# Fiber-Optic Lattice Signal Processing 

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Invited Paper


#### Abstract

We discuss the implementation of fiber-optic lattice structures incorporating single-mode fibers and directional couplers. These fiber structures can be used to perform various high-speed timedomain and frequency-domain functions such as matrix operations and frequency filtering. In this paper we mainly consider systems in which the signals (optical intensities) and coupling coefficients are nonnegative quantities; these systems fit well in the theory of positive systems. We use this theory to conclude, for example, that for such systems the pole of the system transfer function with the largest magnitude is simple and positive-valued (in the $Z$-plane), and that the magnitude of the frequency response can nowhere exceed its value at the origin. We also discuss the effects of various noise phenomena on the performance of fiber-optic signal processors, particularly considering the effects of laser source phase fluctuations. Experimental results are presented showing that the dynamic range of the fiber systems, discussed in this paper, is limited, not by the laser source intensity noise or shot noise, but by the laser phase-induced intensity noise. Mathematical analyses of lattice structures as well as additional applications are also presented.


## Introduction

The low-loss (fractional decibels/kilometer) and large bandwidth-length product (on the order of $100 \mathrm{GHz} \cdot \mathrm{km}$ for single-mode fibers) of optical fibers, together with advances in the manufacturing of fiber-optic components provide an attractive technology for processing broad-band signals [1], [2].

Fiber-optic signal processing devices can be constructed to perform various functions, such as convolution, correlation, matrix operations [3], and frequency filtering [3]-[5]. Pulse-train generation, data-rate transformation [6], code generation, pulse compression, and matched filtering [7]-[10] are also achievable.

Previous work has concentrated on classical tapped-delay-line forms (transversal filters). Various mechanisms, such as radiation due to bending [4] and evanescent coupling by polishing the cladding down very close to the fiber core [11], have been used to realize the taps. In this paper, attention is on different fiber-optic structures, namely lattice (or ladder) forms, which can be used as alternatives for performing optical signal processing.

[^0]Lattice structures have found increasing applications in signal processing during the past decade [12]. This interest has been for the following reasons:
i) They allow a systematic approach to filter synthesis.
ii) They have a very straightforward implementation.
iii) They are both regular and modular. That is, the implementation is accomplished by interconnecting sections having the same basic topology, making the assembly easily expandable.
iv) They have convenient numerical properties which make them suitable for applications to digital signal processing.
v) They provide a natural model for many important physical and mathematical phenomena such as speech signal generation [13] and are useful in estimation problems.

In fiber-optic form, we use single-mode fibers and directional couplers in conjunction with semiconductor laser sources and optical detectors to realize a fiber-optic lattice processor. Higher order lattices are simply linear extensions of the basic sections. These basic sections, as well as the operation of directional couplers, are described in Section H.

The signals to be processed are modulated as intensity variations on optical carriers whose coherence time is less than the shortest relevant time delay in the system. This restriction is to avoid environmentally sensitive optical interference (phase) effects. In this paper we consider only fiber-optic systems which are linear in the intensities of the propagating waves. It should be mentioned that in all our experiments we used a short coherence length (a few millimeters) multimode semiconductor laser as the optical source. This particular optical source could be amplitude modulated from 1 MHz to 1.3 GHz . Furthermore, the output light intensities were detected by photodiodes whose outputs can be connected to measurement systems such as an oscilloscope, a network analyzer, or a spectrum analyzer, and for which the output voltage is proportional to input intensity.

## Paper Outline

In Section I we introduce the elementary concepts that we need to construct fiber-optic signal processing devices. Using the concepts explained in Section I, in Section II we demonstrate two structures that are basic to fiber signal processing: two-coupler nonrecirculating and two-coupler recirculating delay lines. These basic fiber structures are employed in Section III to implement two different types (feed-forward and feed-backward configurations) of socalled lattice structures in their fiber-optic forms.

Section IV discusses mathematical tools for the analysis of lattice structures. These tools include transformation techniques, the transfer-matrix and chain-matrix formulations [14], and a modern control theory formulation [15]. The transfer-matrix and chain-matrix characterizations are useful for the analysis of cascaded two-port systems which include our fiber lattice structures. We utilize the Z-transform method, together with the transfer-matrix and chainmatrix formulations, to describe these structures for filtering applications.

The modern control theory formulation for these lattice structures uses the notion of a state vector to represent the system with two matrix equations: a state update equation and an output equation. This formulation is well suited for the theory of positive systems that will be used in Section IV to draw some conclusions about the behavior of fiberoptic systems incorporating positive-valued quantities. We shall see that this knowledge alone, namely, the fact that all the system quantities are nonnegative in value, enables us to make some very strong statements about the behavior of the system, such as pole-zero positions, system stability, and sensitivity.

Section $V$ contains some time-domain signal processing applications of fiber lattice structures, such as the realization of a high-speed matrix-vector multiplier using the lattice concept. In this form, our lattice structure resembles systolic array architectures [16] which were originally designed for VLSI implementation of various matrix operations. Experimental results are also presented showing how these lattices can be used to perform some useful matrix operations.

Frequency-domain signal processing applications of fiber lattice structures are presented in Section VI. In this section we use Z-transform methods to study the properties of fiber-optic lattice structures when used as frequency filters. Possible pole-zero patterns of various lattice orders are shown with a discussion of their implications for the analysis and design of various filter types. We also examine a few fiber-optic filtering devices, such as all-pole, all-zero, pole-zero, and all-pass (phase-equalizer) filters, together with the presentation of some experimental and theoretical results and optical power efficiency calculations.

Finally, Section VII concludes the paper with a discussion of relevant noise phenomena. We particularly consider the effects of laser source phase fluctuations on the performance of fiber-optic filtering systems. Experimental results are also presented showing that the dynamic range of the fiber systems discussed is limited by this laser phaseinduced intensity noise [17], [18].

## I. Elementary Concepts

To perform the various signal processing operations described, we need to employ the following elements:

1) Time delay:

The output of the delay element is the input delayed by a unit of time. That is, for a discrete-time input signal $f(k)$, the output is $f(k-1)$; where $k$ represents the time index. A block diagram representation and the impulse response (response to an intensity impulse input) of this element are shown in Fig. 1(a). Note that the $z^{-1}$ inside the block diagram represents a unit time delay. Here, we have used a notation from the $Z$-transform which will be described later. Fiber lengths provide precise time delays that can be


Fig. 1. Elementary concepts for signal processing. (a) Timedelay element. (b) Tapping element. (c) Time-advance element.
employed to perform the time-delay function. Assuming a refractive index of about 1.5 for the fiber, the propagation delay is $\approx 5 \mathrm{~ns} / \mathrm{m}$.
2) Tapping:

Various mechanisms, such as evanescent directional couplers or bends ("kinks"), can be used to accomplish tapping. Mathematically, the output of the tapping element is a constant multiple, $K$, of the input (see Fig. 1(b)).
3) Time advance:

The signal is advanced one unit of time rather than delayed (see Fig. 1(c)). Note that this element has only mathematical meaning, and that systems requiring unit advance subsystems in their construction are not physically realizable.
4) Summing element:

This subsystem sums incoming signals (Fig. 2(a)). In fiber-optic systems a directional coupler can be used to do the summing operation.

(a)

(b)

Fig. 2. Summing and branching elements. (a) Summing element. (b) Branching element.

## 5) Branching element:

This subsystem divides the incoming signal into several parts (Fig. 2(b)). A directional coupler can also be used to divide the incoming light signal into two parts.

## II. Basic Fiber Structures

Recirculating (feed-backward) and nonrecirculating (feed-forward) delay lines are two structures that are basic to fiber-optic signal processing. Using single-mode fibers and directional couplers in conjunction with semiconductor laser sources, one can construct a two-coupler recirculating delay line (see Fig. 3(a)) and a two-coupler nonrecirculating delay line (see Fig. 3(b)).


Fig. 3. Basic fiber structures. (a) Two-coupler recirculating (feed-backward) delay line. (b) Two-coupler nonrecirculating (feed-forward) delay line. (c) Impulse response of the twocoupler recirculating delay line. (d) Impulse response of the two-coupler nonrecirculating delay line.

In the two-coupler recirculating delay line a single-mode fiber loop (with loop delay $T$ ) is closed upon itself by two directional couplers, with the second coupler feeding some portion of the light back to the first coupler through the fiber loop. Thus optical signals sent into the input port, say at $X_{1}$, circulate repeatedly around the loop, sending a portion of the recirculating intensity to the output ports $Y_{1}$ and $Y_{2}$. The impulse response of the system consists of a series of decaying impulses equally spaced in time by the loop delay $T$, as shown in Fig. 3(c). A two-coupler recirculating delay line has already been used for frequency filtering [5].

The two-coupler nonrecirculating delay line also consists of two directional couplers, but in this case the outputs of the first coupler are both fed forward to the second coupler where they are recombined after a time delay $T$ which is the time-delay difference between the two feed-forward fiber lines. The impulse response of the system is composed of just two impulses spaced by the amount $T$ in time (see Fig. 3(d)).

The input-output relationship of the directional couplers [19] can be described by a $2 \times 2$ complex transfer matrix (see Fig. 4)

$$
\binom{E_{3}}{E_{4}}=(\gamma)^{1 / 2}\left(\begin{array}{ll}
A & B  \tag{1}\\
C & D
\end{array}\right) \cdot\binom{E_{1}}{E_{2}}
$$

where ( $E_{1}, E_{2}$ ) and ( $E_{3}, E_{4}$ ) are, respectively, the input and


Fig. 4. Schematic diagram of a fiber-optic directional coupler.
output complex amplitudes of the vector fields $\boldsymbol{E}_{1}, \boldsymbol{E}_{2}, \boldsymbol{E}_{3}$, and $\boldsymbol{E}_{4}$ which are assumed, for simplicity, to have the same state of polarization. It should be mentioned that the coupler itself exhibits very little dependence on the state of polarization of the input fields [20]. Since backward reflections are negligible, the coupler matrix can be approximated [21] by the product of an overall amplitude transmission factor $(\gamma)^{1 / 2}(0.9<\gamma<1)$ and a unitary matrix. This property of the matrix implies that $|A|^{2}+|B|^{2}=|C|^{2}+|D|^{2}$ $=1$ and $C A^{*}+D B^{*}=0$ (the star denotes complex conjugation).

