FERMION MASS SPECTRUM IN $SU(2)_L \times U(1)$

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ABSTRACT

We investigate the possibility of gene-
rating fermion masses through radiative
corrections in the well-defined frame-
work of $SU(2)_L \times U(1)$. No-go theorems,
however, imply a complete tree-level
description of the spectrum with at
least one electroweak singlet scalar
and suggest therefore the need of new
gauge interactions.

1. - INTRODUCTION

In the standard framework of the electroweak gauge interactions, the
Lagrangian contains two important pieces, namely the gauge and the so-called
Higgs sectors. While the first describes the local interactions quite well,
the second provides us with a unique mechanism for mass splitting without, how-
ever, giving any explanation about the fermion mass spectrum and its intriguing
family structure:

$$
\begin{pmatrix} m_e \\ m_u \\ m_d \end{pmatrix} \ll \begin{pmatrix} m_\mu \\ m_c \\ m_s \end{pmatrix} \ll \begin{pmatrix} m_\tau \\ m_t \\ m_b \end{pmatrix} \tag{1.1}
$$

We will call this the second hierarchy problem.

The spectrum is therefore the only present open window on the scalar sector
which after all can be considered as an effective one [see for example the
"extended technicolour" approach which seems precisely to be ruled out by $(1.1)^{-1)}$].
In this paper, we do not try to understand the spontaneous breaking mechanism
but rather to take off some informations starting from the assumption that masses
are <u>calculable</u> in a more fundamental theory. At the present stage of our under-
standing of gauge theories, one simple way to ensure calculability is provided
by the so-called natural, zero-order relations[2] (i.e., independent of free para-
meters of the Lagrangian) which can be simply carried out by imposing new sym-
metries on the $SU(2)_L \times U(1)$ invariant Lagrangian. We want here to make a
clear distinction between two orthogonal points of view. The vertical one con-
sists in introducing new (local) symmetries in order to link fermions of the
same family: this is the well-known "grand unification approach"[3]. In this
case the first $(10^2/10^{15})$ as well as the second hierarchy problems remain
unexplained and we "only" find, for example, the $SU(4)$ invariant successful
relation $m_b/m_\tau = 1$ in the Georgi-Glashow $SU(5)$ model[3]. Although this
vertical approach gives rise to spectacular results, it seems more urgent for
us to understand first the structure described by (1.1). The horizontal point
of view is therefore more promising since fermions with the same electrical
charge Q are now connected. The basic idea consists indeed in extending at
least a subgroup of the $G_{flavour} \equiv U(n)_L^{quarks} \times U(n)_L^{leptons} \times U(n)_R^Q$ global sym-
metry arising in the gauge sector (n being the number of families) to the
full Lagrangian. As indicated by the mass spectrum, this subgroup must by
spontaneously broken. In other words we have to introduce more than one $SU(2)_L \times U(1)$
scalar doublet in order to form a non-trivial representation under
this horizontal group. Moreover only continuous but local, or discrete

symmetries are allowed if we want to avoid dangerous massless scalars[*]. If for natural[2] reasons[5], one family of fermions is massless at the tree level. radiative corrections provide it by definition with some finite mass[6] in naïve agreement with the $m_i^Q/m_{i+1}^Q \sim \alpha$ picture suggested by (1.1). In the local option, self-energy diagrams (Fig. 1) are important but need again a new scale in order to respect the observed conservation of flavour in neutral currents. For this reason we will confine ourselves to studying here only discrete subgroups of G_F. In the $SU(2)_L \times U(1)$ gauge framework, radiative corrections to a natural relation for masses arise from tadpole-like diagrams with fermion loops (Fig. 2)[7].

Although we are concerned with a simple but non-trivial subgroup of G_F, namely the permutation group S_n, we would like to stress that the main point of this paper is not the proposition of an additional model with specific discrete symmetries but rather an analysis of mass calculability in $SU(2)_L \times U(1)$. In this well-defined framework we first show the impossibility of generating some finite mass by radiative corrections, all the vacua leaving at least a S_2 residual symmetry unbroken. The argument based on the powerful Georgi-Pais theorem[8] can be easily extended to any larger horizontal group, as far as the scalar potential does not exhibit so-called "accidental symmetries"[2]. In this case indeed, the above quoted theorem does not apply and the original permutation symmetry can be completely broken in a natural way. This possible loop-hole unfortunately gives rise to an unrealistic mass spectrum. We can also avoid such a residual discrete symmetry in conflict with experimental data, by introducing a new scalar representation, singlet under the electroweak gauge group. This, however, implies a tree level description in which quark masses can be strongly constrained and mixing angles predicted.

Despite the fact that these results appear more modest than our previous scenario, we would like to emphasize that the necessity for singlet scalars suggests in itself the presence of new gauge interactions and moreover, is in apparent conflict with a dynamical breaking picture in absence of right-handed neutrinos.

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[*] A broken continuous global symmetry also needs a large scale in order to make the Goldstone bosons invisible[4].

