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ABSTRACT

The performance of frequency-hopped (FH) M-ary frequency-shift keyed (MFSK) signals in partial-band noise has been extensively analyzed in the open literature. This report extends the previous research to the usually more effective class of multitone jamming. Specifically, this report will:

(1) Categorize several different multitone jamming strategies.

(2) Analyze the performance of FH/MFSK signaling, both uncoded and with diversity, assuming a noncoherent energy detection metric with linear combining and perfect jamming state side information, in the presence of worst case interference for each of these multitone categories.

(3) Compare the effectiveness of the various multitone jamming techniques, and contrast the results with the partial-band noise jamming case.

CONTENTS

I. Introduction

The effectiveness of partial-band noise jamming as an electronic countermeasure (ECM) against frequency-hopped (FH) M-ary frequency-shift keyed (MFSK) signals has been widely documented. Houston [1] demonstrated that an optimized partial-band duty factor can severely degrade uncoded FH/MFSK transmissions, resulting in an inverse-linear relationship between the bit error rate (BER) and the signal-to-noise ratio (SNR). Viterbi and Jacobs [2] showed that most of this jamming advantage can be recovered (and an exponential BER-SNR dependence restored) through the use of optimized time diversity, which is a simple repetition code. Later articles explored the improvements afforded by more sophisticated block and convolutional codes [e.g. 3-5].

By comparison, the often more effective class of multiple CW (multitone) jamming of FH/MFSK signals has been sparsely treated in the open literature. Houston [1] and Trumpis [6] did analyse the performance of uncoded FH/MFSK communications in two types of multitone interference, and this work was later extended to include optimum diversity [7]. That is approximately the extent of the published information available on this subject in the unclassified arena. Recently, however, the author had the opportunity to contribute to a major new reference text on spread-spectrum communications [8], including previously unpublished results on the performance of coded FH/MFSK signals noncoherently detected in a variety of multitone jamming environments; this report is a compendium of some of that research.

II. Multitone Jamming Strategies

A practical multitone jammer partitions its total available power J (referenced to the receiver input) into Q distinct, equal power, random phase

CW tones. These are distributed over the spread-spectrum bandwidth W_{ss} according to one of several strategies illustrated in Fig. 1. It is assumed that the jamming tones coincide in frequency with the FH slots, with at most one tone per slot, and thermal noise is neglected. The power J/Q in each received jamming tone is related to the received signal power S by

$$J/Q = S/\alpha \tag{1}$$

where, for a given strategy and system parameters, the jammer will optimize α to maximize the BER. Although one might believe that in the absence of thermal noise, each jamming tone power must be slightly larger than S to be effective, corresponding to $\alpha = 1_{-}$ in (1), we will see that there are many cases where this is not optimum from the jammer's viewpoint.

In a conventional FH/MFSK implementation, a single carrier frequency is hopped over W_{ss} and the M-ary modulation is effected by a deviation about this carrier. Thus we can talk about M-ary bands in which all M possible signals on a given hop occupy adjacent, uniformly-spaced FH slots. For ease of representation, Fig. 1 restricts this structure even further to non-overlapping, contiguous M-ary bands, although this restriction does not impact the analysis. Since we will see later that this M-ary band structure can be exploited by a smart multitone jammer, a more sophisticated (and expensive) FH/MFSK system might use not one but M frequency synthesizers to independently hop each MFSK signal [9]; we will assume that independent hopping is not used in this analysis.

Under the so-called "band-multitone" strategy of Fig. 1, a jammed M-ary band contains exactly n jamming tones,[1] $1 \leq n \leq M$, with the implied assumption that Q/n is an integer.[2] In the less structured "independent-multitone" implementation, the Q jamming tones are pseudorandomly distributed uniformly over the available FH slots, without regard for the location of the M-ary bands; this strategy is equally effective against independently hopped FH/MFSK systems with no change in the analysis.

For both multitone jamming strategies we have to consider the possibility that the transmitted signal frequency will itself be jammed on a given hop. If the phase offset between the signal and jamming tones is ϕ, the phasor diagram of Fig. 2 shows that the resultant power into the corresponding energy detector is

$$S^* = S(1 + 2 \cos \phi / \sqrt{\alpha} + 1/\alpha) \tag{2}$$

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[1]Houston and Trumpis both restricted their analyses to the special case of band-multitone jamming with n = 1 and n = M. In particular, Trumpis referred to the n = M band-multitone case as "partial-band multitone jamming" by analogy to the partial-band noise scenario.

[2]In practice, if Q/n is not integral, int(Q/n) of the M-ary bands will each contain exactly n jamming tones, while one band will contain Q mod n jamming tones. Assuming Q >> n, the performance for this structure is essentially the same as that for n = M band multitone jamming with $Q' \sim Q$ jamming tones such that Q'/n is an integer.

Extending (2) to other cases of interest, the energy detector outputs for the transmitted and untransmitted M-ary symbols, normalized by the signal energy and conditioned on whether they are tone jammed, are given in Table 1. These expressions will be instrumental in the performance analysis below, particularly with regard to the range over which the jammer should optimize the power distribution parameter α.

For example, consider the case of uncoded FH/MFSK signaling: if the data symbol is not jammed and any of the other M-1 symbols is, an error will always be made if $\alpha < 1$ but never for $\alpha > 1$. (Ties that occur for the singular case $\alpha = 1$ can be resolved by an M-sided coin or pessimistically assigned to the error side of the ledger.) The only other condition under which an error can occur is if the data and any other symbol are simultaneously jammed: then, assuming ϕ is uniformly distributed, an error will occur with probability

$$\Pr\left[\cos\phi < -\sqrt{\alpha}/2\right] = \cos^{-1}(\sqrt{\alpha}/2)/\pi \tag{3}$$

which is positive for $0 \leq \alpha < 4$. Here then is an example where an error can occur when each of the jamming tones have up to 6 dB less power than the received signal.

III. Uncoded Performance

The n = 1 band-multitone scheme is the simplest to analyze, and the uncoded case has been adequately treated by Houston, so we will simply restate his results here. The worst case (WC) performance and corresponding value of α are given by [1, (34)]

$$\text{BER} = 1/2, \quad \alpha_{wc} = KE_b/MN_J; \quad E_b/N_J \lesssim M/K$$

$$\text{BER} = M/(2KE_b/N_J), \quad \alpha_{wc} = 1_-; \quad E_b/N_J \gtrsim M/K \tag{4}$$

where $E_b = S/R_b$ is the received bit energy when the data bit rate is R_b, $N_J \equiv J/W_{ss}$ (so defined for comparison with the broadband noise jamming case where N_J is the effective noise power spectral density neglecting thermal noise), and E_b/N_J is the common SNR that all of our performance results will be referenced to; also, $K = \log_2 M$ is the number of information bits per uncoded M-ary symbol. The WC n = 1 band-multitone performance of (4) is contrasted with broadband noise [10, (8.14)] and WC partial-band noise jamming [1, (15)-(16)] in Fig. 3. In the WC partial-band noise scenario, the BER-SNR dependence is inverse-linear for SNRs below a threshold that varies with K; with WC n = 1 band-multitone jamming, that same type of relationship arises for all BERs < 1/2 independent of K. For SNRs below the threshold specified in (4), the entire SS band W_{ss} is saturated with exactly 1 jamming tone per M-ary band, and the jamming tone power rises above S inversely with E_b/N_J while the BER is pegged at 1/2. It is evident that the multitone strategy is significantly more effective than partial-band noise, particularly for larger values of K (e.g. 4.3 dB better when K = 1 versus 10.5 dB for K = 4). This last observation reflects the fact that the multitone performance degrades with increasing K unlike the noise jamming cases.

Next we consider the performance of uncoded FH/MFSK signaling in band-multitone jamming with n > 1 tones per jammed M-ary band. Since the spacing between adjacent FH slots is the M-ary symbol rate $R_s = R_b/K$, there are $N_t = W_{ss}/R_s$ available FH frequencies, and N_t/M adjacent M-ary bands in the FH/MFSK structure of Fig. 1. With Q/n of these bands jammed, the probability that a given band is in fact jammed is

$$\mu = (Q/n)\ (N_t/M) = \alpha M/(nKE_b/N_J) \tag{5}$$

where we have used (1) and the definition of E_b/N_J above. If the M-ary band containing the data symbol on a given hop is jammed, the conditional probability that one of the n tones hits the data symbol is

$$\binom{M-1}{n-1} \Big/ \binom{M}{n} = n/M \tag{6}$$

On a given hop, a symbol error can occur only if the M-ary band containing the data symbol is jammed (since $n > 1$, this implies that at least one of the M-1 untransmitted symbols is hit), and

(i) the data symbol is not hit and $\alpha < 1$, or

(ii) the data symbol is hit and the phase of the jamming tone lies in the range defined by (3).

Expressing these conditions mathematically, the symbol error rate (SER) is given by

$$SER = \mu \left[(1 - n/M)\ u_{-1}(1 - \alpha) + n \cos^{-1}(\sqrt{\alpha}/2)\ M\pi\right] \tag{7}$$

where $u_{-1}(\bullet)$ is the standard unit step function.

The WC jammer chooses $\alpha \in (0,4)$ to maximize the SER subject to the constraint that the probability $\mu \leq 1$ in (5). It can be verified that the term $\alpha \cos^{-1}(\sqrt{\alpha}/2)$ has the unique interior maximum of 0.525 at $\alpha = 2.52$

over $0 < \alpha < 4$.[3] With this result and the relationship $BER = M\ SER/2(M-1)$, the worst case performance must be specified for three distinct range combinations of SNR and n/M:

(i) $E_b/N_J \leq M/nK$

$$\alpha_{wc} = nKE_b/MN_J \leq 1$$
$$BER = [M/2(M-1)] \left\{ 1 - (n/M)[1 - \cos^{-1}(\sqrt{\alpha_{wc}}/2)/\pi] \right\} \tag{8}$$

(ii) $E_b/N_J \geq M/nK$ and $n/M \leq (\beta + 2/3)^{-1}$

$$\alpha_{wc} = 1_-$$
$$BER = M(M - 2n/3)/2nK(M-1)(E_b/N_J) \tag{9}$$

[3]Note that for given received powers S and J, α depends on Q via (1). Furthermore, if we rigorously demand that Q/n be an integer, then α is restricted to a set of discrete values. Pragmatically, we have already argued in footnote 2 that if $Q >> n$ (this holds for typical scenarios with small values of n and large J/S ratios), Q need not be an integer multiple of n for the analytical results presented here to be valid. Also, although $\alpha \cos^{-1}(\sqrt{\alpha}/2)$ is mathematically maximized at $\alpha = 2.52$, the maximum is broad enough to allow nearby values of α to be almost as effective. Consequently, we need not be concerned that the optimization regards α as a continuous parameter.

(iii) $E_b/N_J \geq M/nK$ and $(\beta + 2/3)^{-1} \leq n/M \leq 1$

$$\alpha_{wc} = \min(2.52,\ nKE_b/MN_J) \geq 1$$

$$BER = M\,\beta/2K(M-1)(E_b/N_J) \tag{10}$$

where

$$\beta \equiv \begin{cases} (nKE_b/M\pi N_J)\cos^{-1}(\sqrt{nKE_b/4MN_J});\ M/nK \leq E_b/N_J \leq 2.52\ M/nK \\ .525;\ E_b/N_J \geq 2.52\ M/nK \end{cases} \tag{11}$$

which implies that $(\beta + 2/3)^{-1} \in [.84,1]$. Note that in the third region above, (11) says that the BER is independent of n.

So for $n/M \geq .84$, there are some conditions under which it is advantageous for the jammer to allocate less power to each tone than the received signal power (i.e. $\alpha > 1$). However, practically speaking, band-multitone jammers with values of n in this range are not very effective (e.g. see Fig. 4 for M = 16, which is characteristic of the relative jamming effectiveness for other values of M). In general, the best band-multitone strategy is to use n = 1, subject to the assumptions underlying this analysis. For a more complete discussion of the ramifications of (8)-(11), the reader is referred to [8].

Finally we consider the relatively simplistic independent-multitone jamming strategy, which requires no knowledge of the M-ary band structure. In this category, the probability that a given FH frequency is hit by a jamming tone is

$$\rho = Q/N_t = \alpha/K(E_b/N_J) \tag{12}$$

If $Q >> M$, each symbol in a given M-ary band is essentially independently[4] tone jammed with probability ρ (hence the name for this class of multitone jamming), so that the probability that a particular M-ary band contains at least one jamming tone is given by

$$\mu = 1 - (1 - \rho)^M \cong \alpha M/K(E_b/N_J); \; \rho << 1 \qquad (13)$$

Note that ρ in (12) is very small for large E_b/N_J, which justifies the approximation for μ in (13); furthermore, this approximation is identical to the expression in (5) for band-multitone jamming with $n = 1$. It is argued in [8] that it is to the jammer's advantage to hit as many M-ary bands as possible, implying that μ is a measure of the jammer's effectiveness. Consequently, we might expect that the independent- and $n = 1$ band-multitone strategies have the same asymptotic effectiveness against uncoded FH/MFSK signals for large SNRs, and this observation is confirmed below.

