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Research Article

Fermion's Tunnelling with Effects of Quantum Gravity

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In this paper, using Hamilton-Jacobi method, we address the tunnelling of fermions in a 4-dimensional Schwarzschild spacetime. Based on the generalized uncertainty principle, we introduce the influence of quantum gravity. After solving the equation of motion of the spin-1/2 field, we derive the corrected Hawking temperature. It turns out that the correction depends not only on the black hole's mass but also on the mass (energy) of emitted fermions. It is of interest that, in our calculation, the quantum gravity correction decelerates the temperature increase during the radiation explicitly. This observation then naturally leads to the remnants in black hole evaporation. Our calculation shows that the residue mass is $\gtrsim M_p/\beta_0$, where M_p is the Planck mass and β_0 is a dimensionless parameter accounting for quantum gravity effects. The evaporation singularity is then avoided.

1. Introduction

Hawking radiation is described as a quantum tunnelling effects of particles at horizons of black holes [1–9]. With the consideration of the background variation in black hole evaporation, Parikh and Wilczek studied the tunnelling behaviors of massless scalar particles [3]. They derived the modified emission spectra for spherically symmetric black holes. The leading corrections to the Hawking temperature are found to be also dependent on the energy of emitted particles. This work was extended to massive and charged scalar particles. The Hawking radiation of general black holes was studied in [4, 5]. For an outgoing massive particle, the equation of motion is different from that of a massless particle. The trajectory of massless particles is a null geodesic, while the massive particle's motion satisfies de Broglie wave and is the phase velocity of outgoing particles. Subsequently, the tunnelling behaviors of fermions were carefully investigated with Hamilton-Jacobi method by Kerner and Mann [10, 11]. The Hawking temperatures were recovered by the fermions tunnelling. The extension of this work to complicated spacetimes is referred to in [12–15]. The noncommutative spacetimes bring a similar consequences in the spirit of black holes thermodynamics. In [16], the tunnelling process in the noncommutative Schwarzschild black holes was researched by Parikh-Wilczek tunnelling method. They derived an interesting result that information might be preserved by a stable black hole remnant.

Various theories of quantum gravity predict the existence of a minimum measurable length [17–21]. This length can be approached from the generalized uncertainty principle (GUP). Through the modified fundamental commutation relation [22]

$$\left[x_i, p_j\right] = i\hbar\delta_{ij}\left[1 + \beta p^2\right], \tag{1}$$

the expression of GUP is derived as $\Delta x \Delta p \geq (\hbar/2)[1 + \beta(\Delta p)^2]$, where $\beta = \beta_0/M_p^2$. M_p is the Planck mass. β_0 is a dimensionless parameter. From simple electroweak consideration, it is readily to find an upper limit $\beta_0 < 10^{34}$. x_i and p_i are defined by $x_i = x_{0i}$ and $p_i = p_{0i}(1 + \beta p^2)$, respectively. x_{0i} and p_{0j} satisfy the canonical commutation relations $[x_{0i}, p_{0j}] = i\hbar\delta_{ij}$. The modification of the fundamental commutation relation is not unique. Other modifications are referred to in [23–25] and references therein.

These modifications are widely applied to gain some information about the quantum properties of gravity. Black holes are effective modes to explore the effects of quantum gravity. Incorporating effects of quantum gravity into black hole physics by GUP, some interesting implications and results were achieved [26–32]. It is shown in [26] that a small black hole is unstable. Moreover, the constraint for a large black hole comparable to the size of the cavity in connection with the critical mass is needed. In [27], the tunnelling radiation in the noncommutative higher spacetime was discussed with GUP. They found that information may be preserved in a stable black hole remnant. The characteristic size in the absorption process, represented by the black hole irreducible mass, was gotten in [28]. The remnant mass and corrections to the area law and heat capacity were obtained in [29]. In [30], following Parikh-Wilczek tunnelling method, based on GUP, the radiation of massless scalar particles in the Schwarzschild black hole was discussed. The commutation relation between the radial coordinate and the conjugate momentum are modified with GUP. The authors treat the natural cutoffs as a minimal length, a minimal momentum, and a maximal momentum. They addressed the tunnelling rate of black holes. The corrected Hawking temperature was obtained and related to the energy of emitted particles.