2. - RADIATIVE CORRECTIONS TOWARDS THE HIERARCHY PROBLEM

In the standard model, a flavour symmetry G_F arises from the gauge sector, expressing the fact that fermions with the same electroweak quantum numbers are not distinguished by gauge interactions:

$$G_F = U(n)_L^{quarks} \times U(n)_R^{2/3} \times U(n)_R^{-1/3} \times U(n)_L^{leptons} \times U(n)_R^{-1} \qquad (2.1)$$

The observed non-degenerated mass spectrum is therefore entirely related to an explicit breaking of G_F by the $3n^2$ Yukawa couplings. However, if we want to say something about the fermion masses, we have to extend some subgroup of G_F to the complete Lagrangian in order to relate these free parameters. The purpose of this paper consists in restricting ourselves to the simplest but large enough S_n permutation subgroup such that fermions transform like the n-dimensional reducible representation. In order to avoid (n-1) degenerated eigenvalues for each fermion mass matrix, we straightforwardly extend the family structure to the scalar sector by taking n doublets ϕ_i of spin 0 fields[9]. In this now well-defined framework, we can write the non-gauge interactions[10]:

$$\mathcal{L}_{Yukawa} = a\, \overline{\Psi}_{i_L} \phi_i \Psi_{i_R} + b\, \overline{\Psi}_{i_L} \phi_i (\Psi_j + \Psi_k)_R$$
$$+ c\, \overline{\Psi}_{i_L} (\phi_j + \phi_k) \Psi_{i_R} + d(\overline{\Psi}_j + \overline{\Psi}_k)_L \phi_i \Psi_{i_R} + e\, \overline{\Psi}_{i_L} \phi_j \Psi_{k_R} \qquad (2.2)$$

$$\mathcal{L}_{potential} = -\lambda (\phi_i^\dagger \phi_i) - \frac{\gamma}{2}(\phi_i^\dagger \phi_j) + A(\phi_i^\dagger \phi_i)^2 + C(\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j)$$
$$+ \bar{C}(\phi_i^\dagger \phi_j)(\phi_j^\dagger \phi_i) + \frac{D}{2}[(\phi_i^\dagger \phi_j)(\phi_i^\dagger \phi_j) + h.c.]$$
$$+ \frac{E_1}{2}[(\phi_i^\dagger \phi_i)(\phi_i^\dagger \phi_j) + h.c.] + \frac{E_2}{2}[(\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_i) + h.c.]$$
$$+ \frac{E_3}{2}[(\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_k) + h.c.] + \frac{E_4}{2}[(\phi_i^\dagger \phi_j)(\phi_i^\dagger \phi_k) + h.c.]$$
$$+ F[(\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_\ell) + h.c.] \qquad (2.3)$$

a summation on all permutations with $i \neq j \neq k \neq \ell$ being understood (this in particular implies $F = 0$ for $n = 3$). As already explained, we are interested in the possibility of implementing zero-order massless fermions in a technically natural way[2]. From (2.2) and (2.3), this constraint uniquely fixes the number of generations as well as the vacuum expectation values (v.e.v.), namely:

$$n = 3$$
$$\langle \phi_1 \rangle = -\langle \phi_2 \rangle \qquad \langle \phi_3 \rangle = 0 \qquad (2.4)$$

In this particular case indeed, the first generation of fermions is massless at the tree level:

$$m_u = m_d = m_e = 0 \tag{2.5}$$

independently of the bare parameters of the Lagrangian.

Unfortunately, a careful calculation of the radiative corrections to the various v.e.v. at the one-loop level shows that (2.5) remains unchanged. This result can be easily explained and furthermore extended to all orders in perturbation expansion thanks to the well-known Georgi-Pais theorem[8]. This indeed claims that radiative corrections to the zero-order v.e.v. leave the __same__ subgroup of the original Lagrangian unbroken:

$$U_{ij} \langle \phi_j \rangle = \langle \phi_i \rangle$$
$$\mathcal{L}(U\phi) = \mathcal{L}(\phi) \quad \Rightarrow \quad U_{ij} \delta\langle\phi_j\rangle = \delta\langle\phi_i\rangle \tag{2.6}$$

Since the Lagrangian is here obviously invariant under the unbroken transformation

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \longrightarrow -\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \tag{2.7}$$

the first generation of quarks and leptons remains massless to all orders in perturbation expansion.

A simple way to avoid this very limitative theorem consists in looking for models where the scalar potential possesses a so-called natural accidental symmetry A[2] such that

$$\mathcal{L}_{potential}(A\phi) = \mathcal{L}_{potential}(\phi)$$
$$\text{while} \quad \mathcal{L}(A\phi) \neq \mathcal{L}(\phi) \tag{2.8}$$

In the framework of the electroweak model, we can only have

$$\mathcal{L}_{Yukawa}(A\phi) \neq \mathcal{L}_{Yukawa}(\phi) \tag{2.9}$$

since the gauge interaction sector of the scalars is invariant under unitary transformations. Radiative corrections due to the scalar-fermion couplings (Fig. 2) will then break this accidental symmetry, providing us with non-trivial modifications for the various v.e.v.:

$$A_{ij} \; \delta \langle \phi_j \rangle \neq \delta \langle \phi_i \rangle \qquad (2.10)$$

A <u>discrete</u> accidental symmetry can be implemented with again three families since then the scalar potential (2.3) is already obtained by only imposing the discrete d_3 group of cyclic permutations on the Lagrangian[*]. We have seen that the minimum defined by (2.4) is invariant under a reflexion transformation (2.7) which, however, does not belong to d_3. Consequently, a one-loop contribution of the d_3 invariant Yukawa couplings through the tadpole-like diagrams breaks the S_3 accidental symmetry. Unfortunately $\mathcal{L}_{Yukawa}$ contains now a few more free parameters such that additional discrete symmetries are needed in order to keep one vanishing eigenvalue for each mass matrix at the tree level. The Georgi-Pais theorem also applies on these new symmetries in such a way that again (2.5) remains exact to all orders in perturbation expansion.

We can also have a <u>continuous</u> accidental symmetry associated to some flat direction in the classical potential, such that the v.e.v. are only determined by radiative corrections. As local curvature corresponds to scalar mass, a zero-order massless scalar can be used to detect such a phenomenon.