On a given hop, a symbol error can occur only if at least one of the M-1 untransmitted symbols in the M-ary band containing the data symbol is hit, and

(i) the data symbol itself is not hit and $\alpha \leq 1$, or

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[4] Let J_i denote the event that the i^{th} symbol in an M-ary band is tone jammed. Then $Pr[J_1] = Q/N_t = \rho$. And $Pr[J_2 | J_1] = (Q-1)/(N_t-1) \cong \rho = Pr[J_2]$, if $N_t > Q >> 1$, so that J_1 and J_2 are statistically independent. Continuing in this manner, $Pr[J_M | J_1, J_2, \ldots, J_{M-1}] = (Q-M+1)/(N_t-M+1) \cong \rho = Pr[J_M]$ if $N_t > Q >> M-1$. That is, all of the J_i's are mutually independent.

(ii) the data symbol is hit and the jamming tone phase lies in the range defined by (3).

Consequently,

$$SER = \left[1 - (1 - \rho)^{M-1}\right]\left[(1 - \rho)\, u_{-1}(1 - \alpha) + \rho \cos^{-1}(\sqrt{\alpha}/2)/\pi\right] \qquad (14)$$

The WC independent-multitone jammer selects $\alpha \in (0,4)$ to maximize this SER subject to the constraint $\rho \leq 1$ in (12). For small SNRs (i.e. $E_b/N_J < \gamma$ defined in Table 2), we find that $\alpha_{wc} < 1$, but the maximization in (14) must be computed numerically for each combination of K and E_b/N_J. However, for larger SNRs, it can be shown that $\alpha_{wc} = 1_-$ and the performance is specified by

$$BER = [M/2(M-1)] \left\{1 - [1 - 1/K(E_b/N_J)]^{M-1}\right\} [1 - 2/3K(E_b/N_J)]; \; E_b/N_J \geq \gamma \qquad (15)$$

Note that for $KE_b/N_J >> 1$, (15) reduces to the inverse-linear relationship of (4) for n = 1 band-multitone jamming, an observation that is reinforced in Fig. 5. So, as promised above based on the figure of merit μ, for the low BERs that typify most practical applications, these two ECM strategies are equally effective against uncoded, noncoherently detected FH/MFSK signals.

A summary of the relative effectiveness of all of the WC noise and tone jammers for uncoded FH/MFSK signaling is shown in Fig. 4 for M = 16. From the communicator's standpoint, the n = 1 band- and independent-multitone strategies are superior; partial-band noise is on a par with band-multitone jamming for n ~ M/2; and n = M band-multitone jamming is inferior (even worse than broadband noise for low SNRs). Furthermore, all of the WC jammers asymptotically exhibit the inverse-linear performance characteristic for sufficiently large SNRs.

IV. Performance With Diversity

Whether confronted by a WC partial-band noise or multitone jammer, we know that the performance of uncoded FH/MFSK communication systems is severely degraded. The reason is that each M-ary symbol is sent on a single hop, allowing an average power-limited jammer to concentrate its available power over a relatively small portion of the entire spread-spectrum bandwidth W_{ss}: although a correspondingly small fraction of the data transmissions are hit, that data suffers a very high conditional error rate. An effective countermeasure against such jammers is to introduce coding redundancy so that data decisions are based on multiple hops. This causes the jammer to spread its power so as to hit a larger portion of W_{ss}; ultimately the effect is to force the jammer to retreat back towards the original broadband noise jamming strategy, thereby restoring the desired exponential performance characteristic.

One of the simplest albeit effective coding techniques is time diversity or repetition coding. Each M-ary symbol is partitioned into L equal-duration subsymbols or "chips," each with energy $E_c = KE_b/L$. These chips are transmitted on different hops using fast frequency hopping (FFH) or slow frequency hopping (SFH) with pseudorandom interleaving [3, Fig. 2]. (Denoting the hop rate by R_h and the chip rate by $R_c = LR_s = LR_b/K$, our convention is that FFH implies that $R_c = R_h$ while SFH defines the multiple-chip-per-hop condition $R_c > R_h$.) To maintain orthogonality between adjacent energy detectors, the spacing of the FH frequency slots is now R_c instead of R_s in the uncoded case of Fig. 1, and the number of available FH slots is now reduced to $N_t = W_{ss}/LR_s$.

We assume that the receiver has perfect jamming state side information: that is, it can somehow determine with certainty whether a given hop is jammed. A chip is declared to be jammed when two or more of the energy detector outputs is high [6], since, having neglected thermal noise, the M-1 energy detectors not tuned to the received MFSK signal will have outputs that are identically zero. Consequently, if any of the L chips comprising an M-ary symbol is not jammed, an error-free M-ary decision is made; otherwise select the largest of the symbol metrics [7, (18) or 8, (2.57)]

$$\left\{\Lambda_i = \sum_{j=1}^{L} e_{ij};\ 1 \leq i \leq M\right\} \tag{16}$$

where e_{ij} is the energy detector output for the i^{th} M-ary chip on the j^{th} diversity transmission. The suboptimum linear sum metric of (16) produces a noncoherent combining loss for large amounts of divesity L [e.g. 3 and 8]. Since exact BER calculations based on this metric do not generally yield closed-form expressions, our approach is to compute exponentially-tight Chernoff upperbounds [2, 3, 7, and 8]; optimizations of the diversity L and the jammer parameter α based on these bounds should be regarded as close approximations and more accurately identified as "quasi-optimum" [2, p. 289].

Now consider the performance of FH/MFSK signals with diversity in band-multitone jamming. Since N_t is reduced by a factor of L, the probability that a given M-ary band is jammed on a particular diversity transmission must be an appropriately modified version of (5); i.e.

$$\mu = \alpha\ LM/(nKE_b/N_J) \tag{17}$$

Restricting our attention initially to the n = 1 band-multitone strategy, a necessary set of conditions for a symbol error to occur when the square-law metric of (16) is used in conjuction with perfect jamming state side information is that[5]

(i) all L hops are tone jammed,

(ii) the data frequency is not hit by the single jamming tone on each hop (actually, this constraint is redundant with the convention we have adopted that more than one energy detector output must be high for a hop to be considered jammed),

(iii) and $\alpha < 1$.

The probability of this event, denoted by $\underline{H} = (H_1, H_2, \ldots, H_L)$ where H_j is the event that the j^{th} hop is jammed, is

$$\Pr[\underline{H}] = [\mu(M-1)/M]^L = [\alpha L(M-1)/(KE_b/N_J)]^L; \quad \alpha < 1 \tag{18}$$

For the special case of binary (K = 1 or M = 2) signaling, we can still compute the exact performance. The BER is synonymous with $\Pr[\underline{H}]$ above, and the WC jammer wants to maximize α subject to $\mu \leq 1$:

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[5]In fact, we know that the correct symbol will produce a high energy detector output on all L diversity transmissions. If we were to incorporate this criterion into the symbol decision process instead of simply using the detection metric of (16), we would create an additional necessary condition for a symbol error to occur:

(iv) the same incorrect symbol would have to be tone jammed on each diversity transmission.

$$BER = 2^{-L}, \quad \alpha_{wc} = (E_b/N_J)/2L; \quad E_b/N_J < 2L$$

$$BER = [L/(E_b/N_J)]^L, \quad \alpha_{wc} = 1_-; \quad E_b/N_J \geq 2L \tag{19}$$

This is plotted in Fig. 6 for various values of L. The horizontal portion of each piecewise linear curve for a particular value of L (specified by the first line in (19)) is the so-called "saturation region" for which each available M-ary (binary) band contains its quota of one jamming tone (i.e. $\mu = 1$) and the power in each received jamming tone exceeds S (i.e. $\alpha < 1$). Notice that the BER in the saturation region can be made arbitrarily small by choosing a sufficiently large L for a given E_b/N_J. However, in practical implementations, the optimum diversity L_{opt} is determined by minimizing the unsaturated expression for the BER (i.e. the second line) in (19). (Please refer to [8] for an extensive discussion of this consideration.) Furthermore, although L clearly should be restricted to integer values, it is more convenient to regard it as a continuous variable and to perform the minimization by the usual technique of differentiating the BER with respect to L and setting the result to zero. Since it can be shown that the corresponding minimum is relatively insensitive to small deviations in L about L_{opt} as in the case of partial-band noise jamming [e.g. 2, 3 , and 8], there is a negligible performance degradation when the continuous parameter L_{opt} is truncated to the nearest integer. Adopting this approach with (19), we find that the performance optimized from both the ECM and ECCM (electronic counter-countermeasure) vantage points is specified by

$$\mathrm{BER} = \exp(-L_{opt}),\ \alpha_{wc} = 1_-,\ L_{opt} = e^{-1}E_b/N_J;\ E_b/N_J \geq e$$

$$\mathrm{BER} = (E_b/N_J)^{-1},\ \alpha_{wc} = 1_-,\ L_{opt} = 1;\ 2 \leq E_b/N_J \leq e \qquad (20)$$

$$\mathrm{BER} = 1/2,\ \alpha_{wc} = E_b/2N_J,\ L_{opt} = 1;\ E_b/N_J < 2$$

Note in (20) that for $E_b/N_J \leq e$ the optimum communication strategy for the given scenario is to have no diversity (i.e. $L_{opt} = 1$), and the saturation region is reached for $E_b/N_J \leq 2$. We have seen before how effective simple time diversity can be for FH/MFSK signals against WC partial-band noise jammers, restoring the desired exponential BER-SNR relationship [2]. Based on (20), Fig. 7 demonstrates that optimum diversity provides the same dramatic performance gains for FH/BFSK (binary FSK) communications in a WC n = 1 band-multitone environment. For example, at BER = 10^{-5}, the improvement relative to L = 1 is 35 dB, and the performance is only 1.6 dB worse than in broadband noise.

Next we consider larger size alphabets (K > 1), still with n = 1 band-multitone jamming. We noted previously that with perfect jamming state side information, an error can only be made if all L diversity chips are jammed, a condition we denoted by the event H. Conditioned on H, which implies that one of the M-1 untransmitted symbols is hit on each of the L diversity hops, the linear sum energy detection metric for the correct symbol has the value LE_c. For an error to occur, one of the other M-1 metrics must exceed this value, which requires that $\alpha < 1$. If we operated under the common belief that each jamming tone must have a received power slightly in excess of S to be effective (i.e. $\alpha = 1_-$), an error could be made only if the same untransmitted symbol was hit on all L hops (see footnote 5 for another case where this condition would be required); the probability of this event could be computed exactly. However, by allowing the jammer the additional freedom to optimize α over (0,1), there are many more error events and the pragmatic approach is to use the union/Chernoff bound technique [2, 3, or 8].

Without loss of generality, suppose symbol 1 is sent, and, for simplicity, assume an error is made if any of the M-1 other metrics <u>equals</u> or exceeds Λ_1. Conditioned on $\underline{H}$, the probability of this occurrence is

$$\begin{aligned} \Pr\left[\bigcup_{i=2}^{M}\left(\Lambda_i > \Lambda_1\right)\Big|\underline{H}\right] &\leq (M-1)\Pr\left[\Lambda_2 - \Lambda_1 \geq 0 \big|\underline{H}\right] \\ &= (M-1)\Pr\left[\sum_{j=1}^{L}(e_{2j}-e_{1j}) \geq 0 \Big|\underline{H}\right] \\ &\leq (M-1)\left(E\left\{\exp[\lambda(e_{2j}-e_{1j})]\big|\underline{H}_j\right\}\right)^L;\ \lambda \geq 0 \end{aligned} \tag{21}$$

The first line of (21) is an application of the union bound, while the third line uses the Chernoff bound with Chernoff parameter λ and recognizes that the e_{ij}'s are identically distributed for $i > 1$. Conditioned on the j^{th} hop being jammed (H_j), the normalized chip energy detector outputs e_{1j} and e_{2j} have the following probabilities:

$$\Pr[e_{1j} = 1 | H_j] = 1$$

$$\Pr[e_{2j} = X | H_j] = \begin{cases} 1/(M-1);\ X = 1/\alpha \\ (M-2)/(M-1);\ X = 0 \end{cases} \tag{22}$$

so that

$$\Pr[e_{2j}-e_{1j} = X|H_j] = \begin{cases} 1/(M-1); & X = 1/\alpha - 1 \\ (M-2)/(M-1); & X = -1 \end{cases} \tag{23}$$

Applying (18), (21), and (23), and the usual relationship between SER and BER, we have

$$\begin{aligned} BER &= [M/2(M-1)] \Pr[\underline{H}] \Pr\left[\bigcup_{i=2}^{M} \left(\Lambda_i > \Lambda_1\right) | \underline{H}\right] \\ &\leq (M/2) \left[(M - 2 + e^{\lambda/\alpha}) \, \alpha L e^{-\lambda} \, (KE_b/N_J)\right]^L \end{aligned} \tag{24}$$