The purpose of this paper is to investigate fermions' tunnelling behavior cross the event horizon of a 4-dimensional Schwarzschild black hole, where the effects of quantum gravity are taken into account. We first modify the Dirac equation in curved spacetime to reflect the influence of quantum gravity. The model we adopt is the generalized uncertainty principle. We use the Hamilton-Jacob method to solve the equation of motion of the spinor field. Then, the tunnelling rate and Hawking temperature are calculated. Our results show that the quantum correction to the Hawking temperature is dependent not only on the black hole's mass but also on the mass and energy of emitted fermions. Moreover, the correction slows down the temperature increase during the evaporation. This in turn leads to the remnants in black hole evaporation and prevents the existence of the thermodynamics singularity.

The rest of the paper is organized as follows. In Section 2, taking into account the effects of quantum gravity, we modify the Dirac equation in curved spacetime by GUP and get a generalized Dirac equation. In Section 3, the fermion tunnelling behavior in the Schwarzschild black hole is addressed and the corrected Hawking temperature is derived. Section 4 is devoted to our discussion and conclusion.

2. Generalized Dirac Equation in Curved Spacetime

To take into account the effects of quantum gravity, we adopt the generalized commutation relation in [22] to modify the Dirac equation. In (1), the momentum operators are defined by

$$p_i = p_{0i}\left(1 + \beta p^2\right). \tag{2}$$

The square of momentum operators is

$$\begin{aligned} p^2 = p_i p^i &= -\hbar^2\left[1 - \beta\hbar^2\left(\partial_j\partial^j\right)\right]\partial_i \cdot \left[1 - \beta\hbar^2\left(\partial^j\partial_j\right)\right]\partial^i \\ &\simeq -\hbar^2\left[\partial_i\partial^i - 2\beta\hbar^2\left(\partial^j\partial_j\right)\left(\partial^i\partial_i\right)\right], \end{aligned} \tag{3}$$

where in the last step, we only keep the leading order term of β. To account for the effects from quantum gravity, the frequency is generalized as [33]

$$\overline{\omega} = E\left(1 - \beta E^2\right), \tag{4}$$

with the energy operator $E = i\hbar\partial_0$. Substituting the mass shell condition $p^2 + m^2 = E^2$, we get the generalized expression of energy [34, 35]:

$$\overline{E} = E\left[1 - \beta\left(p^2 + m^2\right)\right]. \tag{5}$$

The tunnelling of massless scalar particles in the Schwarzschild black hole was studied in detail and the corrected Hawking temperature was derived in [30]. On the other hand, the Dirac equation with the consequence of GUP in flat spacetime has been investigated in [34].

We start with the Dirac equation in curved spacetime,

$$i\gamma^\mu\left(\partial_\mu + \Omega_\mu\right)\psi + \frac{m}{\hbar}\psi = 0, \quad \Omega_\mu \equiv \frac{i}{2}\omega_\mu{}^{ab}\Sigma_{ab}, \tag{6}$$

where $\omega_\mu{}^{ab}$ is the spin connection defined by the tetrad $e^\lambda{}_b$ and ordinary connection

$$\omega_\mu{}^a{}_b = e_\nu{}^a e^\lambda{}_b \Gamma^\nu_{\mu\lambda} - e^\lambda{}_b \partial_\mu e_\lambda{}^a. \tag{7}$$

The Latin indices live in the flat metric η_{ab} while Greek indices are raised and lowered by the curved metric $g_{\mu\nu}$. The tetrad can be constructed from

$$\begin{aligned} g_{\mu\nu} &= e_\mu{}^a e_\nu{}^b \eta_{ab}, \qquad \eta_{ab} = g_{\mu\nu} e^\mu{}_a e^\nu{}_b, \\ e^\mu{}_a e_\nu{}^a &= \delta^\mu_\nu, \qquad e^\mu{}_a e_\mu{}^b = \delta^b_a. \end{aligned} \tag{8}$$