For $n = 3$, we find [see (A.6) and (A.7)] three independent sets of v.e.v.[10] given respectively by

$$\langle \phi_1 \rangle = \langle \phi_2 \rangle = \langle \phi_3 \rangle \qquad (2.11a)$$

$$\langle \phi_1 \rangle = \langle \phi_2 \rangle \neq \langle \phi_3 \rangle \qquad (2.11b)$$

$$\langle \phi_1 \rangle = -\langle \phi_2 \rangle \qquad \langle \phi_3 \rangle = 0 \qquad (2.11c)$$

Only the two last solutions present two neutral massless scalars in addition to the unphysical Goldstone bosons if

$$4A - 2(C + \bar{C}) - E_1 + E_2 + E_3 + E_4 = 0 \qquad (2.12)$$

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[*]This is no more true for $n = 4$ where the following quadratic terms
$$\phi_1^+ \phi_2 + \phi_2^+ \phi_3 + \phi_3^+ \phi_4 + \phi_4^+ \phi_1 + h.c.$$
$$\phi_1^+ \phi_3 + \phi_2^+ \phi_4 + h.c.$$
are allowed by the d_4 invariance.

This can be understood by choosing a more suitable basis for the scalar doublets, namely:

$$\begin{pmatrix} M_1 \\ M_2 \\ S \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \tag{2.13}$$

where $\vec{M} = \begin{pmatrix} M_1 \\ M_2 \end{pmatrix}$ and S transform respectively like the mixed and symmetric irreducible representations of S_3. The ten invariants of the potential become in this new basis:

$$S^\dagger S \,, \quad S' \,, \quad (S^\dagger S)^2 \,, \quad S'^2 \,, \quad A'^2 \,,$$

$$\vec{M}^\dagger \cdot \vec{M} \,, \quad (S^\dagger S) S' \,, \quad (\vec{M}^\dagger S) \cdot (S^\dagger \vec{M}) \,,$$

$$(\vec{M}^\dagger S) \cdot (\vec{M}^\dagger S) + h.c. \,, \quad (\vec{M}^\dagger \times \vec{M}) \cdot (\vec{M}^\dagger S + \vec{M} S^\dagger) \tag{2.14}$$

with the decomposition

$$\vec{M}^\dagger \otimes \vec{M} = S' \oplus A' \oplus \vec{M}'$$

The coupling constant in front of the last invariant is proportional to the left-handed side of (2.12) which means that this accidental symmetry naturally arises if we impose an additional discrete symmetry:

$$\vec{M} \longrightarrow -\vec{M}$$

$$\text{or} \quad S \longrightarrow -S \tag{2.15}$$

on the complete Lagrangian. In each case indeed we see that the nine remaining terms in (2.14) are now invariant under a global two dimensional rotation:

$$\begin{pmatrix} M_1 \\ M_2 \end{pmatrix} \longrightarrow \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} M_1 \\ M_2 \end{pmatrix} \tag{2.16}$$

which is only left unbroken by the S_3 symmetric v.e.v. (2.11a). As already mentioned, in order to pick out the true vacuum we have now to consider radiative corrections coming from the Yukawa couplings defined in this new basis:

$$g_1 \, \bar{S}_L S S_R + g_2 (\vec{\bar{M}}_L \times \vec{M}) \cdot \vec{M}_R + g_3 \, \vec{\bar{M}}_L \cdot \vec{M}_R S$$

$$+ g_4 \, \vec{\bar{M}}_L \cdot \vec{M} S_R + g_5 \, \bar{S}_L \vec{M} \cdot \vec{M}_R \tag{2.17}$$

So, we conclude that only the $S \to -S$ transformation still allows an explicit breaking of the rotational symmetry by the Yukawa sector. If we parametrize the v.e.v. as follows:

$$\langle M_1 \rangle = r \, \sin \vartheta \, \sin t$$

$$\langle M_2 \rangle = r \, \sin \vartheta \, \cos t$$

$$\langle S \rangle = r \, \cos \vartheta \tag{2.18}$$

the t angle will be fixed[*] by the extremal condition (B.6) namely

$$\frac{\partial}{\partial t} \, \delta V (r, \vartheta, t) = 0 \tag{2.19}$$

where δV is the effective potential[12] given in Appendix B. Since in our case the gauge sector as well as the potential are t-independent, (2.19) implies the following constraint on t:

$$\frac{\partial}{\partial t} \, \text{Tr} \, m^4 (t) \, \ln m^2(t) = 0 \tag{2.20}$$

where m is the fermion mass matrix

$$m = \begin{pmatrix} g_2 \langle M_2 \rangle & g_2 \langle M_1 \rangle & g_4 \langle M_1 \rangle \\ g_2 \langle M_1 \rangle & -g_2 \langle M_2 \rangle & g_4 \langle M_2 \rangle \\ g_5 \langle M_1 \rangle & g_5 \langle M_2 \rangle & 0 \end{pmatrix} \tag{2.21}$$

with eigenvalues m_n. Since $\sum_n m_n^2$ and $\sum_n m_n^4$ are t - independent, the constraint (2.20) simply becomes

$$\sum_m \left(\frac{\partial m_m^2}{\partial t} \right) m_m^2 \, \ln m_m^2 = 0 \tag{2.22}$$

A first set of solutions is defined by

$$t = p \, \frac{\pi}{6} \qquad (p : \text{integer}) \tag{2.23}$$

and corresponds to the already known S_2 symmetric minima (2.11b) and (2.11c) for respectively p even and odd. In addition we find a new type of vacuum solution for