First the Chernoff bound in (24) is tightened by minimizing the bound over $\lambda \geq 0$, yielding the expression

$$BER \leq (M/2)\left\{ \left[(M-2) \, \alpha/(1-\alpha)\right]^{1-\alpha} L/(KE_b/N_J) \right\}^L \tag{25}$$

since $M > 2$, and provided that $\alpha \geq 1/(M-1)$ so that the minimizing $\lambda \geq 0$. The WC $n = 1$ band-multitone jammer wants to maximize this BER over $1/(M-1) \leq \alpha < 1$ subject to $\mu \leq 1$; the resulting performance for arbitrary diversity L has the form [8, (2.90)]

$$\begin{aligned} &BER \leq (M/2)[\beta L/(E_b/N_J)]^L, \quad \alpha_{wc} = \alpha_o; \; E_b/N_J \geq \zeta L \\ &BER \leq (M/2) \left\{\left[(M-2)\alpha_{wc}/(1-\alpha_{wc})\right]^{1-\alpha_{wc}} \alpha_{wc} M\right\}^L, \quad \alpha_{wc} = KE_b/LMN_J; \\ &\qquad\qquad LM/K(M-1) \leq E_b/N_J \leq \zeta L \end{aligned} \tag{26}$$

where α_o, β, and $\zeta \equiv \alpha_o M/K$ are listed in Table 3. The upperbound of (26) is illustrated in Figs. 8-9 for several values of K and L (note the impact of the lower limit on the range of E_b/N_J in Fig. 8). Although combinatorially difficult, an exact performance analysis for FH/MFSK signals (K > 1) with arbitrary diversity in WC n = 1 band-multitone jamming is presented in [8]. It proves that the upperbound results of (26) are pessimistic by several dB for small $L \sim 2$, but are accurate to within about 1/2 dB for $L \gtrsim 10$ [8, Figs. 2.42-2.43].

Just as we observed in the binary (K = 1) signaling case, it can be proved from the second line of (26) that the BER can be made arbitrarily small for sufficiently large amounts of diversity L [8, (2.91)]. However, in practice it is more reasonable to choose the value of L that minimizes the first line of the bound in (26):

$$\mathrm{BER} \leq (M/2)\exp(-L_{opt}), \ \alpha_{wc} = \alpha_o, \ L_{opt} = \delta\, E_b/N_J; \ E_b/N_J \geq \gamma \tag{27}$$

where $\delta = 1/\beta e$ and $\gamma = 1/\delta$ (this lower limit on the range of E_b/N_J ensures that $L_{opt} \geq 1$) are also given in Table 3. (Of course, since the parameters α_{wc} and L_{opt} are based on upperbounds, they should more correctly be labelled quasi-optimum, as argued earlier.) For values of E_b/N_J below γ, $L_{opt} = 1$; in this domain, the BER upperbound of (26) can be used with L = 1, although the exact BER is specified by (8)-(11) in the absence of diversity. The effectiveness of optimum diversity against WC n = 1 band-multitone jamming is shown in Figs. 10-11 for K = 2 and 4. As a benchmark, at BER = 10^{-5}, the improvement relative to L = 1 is approximately 36 dB at K = 2 and 38 dB at K = 4, although the performance is significantly worse than in broadband noise for larger values of K. The exact performance analysis for optimum diversity in [8] shows that the upperbound of (27) is accurate to within about 1/2 dB for $2 \leq K \leq 4$ [8, Fig. 2.46].

The performance of FH/MFSK signals with optimum diversity in WC $n = 1$ band-multitone jamming is summarized in Fig. 12 for $1 \leq K \leq 5$. Note that from the communicator's viewpoint, the best performance is achieved with 4-ary FSK.

We now consider band-multitone jamming for $2 \leq n \leq M$. Recall that all L diversity chips must be jammed (event $\underline{H}$) for a symbol error to be made. Recall further that, by convention, a diversity chip is considered to be jammed only if two or more of the M energy detector outputs are high. For $n = 1$ band-multitone jamming, the expression in (18) for the probability that $\underline{H}$ occurs contains the factor $(M-1)/M$ to delete those situations when the single jamming tone hits the data chip on a given hop; with $n \geq 2$, this factor is no longer needed so that

$$\Pr[H] = \mu^L = \left[\alpha LM / \left(nKE_b/N_J\right)\right]^L \tag{28}$$

We will again use the union/Chernoff approach, which requires detection metric statistics only for the transmitted data (assumed to be symbol 1 without loss of generality) and one of the other M-1 symbols (e.g. symbol 2), and only for a single diversity chip transmission (see (21)). Therefore, we need the statistics of the differenced energy detector output $e_{2j}-e_{1j}$ conditioned on H_j. Referring to Table 1 and incorporating the joint likelihood that either or both symbols 1 and 2 are hit when the M-ary band is jammed, we find that

$$Pr[e_{2j}-e_{1j} = X\ H_j,\ \phi]$$

$$= \begin{cases} \binom{M-2}{n-2} \Big/ \binom{M}{n} = \frac{n(n-1)}{M(M-1)};\ X = -1 - \frac{2}{\sqrt{\alpha}} \cos\phi \\[2ex] \binom{M-2}{n-1} \Big/ \binom{M}{n} = \frac{n(M-n)}{M(M-1)};\ X = -1 - \frac{1}{\alpha} - \frac{2}{\sqrt{\alpha}} \cos\phi \\[2ex] \frac{n(M-n)}{M(M-1)};\ X = \frac{1}{\alpha} - 1 \\[2ex] \binom{M-2}{n} \Big/ \binom{M}{n} = \frac{(M-n)(M-n-1)}{M(M-1)};\ X = -1 \end{cases} \qquad (29)$$

As in (21) and (24), the union/Chernoff BER upperbound has the form

$$\begin{aligned} BER &\leq (M/2)\ Pr[\underline{H}]\ \left(E\ \left\{\exp[\lambda(e_{2j}-e_{1j})] \mid H_j\right\}\right)^L \\ &= (M/2)\ F^L \end{aligned} \qquad (30)$$

where, using (28) and (29) and averaging over the uniformly distributed random phase ϕ,

$$\begin{aligned} F = [\alpha L\ e^{-\lambda}/nK(M-1)(E_b/N_J)]\ &\{n(n-1)\ I_0(2\lambda/\sqrt{\alpha}) \\ &+ n(M-n)\ e^{-\lambda/\alpha}\ [I_0(2\lambda/\sqrt{\alpha}) + 1] + (M-n)(M-n-1)\} \end{aligned} \qquad (31)$$

with $I_0(\cdot)$ denoting the zeroeth order modified Bessel function of the first kind. Because F contains Bessel functions, the minimization of the bound over the Chernoff parameter λ cannot be expressed in closed form. However, using numerical techniques to minimize F over $\lambda \geq 0$ and subsequently to maximize it over $\alpha \in (0,4)$, the performance in WC jamming with diversity L is given by [8, (2.111)]

$$BER \leq (M/2)\ [\beta L/(E_b/N_J)]^L,\ \alpha_{wc} = \alpha_o;\ E_b/N_J \geq \zeta L \tag{32}$$

where α_o, β, and $\zeta = \alpha_o M/nK$ are listed in Table 4 for selected values of K and n. It is noted in [8] that in the saturation region $E_b/N_J < \zeta L$ where $\mu = 1$, $\alpha_{wc} = nKE_b/LMN_J$ and F must be minimized numerically over $\lambda \geq 0$ for <u>each</u> value of E_b/N_J. Consequently, a closed form expression for the BER upperbound does not exist in this region (which is not of practical interest in any case). It is also shown in [8] that the performance in (32) improves as n increases for a given combination of K, L, and E_b/N_J, indicating that it is to the jammer's advantage to keep n small so as to jam the largest num r of M-ary bands.

Minimizing (32) over $L \geq 1$, the performance with optimum diversity has the same form as (27) with different values of δ and γ as shown in Table 4 [8, (2.113)]. Since δ increases monotonically with n for each value of K in Table 4, the implication is that jamming effectiveness against FH/MFSK signals with optimum diversity improves as n becomes smaller.

Finally, we consider independent-multitone jamming. With L-diversity, the probability that a given FH slot is hit by a jamming tone is given by a modified version of (12):

$$\rho = \alpha L/(KE_b/N_J) = \alpha L^* \tag{33}$$

where we have introduced the normalized diversity $L^* = L/(KE_b/N_J)$. With perfect jamming state side information, a symbol error can occur only if at least one of the M-1 untransmitted chips is hit by a jamming tone on each diversity hop. The likelihood of this happening on the j^{th} hop is

$$\Pr[H_j] = 1 - (1-\rho)^{M-1} \equiv \varepsilon \tag{34}$$

Independent of H_j, the transmitted symbol on the j^{th} hop is hit with probability ρ; however, conditioned on H_j, a particular untransmitted symbol on that hop is hit with probability ρ/ε. Referring once again to the normalized energy detector outputs in Table 1, and assuming symbol 1 is sent, we can write

$$\Pr[e_{2j}-e_{1j} = X|H_j, \phi]$$

$$= \begin{cases} \rho^2/\varepsilon; \; X = -1 - 2\cos\phi/\sqrt{\alpha} \\ \rho(1 - \rho/\varepsilon); \; X = -1 - 1/\alpha - 2\cos\phi/\sqrt{\alpha} \\ (1-\rho)\,\rho/\varepsilon; \; X = 1/\alpha - 1 \\ (1-\rho)(1 - \rho/\varepsilon); \; X = -1 \end{cases} \tag{35}$$

Note from (34) that

$$\varepsilon - \rho = (1-\rho)\left[1 - (1-\rho)^{M-2}\right] \tag{36}$$

Then the union/Chernoff BER upperbound has the form of (30) with

$$F = e^{-\lambda}\left[\alpha L^* I_0(2\lambda/\sqrt{\alpha}) + (1 - \alpha L^*)\, e^{\lambda/\alpha}\right] \times \left\{\alpha L^* + e^{-\lambda/\alpha}(1 - \alpha L^*)\left[1 - (1 - \alpha L^*)^{M-2}\right]\right\} \tag{37}$$

Now we want to minimize F over $\lambda \geq 0$ and maximize the result over $\alpha \in (0,4)$; unfortunately, since L^* depends on L, K, and E_b/N_J, the joint optimization must be computed numerically for each combination of these parameters. However, we can derive a closed form expression for the BER upperbound with optimum diversity. First we rewrite (30), replacing L by the normalized diversity L^*:

$$\text{BER} \leq (M/2)\exp\left[-L^*K\ln(1/F)\left(E_b/N_J\right)\right] \tag{38}$$

and determine the WC jamming solution with optimum normalized diversity. Operating with the positive exponential coefficient $L^*K\ln(1/F)$, where F depends on K in (37), we want to maximize this expression over $\lambda \geq 0$, then minimize it over $\alpha \in (0,4)$, and finally maximize it over L^*. Denoting the jointly optimized coefficient by δ, it is argued in [8] that the corresponding performance is specified by

$$\text{BER} \leq (M/2)\exp(-\delta E_b/N_J),\ L_{opt} = L^*_{opt}KE_b/N_J;\ E_b/N_J \geq \gamma \tag{39}$$

where α_{wc}, L^*_{opt}, δ, and $\gamma \equiv 1/L^*_{opt}K$ are listed in Table 5. Note that the WC independent-multitone jammer is not uniquely defined for K = 1: [8] examines the details behind this observation.

As with all of the other multitone jamming strategies, optimum diversity is extremely effective at combatting WC independent-multitone jamming; this is illustrated in Fig. 13 for K = 3. The best asymptotic performance for small BERs is achieved with K = 3, which has the largest value of δ in Table 5; this is graphically underscored in Fig. 14.

V. Overview of Multitone Jamming Effectiveness

Having been immersed in the analytical details of the performance of FH/MFSK communications in the presence of a variety of multitone jammers, let us now step back and compare the bottom-line effectiveness of these ECM strategies along with partial-band noise. For sufficiently large SNRs, the performance upperbounds for all of these WC jamming/optimum diversity scenarios have the generic asymptotic form

$$\text{BER} \leq \begin{cases} (M/4)\exp(-\delta E_b/N_J); & \text{noise jamming [2, (16) and (17)]} \\ \\ (M/2)\exp(-\delta E_b/N_J); & \text{multitone jamming} \end{cases} \tag{40}$$

where δ is enumerated in Table 6. With the reminders that the conclusions that follow are based on exponentially tight bounds rather than exact calculations, that they assume a noncoherent chip detection metric with linear combining and perfect jamming state side information, and that we regard a smaller value of δ as a measure of superior jamming effectiveness, it would appear that the WC n = 1 band-multitone jammer is the best (nonadaptive) ECM strategy against FH/MFSK signals with optimum diversity, at least for $K \geq 2$. (By nonadaptive, we are excluding repeat-back and similar classes of jammers that base their responses on real-time intercepted measurements of their target signals.) Even in the $K = 1$ binary signaling case, since the $n = 1$ band-multitone coefficient δ is based on an exact performance calculation while the partial-band noise counterpart is pessimistically low due to the union/Chernoff bound, it is conceivable that the multitone scheme may actually be the winner.