Back in (6), Σ_{ab}'s are the Lorentz spinor generators defined by

$$\Sigma_{ab} = \frac{i}{4}\left[\gamma^a, \gamma^b\right], \qquad \left\{\gamma^a, \gamma^b\right\} = 2\eta^{ab}. \tag{9}$$

Then, one can construct the γ^μ's in curved spacetime as

$$\gamma^\mu = e^\mu{}_a \gamma^a, \qquad \left\{\gamma^\mu, \gamma^\nu\right\} = 2g^{\mu\nu}. \tag{10}$$

To get the generalized Dirac equation in curved spacetime, we rewrite (6) as

$$-i\gamma^0\partial_0\psi = \left(i\gamma^i\partial_i + i\gamma^\mu\Omega_\mu + \frac{m}{\hbar}\right)\psi, \tag{11}$$

where $i = 1, 2, \ldots$ denotes the spatial coordinates. The left-hand side of the equation above is related to the energy. Using the generalized expression of energy (5) and the square of momentum operators (3), only keeping the leading order term of β, we get

$$-i\gamma^0\partial_0\psi = \left(i\gamma^i\partial_i + i\gamma^\mu\Omega_\mu + \frac{m}{\hbar}\right)\left(1 + \beta\hbar^2\partial_j\partial^j - \beta m^2\right)\psi. \tag{12}$$

Therefore, the generalized Dirac equation in curved spacetime can be written as

$$\begin{aligned}&\left[i\gamma^0\partial_0 + i\gamma^i\partial_i\left(1-\beta m^2\right) + i\gamma^i\beta\hbar^2\left(\partial_j\partial^j\right)\partial_i\right.\\&\quad+\frac{m}{\hbar}\left(1+\beta\hbar^2\partial_j\partial^j-\beta m^2\right)\\&\quad\left.+i\gamma^\mu\Omega_\mu\left(1+\beta\hbar^2\partial_j\partial^j-\beta m^2\right)\right]\psi=0.\end{aligned} \tag{13}$$

This is the equation we are going to solve in the next section.

3. Fermion's Tunnelling with Effects of Quantum Gravity

In this section, we address the tunnelling behavior of spin-1/2 fermions across the event horizon of the Schwarzschild black hole. Effects of quantum gravity are taken into account. The metric is given by

$$ds^2 = -f(r)\,dt^2 + \frac{1}{g(r)}dr^2 + r^2\left(d\theta^2+\sin^2\theta d\phi^2\right), \tag{14}$$

with $f(r) = g(r) = 1-(2M/r)$, and M is the black hole's mass. We have set $G = c = 1$. The event horizon is located at $r_h = 2M$. The fermion motion is determined by the generalized Dirac equation (13). For a spin-1/2 particle, there are two states corresponding, respectively, to spin up and spin down. Following the standard ansatz, to describe the motion semiclassically, we assume the wave function of the spin up state as

$$\Psi = \begin{pmatrix} A\\0\\B\\0\end{pmatrix}\exp\left(\frac{i}{\hbar}I(t,r,\theta,\phi)\right), \tag{15}$$

where A, B, and I are functions of coordinates t, r, θ, and ϕ and I is the action of the emitted fermions. The process of spin down is the same as that of spin up. To solve (13), one should choose appropriate gamma matrices by exploiting (8)–(10). It is straightforward to guess a tetrad for the metric (14):

$$e_\mu{}^a = \operatorname{diag}\left(\sqrt{f}, \frac{1}{\sqrt{g}}, r, r\sin\theta\right). \tag{16}$$

Then, our gamma matrices are given by

$$\begin{aligned}\gamma^t &= \frac{1}{\sqrt{f(r)}}\begin{pmatrix} i & 0\\ 0 & -i\end{pmatrix}, &\quad \gamma^\theta &= \sqrt{g^{\theta\theta}}\begin{pmatrix}0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix},\\ \gamma^r &= \sqrt{g(r)}\begin{pmatrix}0 & \sigma^3\\ \sigma^3 & 0\end{pmatrix}, &\quad \gamma^\phi &= \sqrt{g^{\phi\phi}}\begin{pmatrix}0 & \sigma^2\\ \sigma^2 & 0\end{pmatrix},\end{aligned} \tag{17}$$

with $\sqrt{g^{\theta\theta}} = 1/r$ and $\sqrt{g^{\phi\phi}} = 1/r\sin\theta$. σ^i's are the Pauli matrices with $i = 1, 2, 3$.