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[*] The authors of Ref. 11) derived tree level mass relations as well as mixing angles for the special values $t = 0$, $\pi/4$. We will, however, show that the latter solution is not a natural extremum of the potential.

$$t = \frac{1}{3} \text{Arc} \cos\left(\frac{2+\mu}{3}\right)^{3/2} \frac{1}{(\mu^2 - \nu^2)^{1/2}} \left\{ \left[1 - \frac{9\nu^2}{(2+\mu)^2}\right] + \left[1 + \frac{3\nu^2}{(2+\mu)^2}\right]^{3/2} \right\}^{1/2}$$

$$\mu \equiv \frac{g_5^2 + g_7^2}{g_2^2}$$

$$\nu \equiv \frac{g_5^2 - g_7^2}{g_2^2} \tag{2.24}$$

such that the original S_3 invariant group is now completely broken. This provides us with an example where an accidental symmetry generates new extremal solutions which are functions of the Yukawa couplings. It is therefore possible to spontaneously break all the discrete symmetries in the framework of permutation groups. Unfortunately in the above case the new vacuum picked out by radiative corrections implies an unrealistic mass spectrum since we predict now two non-vanishing, degenerated masses for each charge sector. This is due to an unbroken discrete subgroup of the $SO(2)$ accidental rotation in (2.22), namely

$$t \longrightarrow t + \frac{2\pi}{3} \tag{2.25}$$

So we can already conclude that the most general potential invariant under S_n does not give rise to any calculable fermion mass, since the possible loopholes to the Georgi-Pais theorem provided by accidental symmetries are useless in the present context.

It is, however, possible to introduce a new type of minimum by imposing the invariance of the potential under some additional phase transformations. Only the γ and consequently the E_1, E_2 and E_3 self-interacting couplings in (2.7) can be naturally forbidden if we want to avoid an accidental continuous symmetry, namely $\phi_j \rightarrow e^{iy_j} \phi_j$ which would indeed imply very light scalars without solving the hierarchy problem. The new minimum appearing then is defined by

$$\langle \phi_i \rangle = \frac{\sigma_i}{\sqrt{2}} \delta_{im} \tag{2.26}$$

and leaves a S_{n-1} residual symmetry unbroken.

At the fermion mass matrix level, $n - 2$ degenerated eigenvalues are found, displaying once more the three generations case. There are in fact two independent ways to implement (2.26) with discrete phase transformations:

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \rightarrow \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \qquad \text{+ permutations} \qquad (2.27a)$$

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \rightarrow \begin{pmatrix} e^{i\frac{2\pi}{3}} & 0 & 0 \\ 0 & e^{i\frac{4\pi}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \qquad \text{+ permutations} \qquad (2.27b)$$

These additional symmetries also forbid respectively E_4 and D couplings in (2.3). However, from the Georgi-Pais theorem we again assert that radiative corrections will leave (2.26) unchanged. This result will a fortiori remain true for any larger discrete subgroup of G_F <u>as far as only scalar doublets are</u> <u>concerned</u>.

We can of course consider new electroweak representations for the scalar fields. If they couple to the previous doublets in a non-trivial way, the very strong constraints[10] on the allowed minima of the potential can be avoided without introducing new Yukawa parameters. We find therefore interesting to investigate in detail the implications of such new scalars on the fermion masses.

3. - A NATURAL MODEL FOR THREE GENERATIONS

We consider here the most economic solution with only one electroweak singlet ϕ_0^* such that the so-called weak isospin relation will be preserved. The following discrete symmetries:

$$\phi_j \longrightarrow e^{i\frac{2\pi}{3}j} \phi_j \qquad j = 1, 2, 3$$
$$\phi_0 \longrightarrow e^{i\frac{2\pi}{3}} \phi_0 \qquad\qquad\qquad (3.1)$$

together with the cyclic d_3 group allow then one non-trivial coupling between the doublets and this singlet, namely

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*The need of an extra gauge singlet scalar has already been stressed by D. Wyler[13] in the context of a specific model.

$$\supset \left\{ \left(\phi_1^+ \phi_2 + \phi_2^+ \phi_3 + \phi_3^+ \phi_1 \right) \phi_0^2 + h.c. \right\}$$

(3.2)

if we impose for simplicity invariance under $\phi_0 \to -\phi_0$. As shown in Appendix C, this coupling is crucial for our purpose although it implies $\langle \phi_i \rangle \neq 0$ and consequently <u>a mass spectrum now completely described at the tree level</u>. Moreover, we find that a CP violating vacuum characterized by

$$\langle \phi_j \rangle = \frac{v_j}{\sqrt{2}} e^{i \alpha_j}$$

(3.3)

is needed in order to avoid an unwanted residual S_2 symmetry in trouble with experimental information (b-decay).