Although we saw that, in the absence of diversity, independent-multitone jamming is asymptotically equivalent to $n = 1$ band-multitone jamming, Table 6 shows that this equivalence disappears with the addition of optimum diversity. This dichotomy is related to the assumption of perfect jamming state information which forces the independent-multitone jammer to use a larger value of ρ to try to jam all L diversity chips of a given data transmission [8, (2.127) and accompanying discussion]. Table 6 also reiterates the relative impotence of the $n = M$ band-multitone structure.

Figure 15 is a graphical illustration of Table 6 for $K = 3$, which is representative of the relative effectiveness of these jammers for other alphabet sizes.

VI. Conclusions

We have analyzed the performance of FH/MFSK signals, with and without diversity, in a variety of multitone jamming environments, and we have observed that, at least for a receiver that can derive perfect jamming side information, the class of $n = 1$ band-multitone jammers is superior to all other nonadaptive ECM strategies, including partial-band noise. Although we did not consider other detection metrics in this paper, the analytical techniques presented can be readily applied to many others of practical interest; our intention was to be instructive rather than exhaustive.

We have seen that diversity transmission can dramatically reduce the effectiveness of WC jammers, restoring the exponential relationship between the BER and the SNR. Yet, time diversity is only a simple repetition code, and there are many block and convolutional codes that are much more powerful. The interested reader is referred to [4], which examines the performance of FH/MFSK modulation with a variety of channel codes in WC partial-band noise, and [8], which considers WC multitone jammers as well.

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TABLE 1

NORMALIZED ENERGY DETECTOR OUTPUTS FOR FH/MFSK SIGNALS
IN MULTITONE JAMMING

	Normalized Energy Detector Outputs	
	If Tone Jammed	If not Jammed
Transmitted M-ary Symbol	$1 + 2\cos\phi/\sqrt{\alpha} + 1/\alpha$	1
Any of the M-1 Other Symbols	$1/\alpha$	0

TABLE 2

LOWER LIMIT ON SNR IN (15)

K	γ, dB
1	1.54
2	0.45
3	0.54
4	1.26

TABLE 3

PARAMETERS ASSOCIATED WITH PERFORMANCE UPPERBOUNDS OF (26) AND (27)
FOR FH/MFSK SIGNALS WITH DIVERSITY IN WORST CASE N = 1
BAND MULTITONE JAMMING

K	α_0	β	ζ	δ	γ, dB
2	.683	.7945	1.366	.4631	3.34
3	.527	.8188	1.405	.4493	3.48
4	.427	.9583	1.708	.3839	4.16
5	.356	1.2204	2.278	.3014	5.21

TABLE 4

PARAMETERS ASSOCIATED WITH PERFORMANCE UPPERBOUNDS OF (27) AND (32) FOR FH/MFSK SIGNALS WITH DIVERSITY IN WORST CASE BAND-MULTITONE JAMMING WITH $n \in [2,M]$ TONES PER JAMMED M-ARY BAND

K	n	α_0	β	ζ	δ	γ, dB
1	2	2.395	1.1381	2.395	0.3232	4.91
	2	1.072	0.6305	1.072	0.5835	2.34
2	3	1.745	0.5784	1.163	0.6361	1.96
	4	2.395	0.5691	1.197	0.6465	1.89
	2	0.701	0.5767	0.935	0.6379	1.95
	3	0.898	0.4723	0.798	0.7790	1.08
	4	1.169	0.4237	0.779	0.8682	0.61
3	5	1.488	0.4009	0.794	0.9177	0.37
	6	1.804	0.3894	0.802	0.9446	0.25
	7	2.106	0.3832	0.802	0.9601	0.18
	8	2.394	0.3794	0.798	0.9697	0.13
	2	0.535	0.6354	1.070	0.5790	2.37
	3	0.625	0.5023	0.833	0.7324	1.35
	4	0.716	0.4297	0.716	0.8560	0.68
	5	0.816	0.3844	0.653	0.9571	0.19
4	6	0.931	0.3541	0.621	1.0388	-0.17
	7	1.064	0.3335	0.608	1.1031	-0.43
	8	1.213	0.3193	0.607	1.1523	-0.62
	12	1.827	0.2933	0.609	1.2545	-0.98
	16	2.394	0.2845	0.598	1.2929	-1.12
5	2	0.430	0.7771	1.376	0.4734	3.25
	32	2.395	0.2276	0.479	1.6162	-2.08

TABLE 5

PARAMETERS ASSOCIATED WITH PERFORMANCE UPPERBOUND OF (39) FOR FH/MFSK SIGNALS WITH OPTIMUM DIVERSITY IN WORST CASE INDEPENDENT-MULTITONE JAMMING

K	α_{wc}	L^*_{opt}	δ	γ, dB
1	1.283 or 2.552	.291	.3679	5.36
2	0.793	.354	.5495	1.50
3	0.537	.282	.5760	0.73
4	0.395	.213	.5243	0.70
5	0.298	.158	.4379	1.02

TABLE 6

OVERVIEW OF BER UPPERBOUND EXPONENTIAL COEFFICIENTS FOR FH/MFSK SIGNALS WITH OPTIMUM DIVERSITY IN DIFFERENT TYPES OF WORST CASE JAMMING

Type of Jammer	BER Bound Exponential Coefficient δ				
	K = 1	K = 2	K = 3	K = 4	K = 5
Broadband Noise	.5000	1.0000	1.5000	2.0000	2.5000
Partial-Band Noise	.2500	.5000	.7500	1.0000	1.2500
Independent-Multitone	.3679	.5495	.5760	.5242	.4379
n=1 Band-Multitone	.3679*	.4631	.4493	.3839	.3014
n=2 Band-Multitone	.3232	.5835	.6379	.5790	.4734
n=M Band-Multitone	.3232	.6465	.9697	1.2929	1.6162

* coefficient for exact BER

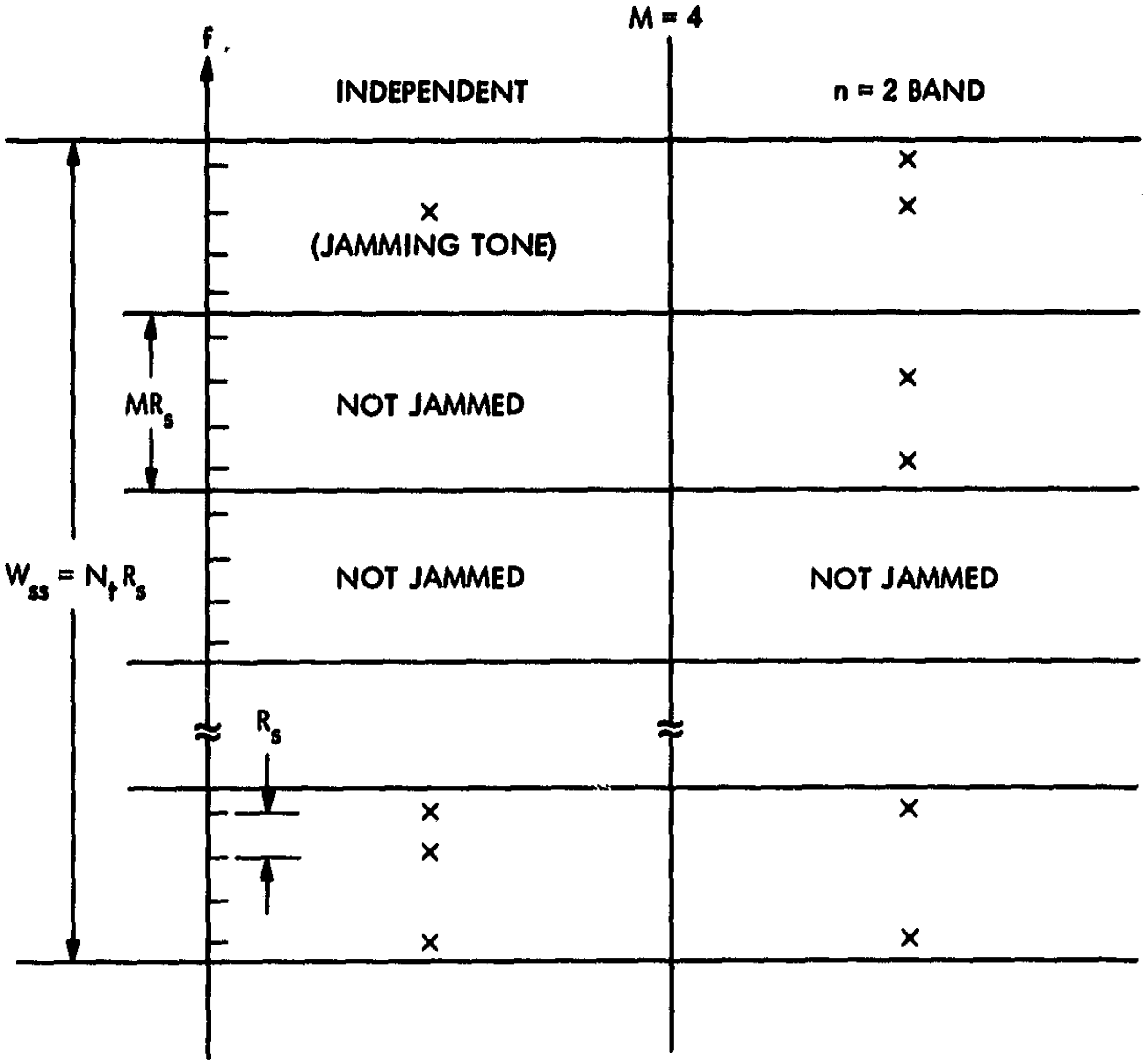

Fig. 1. Multitone jamming strategies: the "independent-multitone" scheme pseudorandomly distributes the jamming tones uniformly over the N_t available FH slots within the spread-spectrum bandwidth W_{ss}; the "band-multitone" structure places exactly n tones (illustrated above for n = 2) in each jammed M-ary band (shown for M=4) of bandwidth MR_s, where R_s is the M-ary symbol rate.

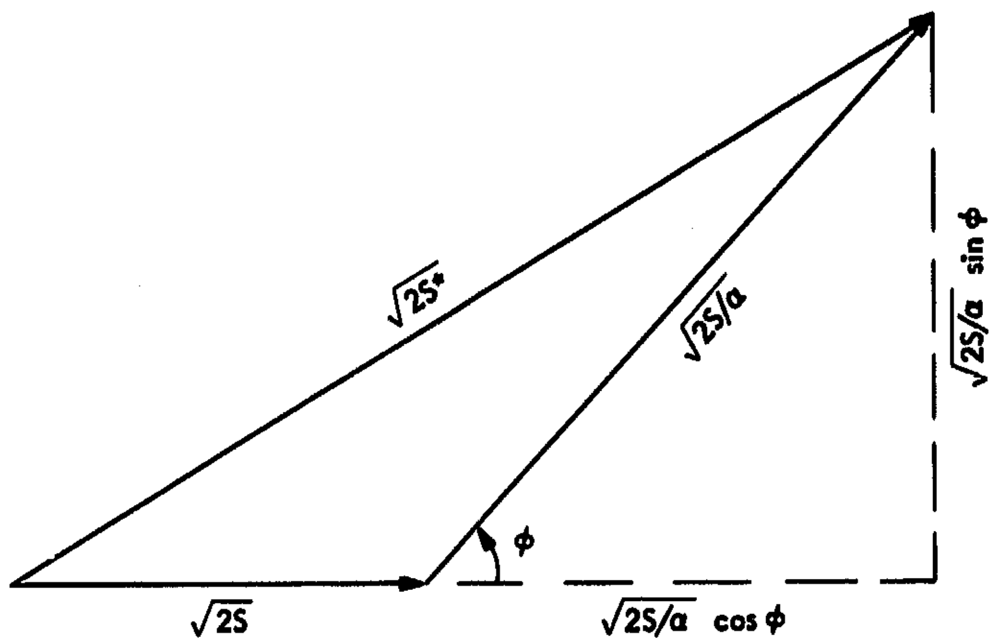

Fig. 2. Phasor representation of the situation in which an FH/MFSK data signal with received power S is hit by a jamming tone with received power S/α and uniformly distributed relative phase ϕ, producing a resultant CW signal with power S^*.