For a symmetric coupler ( $A=D$ and $B=C$ ), (1) can be rewritten as

$$
\binom{E_{3}}{E_{4}}=(\gamma)^{1 / 2}\left(\begin{array}{cc}
\sqrt{1-K} & j \sqrt{K}  \tag{2}\\
j \sqrt{K} & \sqrt{1-K}
\end{array}\right) \cdot\binom{E_{1}}{E_{2}}
$$

where $\sqrt{K}$ is the amplitude coupling coefficient, and $j$ $=\sqrt{-1}$ indicates a $90^{\circ}$ phase shift for the coupled signal.

Furthermore, in the case when a temporally incoherent light source is used, the input and output light intensities can be related to each other by the following expression:

$$
\binom{I_{3}}{I_{4}}=(\gamma)\left(\begin{array}{cc}
1-K & K  \tag{3}\\
K & 1-K
\end{array}\right) \cdot\binom{I_{1}}{I_{2}}
$$

where $\left(I_{1}, I_{2}\right)$ and $\left(I_{3}, I_{4}\right)$ are the input and output intensities, respectively, $\gamma$ is the overall intensity transmission factor, and $K$ is the intensity coupling coefficient. Equations (1)-(3) will be used in later sections for our mathematical analyses.

The basic recirculating and nonrecirculating delay lines described previously are used as elementary sections of feed-backward and feed-forward lattice structures, respectively. These structures are the subject of our discussion in the next section.

## III. Lattice Structures

Lattice structures are suitable forms for performing the various signal processing operations described above. Compared to other forms, lattice structures have some advantageous characteristics, such as modularity, regularity, ease of implementation, and good sensitivity.

In this section we introduce two fiber-optic lattice configurations; recirculating (feed-backward) and nonrecirculating (feed-forward) forms. Fig. 5(a) shows an Nth-order feed-backward (recirculating) fiber-optic lattice structure. In this paper we are interested in cases where $T$ is the same for all sections, but this restriction would not be necessary

(a)

(b)

Fig. 5. Schematic diagram of the Nth-order fiber-optic lattice structure with $N+1$ couplers and $N$ delay elements. (a) Feed-backward configuration. (b) Feed-forward configuration.
in general. This structure is obtained by interconnecting $N$ two-coupler recirculating delay lines (see Fig. 3(a)) in tandem. As seen in Fig. 5(a), this structure has one feed-forward and one feed-backward fiber line.

In contrast, Fig. 5(b) illustrates an Nth-order feed-forward (nonrecirculating) fiber lattice structure which is implemented by cascading $N$ two-coupler nonrecirculating delay lines (see Fig. 3(b)). In this configuration, there are two feed-forward tines with no feed-backward line.

Both of these lattice configurations have two ports with two terminals at each port (two inputs, two outputs). Fig. $6(a)$ and (b) shows the equivalent diagrams of the feed-

(a)

(b)

Fig. 6. Equivalent diagram of the $N$ th-order fiber-optic lattice structure with $N+1$ couplers and $N$ delay elements.
(a) Feed-backward configuration. (b) Feed-forward configuration. ( $\ell_{i}$ is the intensity transmittance of the $i$ th section.)
backward and feed-forward lattice structures. These two lattice forms can also be combined to construct more sophisticated fiber-optic structures.
In later sections, we discuss the applications of the various structures introduced here. Before that, however, we present different methods for mathematical analysis of the lattice structures.

## IV. Mathematical Analysis

This section is devoted mainly to establishing the theoretical background needed to analyze our fiber-optic systems (in particular, lattice structures). In Section IV-A we explain, very briefly, the Z-transform technique, and show its applications using a few simple fiber-optic systems as examples. In Section IV-B we introduce the transfer-matrix and chainmatrix formulations which we use to simplify the mathematical manipulations. Section IV-C presents the modern control theory formulation, while its usage in the theory of positive fiber-optic systems is shown in Section IV-D.

## A. Z-Transform Techniques

In order to increase our ability to handle more complex systems problems, we utilize transform techniques for describing the signals. We particularly use the $Z$-transform [22]. This usage is justifiable since all the systems of interest here are, first, linear and time-invariant and, second, discrete in time. The latter is due to the fact that we can define a basic time delay $T$ such that any other relevant system delays are integer multiples of this basic time delay. That is, the impulse response of the system is a series of impulses which are equally spaced in time. As a result, we can simplify the mathematical analysis of such systems by
considering the values of the system signals only at discrete instants in time. Since the input and output are described in terms of discrete samples, these systems are called sam-pled-data systems [22].

The mathematical analysis of discrete systems is essentially similar to that of continuous data systems, but it is simpler, and more intuitive in terms of physical interpretations.

The $Z$-transform, $f(z)$, of a signal, $f(k)$, is defined by the following expression:

$$
\begin{equation*}
F(z) \triangleq \sum_{k=-\infty}^{\infty} f(k) z^{-k} \tag{4}
\end{equation*}
$$

where $k$ is the time index, which is an integer multiple of the basic time delay ( $k=n T$ ), and $z$ is the transform variable which represents a unit time advance ( $z^{-1}$ represents a unit time delay).

We also describe the input-output relationships by system transfer functions (the ratio of the $Z$-transform of the output to the $Z$-transform of the input) whose poles and zeros are very important in the design and analysis of frequency-selective filters. The frequency response is obtained when we evaluate the transfer function at $z=e^{j \omega T}$. This $e^{j \omega T}$ describes a unit circle centered at the origin of the Z-plane (Fig. 7). The unit circle also plays an important role in problems regarding stability.


Fig. 7. Z-plane with the unit circle.
To insure stability, all the poles of the system transfer function must be inside the unit circle. Fiber lattice filters incorporating passive elements (no gain) are sure to be stable. Furthermore, if all the zeros are also inside the unit circle, the system is called a minimum phase system, while if any zero lies outside the unit circle, the system is called a nonminimum phase system. It should be mentioned that for a minimum phase filter, the phase response and the logarithm of the magnitude response are a Hilbert transform pairs. This is in addition to the property of all rea stable causal systems that the real and imaginary parts of the frequency response are Hilbert transformations of each other. Oppenheim and Schafer [22] give a more complete description of the $Z$-transform theory.

Using the definition of the Z-transform, (4), we find the following expressions for the system transfer functions, $H(z)$, of the two-coupler delay lines introduced earlier (see Fig. 3):
a) Two-coupler recirculating delay line, with $X_{1}$ as the input and $\gamma_{1}$ as the output (see Fig. 3(a))

$$
\begin{equation*}
H_{11}(z)=\frac{a_{1}+\left(1-2 a_{1}\right) a_{0} \ell_{1} z^{-1}}{1-a_{1} a_{0} \ell_{1} z^{-1}} \tag{5}
\end{equation*}
$$

where $a_{1}$ and $a_{0}$ are the intensity coupling coefficients of the couplers, and $\ell_{1}$ is the loop intensity transmittance of
the system. As (5) shows, this system has one zero at

$$
z=\frac{\left(2 a_{1}-1\right)}{a_{1}} a_{0} \ell_{1}
$$

and one pole at $z=a_{1} a_{0} b_{1}$.
b) Two-coupler recirculating delay line, with $X_{1}$ as the input and $\gamma_{2}$ as the output (see Fig. 3(a))

$$
\begin{equation*}
H_{21}(z)=\frac{\left(1-a_{1}\right)\left(1-a_{0}\right) \ell_{11}}{1-a_{1} a_{0} \ell_{1} z^{-1}} \tag{6}
\end{equation*}
$$

where $\ell_{11}$ is the intensity transmission factor of the forward fiber line. In this case, the system has one zero at the origin and one pole at $z=a_{1} a_{0} \ell_{1}$.
c) Two-coupler nonrecirculating delay line, with $X_{1}$ as the input and $Y_{1}$ as the output (see Fig. 3(b))

$$
\begin{equation*}
H_{11}(z)=\left(1-b_{1}\right)\left(1-b_{0}\right) \ell_{11}+b_{1} b_{0} \ell_{12} z^{-1} \tag{7}
\end{equation*}
$$

where $b_{1}$ and $b_{0}$ are the intensity coupling coefficients of the couplers, and $\ell_{11}$ and $\ell_{12}$ are the intensity transmittances in the two feed-forward fiber lines. This system has one pole at the origin and one zero at

$$
z=-\frac{b_{1} b_{0} l_{12}}{\left(1-b_{1}\right)\left(1-b_{0}\right) \ell_{11}}
$$

Note that those zeros or poles which are located at the origin do not affect the frequency response of the system (except for a linear phase factor). For this reason, we refer to the two-coupler recirculating delay line, with $\gamma_{2}$ as the output, as a first-order (one-pole) all-pole system. The twocoupler nonrecirculating delay line and the two-coupler recirculating delay line with $Y_{1}$ as the output, are referred to as first-order (one-zero) all-zero and first-order (one-pole) pole-zero systems, respectively. The pole-zero diagrams of the systems discussed in this section are illustrated in Fig. 8.


Fig. 8. Pole-zero diagrams of the two-coupler delay lines (see Fig. 3), with $X_{1}$ as the input. (a) Two-coupler recirculating delay line, with $Y_{1}$ as the output. (b) Two-coupler recirculating delay line, with $\gamma_{2}$ as the output. (c) Two-coupler nonrecirculating delay line, with $Y_{1}$ as the output.

These diagrams contain information about the frequency responses of the corresponding systems.

## B. Transfer-Matrix and Chain-Matrix Formulations

All the structures of interest in this paper have, in general, two inputs and two outputs (two-pair systems). Trans-fer-matrix and chain-matrix formulations are two methods of characterization which are suitable for cascaded two-pair systems. These methods are first explained very briefly, and then applied to our lattice structures to obtain the total
transfer functions. A more detailed explanation of these methods can be found in [14].

Consider a linear and time-invariant two-port system, with each port having two terminals. In Fig. $9(a)$, the first and second ports consist of the variables ( $X_{1}, Y_{1}$ ) and ( $X_{2}, Y_{2}$ ),


Fig. 9. A two-pair system. (a) Feed-backward configuration. (b) Feed-forward configuration.
respectively; we refer to this configuration as a feed-backward two-pair system. Alternatively, if ( $X_{1}, X_{2}$ ) and ( $Y_{1}, Y_{2}$ ) constitute the first and second ports, then the system is referred to as a feed-forward two-pair system (see Fig. $9(b)$ ). Note that the input variables are shown by $X$, and the output variables by $Y$.

The inputs and the outputs are related to each other by the transfer matrix

$$
\binom{Y_{1}(z)}{Y_{2}(z)}=\left(\begin{array}{ll}
H_{11}(z) & H_{12}(z)  \tag{8}\\
H_{21}(z) & H_{22}(z)
\end{array}\right) \cdot\binom{X_{1}(z)}{X_{2}(z)}
$$

where $z$ is the $Z$-transform variable. Note that $H_{m n}(z)$ is the transfer function from input $X_{n}$ to output $Y_{m}$.