Our task is to find the solutions of (13). First substitute the wave function (15) and the matrices (17) in the generalized Dirac equation (13) and cancel the exponential factor. Since we are working with WKB approximation, the contributions from ∂A, ∂B, and high orders of $\hbar$ are neglected. We finally obtain decoupled four Hamilton-Jacobi equations:

$$\begin{aligned}&-iA\frac{1}{\sqrt{f}}\partial_t I - B\left(1-\beta m^2\right)\sqrt{g}\partial_r I\\&-Am\beta\left[g^{rr}(\partial_r I)^2 + g^{\theta\theta}(\partial_\theta I)^2 + g^{\phi\phi}\left(\partial_\phi I\right)^2\right]\\&+B\beta\sqrt{g}\partial_r I\left[g^{rr}(\partial_r I)^2 + g^{\theta\theta}(\partial_\theta I)^2 + g^{\phi\phi}\left(\partial_\phi I\right)^2\right]\\&+Am\left(1-\beta m^2\right) = 0,\end{aligned} \tag{18}$$

$$\begin{aligned}&iB\frac{1}{\sqrt{f}}\partial_t I - A\left(1-\beta m^2\right)\sqrt{g}\partial_r I\\&\quad-Bm\beta\left[g^{rr}(\partial_r I)^2 + g^{\theta\theta}(\partial_\theta I)^2 + g^{\phi\phi}\left(\partial_\phi I\right)^2\right]\\&\quad+A\beta\sqrt{g}\partial_r I\left[g^{rr}(\partial_r I)^2 + g^{\theta\theta}(\partial_\theta I)^2 + g^{\phi\phi}\left(\partial_\phi I\right)^2\right]\\&\quad+Bm\left(1-\beta m^2\right) = 0,\end{aligned} \tag{19}$$

$$\begin{aligned}&A\left\{-\left(1-\beta m^2\right)\sqrt{g^{\theta\theta}}\partial_\theta I\right.\\&\quad+\beta\sqrt{g^{\theta\theta}}\partial_\theta I\left[g^{rr}(\partial_r I)^2 + g^{\theta\theta}(\partial_\theta I)^2 + g^{\phi\phi}\left(\partial_\phi I\right)^2\right]\\&\quad-i\left(1-\beta m^2\right)\sqrt{g^{\phi\phi}}\partial_\phi I + i\beta\sqrt{g^{\phi\phi}}\partial_\phi I\\&\quad\left.\times\left[g^{rr}(\partial_r I)^2 + g^{\theta\theta}(\partial_\theta I)^2 + g^{\phi\phi}\left(\partial_\phi I\right)^2\right]\right\} = 0,\end{aligned} \tag{20}$$

$$\begin{aligned}&B\left\{-\left(1-\beta m^2\right)\sqrt{g^{\theta\theta}}\partial_\theta I\right.\\&\quad+\beta\sqrt{g^{\theta\theta}}\partial_\theta I\left[g^{rr}(\partial_r I)^2 + g^{\theta\theta}(\partial_\theta I)^2 + g^{\phi\phi}\left(\partial_\phi I\right)^2\right]\\&\quad-i\left(1-\beta m^2\right)\sqrt{g^{\phi\phi}}\partial_\phi I + i\beta\sqrt{g^{\phi\phi}}\partial_\phi I\\&\quad\left.\times\left[g^{rr}(\partial_r I)^2 + g^{\theta\theta}(\partial_\theta I)^2 + g^{\phi\phi}\left(\partial_\phi I\right)^2\right]\right\}.\end{aligned} \tag{21}$$

