

# Fiber Bragg grating sensor for simultaneous measurement of displacement and temperature

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A new approach to measuring displacement and temperature simultaneously by use of a specially designed isosceles triangular cantilevered beam as a strain agent is demonstrated. A fiber Bragg grating epoxied onto the beam surface is experimentally demonstrated to have a temperature sensitivity of  $\sim 0.113$  nm/°C below 60 °C and a displacement sensitivity of  $9.24 \times 10^{-2}$  nm/mm. © 2000 Optical Society of America

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Sensors based on fiber Bragg gratings (FBG's) have attracted considerable interest because of their wavelength-encoded operation and suitability for multiplexed and distributed applications. FBG sensors exhibit many other distinguishing features, such as immunity from electromagnetic interference, suitability for remote measurement, small size, and high resolution. Strain and temperature sensors based on FBG's have been widely reported.<sup>1,2</sup> FBG's are sensitive to both strain and temperature, and a concern associated with the use of FBG sensors for strain measurement is removal of the thermal effect. In applications in which the primary measurand is displacement, the thermal effect can interfere with the measurement results. Therefore, developing a technique for monitoring temperature while measuring displacement is important. Many techniques for measuring strain and temperature simultaneously have been reported.<sup>3-6</sup> Most of those methods used two or more gratings with different strain and temperature characteristics in the sensor heads and calculated both quantities from the different wavelength shifts of the gratings. Those techniques involved writing FBG's that were superimposed in the same section of a fiber with widely different Bragg wavelengths<sup>3,4,6</sup> and required the use of at least two phase masks. Phase masks are quite expensive; furthermore, exposing the bare fibers to UV light for long periods will greatly weaken the fibers' mechanical strength. Therefore it is highly desirable to use a single FBG to make independent strain and temperature measurements. One reported technique<sup>5</sup> used a single FBG written at the spliced joint formed between two dissimilar fibers. This technique had the disadvantage of introducing large splice loss to the sensor and created two reflection peaks of different intensities.

In this Letter we demonstrate a simple approach to measuring displacement and temperature simultaneously by using a specially designed cantilevered beam. The beam has the shape of an isosceles triangle and consists of two sections of different uniform thicknesses. A FBG sensing element is epoxied onto the beam along its axis, crossing the line between the two sections but on the flat surface.

A uniform isosceles cantilevered beam can be used as a uniform-strain agent.<sup>7</sup> The axial strain  $\epsilon_x$  of the beam is given by

$$\epsilon_x = \frac{6LF}{Ech^2}, \quad (1)$$

where  $h$ ,  $L$ , and  $c$  are the thickness, the length, and the base length of the beam, respectively,  $F$  is the force applied vertically to the beam surface at the free end Q, and  $E$  is the Young modulus of the beam material.

The beam shown in Fig. 1 has two sections, with thicknesses  $h_l$  and  $h_r$ . If a FBG is epoxied axially across the sudden change in thickness  $AB$  on the top surface of the beam, the Bragg wavelengths of both grating sections are affected by strain induced by temperature and deflection. Assuming that the weight of the beam and the influence of the deflection at point Q on the bending moment at position  $x$  can be neglected, we can express the fractional change in the Bragg wavelength as<sup>8</sup>

$$\frac{\Delta\lambda_{Bl}}{\lambda_B} = 2 \frac{6F(1 - P_e)(a + b)}{Ech_l^2} + (\alpha + \xi)\Delta T \quad (x < 0), \quad (2a)$$

$$\frac{\Delta\lambda_{Br}}{\lambda_B} = \frac{6F(1 - P_e)(a + b)}{Ech_r^2} + (\alpha + \xi)\Delta T \quad (x > 0), \quad (2b)$$

where  $P_e$  is the effective photoelastic constant,  $\lambda_{Bl}$  and  $\lambda_{Br}$  are the Bragg wavelengths of the left and the right

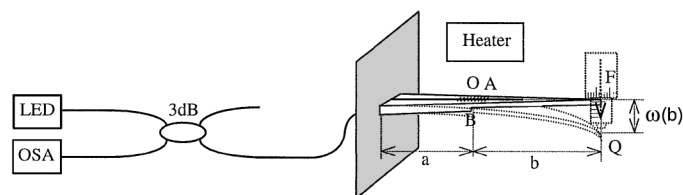


Fig. 1. Experimental setup for simultaneous measurement of displacement and temperature: LED, light-emitting diode; OSA, optical spectrum analyzer.