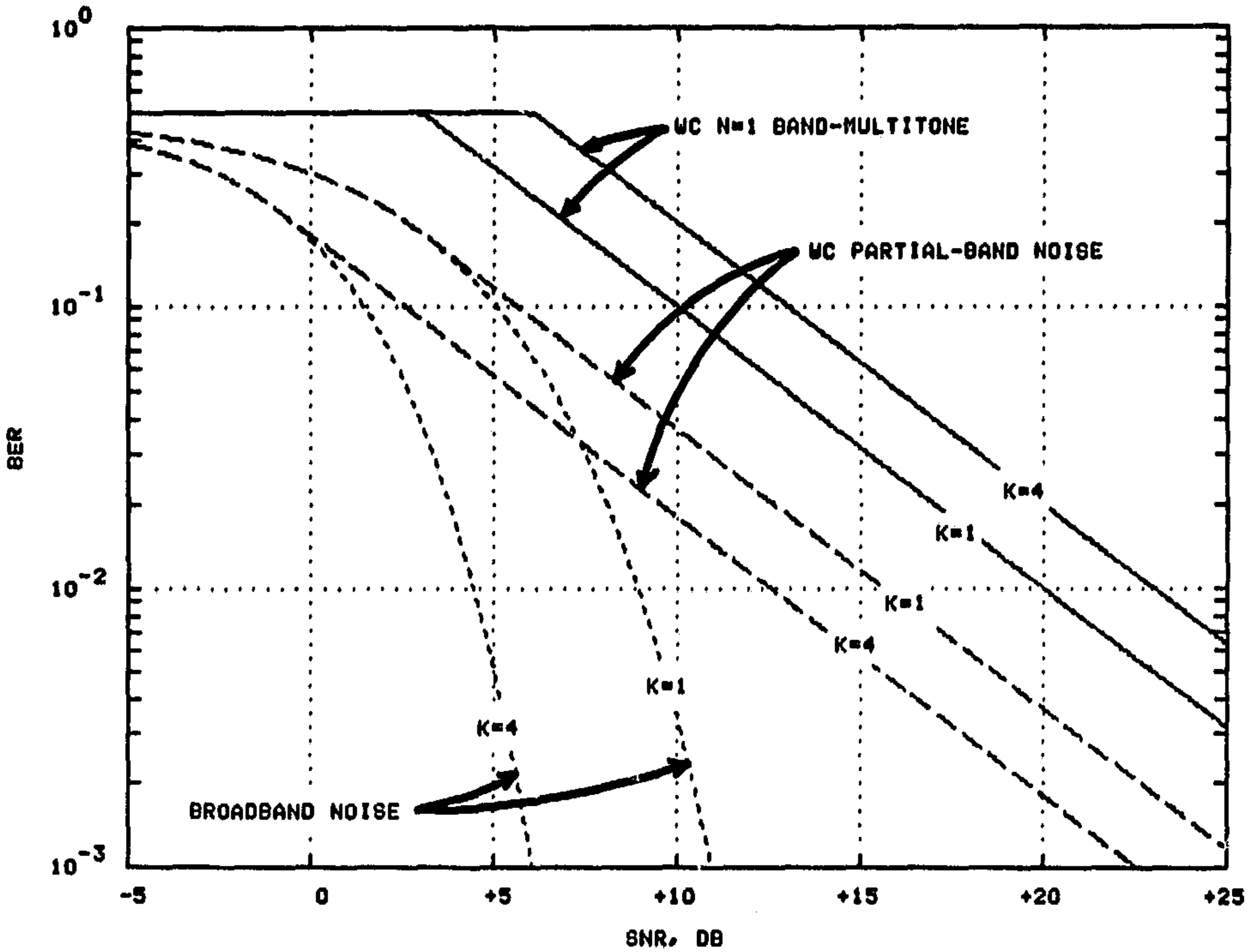

Fig. 3. Advantage of worst case (WC) n = 1 band-multitone strategy over WC partial-band noise and broadband jamming of uncoded FH/MFSK signals.

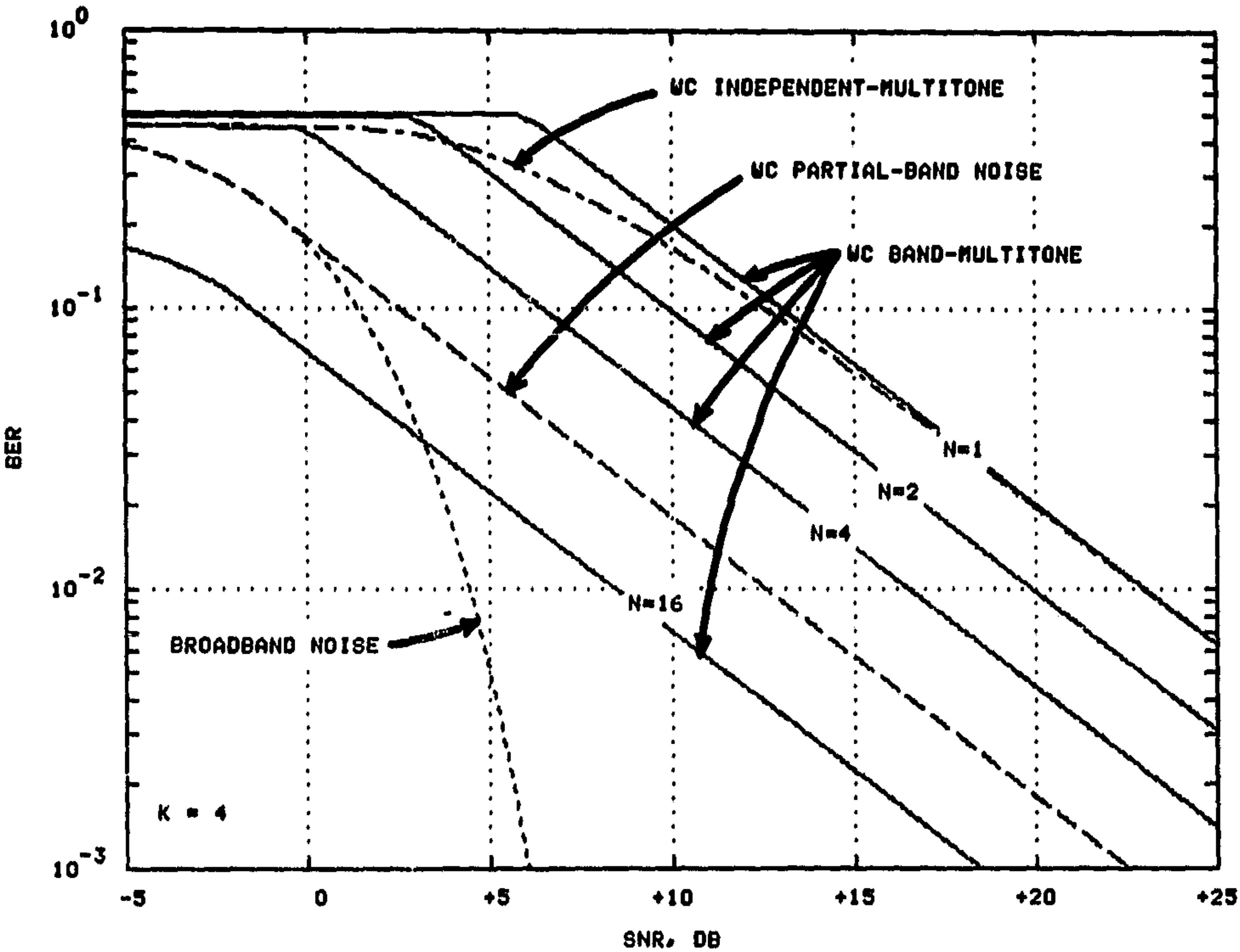

Fig. 4. Comparison of relative effectiveness of various WC jamming strategies against uncoded, noncoherently detected FH/MFSK signals (illustrated for M = 16 or $K = \log_2 M = 4$).

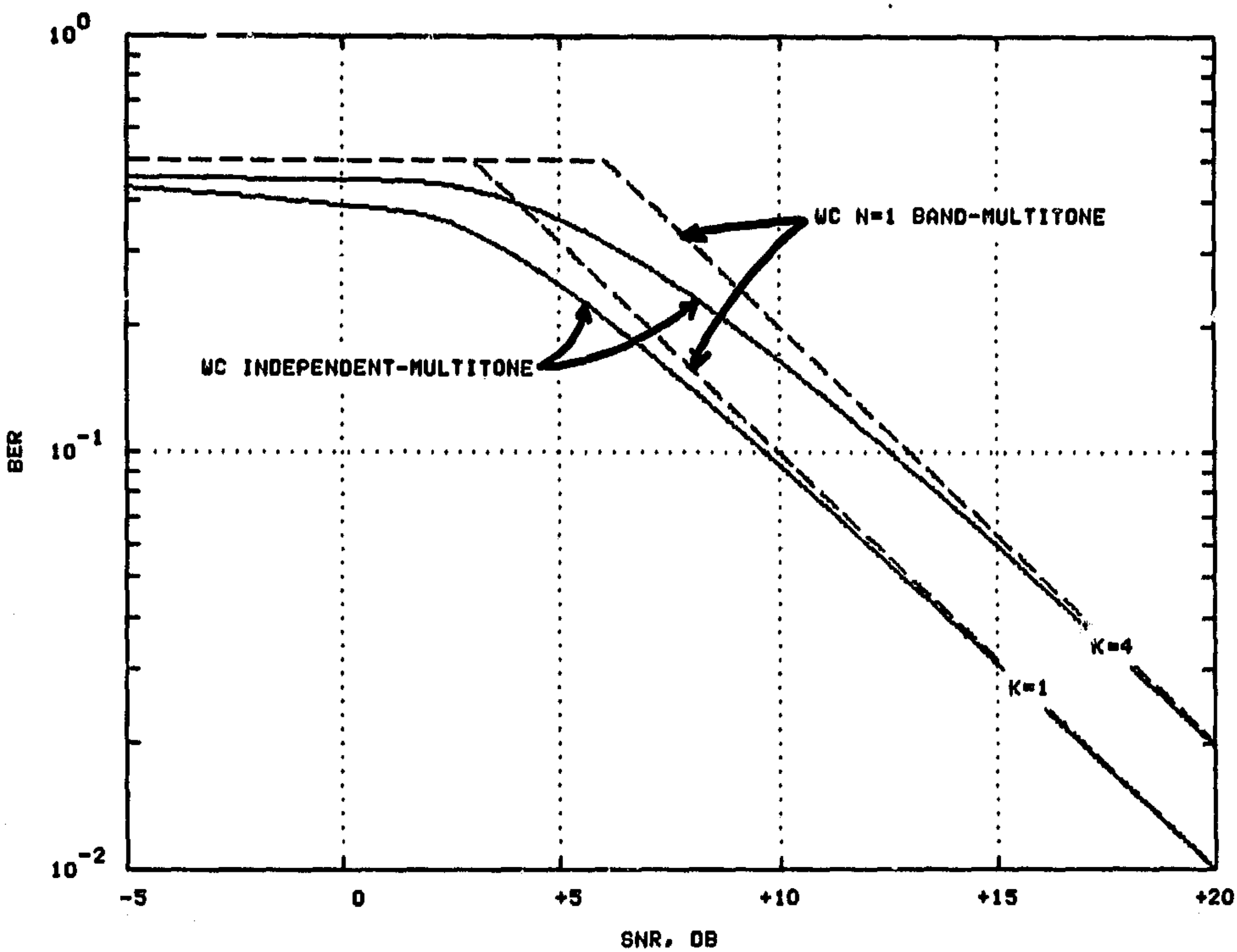

Fig. 5. Comparison of WC independent- and n = 1 band-multitone jamming strategies against uncoded FH/MFSK signals; the two schemes are equally effective for high SNRs (low BERs), where the SNR above is the ratio of the received bit energy E_b to the effective jamming noise power spectral density N_J.