To find the relevant solution, since the metric has a timelike killing vector, we perform the separation of variables as follows:

$$I = -\omega t + W(r) + \Theta(\theta,\phi), \tag{22}$$

where ω turns out to be the energy of the emitted particle. We insert (22) into (18)–(21) and first focus on the last two equations. They are identical after dividing, respectively, by A and B and can be rewritten as follows:

$$\begin{aligned}&\left(\sqrt{g^{\theta\theta}}\partial_\theta\Theta + i\sqrt{g^{\phi\phi}}\partial_\phi\Theta\right)\\&\quad\times\left[\beta g^{rr}(\partial_r W)^2 + \beta g^{\theta\theta}(\partial_\theta\Theta)^2\right.\\&\quad\left.+\beta g^{\phi\phi}\left(\partial_\phi\Theta\right)^2 - \left(1-\beta m^2\right)\right] = 0.\end{aligned} \tag{23}$$

In the equation above, the value in the square bracket cannot vanish since β is a small quantity representing the effects from quantum gravity. Therefore, the expression in the round brackets is zero and yields the solution of Θ. In the previous work, though Θ has a complex solution (other than the trivial one $\Theta = \text{constant}$) and gives rise to a contribution to the imaginary part of the action, it has no contribution to the tunnelling rate. Therefore, (23) is simplified as

$$\sqrt{g^{\theta\theta}}\partial_\theta\Theta + i\sqrt{g^{\phi\phi}}\partial_\phi\Theta = 0. \tag{24}$$

After cancelling A and B, (18) and (19) are identical and give rise to

$$A_6(\partial_r W)^6 + A_4(\partial_r W)^4 + A_2(\partial_r W)^2 + A_0 = 0 \tag{25}$$

with

$$\begin{aligned} A_6 &= \beta^2 g^3 f, \\ A_4 &= \beta g^2 f\left(m^2\beta + 2\beta Q - 2\right), \\ A_2 &= gf\left[\left(1-\beta m^2\right)^2 \right. \\ &\qquad \left. + \beta\left(2m^2 - 2m^4\beta - 2Q + \beta Q^2\right)\right], \\ A_0 &= -m^2\left(1-\beta m^2 - \beta Q\right)^2 f - \omega^2, \\ Q &= g^{\theta\theta}(\partial_\theta\Theta)^2 + g^{\phi\phi}(\partial_\phi\Theta)^2. \end{aligned} \tag{26}$$

Using (24), we find $Q = 0$. Neglecting the higher orders of β and solving the above equations at the event horizon yields (In [36–38], the authors argued that the invariance under canonical coordinate transformation requires that the integral to calculate the imaginary part of $W(r)$ should be a loop rather than an open one-way integral. However, in our calculation, only the difference between the imaginary parts matters.)

$$\begin{aligned} W(r) &= \pm\int \frac{1}{\sqrt{gf}}\sqrt{m^2\left(1-2\beta m^2\right)f + \omega^2} \\ &\qquad \times\left(1 + \beta\left(m^2 + \frac{\omega^2}{f}\right)\right)dr \\ &= \pm i\pi 2M\omega\left(1 + \frac{1}{2}\beta\left(3m^2 + 4\omega^2\right)\right) + (\text{real part}). \end{aligned} \tag{27}$$

In the above equation, $f = g = 1 - (2M/r)$. The real part is irrelevant to the tunnelling rate. The $+/-$ sign corresponds to outgoing/ingoing wave. Then, the tunnelling rate [39] of the spin-1/2 fermion crossing the horizon is

$$\begin{aligned} \Gamma &= \frac{P_{(\text{emission})}}{P_{(\text{absorption})}} = \frac{\exp\left(-2\,\text{Im}\,I_+\right)}{\exp\left(-2\,\text{Im}\,I_-\right)} \\ &= \frac{\exp\left(-2\,\text{Im}\,W_+ - 2\,\text{Im}\,\Theta\right)}{\exp\left(-2\,\text{Im}\,W_- - 2\,\text{Im}\,\Theta\right)} \\ &= \exp\left[-8\pi M\omega\left(1 + \frac{1}{2}\beta\left(3m^2 + 4\omega^2\right)\right)\right]. \end{aligned} \tag{28}$$