grating sections, respectively, and  $\Delta T = T - T_0$  is the temperature variation.  $\alpha$  and  $\xi$  are the linear expansion and thermo-optic coefficients, respectively. The relationship between  $F$  and the deflection at the free end,  $\omega(b)$ , can be expressed as

$$F = \frac{Ec\omega(b)}{6(a+b)\left(\frac{b^2}{h_r^3} + \frac{2ab}{h_l^3} + \frac{a^2}{h_l^3}\right)}. \quad (3)$$

Defining  $\Delta\lambda_{\text{BrI}}$  as  $\lambda_{\text{Br}} - \lambda_{\text{Bl}}$  and combining Eqs. (2) and (3), we get

$$\begin{bmatrix} \Delta\lambda_{\text{BrI}} \\ \Delta\lambda_{\text{Br}} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} \omega(b) \\ \Delta T \end{bmatrix}, \quad (4)$$

where

$$K_{11} = \frac{(1 - P_e)\left(\frac{1}{h_r^2} - \frac{1}{h_l^2}\right)\lambda_B}{\frac{b^2}{h_r^3} + \frac{2ab}{h_l^3} + \frac{a^2}{h_l^3}},$$

$$K_{12} = 0,$$

$$K_{21} = \frac{(1 - P_e)\lambda_B}{h_r^2\left(\frac{b^2}{h_r^3} + \frac{2ab}{h_l^3} + \frac{a^2}{h_l^3}\right)},$$

$$K_{22} = (\alpha + \xi)\lambda_B.$$

Equation (4) shows that the wavelength shifts of the epoxied grating sections have different responses to deflection-induced strain but the same responses to temperature change. Therefore the wavelength difference is insensitive to temperature variation and is affected only by deflection. Taking  $\omega(b)$  as the displacement to be measured, we can obviously determine the displacement by motoring  $\Delta\lambda_{\text{BrI}}$ . The temperature effects are incorporated into  $\Delta\lambda_{\text{Br}}$  (or  $\Delta\lambda_{\text{Bl}}$ ). Therefore by measuring the wavelength shift of each grating section individually one can determine the displacement and the temperature simultaneously by taking the inverse of the coefficient matrix.

The experimental setup for the proposed sensor system is shown in Fig. 1. A 10-mm-long FBG fabricated by the side-writing technique had a reflectivity of 95% and a spectral width of 0.46 nm. Its center was located near the place where the thickness changes suddenly. The central wavelength of the grating was 1548.25 nm at  $T_0 = 18^\circ\text{C}$ . A light-emitting diode with a center wavelength of 1525.2 nm and a bandwidth of 73 nm was used to illuminate the grating via a 3-dB coupler. The beam was made from Perspex and had the dimensions  $a = 30$  mm,  $b = 82.5$  mm,  $c = 12$  mm,  $h_r = 1.2$  mm, and  $h_l = 1.8$  mm. It was deflected at free end Q with a micrometer driver to provide positive-negative displacement.

With these beam dimensions and using the value of 0.22 for  $P_e$ ,<sup>8</sup>  $\alpha_{\text{SiO}_2} \ll \xi_{\text{SiO}_2} = 6.8 \times 10^{-6}/^\circ\text{C}$ ,<sup>2</sup> and  $\alpha_{\text{Perspex}} = 7 \times 10^{-5}/^\circ\text{C}$ ,<sup>9</sup> we found the following coefficient matrix of Eq. (4):

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} = \begin{bmatrix} 0.094 \text{ nm/mm} & 0 \text{ nm}/^\circ\text{C} \\ 0.17 \text{ nm/mm} & 0.119 \text{ nm}/^\circ\text{C} \end{bmatrix}. \quad (5)$$

The elements of this matrix can be determined experimentally. The main diagonal elements of Eq. (4) are related to the sensitivities of this sensor system. To determine them, we require the following experimental conditions: temperature kept constant while the relationships between  $\Delta\lambda_{\text{BrI}}$  with  $\omega(b)$  and  $\Delta\lambda_{\text{Br}}$  with  $\omega(b)$  are observed and temperature varied and  $\Delta\lambda_{\text{Br}}$  or  $\Delta\lambda_{\text{Bl}}$  monitored without any displacement.