If we impose (3.1) and simultaneous phase transformations[14] on the electro-weak quark multiplets:

$$\begin{pmatrix} u_j^\circ \\ d_j^\circ \end{pmatrix}_L \longrightarrow e^{-i \frac{2\pi}{3} j} \begin{pmatrix} u_j^\circ \\ d_j^\circ \end{pmatrix}_L$$

$$u_{jR}^\circ \longrightarrow u_{jR}^\circ$$

$$d_{jR}^\circ \longrightarrow e^{i \frac{2\pi}{3} j} \, d_{jR}^\circ$$

(3.4)

the mass matrix for the up and down quarks are respectively given by:

$$M^u = \begin{pmatrix} a \langle \phi_1 \rangle^* & b_1 \langle \phi_1 \rangle^* & b_2 \langle \phi_1 \rangle^* \\ b_2 \langle \phi_2 \rangle^* & a \langle \phi_2 \rangle^* & b_1 \langle \phi_2 \rangle^* \\ b_1 \langle \phi_3 \rangle^* & b_2 \langle \phi_3 \rangle^* & a \langle \phi_3 \rangle^* \end{pmatrix}$$

(3.5a)

$$M^d = \begin{pmatrix} a' \langle \phi_1 \rangle & e_1 \langle \phi_3 \rangle & e_2 \langle \phi_2 \rangle \\ e_2 \langle \phi_3 \rangle & a' \langle \phi_2 \rangle & e_1 \langle \phi_1 \rangle \\ e_1 \langle \phi_2 \rangle & e_2 \langle \phi_1 \rangle & a' \langle \phi_3 \rangle \end{pmatrix}$$

(3.5b)

We consider a hierarchy in the v.e.v. namely

$$v_1 \ll v_2 \ll v_3$$

(3.6)

which is natural only in a finite but continuous domain of the scalar coupling parameters. The first matrix possesses then interesting eigenvalues:

$$m_u \sim a \left[\frac{r^2+2}{2} \right]^{1/2} \left[\frac{(1-R)(1+2R)}{1+R} \right]^{1/2} v_1$$

$$m_c \sim a \left[\frac{r^2+2}{2} \right]^{1/2} \left[1-R^2 \right]^{1/2} v_2$$

$$m_t \sim a \left[\frac{r^2+2}{2} \right]^{1/2} v_3 \qquad (3.7)$$

with

$$r \equiv 2 \frac{a}{b_1+b_2}$$

$$R \equiv \frac{2r+1}{r^2+2} \qquad -\frac{1}{2} \leqslant R \leqslant 1$$

For the down quark mass matrix we find in the limit $v_1 \to 0$:

$$m_d \sim \frac{1}{\sqrt{2}} \frac{e^2 v_3^2}{a' v_2}$$

$$m_s \sim \frac{1}{\sqrt{2}} a' v_2 \left[1 + 2 \frac{e^2 v_3^2}{a'^2 v_2^2} \right]^{1/2}$$

$$m_b \sim \frac{1}{\sqrt{2}} a' v_3 \qquad (3.8)$$

if

$$e_1 \sim e_2 \equiv e$$

$$a' v_2 > e v_3$$

We can therefore estimate the unitary matrices which define the physical states in terms of the weak ones:

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R} = U^u_{L,R} \begin{pmatrix} u^o \\ c^o \\ t^o \end{pmatrix}_{L,R}$$

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R} = U^d_{L,R} \begin{pmatrix} d^o \\ s^o \\ b^o \end{pmatrix}_{L,R} \qquad (3.9)$$

Since the $SU(2)_L \times U(1)$ models contain only left-handed currents, we only consider the matrices $U^{u,d}_L$ which diagonalize the Hermitean matrices $M^{u,d} M^{u,d^+}$. The Kobayashi-Maskawa (K.M.) mixing matrix[15] is indeed defined by

$$K_L \equiv U^u_L U^{d^+}_L \qquad (3.10)$$

and conventionally parametrized as follows:

$$
\begin{pmatrix}
c_1 & s_1 c_3 & s_1 s_3 \\
-s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\
-s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta}
\end{pmatrix}
\tag{3.11}
$$

where $c_i \equiv \cos\theta_i$, $s_i \equiv \sin\theta_i$. A non-vanishing δ implies a CP violating phase due here to the complex v.e.v. (we consider only real coupling constants). As shown in Appendix C, in this specific model, the extremal conditions on the potential provide us with phases which give rise to a very small δ parameter:

$$
\delta \sim \frac{s_1}{s_2}
\tag{3.12}
$$

As far as the angles are concerned we find from (3.10)

$$
|K_L| \sim
\begin{pmatrix}
1 & \dfrac{e\, s_3}{a'\, s_2} & \dfrac{e\, s_2}{a'\, s_3} \\[2ex]
-\dfrac{e\, s_3}{a'\, s_2} & 1 & -R\, \dfrac{s_2}{s_3} \\[2ex]
\dfrac{e}{a'}\left(R + \dfrac{s_2}{s_3}\right) & -R\, \dfrac{s_2}{s_3} & -1
\end{pmatrix}
\tag{3.13}
$$

such that a direct comparison with (3.11) implies the following relations:

$$
\theta_1 \sim \frac{e\, s_3}{a'\, s_2}
$$

$$
\theta_2 \sim -\frac{s_2}{s_3}\left(R + \frac{s_2}{s_3}\right)
$$

$$
\theta_3 \sim \left(\frac{s_2}{s_3}\right)^2
\tag{3.14}
$$

If we now assume for simplicity that the lepton mass matrix is (nearly) diagonal, namely

$$
\frac{m_e}{m_\mu} \simeq \frac{s_1}{s_2}
$$

$$
\frac{m_\mu}{m_\tau} \simeq \frac{s_2}{s_3}
\tag{3.15}
$$

We find very interesting relations between mass ratios[14]:

$$\frac{m_u}{m_c} \sim \frac{\sqrt{1+2R}}{1+R} \qquad \frac{m_e}{m_\mu}$$

$$\frac{m_c}{m_t} \sim \sqrt{1-R^2} \qquad \frac{m_\mu}{m_\tau}$$

$$\frac{m_s}{m_b} \sim \frac{m_\mu}{m_\tau} \tag{3.16}$$

while the mixing angles are now given by

$$\vartheta_1 \sim \vartheta_c \sim \sqrt{\frac{m_d}{m_s}}$$

$$\vartheta_2 \sim - \frac{m_\mu}{m_\tau} \left(R + \frac{m_\mu}{m_\tau} \right)$$

