|\boldsymbol{p}-\boldsymbol{p}'|/2)(1-\sigma\cdot\sigma'), \tag{1}$$

where δ_0 is the S-wave phase-shift. In deriving (1) we have discarded the phase-shifts of higher angular momentum which are small compared to δ_0 at least by order $(l_0/r_0)^2$.

By the arguments similar to those describing resonance scattering in 3D,[10)] one can show in 2D that, if a shallow S-wave bound state is possible, the S-wave phase-shift for low energy scattering is expressed in terms of the bound state energy $E_0 = -\hbar^2/mb^2$:

$$\delta_0(k) \simeq \tan^{-1}[(\pi/2)\ln(kb)], \tag{2}$$

where terms of $O(kl_0)^2$ or $O(l_0/b)^2$ have been neglected compared to those of order $1/\ln(kl_0)$ or $1/\ln(b/l_0)$.

On the basis of (1) and (2), the Fermi liquid properties of the normal phase are expressed in terms of a single parameter $\xi \equiv 1/k_F b (k_F \equiv (2\pi n)^{1/2} = (4\pi/\sqrt{3})^{1/2}\gamma_0^{-1})$: The Fermi liquid parameters are given in the form (notations are the 2D version of Ref. 4))

$$-F_m^s = F_m^a = \frac{2}{\pi}\frac{m^*}{m}\int_0^{2\pi}\frac{d\theta}{2\pi} \times \tan^{-1}\left[\frac{\pi}{2\ln|\xi^{-1}\sin(\theta/2)|}\right]\cos m\theta, \tag{3}$$

where $m^*/m = 1+1/2 \cdot F_1^s$. The relation (3) determines the Fermi liquid parameters $F_m^{s,a}$ in terms of ξ, which in turn enables us to determine ξ, i. e., E_0, from experiments. We have to keep in mind that the above arguments are valid only in the normal degenerate state, i. e., at temperatures higher than the superfluid transition but lower than $T_F \equiv \hbar^2 k_F^2/2m^* = \varepsilon_F$.*) As will be discussed later, such regions are possible only for the case $\xi \ll 1$.

It is straightforward to calculate the experimentally observable quantities from $F_m^{s,a}$. The specific heat is given by $C = \pi m^* T/3\hbar^2$. The inverse ratio of the spin susceptibility to that calculated for an ideal gas with effective mass m^* is (as in 3D) $\chi_{\text{ideal}}/\chi = 1+F_0^a$. The spin diffusion coefficient D in 2D is given by[12)]

$$D = \frac{\hbar}{\pi m^*}\frac{(1+F_0^a)C(\lambda)}{[\sum_{m=0}^{\infty}(-1)^m A_m^a]^2}\left(\frac{T_F}{T}\right)^2\frac{1}{\ln(T_F/T)}, \tag{4}$$

where the scattering amplitude parameters $A_m^a \equiv F_m^a/(1+c_m F_m^a)$, ($c_m = 1$ if $m=0$ and $1/2$ otherwise), and $C(\lambda)$ is the Sykes-Brooker correction factor with $\lambda(<1)$ being expressible in terms of A_m^s and A_m^a. We will not give here the complicated expression for λ. However, $C(\lambda)$ is a slowly decreasing function with $C(-\infty)=1$ and $C(1)=3/4$.

Next let us discuss the superfluid transition

*) The formation of a ^{3}He dimer on the surface of bulk ^{4}He II and related Phenomena were first discussed by Bashkin.[7)]

*) It should be remarked that $F_1^s(<0)$ and $F_1^a(>0)$, obtained from (3), do not satisfy the inequality $F_1^s > F_1^a$ which is derived from a general argument using the sum rule for the normal (paramagnetic) ground state.[4),11)] For the dilute hard disc gas, $F_1^{s,a}$, which are obtained by a method similar to (1)~(3), satisfy this inequality.[9)]

of ^{3}He.*) The nature of such a dilute system in 3D has been discussed by Leggett in recent years.[13] He has discussed, in the mean-field (i.e., BCS) approximation, the transition at $T=0$ from the state of Cooper pairs ($\xi \ll 1$) to that of a Bose condensation of diatomic molecules ($\xi \gg 1$) as the parameter $\xi = 1/k_F b$ is varied.**)