When the condition  $T = T_0 = 18^\circ\text{C}$  was maintained, both  $\lambda_{\text{Br}}$  and  $\lambda_{\text{Bl}}$  increased with displacement but at different rates. The value of  $\Delta\lambda_{\text{BrI}}$  in Fig. 2 shows a linear response to displacement. The slope of the curve corresponds to the displacement sensitivity of this sensor and is  $9.24 \times 10^{-2}$  nm/mm, in reasonably good agreement with predicted value of  $9.4 \times 10^{-2}$  nm/mm.

Figure 3 shows the experimental relationship of  $\Delta\lambda_{\text{Br}}$  to  $\omega(b)$  when the temperature was kept at  $18^\circ\text{C}$ .  $\Delta\lambda_{\text{Br}}$  is proportional to the measured displacement but is affected by temperature. However, the value of  $\Delta\lambda_{\text{BrI}}$  was not affected by the temperature change. Figure 4 shows the reflected spectra of the sensing element recorded at different temperatures while  $\omega(b)$  was maintained at 11 mm. We observed that  $\Delta\lambda_{\text{BrI}}$  remained at 1.04 nm over a temperature range from 0 to  $60^\circ\text{C}$ , whereas the variation of  $\Delta\lambda_{\text{Br}}$  was as much as 5.5 nm. This result demonstrates that the value of  $K_{12}$  should be 0 and that this setup has the function of temperature compensation when displacement is measured.

To determine the value of  $K_{22}$ , which is related to the temperature sensitivity of the sensor, we kept the deflection at a fixed value and observed the relationship between  $\Delta\lambda_{\text{Br}}$  and  $T$ . In these conditions  $\Delta\lambda_{\text{Br}}$  and  $\Delta\lambda_{\text{Bl}}$  had the same relationship to  $T$ , and Fig. 5 shows plots of  $\Delta\lambda_{\text{Br}}$  versus  $T$  for  $\omega(b) = 0$  mm. The dashed curve becomes nonlinear at temperatures above  $60^\circ\text{C}$ . The linear part of the dashed curve has a slope of  $0.113 \text{ nm}/^\circ\text{C}$ , which is close to the theoretical value of  $0.119 \text{ nm}/^\circ\text{C}$ . The discrepancy between the theoretical and the experimental results at high temperatures in the measured range is likely

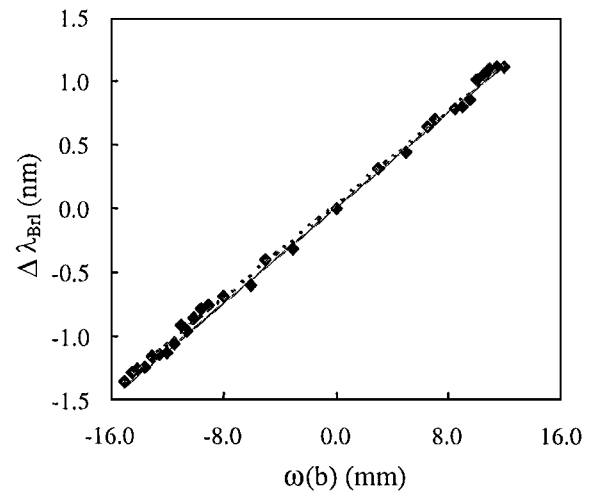


Fig. 2. Relationship between the difference in wavelength of the two sides and the measured displacement. Dashed curve, experiment; solid curve, theory.

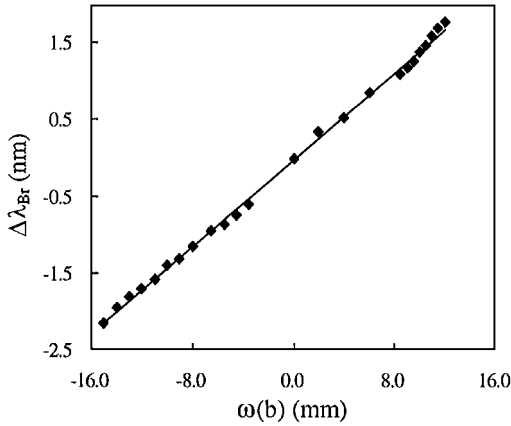


Fig. 3. Experimental plot of  $\Delta\lambda_{Br}$  versus  $\omega(b)$  for  $T = 18^\circ\text{C}$ .