$$\vartheta_3 \sim \left(\frac{m_\mu}{m_\tau} \right)^2 \tag{3.17}$$

Moreover, the parameter R is strongly constrained by the experimental limit on the $B-$ meson time-life

$$\tau_B^{exp.} \leq 1.4 \ 10^{-12} \ s. \tag{3.18}$$

Indeed, assuming a spectator model[16] for this decay, we find

$$\tau_B^{th.} \sim \frac{8}{R^2} \ 10^{-13} \ s. \tag{3.19}$$

which implies another bound on R, namely

$$0.75 \leq R < 1 \tag{3.20}$$

From (3.16) and (3.19) we deduce respectively a lower bound for the t-quark mass and the $B-$ meson time-life

$$m_t \geq 26 \ m_c$$

$$\tau_B > 8 \ 10^{-13} \ s. \tag{3.21}$$

This model therefore provides us with a nice _description_ of the present situation for masses and angles. The CP problem remains, however, an open question since the contribution of the K.M. mixing matrix to the ε parameter for the $K^o - \bar{K}^o$ system is found to be of the order of 10^{-6} from (3.12) and (3.14).

Consequently in this model the CP violation must be carried by neutral scalars in flavour-changing processes[17]. We just want to stress that the problem pointed out by Branco and Sanda[18] concerning the need of too heavy scalars spoiling the possibility for a perturbative expansion does not arise here because the singlet scalar can have a large v.e.v. (C.9) since it decouples from the gauge sector.

4. - COMMENTS AND CONCLUSION

Instead of (3.4) we can impose the following discrete transformations

$$\begin{pmatrix} u^o_{\partial} \\ d^o_{\partial} \end{pmatrix}_L \longrightarrow \begin{pmatrix} u^o_{\partial} \\ d^o_{\partial} \end{pmatrix}_L$$

$$u^o_{\partial R} \longrightarrow e^{i \frac{2\pi}{3} \partial} u^o_{\partial R}$$

$$d^o_{\partial R} \longrightarrow e^{-i \frac{2\pi}{3} \partial} d^o_{\partial R} \tag{4.1}$$

such that the mass matrices become now

$$M^u = \begin{pmatrix} a \langle \phi_1 \rangle^* & b_1 \langle \phi_2 \rangle^* & b_2 \langle \phi_3 \rangle^* \\ b_2 \langle \phi_1 \rangle^* & a \langle \phi_2 \rangle^* & b_1 \langle \phi_3 \rangle^* \\ b_1 \langle \phi_1 \rangle^* & b_2 \langle \phi_2 \rangle^* & a \langle \phi_3 \rangle^* \end{pmatrix}$$

$$M^d = \begin{pmatrix} a' \langle \phi_1 \rangle & b'_1 \langle \phi_2 \rangle & b'_2 \langle \phi_3 \rangle \\ b'_2 \langle \phi_1 \rangle & a' \langle \phi_2 \rangle & b'_1 \langle \phi_3 \rangle \\ b'_1 \langle \phi_1 \rangle & b'_2 \langle \phi_2 \rangle & a' \langle \phi_3 \rangle \end{pmatrix} \tag{4.2}$$

These mass matrices are interesting in the sense that we have no CP violation at all in the gauge sector, for any v.e.v., because the phases can be absorbed in the right-handed sector.* Moreover, if we again assume (3.6) then the second hierarchy problem is "transposed" in the scalar sector [see (3.7)]:

---

*In the framework of $SU(2)_L \times SU(2)_R \times U(1)$ gauge models, this ensures a small ε'/ε ratio[19]. Note also that (4.2) implies $\arg \det M^u M^d = 0$, such that the strong CP violating angle θ_{QCD} vanishes at the tree level.

$$\frac{m_i}{m_{i+1}} \sim \frac{\sigma_i}{\sigma_{i+1}} \ll 1 \qquad (4.3)$$

If we extend (4.1) to the leptonic sector in order to obtain the same type of of matrix, the Yukawa interaction is then invariant under a global U(1):

$$\phi_j \longrightarrow e^{i\gamma_j} \phi_j$$

$$u^o_{jR} \longrightarrow e^{i\gamma_j} u^o_{jR}$$

$$d^o_{jR} \longrightarrow e^{-i\gamma_j} d^o_{jR} \qquad (4.4)$$

which can be gauged since it is anomaly-free.

This, of course, put us on the track of the horizontal interactions[20], where we need SU(2) x U(1) scalar singlets with rather large v.e.v. in order to provide the new gauge bosons with a large mass.

In conclusion we have shown that in the framework of electroweak SU(2) x x U(1) gauge theories, fermion masses are not <u>calculable</u> owing to the Georgi-Pais theorem which strongly constrained so-called natural models with residual unbroken global symmetries. Accidental symmetries, although providing a possible loop-hole, do not seem to avoid this negative result. The introduction of at least one weak singlet scalar implies a tree level <u>description</u> of the mass spectrum. This new field calls for right-handed, neutrino-like particles in order to keep the possibility of a dynamical breaking by some condensates, as well as for new (horizontal) gauge interactions[*]. We want, however, to stress that our analysis is essentially based on naturalness which provides a way to calculate finite physical quantities. A new door seems now open on this problem by supersymmetry. Indeed, just as the introduction of relativistic quantum mechanics weakened the singularity of the electromagnetic mass for the classical electron[21], the introduction of (global) supersymmetry low energy theories leads to a cancellation of the quadratic divergences in fermion (and gauge boson) masses[22] carried by the scalar bosons needed at the present time. Moreover, as long as supersymmetry is unbroken, masses are unrenormalized[22], a fact which of course upsets our previous approach.