While the transfer matrix formulation is suitable for the analysis of cascaded feed-forward two-pair systems, there is another way of characterization which is more suitable for the analysis of cascaded feed-backward two-pair systems. This method makes use of the chain matrix which relates the pair $\left(X_{1}, Y_{1}\right)$ to the pair $\left(X_{2}, Y_{2}\right)$

$$
\binom{x_{1}(z)}{y_{1}(z)}=\left(\begin{array}{ll}
G_{11}(z) & G_{12}(z)  \tag{9}\\
G_{21}(z) & G_{22}(z)
\end{array}\right) \cdot\binom{y_{2}(z)}{x_{2}(z)}
$$

Note that both the transfer matrix and the chain matrix are $2 \times 2$ matrices which are uniquely determined for specified input and output variables.

It can be easily shown that elements of the transfer matrix of a feed-backward two-pair system are related to the elements of its chain matrix through the following expressions:

$$
\begin{align*}
& H_{11}=\frac{G_{21}}{G_{11}}  \tag{10}\\
& H_{12}=\frac{G_{11} G_{22}-G_{12} G_{21}}{G_{11}}  \tag{11}\\
& H_{21}=\frac{1}{G_{11}}  \tag{12}\\
& H_{22}=-\frac{G_{12}}{G_{11}} \tag{13}
\end{align*}
$$

If $G_{11} G_{22}-G_{12} G_{21}=1$, the system is called a reciprocal ( $H_{12}=H_{21}$ ) two-pair. A very important property of the
chain-matrix formulation is that for cascaded feed-backward two-pair systems (see Fig. $9(a)$ ), the chain matrix of the composite system is the product of the individual chain matrices, multiplied in the same order as the systems are cascaded, i.e.,

$$
\begin{equation*}
G_{\text {total }}=G_{1} G_{2} G_{3} \cdots G_{N} \tag{14}
\end{equation*}
$$

The same multiplicative rule, but with the reverse order, is also valid for the transfer matrices of cascaded feed-forward two-pair systems (see Fig. $9(b)$ ), as shown below

$$
\begin{equation*}
H_{\text {total }}=H_{N} \cdots H_{3} H_{2} H_{1} \tag{15}
\end{equation*}
$$

where $N$ is the number of cascaded systems. These multiplicative properties of the chain-matrix and transfer-matrix formulations greatly simplify the mathematical analysis of cascaded two-pair systems.

Lattice structures are good examples of cascaded two-pair systems. As mentioned before, these structures are obtained by a cascade connection of individual two-coupler delay lines (basic lattice sections) of the types illustrated in Fig. 3.

It can readily be shown that for a directional coupler, with $a$ as the coupling coefficient, the transfer matrix $H_{c}$ (with variables labeled as shown in Fig. $9(b)$ ) and the chain matrix $G_{c}$ (with variables labeled as shown in Fig. $9(a)$ ) are given by the following expressions:

$$
H_{c}=\left(\begin{array}{cc}
1-a & a  \tag{16}\\
a & 1-a
\end{array}\right)
$$

and

$$
G_{c}=\frac{1}{1-a}\left(\begin{array}{cc}
1 & -a  \tag{17}\\
a & 1-2 a
\end{array}\right)
$$

In a similar way, for a simple two-line delay system, with a unit time delay $\left(z^{-1}\right)$ and intensity transmission factors $\ell_{11}$ and $\ell_{12}$, as depicted in Fig. 10, we obtain


Fig. 10. A simple two-line delay system; $\ell_{11}$ and $\ell_{12}$ are intensity transmission factors of the first and second fiber lines, respectively. $z^{-1}$ represents a unit time delay.

$$
H_{d}=\left(\begin{array}{cc}
\ell_{11} & 0  \tag{18}\\
0 & \ell_{12} z^{-1}
\end{array}\right)
$$

and

$$
G_{d}=\frac{1}{\ell_{11}}\left(\begin{array}{cc}
1 & 0  \tag{19}\\
0 & \ell_{11} \ell_{12} z^{-1}
\end{array}\right)
$$

The two-coupler delay lines (see Fig. 3) are both a cascade connection of two couplers and a two-line delay line. Therefore, using (14)-(19), we can calculate the chain matrix and transfer matrix of, respectively, the two-coupler recirculating and nonrecirculating delay lines. The results are shown below.

$$
G_{b}=G_{c 1} G_{d} G_{\mathrm{c} 0}=\frac{1}{\ell_{11}\left(1-a_{1}\right)\left(1-a_{0}\right)} \cdot\left(\begin{array}{ll}
G_{11}^{\prime} & G_{21}^{\prime}  \tag{20a}\\
G_{12}^{\prime} & G_{22}^{\prime}
\end{array}\right)
$$

where

$$
\left\{\begin{array}{l}
G_{11}^{\prime}=1-a_{1} a_{0} \ell_{1} z^{-1}  \tag{20b}\\
G_{12}^{\prime}=-a_{0}-a_{1}\left(1-2 a_{0}\right) \ell_{1} z^{-1} \\
G_{21}^{\prime}=a_{1}+\left(1-2 a_{1}\right) a_{0} \ell_{1} z^{-1} \\
G_{22}^{\prime}=-a_{1} a_{0}+\left(1-2 a_{1}\right)\left(1-2 a_{0}\right) \ell_{1} z^{-1}
\end{array}\right.
$$

and $\ell_{1}=\ell_{11} \ell_{12}$ is the loop intensity transmittance, $G_{c 1}$ and $G_{c 0}$ are coupler chain matrices (see (17)) with coupling coefficients equal to $a_{1}$ and $a_{0}$, respectively, and $G_{d}$ is the chain matrix of the two-line delay system, given in (19). Moreover, in this case, the elements of the corresponding transfer matrix can be found through (10)-(13), which are consistent with those of (5) and (6); also for the nonrecirculating case we have

$$
H_{f}=H_{c 0} H_{d} H_{c 1}=\left(\begin{array}{ll}
H_{11} & H_{12}  \tag{21a}\\
H_{21} & H_{22}
\end{array}\right)
$$

where

$$
\left\{\begin{array}{l}
H_{11}=\left(1-b_{1}\right)\left(1-b_{0}\right) \ell_{11}+b_{1} b_{0} \ell_{12} z^{-1}  \tag{21b}\\
H_{12}=b_{1}\left(1-b_{0}\right) \ell_{11}+\left(1-b_{1}\right) b_{0} \ell_{12} z^{-1} \\
H_{21}=\left(1-b_{1}\right) b_{0} \ell_{11}+b_{1}\left(1-b_{0}\right) \ell_{12} z^{-1} \\
H_{22}=b_{1} b_{0} \ell_{11}+\left(1-b_{1}\right)\left(1-b_{0}\right) \ell_{12} z^{-1} .
\end{array}\right.
$$

Note that the $H_{11}$ element is the same as the one given in (7).

The chain and transfer matrices of the basic lattice sections, given in (20) and (21), can be used to calculate the total chain and transfer matrices of higher order lattice structures. Using this approach, together with Z-transform techniques, we can obtain general expressions for the transfer functions of our lattice structures.

## C. Modern Control Theory Formulation

In modern control theory [15], a general (discrete-time) linear $n$ th-order system is described in a state space by the state update and the output equations as shown below.

$$
\left\{\begin{align*}
\boldsymbol{x}(k+1) & =\boldsymbol{A}(k) \boldsymbol{x}(k)+\boldsymbol{B}(k) \boldsymbol{u}(k)  \tag{22a}\\
\boldsymbol{y}(k) & =\boldsymbol{C}(k) \boldsymbol{x}(k)+\boldsymbol{D}(k) \boldsymbol{u}(k) .
\end{align*}\right.
$$

We represent the matrices and the vectors by upper and lower case notations, respectively. In (22a), the vector $\boldsymbol{x}(k)$ is an $n$-dimensional state vector, $\mathbf{u}(k)$ is an $m$-dimensional input vector, $\boldsymbol{y}(k)$ is a $p$-dimensional output vector, and $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ are $(n \times n),(n \times m),(\rho \times n)$, and $(\rho \times m)$ matrices, respectively. In the case of shift-invariant (time-invariant) systems the above matrices are independent of time. However, these matrices are, in general, functions of system parameters such as coupling coefficients and system losses or gains.

It should also be noted that the state of a dynamic system is usually defined as the smallest set of variables (called state variables) such that the knowledge of these variables at the initial time $t=t_{0}$ together with the inputs for $t \geqslant t_{0}$, completely determines the behavior of the system for any time $t \geqslant t_{0}$. Usually, the state variables are taken to be the outputs of the delay elements.
In the mathematical model described by (22a) we have assumed the general case of a multiple-input ( $m$ inputs), multiple-output ( $p$ outputs) system ("MIMO" system). If
the system has only a single input and a single output ("SISO" system), the equations can be expressed as

$$
\left\{\begin{align*}
\boldsymbol{x}(k+1) & =\boldsymbol{A}(k) \boldsymbol{x}(k)+\boldsymbol{b}(k) \boldsymbol{u}(k)  \tag{22b}\\
\boldsymbol{y}(k) & =\boldsymbol{c}^{T}(k) \boldsymbol{x}(k)+d(k) \boldsymbol{u}(k)
\end{align*}\right.
$$

where now $\boldsymbol{b}$ and $\boldsymbol{c}$ are vectors and $d$ is a scalar.
As an example, taking the outputs of the delay elements as the state variables, we get the following expressions for the $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D})$ matrices of the $\boldsymbol{N}$ th-order feed-backward lattice structure:

$$
\begin{align*}
& \mathbf{A}=\left(\begin{array}{cccc}
a_{1} a_{0} \ell_{1} & a_{2}\left(1-a_{1}\right) a_{0} l_{1} & \cdots & a_{N} \prod_{k=1}^{N-1}\left(1-a_{k}\right) a_{0} l_{1} \\
\left(1-a_{1}\right) \ell_{2} & a_{2} a_{1} l_{2} & \cdots & a_{N} \prod_{k=2}^{N-1}\left(1-a_{k}\right) a_{1} l_{2} \\
& \ddots & & \vdots \\
0 & \ddots & & a_{N}\left(1-a_{N-1}\right) a_{N-2} l_{N-1} \\
& & & \left(1-a_{N-1}\right) \ell_{N} a_{N} a_{N-1} l_{N}
\end{array}\right)  \tag{23a}\\
& \boldsymbol{B}=\left(\begin{array}{cc}
\prod_{k=1}^{N}\left(1-a_{k}\right) a_{0} \ell_{1} & \left(1-a_{0}\right) \ell_{1} \\
\prod_{k=2}^{N}\left(1-a_{k}\right) a_{1} \ell_{2} & 0 \\
\vdots & \vdots \\
\left(1-a_{N}\right) a_{N-1} l_{N} & 0
\end{array}\right)  \tag{23b}\\
& C=\left(\begin{array}{cccc}
0 & 0 & \cdots & 1-a_{N} \\
a_{1}\left(1-a_{0}\right) & a_{2}\left(1-a_{1}\right)\left(1-a_{0}\right) & \cdots & a_{N} \prod_{k=0}^{N-1}\left(1-a_{k}\right)
\end{array}\right) \tag{23c}
\end{align*}
$$

and

$$
\boldsymbol{D}=\left(\begin{array}{cc}
a_{N} & 0  \tag{23d}\\
\prod_{k=0}^{N}\left(1-a_{k}\right) & a_{0}
\end{array}\right)
$$

where the input, output, and state vectors are as shown below

$$
\begin{align*}
& \boldsymbol{u}=\binom{x_{1}}{x_{2}}  \tag{23e}\\
& \boldsymbol{y}=\binom{y_{1}}{y_{2}} \tag{23f}
\end{align*}
$$

and

$$
x=\left(\begin{array}{c}
x_{1}  \tag{23g}\\
x_{2} \\
\vdots \\
x_{N}
\end{array}\right)
$$

Note that all the indices are numbered increasingly from the right to the left of the lattices' schematic diagrams.