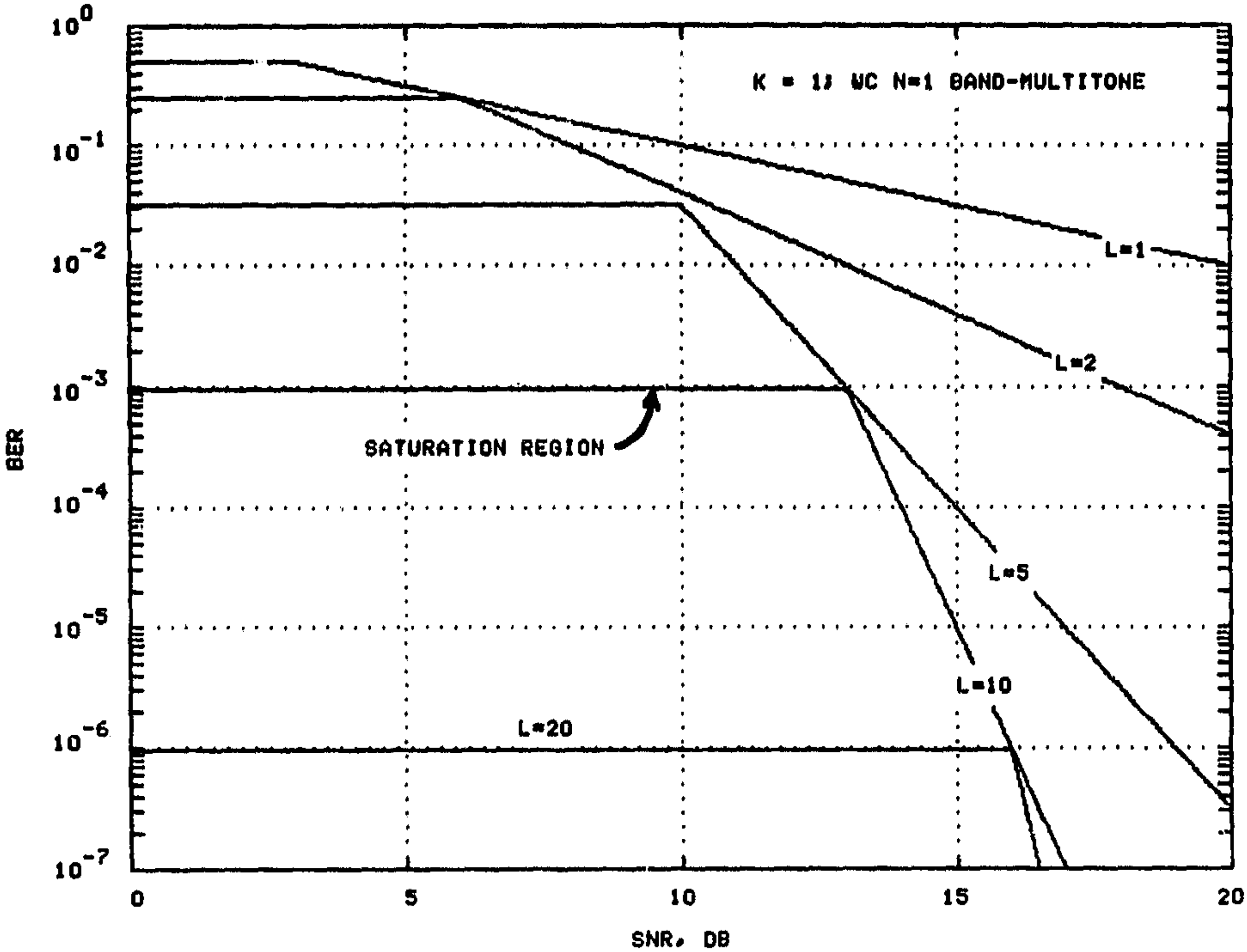

Fig. 6. Performance of FH/BFSK signals with diversity L chips/bit in WC n = 1 band-multitone jamming. In the (horizontal) "saturation regions", <u>every</u> binary band contains its quota of one jamming tone, and the probability μ that a band is jammed is precisely 1.

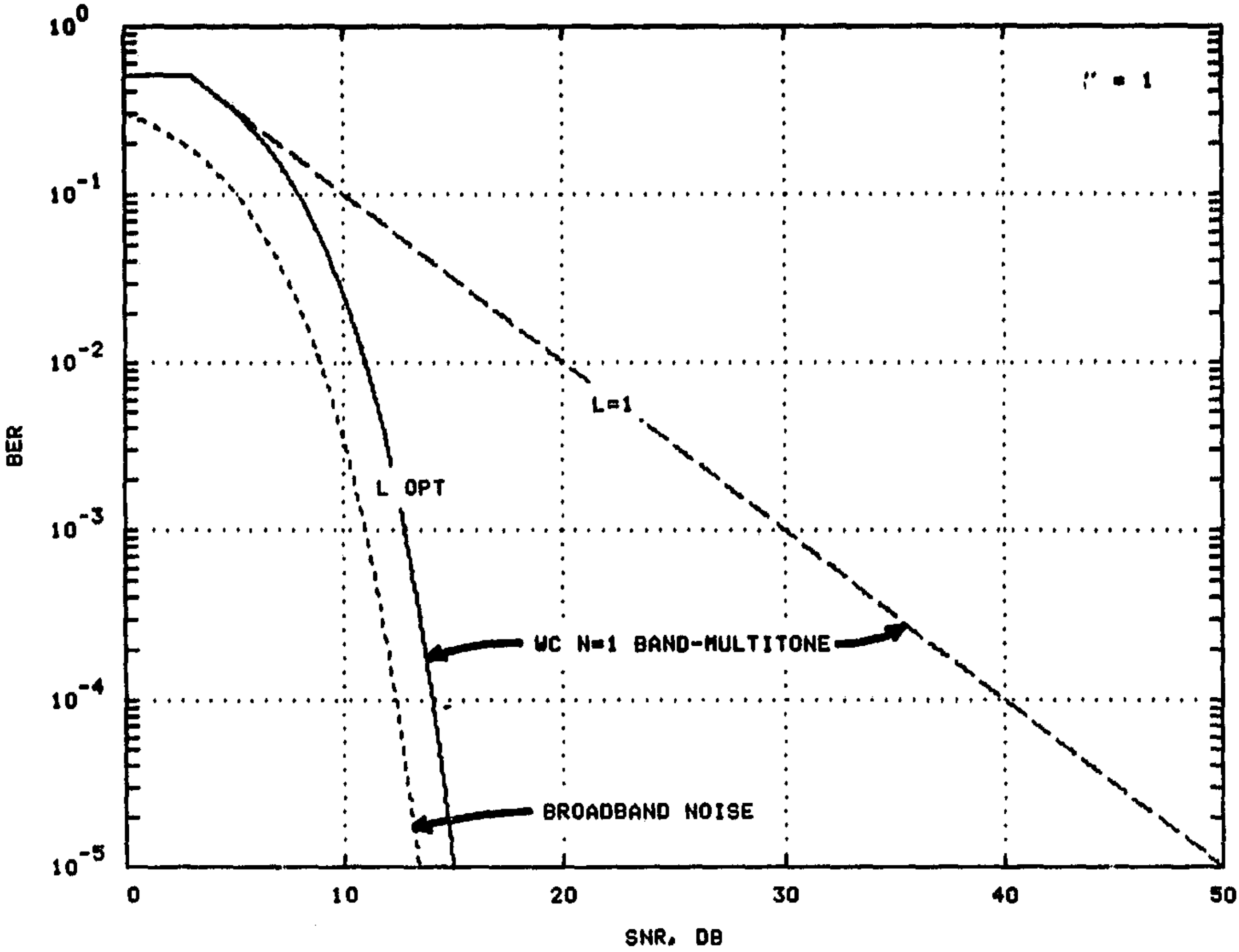

Fig. 7. Effectiveness of optimum diversity (L_{opt}) against WC n = 1 band-multitone jamming for FH/BFSK signaling. For example, at BER = 10^{-5}, improvement relative to no-diversity (L = 1) system is 35 dB, while performance is only 1.6 dB worse than in broadband noise.

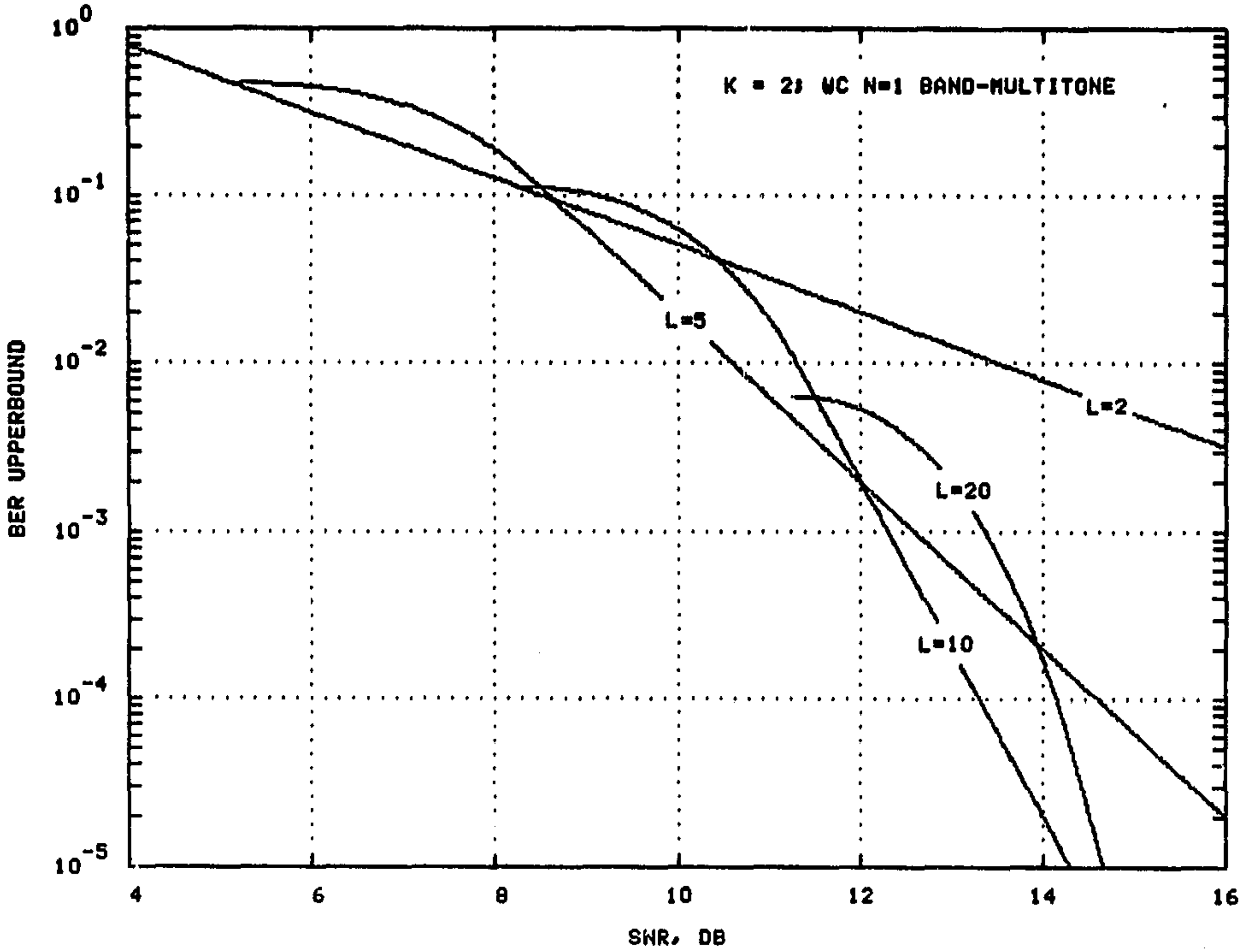

Fig. 8. Performance upperbounds for FH/4-ary FSK signals with diversity L chips/4-ary symbol in WC n = 1 band-multitone jamming.

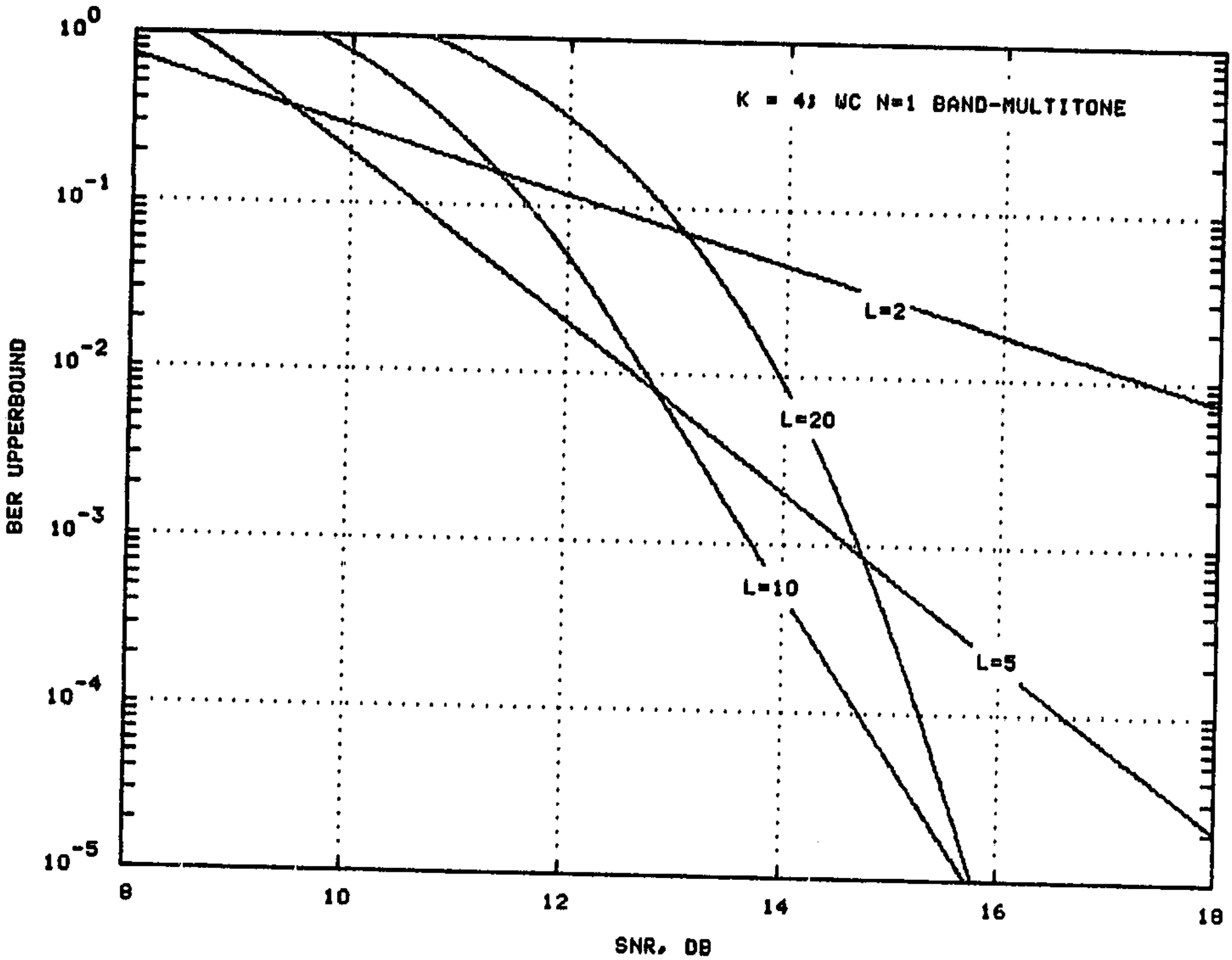

Fig. 9. Same as Fig. 8, except for FH/16-ary signals.