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[*] In fact, for SU(n)$_H$ horizontal gauge groups (n>2) right-handed neutrinos are needed in order to fulfil the anomaly constraints [see T. Yanagida Ref. 20) for n = 3].

ACKNOWLEDGEMENTS

APPENDIX A – EXTREMAL SOLUTIONS IN THE $SU(2)_L \times U(1) \times S_3$ MODEL

If we denote the Q and CP conserving v.e.v. of the scalar doublets by

$$\langle \phi_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_i \end{pmatrix} \tag{A.1}$$

the value of the potential (2.3) at the minimum is given by

$$V(v_i) = -\frac{\lambda}{2} \sum_i v_i^2 + \frac{\gamma}{4} \sum_{i \neq j} v_i v_j + \frac{A}{4} \sum_i v_i^4$$

$$+ \frac{C'}{4} \sum_{i \neq j} v_i^2 v_j^2 + \frac{E_1}{4} \sum_{i \neq j} v_i^3 v_j + \frac{E}{4} \sum_{i \neq j \neq k} v_i^2 v_j v_k \tag{A.2}$$

with

$$C' \equiv C + \bar{C} + D$$

$$E \equiv E_2 + E_3 + E_4$$

In order to find all the possible extremal solutions, we just mimic Derman's approach[10]:

$$\frac{\partial V}{\partial v_1} - \frac{\partial V}{\partial v_2} = (v_1 - v_2) \, \tilde{V}(v_1, v_2 ; v_3) = 0$$

$$\frac{\partial V}{\partial v_2} - \frac{\partial V}{\partial v_3} = (v_2 - v_3) \, \tilde{V}(v_2, v_3 ; v_1) = 0 \tag{A.3}$$

with

$$\tilde{V}(v_i, v_j ; v_k) = \tilde{V}(v_j, v_i ; v_k)$$

If $v_1 \neq v_2$ and $v_2 \neq v_3$, we consider the equation

$$\tilde{V}(v_1, v_2 ; v_3) - \tilde{V}(v_2, v_3 ; v_1) = 0 \tag{A.4}$$

which implies the interesting relation

$$(4A - 2C' - E_1 + E)(v_1 + v_2 + v_3)(v_1 - v_3) = 0 \tag{A.5}$$

From this we conclude that only three natural and independent solutions are allowed, namely:

$$v_1 = v_2 = v_3$$

$$v_1 = v_2 \neq v_3$$

$$v_1 + v_2 + v_3 = 0 \tag{A.6}$$

The last one reduces to

$$v_1 = -v_2 \quad \text{and} \quad v_3 = 0$$

<div align="right">(A.7)</div>

by using the additional constraint given by $\sum_i \partial V/\partial v_i = 0$ and implies in this case, $v_1 v_2 v_3 = 0$. It is, however, worth noting that for

$$4A - 2C' - E_1 + E = 0$$

<div align="right">(A.8)</div>

the v.e.v. v_i are undetermined at the tree level, a typical signal of a continuous accidental symmetry [see Eq. (2.12)].

APPENDIX B .- EFFECTIVE POTENTIAL AND CONTINUOUS ACCIDENTAL SYMMETRY

We briefly review[6] here the way radiative corrections pick out the true vacuum in presence of an accidental symmetry. Let $V(\phi)$ be a scalar potential whose local minimum is defined by the extremal conditions

$$\frac{\partial V}{\partial \phi_i} \left[\langle \vec{\phi} \rangle = \vec{v} \right] = 0 \qquad (B.1)$$

In the Landau gauge, the one-loop radiative corrections to the v.e.v. can be computed just by analyzing the effective potential $(V + \delta V)(\phi)$ where

$$\delta V(\vec{v}) = \frac{1}{64\pi^2} \left[3\, Tr\, \mu^4 \ln \mu^2 + Tr\, M^4 \ln M^2 - 4\, Tr\, m^4 \ln m^2 \right] \qquad (B.2)$$

with μ^2, M^2 and m^2 respectively the squared mass matrices for the gauge bosons, scalars and fermions. The new, slightly shifted vacuum is indeed fixed by the following extremal conditions:

$$\frac{\partial}{\partial \phi_i} \left[V + \delta V \right] (\vec{v} + \delta \vec{v}) = 0 \qquad (B.3)$$

A straightforward expansion implies

$$\frac{\partial \delta V}{\partial \phi_i} (\vec{v}) = - M^2_{ij}\, \delta v_j \qquad (B.4)$$

where $M^2_{ij} \equiv \partial^2 V/(\partial\phi_i \partial\phi_j)\ (\vec{v})$ is a component of the squared mass matrix for the scalars. If (B.1) is now invariant under some continuous (local or global) transformations whose infinitesimal generators are θ_α, we find again by an expansion

$$M^2_{ij}\, (\theta_\alpha v)_i = 0 \qquad (B.5)$$

which is the mathematical expression of the Goldstone theorem. From (B.4) and (B.5) we finally find the necessary additional contraints

$$\frac{\partial \delta V}{\partial \phi_i} (\vec{v})\, (\theta_\alpha v)_i = 0 \qquad (B.6)$$

which allow a first order determination of the true vacuum in the presence of an accidental symmetry.