As seen, all elements of the above matrices and vectors are nonnegative in value. It can be shown that if the coupling coefficients of all the middle couplers (the first and last couplers excluded) are greater than 0.5 , then all the system poles are real and nonnegative. The reason is that, in this case, all the minors of the $\boldsymbol{A}$ matrix which is an upper Hessenberg matrix, are nonnegative.

## D. Positive Optical Systems

In this paper we have considered only systems in which the signals (optical intensities) and coupling coefficients are nonnegative quantities. These systems form a subclass which is well matched to the requirements of several fields of application. These and many other linear systems of interest can be represented by a set of linear matrix-vector equations in which all the elements of the matrices and vectors involved are (real) nonnegative in value. Real matrices with nonnegative elements have important applications in the theory of probability for the study of Markov chains [23] (stochastic matrices), and in the theory of oscillatory properties of elastic vibrations in mechanics [24] (oscillation matrices).

The theory of positive systems is closely associated with the theory of positive matrices, which is remarkably well
developed. This enables us to use the results of the theory of positive matrices in the study of positive systems. After some basic familiarity with this important theory, our goal is to explore its applicability to optical systems which incorporate positive-valued parameters.

A very interesting and advantageous feature of positive systems theory is that it provides us with some fairly strong and important conclusions about the behavior of a positive system. In addition, it is usually consistent with intuition.

In this section, we first define a positive system, using the state-space model, described in Section IV-C to represent that system. Then, we state some definitions and important theorems of positive systems theory. These theoretical results are then used to prove that the magnitude of the frequency response of a linear shift-invariant positive system can nowhere exceed its value at the origin ( $\omega=0$ ). Finally, after discussing the results obtained and verifying their consistency with intuition, we conclude this section with a few remarks regarding some of the effects of the positivity feature on positive optical signal processing systems.

Based on the state-space representation of a linear system, a linear positive system is defined as a linear system in which the state variables (and the outputs) are always positive (or at least nonnegative) in value. Such kinds of systems arise frequently in practice since the state variables of many real systems describe quantities which are always nonnegative. As an example, in many economic systems the variables which describe quantities of goods remain nonnegative. The same is true for the subset of linear optical systems for which the quantities involved are positive-valued, such as incoherent optical processors which have applications in signal processing and filtering operations. In the following we present some facts from positive systems theory.
7. Some Theoretical Background: If $A=\left[a_{i j}\right]$ and $B=$ $\left[b_{i j}\right]$, we shall agree on the following notations for our later discussions:

For two real matrices $A$ and $B$ we write
i) $\boldsymbol{A}>\boldsymbol{B}$ (or $\boldsymbol{B}<\boldsymbol{A}$ ) if and only if $a_{i j}>b_{i j}$ for all $i, j$.
ii) $\boldsymbol{A} \geqslant \boldsymbol{B}$ (or $\boldsymbol{B} \leqslant \boldsymbol{A}$ ) if and only if $a_{i j} \geqslant b_{i j}$ for all $i, j$.
iii) $\boldsymbol{A} \geqq \boldsymbol{B}$ (or $\boldsymbol{B} \leqq \boldsymbol{A}$ ) if and only if $a_{i j} \geqslant b_{i j}$ for all $i, j$ with $a_{i j} \neq b_{i j}$ for at least one ( $i, j$ ).
iv) For a complex-valued matrix $C$ we denote by $\mathrm{C}^{+}$the matrix modulus $C$ which is obtained from $C$ when all the elements are replaced by their moduli. For the special case when $B=0$ we call $A$ strictly positive, positive (or strictly nonnegative), and nonnegative, when $A>0, A \geqslant 0$, and $A \geqq 0$, respectively.

Now we proceed to describe three of the major properties of linear positive systems [15].
i) The first important property, which is stated by the Frobenius [25] and Perron [26] theorems, is related to the fact that for any linear-positive system there exists a dominant eigenvalue and eigenvector. This important property will be described in more detail later in the discussion of the Frobenius-Perron Theorem.
ii) The second property of positive systems is concerned with the connection between stability and positivity. It can be shown [15] that for positive systems there is a direct correspondence between the existence of a positive equilibrium point and stability.
iii) The third major property is concerned with the effects produced by changes in the values of the parameters of the system (comparative statics). For positive systems positive changes (such as increasing an element of the system matrix A) produce corresponding positive changes (increases) in the components of the equilibrium points (the steady-state state vector).

Therefore, even if the presise values of the parameters of a linear-positive system are not known in advance, we can draw some significant qualitative conclusions about the behavior of the system. Here it should also be noted that the structure of continuous-time-positive systems is slightly different from that of discrete-time-positive systems because in the former case the system matrix relates the state to the derivative of the state and the derivative need not be positive, but in both cases the results are virtually the same in character [15].

By using the theory of positive systems it can be shown that the magnitude of the frequency response of a positive system can nowhere exceed its value at the origin, i.e.,

$$
\begin{equation*}
\left[\boldsymbol{H}\left(e^{j \omega T}\right)\right]^{+} \leqslant \boldsymbol{H}(1) \tag{24}
\end{equation*}
$$

Some related theorems, together with the proof of (24), are presented in the Appendix. For more complete information on this topic, see [15] and [27].
2. Discussion and Conclusions: The theory of positive systems has the attractive feature of being consistent with intuition. For example, for the conclusion given in (24), we can have the following explanations:
i) At frequencies for which $\omega T=2 n \pi(n=0,1,2, \cdots)$ the signals add constructively in the system, whereas at other frequencies this is not the case. Therefore, the magnitude of the transfer function should take its maximum value at frequencies which are integer multiples of $1 / T$.
ii) Since the impulse response of a positive system is real and positive-valued for all time, it can be considered analogous to a power spectrum (which is positive for all frequencies); therefore, the Fourier transform of the impulse response (or the system transfer function) can be regarded as an autocorrelation function, and it is well known that the value of an autocorrelation function at the origin is maximum and positive [28].

In conclusion, we have briefly described the theory of the positive systems and used it to prove that the magnitude of the frequency response of a positive system can nowhere exceed its value at the origin. This implies that, for instance, we cannot make high-pass filters with positive systems, since these filters should be able to reject the low-frequency components of the input signals. Further flexibility is possible by combining positive sections with sections allowing negativity. One such example (all-pass filter) is given in Section VI (frequency-domain applications). It should be mentioned that information about the locations of the poles and zeros can also be obtained for positive systems. For example, it can be shown that, for a positive system, there cannot be an odd number of zeros to the right hand of the pole with the largest magnitude (which is real and positive). This result is not proved in this paper.

Finally, we emphasize the point that just knowing the fact that the system is positive enables us to make some fairly strong statements about the behavior of the system
regardless of the precise values of the system parameters. More detailed studies are necessary to reveal other potential applications of positive systems theory in incoherent optical systems, such as iterative optical processors and optical baseband filters.

## V. Time-Domain Signal Processing Applications

In this section, we explain the operation of a new fiberoptic processor, the fiber-optic systolic matrix-vector multiplier [3], which is capable of performing high-speed matrix operations.

To realize this processor, a feed-backward fiber-optic lattice structure has been used. In this configuration, our processor also shares important features of Kung's systolic architecture [16]. Our experimental results are for a $2 \times 2$ (Toeplitz) matrix-vector multiplier, with $100-\mathrm{MHz}$ clock rate (2-m fiber loops). However, it should be noted that, by incorporating smaller fiber loops, and designing smaller components and a more compact geometry, the effective clock rate of the device might be readily increased to well over 10 GHz . Also, higher order lattices can be used to operate on matrices of higher dimension. This extension requires only additional couplers and fiber length. In general, this fiber structure has all the advantages of the lattice structures, described previously. Furthermore, in this application, it also simplifies (compared to the transversal filters) the data flow and detection by performing the summations optically within a single strand of fiber.

The operation of the systolic multiplier is described in connection with Figs. 11 and 12. As is seen in Fig. 11(a), this


Fig. 11. The scattering processor. (a) The counter-propagating guided waves in the two fibers interact in the couplers. $T_{d}$ is the round trip loop delay between adjacent couplers. (b) A directional coupler as the systolic inner product step processor. (c) Operation of the inner product step processor.
processor consists of two parallel fiber lines with several couplers (with the intensity coupling coefficients represented by $a_{i}$ ) distributed along the fibers. This device may also be referred to as a (single-mode) fiber-optic scattering processor (FOSP). The term "scattering" is used here in the signal processing context of [29]. Each coupler (see Fig. 11(b)) operates on the upper left and lower right inputs ( $X_{\text {up }}$ and $X_{\text {down }}$ ) to produce the upper right and lower left outputs ( $Y_{\text {up }}$ and $Y_{\text {down }}$ ). Mathematically, a general (not necessarily passive) linear coupler of this type can be described by (linear) transmission and reflection operators $t$,


Fig. 12. A systolic Toeplitz matrix-vector multiplier for $2 \times$ 2 matrices. ( $a_{i}$ ) are the coupling ratios of the three couplers. The output is the sum of the contributions made by each of the vector components.