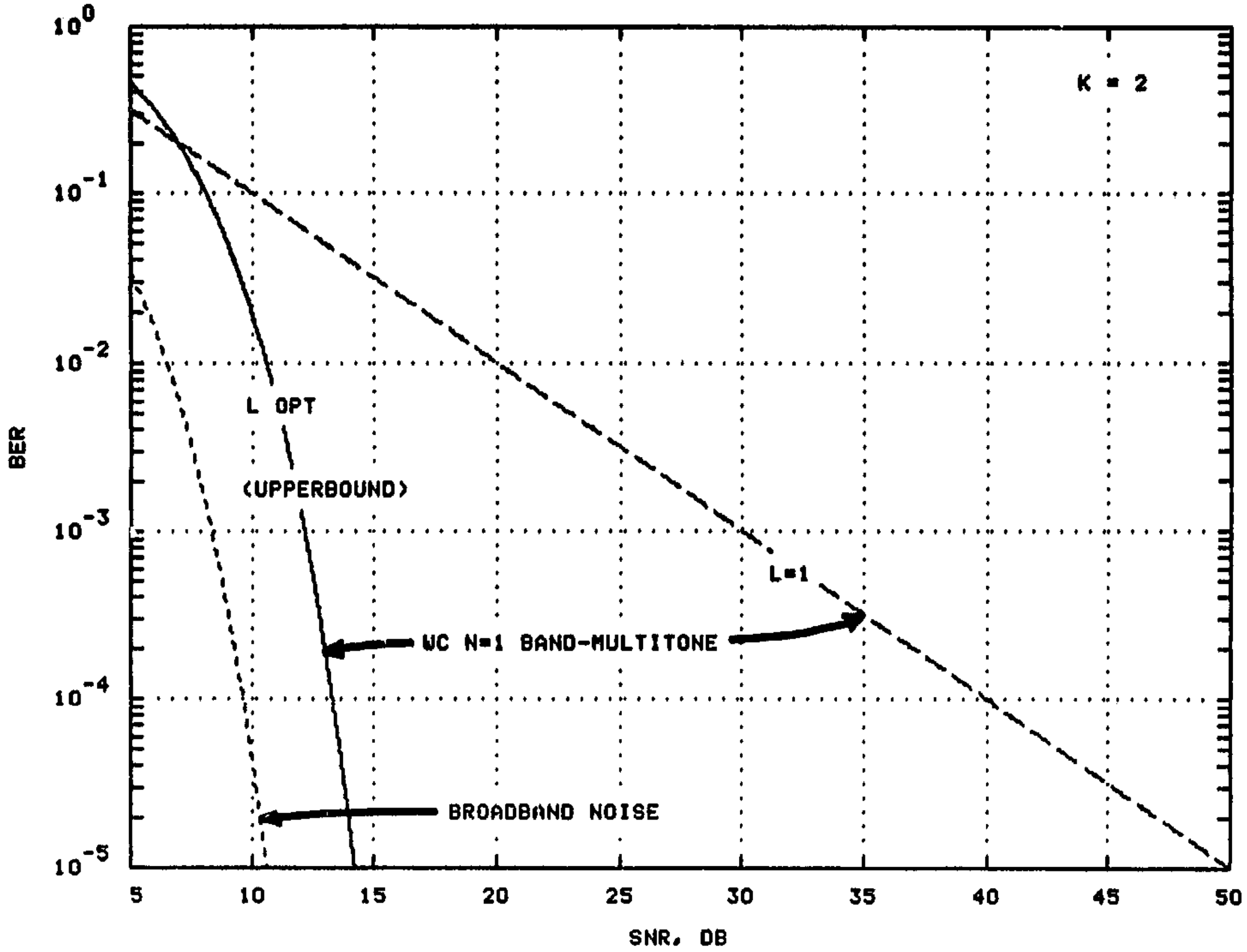

Fig. 10. Effectiveness of (quasi-) optimum diversity (BER upperbound labelled L_{opt} above) against WC n = 1 band-multitone jamming for FH/4-ary FSK signals. At the benchmark BER of 10^{-5}, the gain relative to L = 1 is at least 35.8 dB (since we are comparing a BER upperbound at L_{opt} with an exact BER at L = 1), and the performance is degraded less than 3.6 dB relative to broadband noise.

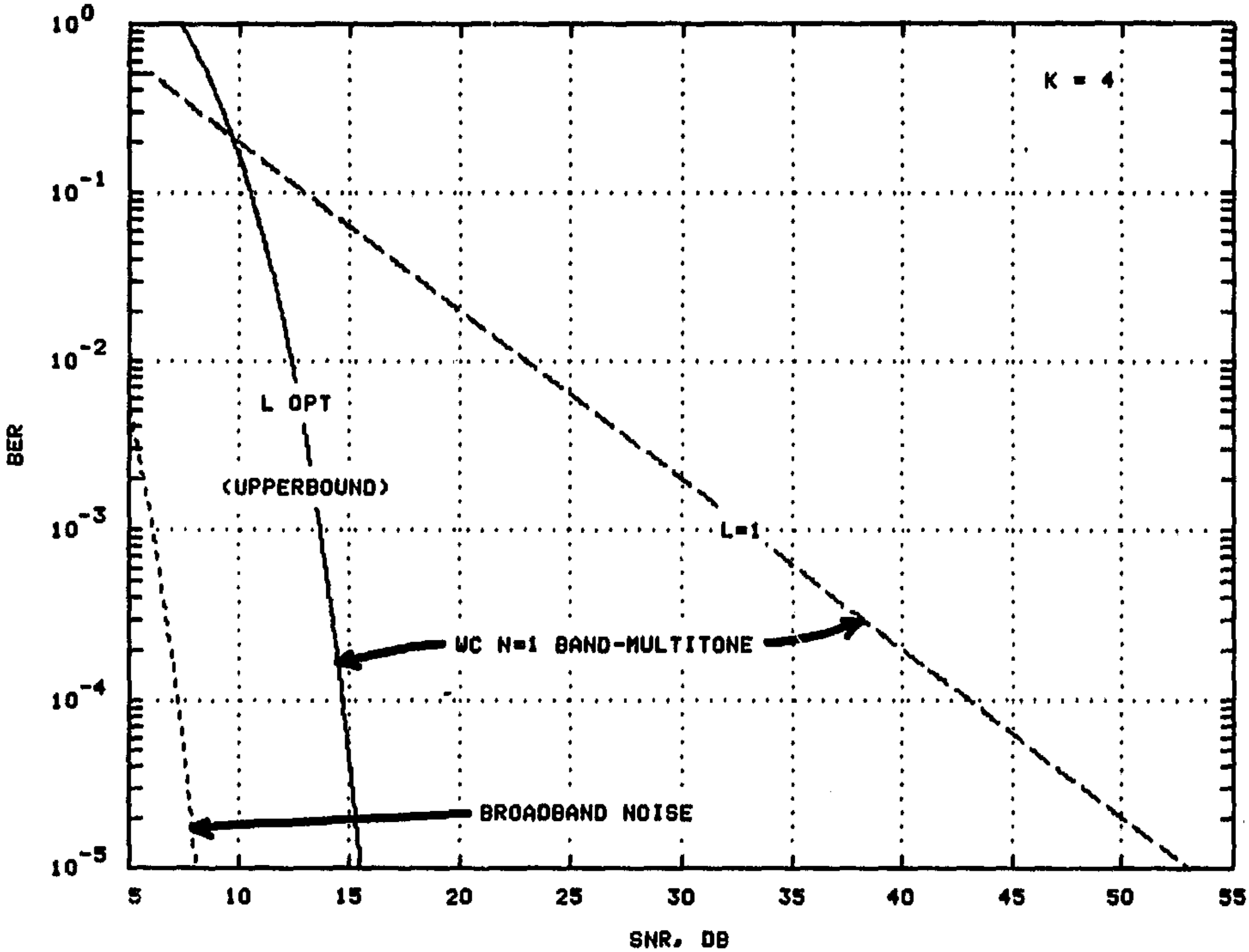

Fig. 11. Same as Fig. 10, but for FH/16-ary FSK signals. Optimum diversity provides at least 37.5 dB performance improvement over L = 1 implementation, and is no more than 7.4 dB worse than in broadband noise at BER = 10^{-5}.

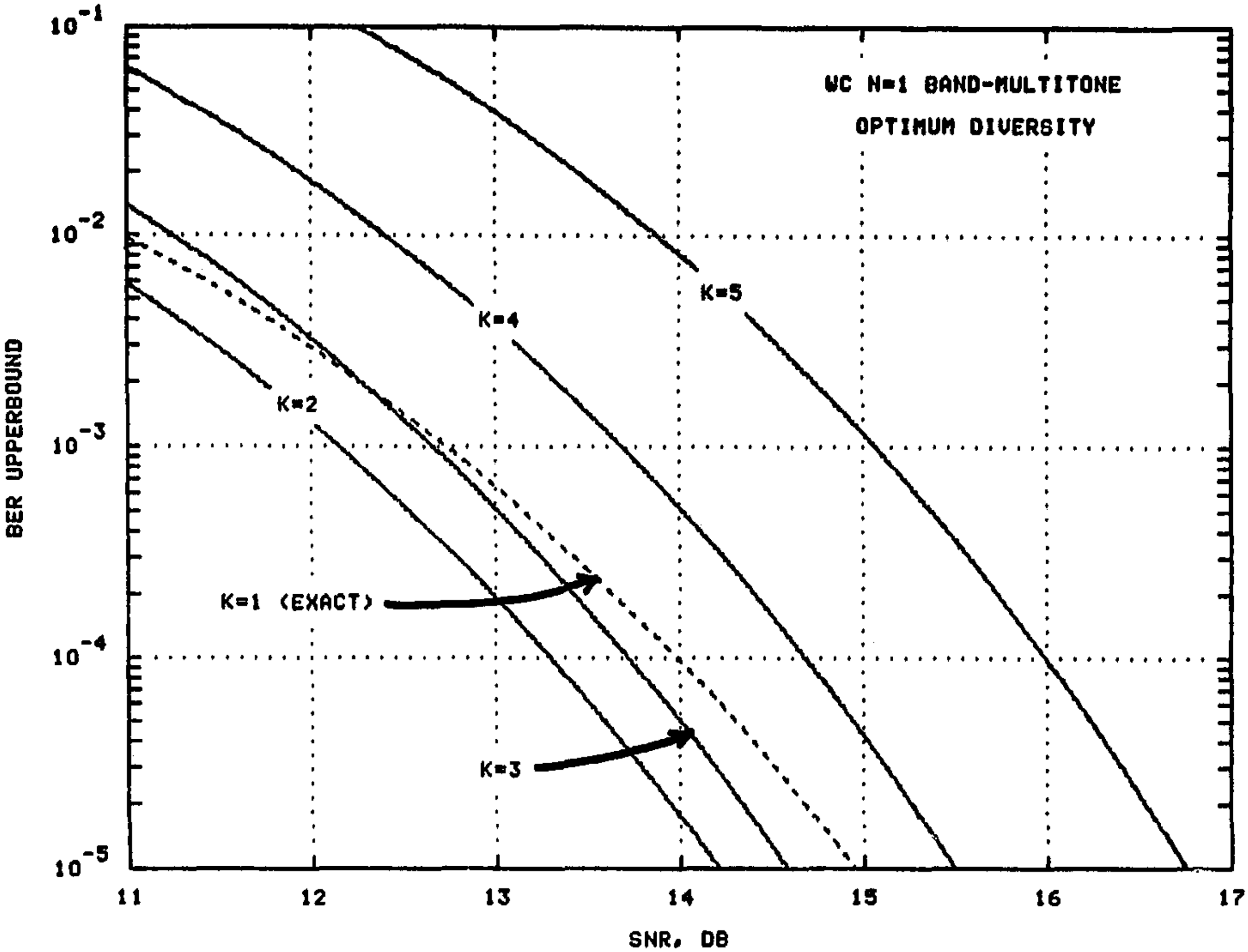

Fig. 12. Overview of performance upperbounds for FH/MFSK signals with optimum diversity in WC n = 1 band-multitone jamming (BER for $K = \log_2 M = 1$ case is exact). Note that the best performance is achieved for K = 2.