This is the Boltzmann factor with Hawking temperature:

$$\begin{aligned} T &= \frac{1}{8\pi M\left(1 + (1/2)\beta\left(3m^2 + 4\omega^2\right)\right)} \\ &= \left[1 - \frac{1}{2}\beta\left(3m^2 + 4\omega^2\right)\right]T_0, \end{aligned} \tag{29}$$

where $T_0 = 1/8\pi M$ is the original Hawking temperature. It shows that there is a small correction to the Hawking temperature and the correction value is dependent not only on the black hole's mass but also on the mass and energy of emitted fermions. This property has been obtained in the literature. In [3], energy conservation is enforced by dynamical geometry and the tunnelling rate is found to be $\Gamma = \exp[-8\pi\omega(M - \omega/2)]$. Then, the corrected Hawking temperature is $T = 1/(8\pi M - 4\pi\omega)$, where the leading correction to the Hawking temperature is related to the energy of emitted particles. To address the effects of quantum gravity, the authors of [30, 40] adopted the modified commutation relation between the radial coordinate and the conjugate momentum. They studied the quantum tunnelling of scalar particles in the Schwarzschild black hole. The tunnelling rate was derived as $\Gamma = \exp[-8\pi M\omega + 4\pi\omega^2(3Mal_p + 1) - 8\pi\omega^3((7/3)Ma^2l_p^2 + (4/3)al_p) + 20\pi a^2 l_p^2\omega^4 + 0(a^2l_p^4)]$. Thus, the correction to the Hawking temperature is also related to the black hole's mass and the particle's energy.

It is of interest to note that in (29) the quantum correction slows down the increase of the temperature during the radiation. This correction therefore causes the radiation cease at some particular temperature, leaving the remnant mass. To estimate the residue mass, it is enough to consider massless particles. The temperature stops increasing when

$$(M - dM)\left(1 + \beta\omega^2\right) \simeq M. \tag{30}$$

Then with the observation $dM = \omega$ and $\beta = \beta_0/M_p^2$, where M_p is the Planck mass and $\beta_0 < 10^{34}$ [41, 42] is a dimensionless parameter marking quantum gravity effects, we can get

$$M_{\text{Res}} \simeq \frac{M_p^2}{\beta_0\omega} \gtrsim \frac{M_p}{\beta_0}, \qquad T_{\text{Res}} \lesssim \frac{\beta_0}{8\pi M_p}, \tag{31}$$

where we have assumed that the maximal energy of the radiated particle is $\omega \simeq M_p$. This result is consistent with those obtained in [29, 43–45]. Compared with previous results, our calculation explicitly shows how the residue mass of black holes arises due to quantum gravity effects. The singularity of black hole evaporation is then prevented by the quantum gravity correction.

4. Discussion and Conclusion

In this work, we modified the Dirac equation in curved spacetime to include the quantum gravity influence. To fulfill this purpose, we employed the generalized uncertainty principle model. This model is derived from the existence of minimal length which arises when combining quantum and gravity.

We calculated the radiation of spin-1/2 particles in the 4-dimensional Schwarzschild spacetime with Hamilton-Jacob method. The tunnelling rate and Hawking temperature were presented.

We found that the quantum gravity correction is related not only to the black hole's mass but also to the mass (energy) of emitted fermions. More interestingly, our result shows that the quantum gravity correction explicitly retards the temperature rising in the black hole evaporation. Therefore, at some point during the evaporation, the quantum correction balances the traditional temperature rising tendency. This leads to the existence of the remnants. We showed that the remnants is $M_{\text{Res}} \gtrsim M_p/\beta_0$, where M_p is the Planck mass and $\beta_0 < 10^{34}$ from simple electroweak consideration. Therefore, the classical thermodynamics singularity can be avoided and a residue temperature $T_{\text{Res}} \lesssim \beta_0/8\pi M_p$ of black holes exists.

We use the 4-dimensional Schwarzschild metric in this work. It is known that the WKB type of approximation is basically the same as working with a 1+1-dimensional spacetime [46–48]. As a consequence, all large nonextremal black holes look basically the same (like Rindler space). It is also true for the charged and rotating black holes. Therefore, the remnants can be also found in other geometries. It is of interest to employ these geometries in the studies. In our calculation, we keep only the leading order of $\hbar$ and $\beta = \beta_0/M_p^2$. It is expected that higher orders of corrections may give more information in the future work.

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