## $\tau, r$, and $\rho$ according to

$$
\binom{Y_{\mathrm{up}}}{Y_{\mathrm{down}}}=\left(\begin{array}{ll}
t & \rho  \tag{25}\\
r & \tau
\end{array}\right) \cdot\binom{X_{\mathrm{up}}}{X_{\text {down }}}
$$

Since the delay between successive couplers can be incorporated easily into their characteristic matrices, our structure comprises a series of scatterers whose cascaded operation is described by Redheffer's scattering formalism [29]. Therefore, with appropriate couplers, the FOSP can simulate (with a very fast speed) many physical phenomena which are also described by the same scattering formalism [29].

As mentioned before, with conventional (passive) directional couplers [20], the FOSP can function as an analog matrix-vector multiplier which operates very much like a digital systolic array processor [16]. (See [30], [31], for a different optical systolic processor, which uses light-emitting diodes (LEDs) and a Bragg cell for input and a chargecoupled device (CCD) detector for output.) In a digital systolic system [16], data flow from the computer memory, passing through many processing elements before they return to memory. The interactions of the input data with the flowing partial results permit multiple computations for each 1/O memory access. The building block of systolic processors is the inner product step processor [16]. In the limit of weak coupling ( $0<a_{i} \ll 1$ ): $a_{i} \cdot X_{u p}+X_{\text {down }} \rightarrow$ $Y_{\text {down }}$, and $X_{\mathrm{up}} \rightarrow Y_{\mathrm{up}}$, so each coupler is an inner product step processor (see Fig. 11(b), (c)) and the basic time unit of the FOSP is determined by the loop delay $T_{d}$, which should match the switching time of the coupler. While the general systolic architecture can be two-dimensional, a FOSP, which uses mechanically or electronically controlled couplers, is one-dimensional, since only two of the three ports of the basic systolic step processor are optical channels. One of the more important applications of one-dimensional systolic arrays is in matrix-vector multiplication. As described in [16], the operation of the multiplier involves $2 N-1$ couplers, corresponding to the $2 N-1$ main- and off-diagonals of the given $N \times N$ matrix (in the special but important case of a banded matrix [16], less than $2 N-1$ will be
required [30]). As the components of the input vector are serially fed into the input line of Fig. 11(a), the matrix elements enter the couplers progressively in such a way that each coupler sees matrix elements from a single off- (or main-) diagonal. However, as long as fast switchable couplers are not available, the high inherent bandwidth of our fiber processor can be fully utilized only for matrix-vector multiplications which involve Toeplitz matrices (i.e., matrices in which all the elements along each diagonal are equal). Since convolutions and correlations are described by Toeplitz operators, this last limitation is not too severe.

The operation of the systolic multiplier with fixed weights is described in Fig. 12, for a $2 \times 2$ Toeplitz matrix. The matrix elements are represented by the coupling ratios $\left(a_{i}\right)$ which are assumed to be weak, thus eliminating recirculation of the input pulses. The components of the input vector enter the input line as a time sequence of light pulses whose spacings are equal to the round-trip loop delay $T_{d}$ between adjacent couplers. With incoherent light, the device is linear in the intensity of the optical guided waves, and the intensity of the light at the output line is the sum of the responses to the two components of the input vector. Referring to Fig. 12, we see that the matrix-vector product appears in a time slot that follows the first pulse. A similar result holds for general (Toeplitz) N -dimensional ma-trix-vector multiplication. Since such a matrix has $2 N-1$ independent elements, $2 N-1$ couplers will be required, and the $N$ components of the output vector will follow the ( $N-1$ )th output pulse.

In the above discussion, we have neglected recirculations and loop losses. However, as is evident from Fig. 12, proper operation of the multiplier depends only on the device impulse response rather than on the individual coupling ratios and loop losses. Therefore, for a given $N \times N$ (Toeplitz) [ $a_{i j}$ ] matrix, the $2 N-1$ couplers can be adjusted to yield an impulse response with pulse heights proportional to $a_{1, N}, a_{2, N-1}, \cdots, a_{i, N-i+1}, \cdots, a_{N, 1}$, respectively.

Although the outlined operation of the device admits only positive vector components and matrix elements, the extension to the general complex case is straightforward if one uses the previously developed techniques for bipolar operation of incoherent optical data processing [32]. As in the digital systolic case, the main advantage of the FOSP over current transversal matrix-vector multipliers [2], is its single-output nature which permits the use of small, and therefore fast detectors without sacrificing light-collection efficiency.

An experimental device, shown in Fig. 13, was built from


Fig. 13. The experimental device. The coupling ratio, $a_{i}$, of the $i$ th coupler was adjusted by sliding the two half-couplers with respect to each other. $T_{d}$ was approximately 10 ns . Negative electronic pulses were required for positive modulation of the laser light.
three directional couplers. After the three upper and three lower half-couplers [20] were spliced separately (by using a fusion splicer), the two sets were assembled to yield three (backwardly) interconnected couplers. The three couplers were manually adjusted to yield an impulse response with the first three pulse heights proportional to $a_{12}, a_{11}\left(=a_{22}\right)$, and $a_{21}$, respectively. The experimental output vectors, which are shown in Fig. 14(a), (b), follow the theoretical


Fig. 14. Experimental matrix-vector products in two different cases (arbitrary vertical scales). The upper traces show the electronic input pulses which are of negative polarity. The bias point of the laser source was set such that negative pulses were required for positive modulation of the laser light. The undershoots that follow the pulses result from the high-pass characteristics of the electronic circuitry. Note that the first and the fourth pulses are proportional to $a_{12} x_{1}$ and $a_{21} x_{2}$, respectively.
predictions quite accurately. Among the factors that determine the ultimate accuracy of the multiplier are incorrect settings of the impulse response, incorrect time delay between the input pulses, unequal loop delays, residual recirculation (for high-data-rate applications), and insufficient frequency bandwidth of the electronic components. (The undershoots that appear after the output pulses in Fig. 14 are due to the high-pass characteristics of the power splitters and the amplifier.)

The extension of time-domain signal processing applications of fiber lattice structures to such operations as code generation, correlation, and matched filtering, are relatively straightforward (although not experimentally demonstrated).

## VI. Frequency-Domain Signal Processing Applications

A common problem in the systems theory and signal processing areas is that of frequency filtering an input signal. By filtering a signal one attempts to remove unwanted frequency components from the input signal, so that the filtered signal closely resembles a desired signal.

There are different methods for filtering a signal [22], [33]. One method is through the use of a transversal filter (tapped-delay line [4]) whose tap weights are adjusted to realize the desired filtering operation. This transversal filter
is sometimes called a nonrecursive system or a finite-duration impulse response (FIR) filter. It should be mentioned that a system is called a nonrecursive system if its present output is a function of past and present inputs [33]. (In contrast, a system is called a recursive system if its output is a function of past outputs as well as past and present inputs.) Furthermore, nonrecursive filters introduce only zeros in the system transfer function, and therefore are known as all-zero filters.

In contrast to FIR filters, there is another class of filters whose impulse response has infinite duration in time. These filters are called infinite-duration impulse response (IIR) or recursive filters. Recursive filters introduce, in general, both poles and zeros (pole-zero filter). In a special case, recursive filters can introduce only poles; the name "all-pole filter" is used in this case.

As mentioned earlier, lattice structures are alternative filter forms. By varying the coupling coefficients of the lattice couplers, we can adjust the system transfer function and, therefore, shape the frequency response of the filter.

Here, we examine filtering properties of both nonrecursive and recursive fiber lattice filters. These two filter types can be combined to realize more complex filtering operations. An example of this combined form which is comprised of all-pole and all-zero lattice filters, in cascade, is also presented with a discussion of its use as an all-pass filter [33] (phase equalizer). This cascaded system has the feature that its poles and zeros can be adjusted independently of each other. Optical power efficiency calculations are also discussed.

## A. Nonrecursive (FIR) Lattice Filters

Fig. 5(b) (or Fig. 6(b)) shows an Nth-order nonrecursive fiber-optic lattice filter made of directional couplers. This nonrecursive structure has two feed-forward lines. This is in contrast with the recursive (IIR) lattice which has one feed-forward line and one feed-backward line.
Using Z-transform techniques, we obtain the following expression for the transfer function of the nonrecursive fiber lattice structure (the coefficient of the zeroth-order delay is taken equal to 1 ):

$$
\begin{align*}
H_{a z}(z) & =1+c_{1} z^{-1}+c_{2} z^{-2}+\cdots+c_{N} z^{-N} \\
& =\left(1-z_{1} z^{-1}\right)\left(1-z_{2} z^{-1}\right) \cdots\left(1-z_{N} z^{-1}\right) \tag{26}
\end{align*}
$$

where the $c_{i}^{\prime}$ 's are positive-valued and are functions of the coupling coefficients, the power loss or gain, and the $z_{i}^{\prime}$ s are the zeros of the transfer function. This transfer function is an all-zero type, with its poles located at the origin.
By expanding (26) into its real and imaginary parts and also using the fact that the $c_{i}$ 's are nonnegative variables, we can conclude that, for a positive nonrecursive filter, there cannot be any zero of the transfer function in the symmetric angular sector $|\Theta|<\pi / N$, where $\theta$ represents the polar angle in the $Z$-plane, and $N$ is the order of the system. This inequality can also be used to estimate the minimum order of the system required to realize a given zero pattern. Moreover, it is easy to show that, for such systems, not all the zeros can be located in the right-hand $Z$-plane, unless sections allowing negativity are incorporated.
Frequency responses of nonrecursive fiber lattice filters are similar to those of a transversal filter (tapped-delay line


Fig. 15. Frequency response of a two-coupler nonrecursive lattice filter, for $b_{0}=b_{1}=0.5$ (with no power loss or gain). (a) Log-magnitude response. (b) Phase response.
[4]). As an example, frequency responses (in one basic period) of a first-order nonrecursive fiber lattice filter, when coupling coefficients of both couplers are 0.5 , for equal intensity transmittances in the two fiber lines, are shown in Fig. 15. This filter can be used as a notch filter to block frequencies around $\omega T=\pi$, where $T$ is the time-delay difference between the two feed-forward fiber lines.

These nonrecursive fiber lattice filters have all the advantageous features [22], [33] of nonrecursive systems, such as inherent stability, good phase control properties (can design linear phase filters), in addition to all the features of lattice structures mentioned before. In particular, the fact that all the operations are done optically within the fiber, simplifies the detection system (compared to that of a fiber transversal filter), and also increases the light-collection efficiency.

One characteristic of nonrecursive filters, whether of the transversal or the lattice type, is that they cannot provide very sharp filtering unless the filter order, $N$, is very high. This is undesirable, since higher order filters are more difficult to fabricate and also more costly.