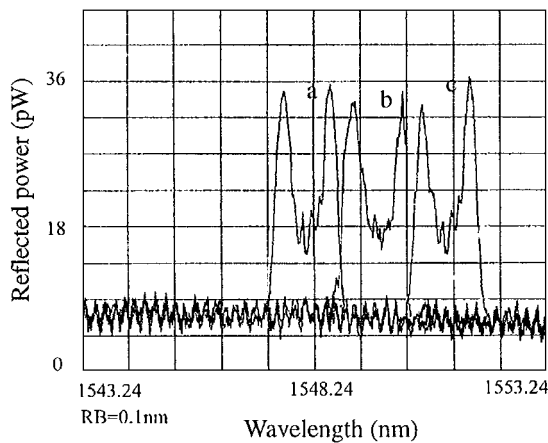


Fig. 4. Reflected spectra of the sensing FBG at three temperatures: a,  $T = 0^\circ\text{C}$ ; b,  $T = 18^\circ\text{C}$ ; c,  $T = 35^\circ\text{C}$ .

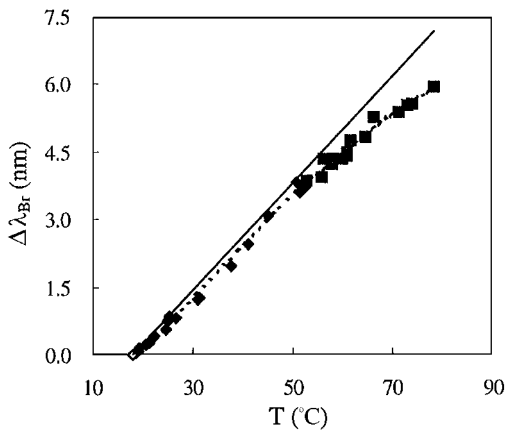


Fig. 5. Relationship between  $\Delta\lambda_{Br}$  and  $T$  for  $\omega(b) = 0$  mm. Solid curve, theory; dashed curve, experiment.

due to the epoxy's softening and thus causing creeping between the FBG and the beam during measurement.

When the free end of the beam was moved down 4 mm and the temperature was kept at  $23^\circ\text{C}$ ,  $\Delta\lambda_{Br}$  was 1.24 nm. The value of  $K_{21}$  was 0.169 nm/mm. Substituting these values into Eq. (4), we obtained

$$\begin{bmatrix} \Delta\lambda_{Br1} \\ \Delta\lambda_{Br} \end{bmatrix} = \begin{bmatrix} 9.24 \times 10^{-2} \text{ nm/mm} & 0 \text{ nm/}^\circ\text{C} \\ 0.169 \text{ nm/mm} & 0.113 \text{ nm/}^\circ\text{C} \end{bmatrix} \times \begin{bmatrix} \omega(b) \\ \Delta T \end{bmatrix}. \quad (4')$$

To verify the validity of Eq. (4') we increased the applied temperature by 15 to  $38^\circ\text{C}$  and the deflection at point Q by 6 mm.  $\Delta\lambda_{Br1}$  and  $\Delta\lambda_{Br}$  were observed to be 0.938 and 3.88 nm, respectively. We then calculated the displacement and the temperature by using Eq. (4') and obtained  $\omega(b) \approx 10.15$  mm and  $\Delta T \approx 19.15^\circ\text{C}$ , which correspond to deviations from the actual values of +1.5% for displacement and -4.25% for temperature. These results demonstrate that the proposed scheme is capable of discriminating the effects of displacement and temperature. The measurement errors were caused primarily by the creeping between the FBG and the beam and by the limited resolution of the optical spectrum analyzer.

In conclusion, we have reported a novel fiber Bragg grating sensor for measuring displacement and temperature simultaneously by use of an isosceles triangular cantilevered beam with a thickness step and uniform density. The temperature sensitivity of the sensor is 0.113 nm/ $^\circ\text{C}$  below  $60^\circ\text{C}$ , and the measured displacement sensitivity is  $9.24 \times 10^{-2}$  nm/mm over a range from -15 to 12 mm. The performance of the sensor could be improved by use of better epoxy, which would extend the linear response region of the sensor. The experimental results demonstrate that this sensing scheme can be used for detecting other physical parameters (such as force and vibration) that can strain the beam.

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