In the Cooper pair limit, the basic equations for $T>0$ (but $T \ll \hbar^2/ml_0^2$) are slightly modified:

$$\Delta_k = -\sum_{k'} V_{k,k'} \frac{\Delta_{k'}}{2E_{k'}} \tanh \frac{E_{k'}}{2T}, \tag{5}$$

$$n = \sum_k \left[1 - \frac{\varepsilon_k - \mu}{E_k} \tanh \frac{E_k}{2T} \right], \tag{6}$$

which are the self-consistent equations both for Δ_k and μ. (For the notation, refer to Ref. 13).) The essence of the 2D version of Leggett's theory is that in the dilute limit, i. e., $(r_0, b) \gg l_0$ (but b/r_0 arbitrary), the interaction $V_{k,k'}$ in (5) can be eliminated (renormalized) comparing (5) with the Schrödinger equation for two-particle bound state and as a result the nature of the system is parametrized by a single parameter ξ. After some manipulations, gap equation (5) is renormalized in the form

$$\sum_K \left[\frac{1}{E_k} \tanh \frac{E_k}{2T} - \frac{1}{\varepsilon_k + (\hbar^2/2mb^2)} \right] = 0, \tag{7}$$

where $E_k = [(\varepsilon_k - \mu)^2 + \Delta^2]^{1/2}$ with constant Δ.

A solution of the self-consistent equations (6) and (7) in 2D at $T=0$*) is given by $\Delta_0 = 2\varepsilon_F \xi$ and $\mu = \varepsilon_F (1-\xi^2)$, which have reasonable forms in the following two limiting cases: In the dimer limit ($\xi \gg 1$) $\mu \simeq -\varepsilon_F \xi^2 = 1/2 \cdot E_0$ and in the Cooper pair limit ($\xi \ll 1$) $\mu \simeq \varepsilon_F$ and $\Delta_0 = 2\varepsilon_F \xi = \hbar^2 k_F/mb$. Note that the latter result for Δ_0 is consistent with the usual BCS formula $\Delta_0 \sim 2\varepsilon_F \exp(1/N(0)v_0)$ if $f_{p\uparrow,-p\downarrow} \simeq 2\pi/m \ln \xi$ is identified with v_0 (cf. (1) and (2)). Here we remark that the Cooper pairing is impossible if the attraction is so weak that no dimer bound states exists; also note that Δ_0 is proportional to $|nE_0|^{1/2}$.

The onset temperature T_c^0 of the Cooper pairing is determined by putting $\Delta = 0$ in (6) and (7):

$$T_c^0 = \frac{\gamma}{\pi} \Delta_0 = \frac{2\pi}{\pi} \varepsilon_F \xi \ll T_F, \tag{8}$$

where $\ln \gamma = 0.577$ (Euler constant). In the opposite dimer limit, the mechanism destroying the order of the low temperature phase is due to the excitations, with motions of the center of mass of the dimer, which are the collective modes in the Fermi liquid language and have not been taken into account in (7). So the above theory fails to determine the transition temperature in the dimer limit. However, we believe the Bose condensation of dimers will set in at $T \sim \varepsilon_F \ll |E_0|$.

To determine an actual transition temperature T_c we may have to discuss the problem in the Kosterlitz-Thouless context[14] starting with the mean field result. However, in the case $T_F \gg T_c^0$, a difference between T_c and T_c^0 is small compared to T_c^0 itself: Since crudely speaking T_c is determined by $n_s(T_c)/n \simeq 2T_c/T_F$ with the superfluid density $n_s(T)$ being calculated by mean field theory, $(T_c^0 - T_c)/T_c^0 \sim (T_c^0/T_F)$.

As far as we know, there are no data for the Fermi liquid parameters, and then for E_0, of the present system. The value of

*) This problem was recently discussed by Kurihara,[2] who emphasized that the transition temperature is located in the experimentally attainable region as far as only an attractive interaction due to exchange of the third sound quanta of ^{4}He II film is taken into account.

**) In 3D, b is the zero-energy S-wave scattering length. In 2D, however, the scattering length cannot be well defined because the transition matrix behaves like $\sim 1/\ln k$ at low k.

*) At $T=0$, Eqs.(6) and (7) are valid also for the dimer limit.

E_0 predicted from MN's theory[5] is $E_0 \sim -10^{-2}$ K.[6] It is known that two ^{3}He particles in a 2D vacuum interacting with 12-6 LJ potential have a bound state with $E_0 \sim -10^{-7}$ K but the system then undergoes no MPSB transition.[5),6)] In 2D, T_c^0 is not so sensitive to b (or E_0) compared to 3D case.[3),13)] Let us take, as an example, the system with $T_F = 300$ mK(i. e., $n \sim 0.008$ A^{-2} or $r_0 \sim 12$ A).[1)] In this case, if $E_0 = -10^{-2}$K, -10^{-3}K, and -10^{-4}K, then $T_c^0 \sim 40$ mK, 12 mK and 4 mK, respectively.

In conclusion, we remark that this system (if it exists) might be a unique example of a Fermi liquid in which the energy of shallow bound state parametrizes the thermodynamic parameters, transport coefficients and the superfluid transition.

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