## B. Recursive (IIR) Lattice Filters

Fig. 5(a) (or Fig. 6(a)) illustrates an Nth-order fiber-optic recursive lattice filter, with the input specified as $X_{1}$ (or $X_{2}$ ) and two outputs as $\gamma_{1}$ and $\gamma_{2}$.
This filter, which has a structure similar to that of the fiber systolic array multiplier (see Section $V$ ), has one feedforward line and one feed-backward line. By using the output $\gamma_{1}$ on the backward line we can do matrix-vector multiplication operations, as described before.
Using the chain-matrix method (see Section IV-B) or (28) and (29), it can be shown that the transfer function, $H_{1}(z)$,
from $X_{1}$ to $Y_{1}$ has both poles and zeros which are dependent on each other. In other words, the poles and zeros cannot be adjusted independently of each other. This is, in general, an undesired feature that can be removed by using the output $Y_{2}$ on the forward line. It should be noted here that in the weak coupling regime $H_{7}(z)$ approximates an all-zero transfer function, which is desirable for matrix-vector multiplication operations.

Using the chain-matrix approach, we have also shown that the transfer function $H_{2}(z)$ from $X_{1}$ to $Y_{2}$ has zeros only at the origin or infinity and poles whose locations can be adjusted independently from the zeros.

Therefore, for frequency filtering applications it is mostly preferred to use the output $\gamma_{2}$. This allows realization of an all-pole filter. The transfer function of this all-pole filter can be represented, within a constant proportionality factor, by

$$
\begin{equation*}
H_{a p}(z)=\frac{1}{\left(1-p_{1} z^{-1}\right)\left(1-p_{2} z^{-1}\right) \cdots\left(1-p_{N} z^{-1}\right)} \tag{27}
\end{equation*}
$$

where the $p_{i}^{\prime} s$ are the poles of the transfer function.
In general, if the phase response of the filter is not important we can always design better filters using the IIR structures. This is true because the poles of the IIR filter can be arranged in such a way as to keep the magnitude of the frequency response flat in some specified frequency range,
and then the zeros can be used to force the frequency response to zero. As a result, low-order IIR filters can be used to produce extremely sharp frequency responses. This is the most advantageous feature of the IIR structures. Among the disadvantageous features [22], which stem from the existence of feedback loops, are complexity of the design, stability concern, and nonlinear phase response (it can be equalized to be arbitrarily close to linear, but intrinsically because of the closeness of the poles and zeros there can be a big phase change in the transition regions).

We have developed general expressions for the transfer functions of the above fiber lattice configurations. For the recursive fiber lattice case, $H_{1, n}$ and $H_{2, n}$ (the $n$ th-order transfer functions from $X_{1}$ to $Y_{1}$, and from $X_{1}$ to $Y_{2}$, respectively) are related to the transfer functions of the $(n-1)$ thorder filter by the following recursion formulas:

$$
\left\{\begin{array}{l}
H_{1, n}=\frac{a_{n}+\left(1-2 a_{n}\right) H_{1,(n-1)} \ell_{n} z^{-1}}{1-a_{n} H_{1,(n-1)} \ell_{n} z^{-1}}  \tag{28}\\
H_{2, n}=\frac{\left(1-a_{n}\right) H_{2,(n-1)} \ell_{1 n}}{1-a_{n} H_{1,(n-1)} \ell_{n} z^{-1}}
\end{array}\right.
$$

where the initial conditions are

$$
\left\{\begin{array}{l}
H_{1,0}=a_{0}  \tag{29}\\
H_{2,0}=1-a_{0}
\end{array}\right.
$$



Fig. 16. The $Z$-plane maps of possible pole locations for recursive fiber-optic lattice filters. These patterns were generated by randomly varying the coupling coefficient vector, $\boldsymbol{A}_{N}=\left(a_{0}, a_{1}, \cdots, a_{N}\right)$, where $0 \leqq a_{i} \leqq 1$, and $\ell_{i}=1$. (a) First order. (b) Second order. (c) Third order. (d) Fourth order. (e) Fifth order. Note that for the Nth-order lattice the coupling coefficient vector, $\boldsymbol{A}_{N}$, is defined as an $(N+1)$-element vector whose elements are the values of the coupling coefficients, $a_{i}$, i.e., $\boldsymbol{A}_{N}=\left(a_{0}, a_{1}, \cdots, a_{N}\right)$.
and the recursion index $n$ varies from 0 to the filter order $N$. The pole-zero locations of these transfer functions in the $z$-plane depend on the coupling coefficients and power losses or gains in the system.

For the first-through fifth-order recursive cases, the maps of possible pole locations are shown in Fig. 16(a)-(e). These maps were obtained by running a computer program that uses a random number generator to vary the elements of the coupling coefficient vector, $A_{N} \triangleq\left(a_{0}, a_{1}, \cdots, a_{N}\right)$, randomly between zero and one. This program then computes the roots of the corresponding characteristic polynomials. This process was repeated 5000 times for the first- and second-order filters, and 10000 times for the third-, fourth-, and fifth-order cases. As seen in the figure, for the first-order filter the pole is always (real) positive, and for the secondorder system both poles are real (the pole with the larger magnitude is positive-valued). For the third-order case, again, the pole with the largest magnitude is always posi-tive-valued, whereas the other two poles can be either both real or a complex conjugate pair. For the fourth- and fifth-order filters again the pole with the largest magnitude is always positive, and the rest of the poles can be either all real or some real and some complex conjugate pairs. Note that we assumed that the systems corresponding to the above patterns do not have any power loss or gain. In the case of power loss or gain, the patterns contract or expand, respectively. In the latter case the system might become unstable, for some cases. Similar maps were obtained also for other lattice orders. These plots show that the $Z$-plane filter pole patterns have regular characteristic branches angularly positioned at $\pm 2 \pi / n,(n=1,2, \cdots, N)$, where $N$ is the order of the lattice, as well as integer multiples of this expression. Information about pole-zero locations are helpful in filter design. As an example, Fig. 17 illustrates the frequency responses of third- and fifth-order all-pole lattices for two different settings of coupling coefficients. The values of the coupling coefficient vectors are chosen such that for one case all the poles of the corresponding filter transfer function are (real) positive, and for the other case there are both real and complex poles. Note that the presence of peaks in the frequency response curves (see Fig. 17) are indicative of the existence of complex conjugate pole pairs. The pole-zero diagrams of these examples are depicted in Fig. 18. Also, frequency responses of the systolic multiplier,


Fig. 17. Log-magnitude frequency responses of third-order and fifth-order all-pole fiber-optic lattice filters for two different sets (shown on the figure) of the coupling coefficient vector.


Fig. 18. Pole diagrams of the third- and fifth-order recursive lattice filter of the examples shown in Fig. 17.

(b)

(c)

Fig. 19. Frequency responses of the recursive lattice structure, used for matrix-vector multiplication. (a) Log-magnitude response in the frequency range $0-1200 \mathrm{MHz}$, for the case when the first three output pulses of the impulse response are equal. The upper trace shows the frequency response of the electronics. (b) Phase and log-magnitude responses, illustrated in one cycle ( $0-100 \mathrm{MHz}$ ). (c) Log-magnitude response in the frequency range $0-600 \mathrm{MHz}$, for the case when the coupling coefficient of the middle coupler is maximum ( $a_{1} \approx 1$ ).
used for matrix-vector multiplication, are shown in Fig. 19. Note that our experimental systolic multiplier was a sec-ond-order recursive lattice structure in the weak coupling regime (when the coupling coefficients are low). As it is clear from the figure, this regime of operation makes the
frequency responses of the systolic multiplier similar to those of a transversal filter.

The pole and zero diagrams shown above are for positive systems. Fig. 16 indicates that, for such systems, the $Z$-plane cannot be covered completely. This result is in contrast with that of lattice filters of the digital filtering theory. For such systems, the system parameters can assume both positive and negative values, and therefore they can cover more area of the $Z$-plane. For fiber lattice filters, $Z$-plane coverage increases as the order of the system is increased. Further flexibility is possible by combining positive sections with sections allowing negativity [11]. One such example is explained below.

## C. Combined fiber Lattice Filters

By cascading the all-zero nonrecursive lattice with the all-pole recursive lattice, described in previous sections, we can obtain more complex systems which have both poles and zeros. But now, the poles and zeros of the combined system, shown in Fig. 20, can be adjusted independently of


Fig. 20. Combined fiber lattice filter: recursive and nonrecursive sections cascaded. The detection system could allow negativity, if needed.
each other. As a result, we may arrange a desirable pole-zero pattern in the $Z$-plane to realize more complex frequency responses.
The total transfer function is given by the product of the two subsystem transfer functions, i.e., within a constant delay factor, we get

$$
\begin{align*}
H_{\text {iotal }}(z) & =H_{a z}(z) \cdot H_{a \rho}(z) \\
& =K \frac{\left(z-z_{1}\right)\left(z-z_{2}\right) \cdots\left(z-z_{M}\right)}{\left(z-p_{1}\right)\left(z-p_{2}\right) \cdots\left(z-p_{N}\right)} \tag{30}
\end{align*}
$$

where $K$ is a constant, the $\boldsymbol{z}_{i}^{\prime}$ s are the zeros, and the $p_{i}^{\prime} s$ are the poles of the transfer function.