APPENDIX C – EXTREMAL SOLUTIONS IN THE $SU(2)_L \times U(1) \times d_3$ MODEL WITH A SINGLET SCALAR.

The most general scalar potential compatible with (3.1) is

$$
\begin{aligned}
V = {} & -\lambda \, (\phi_i^\dagger \phi_i) + A \, (\phi_i^\dagger \phi_i)^2 + C \, (\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) \\
& + \bar{C} \, (\phi_i^\dagger \phi_j)(\phi_j^\dagger \phi_i) + \frac{E_4}{2} \left[(\phi_i^\dagger \phi_j)(\phi_i^\dagger \phi_k) + h.c. \right] \\
& - \lambda' \, \phi_o^\dagger \phi_o + A' \, (\phi_o^\dagger \phi_o)^2 + B \, (\phi_i^\dagger \phi_i) \, \phi_o^\dagger \phi_o \\
& + S \, \left\{ (\phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_3 + \phi_3^\dagger \phi_1) \, \phi_o^2 + h.c. \right\}
\end{aligned} \tag{C.1}
$$

We note the various v.e.v. as follows:

$$
\langle \phi_j \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} o \\ v_j \, e^{i\alpha_j} \end{pmatrix}
$$

$$
\langle \phi_o \rangle = \frac{1}{\sqrt{2}} \, v_o \, e^{i\alpha_o} \tag{C.2}
$$

where $\alpha \equiv \alpha_1 - \alpha_2$ and $\alpha' \equiv \alpha_2 - \alpha_3$ are the relative phases. The extremal conditions on these phases imply three independent equations

$$
v_2 \, \sin Y - v_1 \, \sin X = \frac{E_4}{2S} \, \frac{v_2 v_3}{v_o^2} \left[2 v_1 \sin (X-Y) + v_2 \sin (-Y+Z) + v_3 \sin (X-Z) \right]
$$

$$
v_2 \, \sin Z - v_1 \, \sin X = \frac{E_4}{2S} \, \frac{v_1 v_2}{v_o^2} \left[v_1 \sin (X-Y) - v_2 \sin (-Y+Z) + 2 v_3 \sin (X-Z) \right]
$$

$$
\tag{C.3}
$$

$$
v_1 v_2 \, \sin Y + v_2 v_3 \, \sin Z + v_1 v_3 \, \sin X = o
$$

with

$$
\begin{aligned}
X &\equiv 2\alpha_o + \alpha + \alpha' \\
Y &\equiv 2\alpha_o - \alpha \\
Z &\equiv 2\alpha_o - \alpha'
\end{aligned} \tag{C.4}
$$

The set of CP invariant solutions defined by

$$
X, Y, Z = o, \pi \tag{C.5}
$$

forbids non-degenerated v.e.v.. So, if we want to solve the second hierarchy problem in this framework we must take a CP violating vacuum solution. Let us consider for example the case

$$X = Y \neq Z \tag{C.6}$$

which gives rise from (C.3) to two independent equations

$$v_1 (v_2 + v_3) \cos X + v_2 v_3 \cos Z = -2 \frac{S}{E_4} v_0^2$$

$$v_1 (v_2 + v_3) \sin X + v_2 v_3 \sin Z = 0 \tag{C.7}$$

We stress here the crucial role played by the S coupling constant in the scalar potential since a vanishing S would imply (C.5). Assuming $v_1 \ll \ll v_2 \ll v_3$, (3.6), we find, from (C.4), (C.6) and (C.7), the phases

$$\alpha \sim \frac{1}{6} (\pi + \frac{v_1}{v_2})$$

$$\alpha' \sim -\frac{1}{3} (\pi + \frac{v_1}{v_2})$$

$$\alpha_0 \sim \frac{1}{3} (\pi + \frac{v_1}{v_2}) \tag{C.8}$$

if

$$v_0^2 \sim \frac{E_4}{2S} v_2 v_3 \tag{C.9}$$

We therefore conclude that the CP violating phase appearing in the K.M. mixing matrix (3.10) is of the order of the small ratio v_1/v_2, while the v.e.v. of the singlet can be large.

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FIGURE CAPTIONS

Fig. 1   A self-energy diagram with a fermion mass insertion;
         the wavy line refers to horizontal gauge fields.

Fig. 2   A typical tadpole-like diagram with fermion mass insertions;
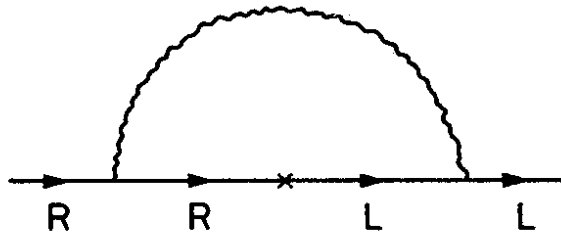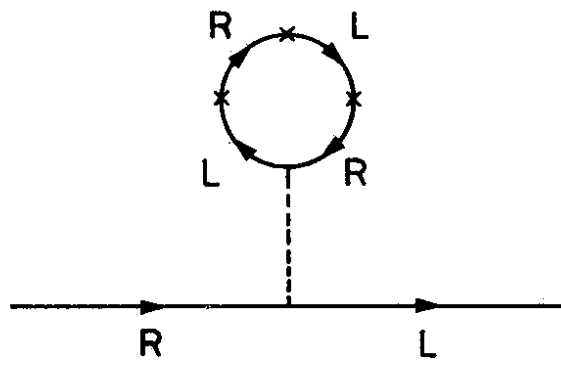   the dashed line refers to scalar fields.

FIGURE CAPTIONS

FIG. 1

FIG. 2