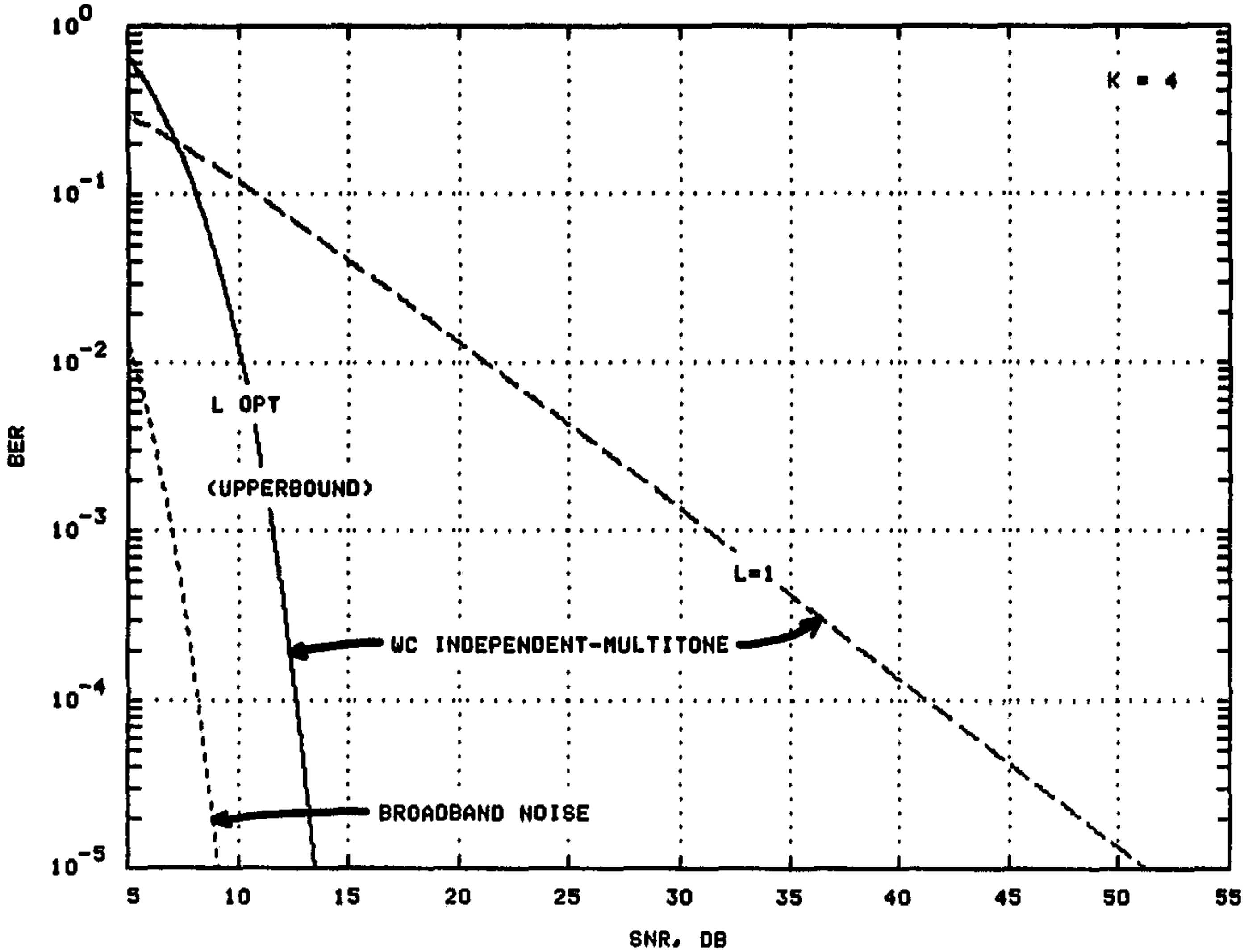

Fig. 13. Effectiveness of diversity against WC independent-multitone jamming for FH/8-ary FSK signals. Improvement with optimum diversity L_{opt} relative to L = 1 exceeds 37.8 dB, and performance is degraded less than 4.4 dB relative to broadband noise jamming, at BER = 10^{-5}.

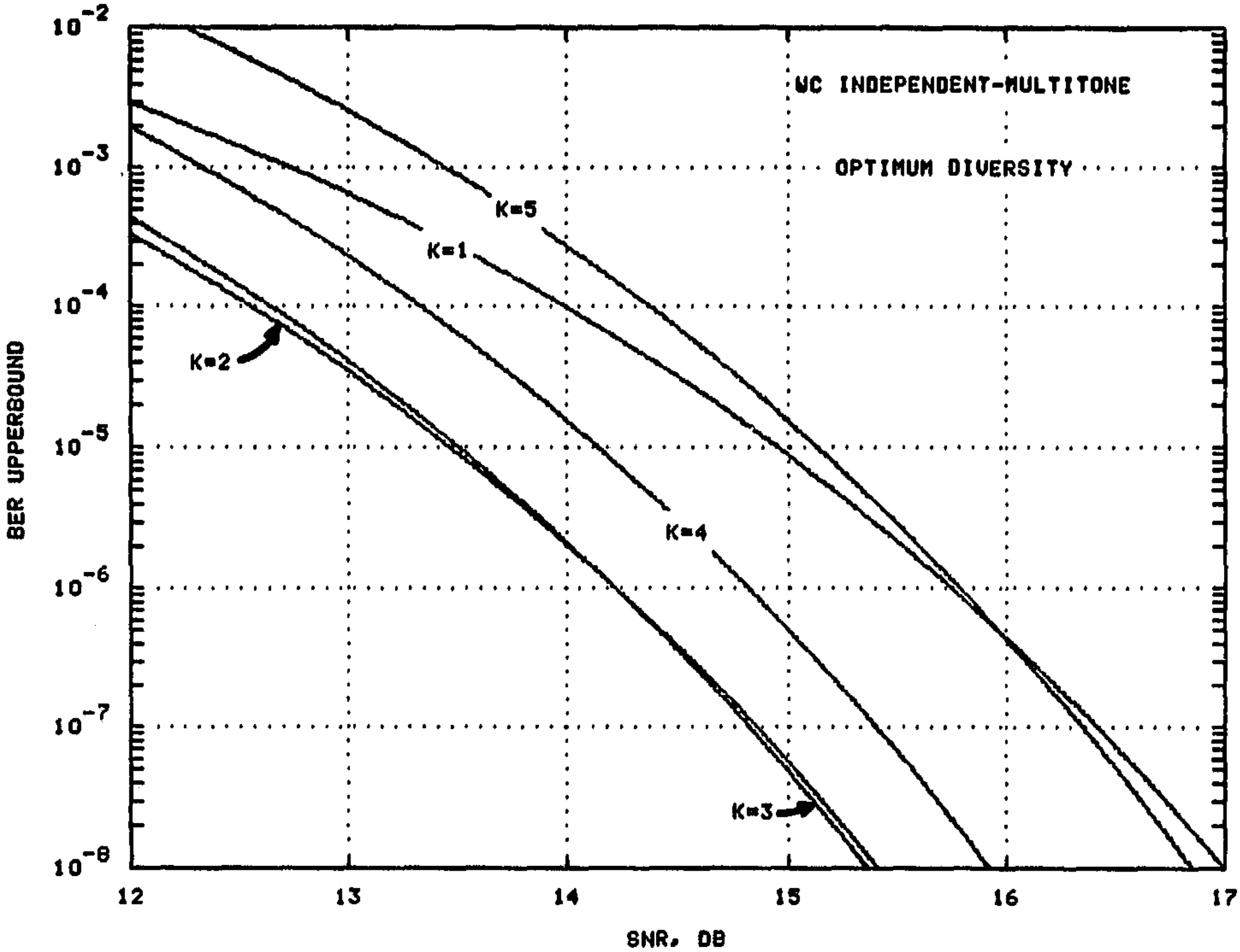

Fig. 14. Comparison of performance of FH/MFSK signals with optimum diversity in WC independent-multitone jamming as a function of $K = \log_2 M$. Best asymptotic performance for small BERs (i.e., large E_b/N_J) is achieved with $K = 8$.

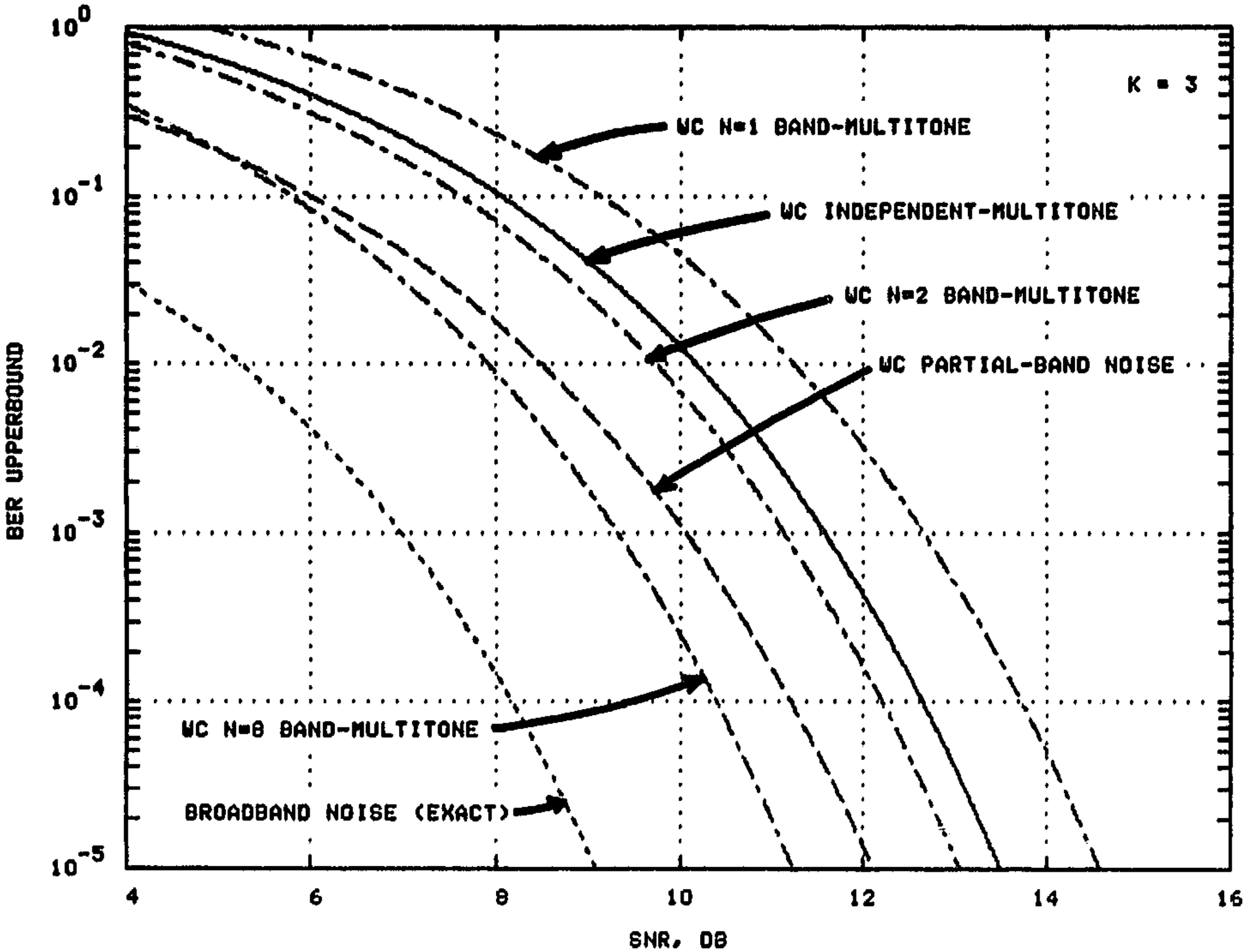

Fig. 15. Relative performance of FH/8-ary FSK signals with optimum diversity in various WC jamming environments. It should be noted that the optimum diversity for a given SNR varies with the type of jamming.

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