Using the idea described above, we have designed a simple first-order all-pass filter which consists of a first-order all-zero lattice and a first-order all-pole lattice, as shown in Fig. 21 (the detection system is described later). An all-pass filter is a special case of an IIR filter in which the magnitude


Fig. 21. Schematic diagram of the first-order all-pass fiber filter.
of the frequency response of the filter is constant and only the phase response changes as the pole and zero positions vary. The impulse response of this filter is shown in Fig. 22(a). From this figure, it is clear that this fiber all-pass filter cannot be realized without incorporating a section allowing negativity. This is due to the fact that the impulse response


Fig. 22. First-order all-pass fiber filter. (a) Impulse response. (b) Pole-zero diagram.
is not positive for all times (it is positive at the origin and zero or negative at other times). In terms of pole-zero locations in the $z$-plane, this means that for every pole at $z=r e^{i \theta}$ there must be a zero at

$$
z=\frac{1}{r} e^{j \theta} .
$$

In other words, the poles and the zeros are reflections of each other with respect to the unit circle, as shown in Fig. 22(b) for the simple first-order case. Therefore, for the first-order all-pass filter, the transfer function is given, within a proportionality factor, by

$$
H(z)=\frac{z-1 /\left(a_{1} a_{0} l_{1}\right)}{z-a_{1} a_{0} l_{1}}
$$

where the parameters are as defined before. From the above transfer function we can conclude that

$$
\left|H\left(e^{j \omega \tau}\right)\right|=\frac{1}{a_{1} a_{0} \ell_{1}}
$$

which is constant for a fixed pole-zero position, whereas the phase is a nonlinear function of $\omega$. The reflection condition was met through the use of a subtractive detection system which allows negativity, and also by adjusting the coupling coefficients. The detection system, as shown in Fig. 23, consists of two photodiodes of opposite polarity


Fig. 23. The electronic subtractive detection system used in the system of Fig. 21.
whose outputs are added in a power combiner; the inputs to the photodiodes are the two fiber output lines of the nonrecursive section. The attenuators can be adjusted to set the ratio of the light intensities which are to be combined in the power combiner. In our experiment, due to high loss in the recursive section resulting from damaged components, we did not observe very good experimental results. However, the idea is feasible and obtaining the desired results would be only a matter of using better components. All-pass filters are important in practice [22], [33] since they can be used to equalize a given phase (or group delay). To show the operation of this all-pass filter, we have plotted, in Fig. 24, the theoretical frequency responses of the first-


Fig. 24. Frequency responses of the first-order all-pass fiber filter. (a) Magnitude response. (b) Phase response. 1) Recursive section. 2) Nonrecursive section. 3) Cascaded system with the subtractive detection system.
order all-zero and all-pole sections, along with those of the cascaded form (with the subtractive detection system).

In general, the fiber-optic lattice forms introduced above provide interesting structures for implementing both all-zero and all-pole filters. The extension of the filter to higher orders can be straightforwardly done by adding more stages to the lattice. Higher order subsystems should provide more degrees of freedom for the design of more complex frequency responses. Also, in-line optical amplifiers [34] can compensate for the propagation and coupling losses, and therefore will allow a very large number of recirculations in the fiber loops. This would improve the performance of the filters.

## D. Optical Power Efficiency Calculations

In general, due to the presence of some power loss associated with the directional couplers or the fiber, and/or multiplicity of the system output, there will be a nonzero
insertion loss. Therefore, for practical reasons, it is important to find out how much optical power would be available at the output.

Here, we show a simple way that uses the system transfer function, $H(z)$, to calculate the output optical power (or the optical power efficiency) for a given input power. For this purpose, it suffices to know that the optical power is proportional to the $\mathrm{dc}(f=0)$ frequency component of the baseband optical intensity signal. Noting that the dc frequency component corresponds to $z=1$ in the transfer function ( $z=e^{j 2 \pi i t}$ ), we obtain the following relation between the input and output optical powers:

$$
\begin{equation*}
P_{o}=[H(1)] P_{i} \tag{31}
\end{equation*}
$$

where $P_{i}$ and $P_{o}$ are the input and output optical powers, respectively. Therefore, the ratio of the output optical power to the input optical power, or the optical power efficiency, $\eta$, is equal to the value of the transfer function at $z=1$, i.e.,

$$
\begin{equation*}
\eta \triangleq \frac{P_{o}}{P_{i}}=H(1) \tag{32}
\end{equation*}
$$

Note that $H(1)$ is a (real) positive quantity for a positive fiber-optic systems of the type described previously.

As an example, for the two-coupler recirculating fiber lattice structure (see Fig. 3(a)) we have

$$
\begin{equation*}
\eta_{1}=\frac{a_{1}+\left(1-2 a_{1}\right) a_{0} l_{1}}{1-a_{1} a_{0} \ell_{1}} \tag{33}
\end{equation*}
$$

and

$$
\begin{equation*}
\eta_{2}=\frac{\left(1-a_{1}\right)\left(1-a_{0}\right) \ell_{11}}{1-a_{1} a_{0} l_{1}} \tag{34}
\end{equation*}
$$

where $\eta_{1}$ is for the case when $X_{1}$ is the input and $Y_{1}$ is the output, and $\eta_{2}$ is for the case when output is $Y_{2}$, rather than $Y_{1}$. Other related parameters are as defined before in (5) and (6). These equations were also used to find the expressions given in (33) and (34). From (33) and (34) we can conclude, for example, that by increasing $a_{0}$ the position of the pole moves closer to the unit circle, while the output power from $Y_{2}$ decreases and the output power from $Y_{1}$ increases.

As previously stated, by varying the parameters of our fiber systems, we can adjust the frequency response. These changes, in turn, affect the optical power efficiency, a factor which is of practical significance. Therefore, it is important to know the corresponding changes in the power efficiency factor, as the filtering characteristics of the system are altered.

## VII. NOise CONsiderations

Many single-mode fiber-optic signal processors [3], [5], such as the fiber recirculating lattice structures introduced before, make extensive use of recirculating delay lines. The simplest recirculating loop, as illustrated in Fig. 26, is constructed by closing a continuous strand of single-mode fiber on itself by using an adjustable coupler. This simple structure is equivalent to the two-coupler recirculating delay line (see Fig. 3(a)), when the coupling coefficient of the second coupler is set to unity.

As mentioned previously, in most fiber-optic signal processing applications an RF signal modulates the intensity of a short coherence length light source (usually a multi-
mode semiconductor laser) and the output is detected by a high-speed detector. Since the laser driving current and the detector output current are, respectively, proportional to the injected and detected light intensities, overall linearity is ensured only if the fiber system is also effectively linear in the intensities of the interacting waves. This requirement is easily met by using low-coherence sources, e.g., multimode diode lasers, with a coherence time much shorter than any relevant loop delay. The use of low-coherence sources has the additional advantage of rendering these devices relatively insensitive to environmental effects such as temperature changes, mild mechanical vibrations, etc.

The dynamic range of these wide-band devices is limited by the available source output power and nonlinearities of the source, detector, or the fiber and by the noise characteristics of the source radiation or the unavoidable detection shot noise. Since a short coherence time is associated with correspondingly large phase fluctuations (see (36)), the output noise of these signal processing devices, when driven by a short coherence laser diode, is expected to be correlated with the laser phase noise. This correlation was first noticed while we were experimenting with the fiber-optic systolic-array multiplier described before. We observed that the output pulses of the processor contain a noise as seen in Fig. 25. The optical source was a CW multimode laser


Fig. 25. Output pulses of the fiber-optic systolic array when inflicted by the phase noise.
diode. Fig. 25 should be compared with Fig. 14 for which the bias point of the laser was decreased and the laser was modulated by positive electronic pulses (rather than negative pulses for the case when the bias point is high). After further study of the above-mentioned observation it was demonstrated [17], [18] that the power spectrum of the output optical intensity of a single-mode fiber recirculating delay line driven by a multimode semiconductor laser exhibits a spectral structure with notches at zero frequency as well as at other integer multiples of the inverse of the loop delay. In addition, the contribution to the output noise due to laser phase fluctuations is much stronger than that due to laser intensity fluctuations.

The role of laser phase noise in nonrecirculating interferometric sensors [35] as well as in homodyne and heterodyne communication systems [36] has been extensively studied by many investigators. As predicted by these studies, the phase-induced output power spectrum attains its maximum at zero frequency, $f=0$ and, when the source coherence time $\tau_{c}$ is much shorter than the device delay $\tau$, the spectrum near $f=0$ is relatively flat within several $1 / \tau$


Fig. 26. Experimental setup for the measurement of the spectrum of the intensity noise at the output of a recirculating delay line, driven by a semiconductor laser. $E_{1}, E_{2}$ and $E_{3}, E_{4}$ are, respectively, the input and output fields. The optical power at the detector was $130 \mu \mathrm{~W}$, the loop length 27 cm , and the amplifier gain $\approx 40 \mathrm{~dB}$.
units [37]. Therefore, it appears that recirculating and nonrecirculating interferometers convert the laser phase noise into intensity noise in very different ways.

Fig. 26 describes the experimental setup used for the phase noise study. As shown, the mechanically polished evanescent field coupler [19] closes the delay line with no splices. The loop length was 27 cm which corresponds to a delay of $\tau=1.35 \mathrm{~ns}$. The continuous-wave GaAlAs multimode laser (GENERAL OPTRONICS Model GO-ANA) runs with no external modulation. The p-i-n photodetector is followed by a $1-1000-\mathrm{MHz}$ amplifier whose output is directly fed to the input of the spectrum analyzer. Curve $A$ of Fig. 27 describes the spectrum of the amplifier output when


Fig. 27. Experimental noise spectra: $\boldsymbol{A}$-the amplifier noise with the laser off; $B$-the laser intensity noise (the coupler is disassembled); and $C$-phase-induced noise with an assembled coupler (see Fig. 26).
the laser is off. Curve $B$ shows the output spectrum when the coupler is disassembled into its two halves, so that the laser light goes directly from the laser through the fiber into the detector. This spectrum is therefore characteristic of the
intensity noise of the laser. Curve $C$ was taken with an assembled coupler and a power coupling ratio of approximately 40 percent. Here we see a much stronger spectrum with two notches (within the $1-1000-\mathrm{MHz}$ range): one at $f=0$ and the other at a frequency of $1 / \tau(=740 \mathrm{MHz})$. Therefore the dynamic range of the loop, when the loop is operating as a filter, is limited not by the laser intensity noise but by a $20-\mathrm{dB}$ stronger noise whose origin was shown to be the laser phase noise. The observed noise spectrum was found to be highly dependent on the coupling ratio. In particular, when this ratio was either 0 or 100 percent the noise reduced to the level of curve $B$ in Fig. 27, and a coupling ratio of 40 percent was found to give the highest noise. Fig. 28 shows the spectrum for a $10-\mathrm{m}$ loop

(a)

(b)

Fig. 28. A recirculating delay line with a $10-\mathrm{m}$ loop. (a) Spectrum of the output intensity noise. The loop delay is 50 ns , and the frequency spacing between successive notches is 20 MHz . (b) Intensity impulse response of the $10-\mathrm{m}$ loop as determined by pulsing the laser with a $35-n s$ pulse. The input pulse is partially coupled out (the first pulse in the picture); after one circulation it is split again, and the second pulse in the picture represents its uncoupled part. The process repeats itself, and this time-domain display clearly shows several recirculations.
( $\tau=50 \mathrm{~ns}$ ) along with the intensity impulse response of the recirculating delay line (Fig. 28(b) was obtained by pulsing the laser with a single $35-n$ s pulse). Again, the spectrum is characterized by multiple notches equally spaced by $1 / \tau$. In this longer loop it was possible to incorporate a manually adjustable polarization controller which can change the state of polarization of the propagating wave without altering its degree of polarization [38]. As the state of polarization of the recirculating waves was varied, we observed up and down vertical shifts of a few decibels in the noise power spectrum, and Fig. 28 was obtained after the polarization controller was optimized to give maximum noise. Note that this spectral picture is complementary to the transfer function of the recirculating delay line when operating as a filter for RF modulation signals applied to the incident light [5]. It was experimentally determined that the above spectral structure was completely insensitive to loop-length variations on the order of an optical wavelength (indicating that $\tau \gg \tau_{c}$ ) and it was not the result of optical feedback from the device into the laser. To the best of our knowledge, laser-phase-induced noise, with the par-
ticular spectral structure [18] noted here, has not been reported previously.

In the following, we present a brief analysis of the phase-induced intensity noise at the output of a singlemode fiber recirculating delay line driven by a single-mode laser with a finite coherence time but negligible amplitude noise. A more comprehensive analysis will be published in a forthcoming paper.

## A. Mathematical Analysis

By using the coherent characteristics of the coupler as described by (1) or (2), and assuming a finite number of recirculations, it is possible to relate the observed spectral structure to the laser phase noise [17], [18]. Here, we analyze mathematically the experimental arrangement illustrated in Fig. 26. In our analysis we assume that the laser source emits a single longitudinal and transverse mode of the form

$$
\begin{equation*}
\boldsymbol{E}(t)=\boldsymbol{E}_{0} \exp \left[j\left(\omega_{0} t+\phi(t)\right)\right] \tag{35}
\end{equation*}
$$

where $E_{0}$ and $\omega_{0}$ denote, respectively, the time-independent laser complex field amplitude (no intensity noise) and the center optical frequency, and $\phi(t)$ represents the laser phase noise.

Laser phase fluctuations $\phi(t)$ generate a finite bandwidth around $\omega_{0}$. It is assumed that this source bandwidth is much smaller than $\omega_{0}$. We also assume that $\phi(t)$ is a Wiener random process with a structure function (defined as the variance of the increments) which is linear with the time increment $t_{2}-t_{1}$, i.e.,

$$
\begin{equation*}
D\left(t_{2}-t_{1}\right) \triangleq\left\langle\left[\phi\left(t_{2}\right)-\phi\left(t_{1}\right)\right]^{2}\right\rangle=\frac{\left|t_{2}-t_{1}\right|}{\tau_{c}} \tag{36}
\end{equation*}
$$

where $\tau_{c}$ is the coherence time of the laser. Note that the Wiener process is a stochastic time-independent increment process with stationary increments such that the increment densities are Gaussian with zero mean. Also, the coupler can be described by (1), where the characteristic matrix is assumed to be unitary.

While the state of polarization of a wave that propagates in the fiber generally changes, there exist two orthogonal eigenmodes $\boldsymbol{E}_{a}$ and $\boldsymbol{E}_{b}$ such that, when propagating from port 1 to port 2 through the loop, their polarizations are conserved. The phase velocities of these waves are different so that the two modes experience different delays $\tau_{a}$ and $\tau_{b}$. These time delays can be effectively changed by inserting a polarization controller [38] in the loop. For simplicity, the two modes are assumed to have the same intensity attenuation factor, $\exp \left[-2 \alpha_{0}\right]$.

As a result, all the fields in the system can be decomposed into their two eigenmode components; the input field can be written as

$$
\begin{equation*}
E(t)=g E_{a}(t)+h E_{b}(t) \tag{37}
\end{equation*}
$$

where $g$ and $h$ are both constants, and $E, E_{a}$, and $E_{b}$ are properly normalized so that

$$
\begin{equation*}
|g|^{2}+|h|^{2}=1 \tag{38}
\end{equation*}
$$

The field amplitude at the output of the recirculating delay line $\left(E_{4}\right)$ is the sum of contributions from an infinite number of recirculations (see Fig. 28(b)); therefore, using (1), for each of the decomposition components of the output field we have

$$
\begin{align*}
E_{4}= & (\gamma)^{1 / 2} C \exp \left(j\left[\omega_{0} t+\phi(t)\right]\right) \\
& +\gamma D A \sum_{n=1}^{\infty}\left((\gamma)^{1 / 2} B\right)^{n-1} \exp \left(-n \alpha_{0} L\right) \\
& \cdot \exp \left(j\left[\omega_{0}(t-n \tau)+\phi(t-n \tau)\right]\right) \tag{39}
\end{align*}
$$

where $\tau$ and $L$ are, respectively, the loop delay for the corresponding component and the loop length, and $\alpha_{0}$ is the fiber (amplitude) attenuation per unit length which was of the order of several decibels per kilometer [39].

The output of the ac-coupled spectrum analyzer is related to its input, $v(t)$, by the Wiener-Kinchine theorem, namely: the autocovariance function of $v(t),\left\langle\left[v\left(t+t^{\prime}\right)-\right.\right.$ $\langle v(t)\rangle][v(t)-\langle v(t)\rangle]\rangle(\rangle$ denote ensemble average) and the displayed spectrum, $S(f)$, are a Fourier transform pair. Since the spectrum analyzer input voltage is proportional to the output light intensity $I(t), S(f)$ is related to the autocovariance function of $l(t)$ by

$$
\begin{align*}
\operatorname{Cov}_{1}\left(t_{1}, t_{2}\right) & \left.=\left\langle I\left(t_{1}\right)-\langle I\rangle\right)\left(I\left(t_{2}\right)-\langle I\rangle\right)\right\rangle \\
& =P \int_{-\infty}^{+\infty} S(f) \exp \left[2 \pi j\left(t_{1}-t_{2}\right) f\right] d f . \tag{40}
\end{align*}
$$

The proportionality factor $P$ depends on the detector responsivity and the amplifier gain, as well as on the resolution setting of the spectrum analyzer. The observed spectrum of Fig. 28(a) can be approximated by $a+b \sin ^{2}(\pi f \tau)$ ( $a>0, b>0$ ), and using (40) we can conclude that the intensity autocovariance function has at least the following components: $(a+0.5 b) \delta\left(t_{1}-t_{2}\right)-0.25 b\left[\delta\left(t_{1}-t_{2}-\tau\right)+\right.$ $\delta\left(t_{1}-t_{2}+\tau\right)$ ], where $\delta(t)$ is the Dirac delta function. Although the above approximation is crude in that it predicts infinite signal energy, it still indicates that the intensity autocovariance function is characterized by a positive narrow peak at the origin ( $t_{1}-t_{2}=0$ ), and two negative narrow peaks at $t_{1}-t_{2}= \pm \tau$ (see Fig. 29). We now proceed to show that this form of the autocovariance function can actually be predicted from a model which takes into consideration both the laser phase noise and the unitary nature of the directional coupler.

The covariance function of the output intensity fluctuations, (40), can be calculated using (39). It should also be noted that since the two eigenmodes are orthogonal the output intensity at port 4 can be written as the sum of contributions from each eigenmode, i.e.,

$$
\begin{equation*}
I(t)=|g|^{2} I_{a}(t)+|h|^{2} I_{b}(t) \tag{41}
\end{equation*}
$$

with $I_{a}$ and $I_{b}$ being, respectively, the output intensities
due to the two eigenmodes $\boldsymbol{E}_{a}$ and $\boldsymbol{E}_{b}$. It can be shown that the autocovariance function of $l(t)$ (which is the inverse Fourier transform of the observed power spectrum) is given by

$$
\begin{align*}
\operatorname{Cov}_{1}\left(t_{1}, t_{2}\right)= & \left\langle\left[I\left(t_{1}\right)-\langle I\rangle\right]\left[I\left(t_{2}\right)-\langle I\rangle\right]\right\rangle \\
= & |g|^{4} C_{a a}\left(t_{1}, t_{2}\right)+2|g|^{2}|h|^{2} C_{a b}\left(t_{1}, t_{2}\right) \\
& +|h|^{4} C_{b b}\left(t_{1}, t_{2}\right) \tag{42}
\end{align*}
$$

where $C_{a a}$ and $C_{b b}$ are, respectively, the autocovariance functions of $I_{a}$ and $I_{b}$, and $C_{a b}$ is the cross-covariance of $I_{a}$ and $I_{b}$.

In order to obtain closed-form analytical results, the following assumptions are made:
a) $\tau_{c} \ll \tau$ [40] ( $\tau=$ the loop delay)
b) $\left|\tau_{b}-\tau_{a}\right| \ll \tau_{c}$ but $\omega_{0}\left|\tau_{b}-\tau_{a}\right| \approx 2 \pi$.

It should be mentioned that all our experimental setups meet these two conditions.

After some mathematical manipulation (to be published in a forthcoming paper) it can be shown that the autocovariance function has the following form:

$$
\begin{equation*}
\operatorname{Cov}_{1}\left(t_{1}, t_{2}\right)=T \cdot C_{a a}\left(t_{1}, t_{2}\right) \tag{43a}
\end{equation*}
$$

with the spectrums related to each other as shown below

$$
\begin{equation*}
S(f)=T \cdot S_{a d}(f) \tag{43b}
\end{equation*}
$$

Here, $T$ (which we call the birefringence factor) is polariza-tion-dependent and independent of $t_{1}$ and $t_{2}$, and is given by

$$
\begin{equation*}
T=|g|^{4}+2|g|^{2}|h|^{2}[\operatorname{Re} S]+|h|^{4} \tag{44a}
\end{equation*}
$$

where

$$
\begin{align*}
S= & \exp \left[j \omega_{0}\left(\tau_{b}-\tau_{a}\right)\right] \\
& \frac{1-\gamma \exp \left[-2 \alpha_{0}\right]|B|^{2}}{1-\gamma \exp \left[-2 \alpha_{0}\right]|B|^{2} \exp \left[j \omega_{0}\left(\tau_{a}-\tau_{b}\right)\right]}, \\
& \left.-1<\operatorname{Re} S \leqslant 1 \quad \text { (equal to } 1 \text { if } \tau_{a}=\tau_{b}\right) \tag{45}
\end{align*}
$$

It can also be shown that

$$
\begin{equation*}
C_{a d}\left(t_{1}, t_{2}\right)=C_{\delta}\left(t_{1}, t_{2}\right) \oplus \exp \left[-|t| / \tau_{c}\right] \tag{46}
\end{equation*}
$$

with $\oplus$ denoting the convolution operation and $C_{\delta}$ defined below.

Thus the adjustment of the polarization controller (which changes only $S$ ), will result only in vertical shifts of the output spectrum, and this fact was observed experimentally. Also, while it is possible to adjust the polarization controller such that the polarization of the first recirculated


Fig. 29. Autocovariance function of the output light intensity generated by the first four terms in the expansion of (39). The power coupling ratio was assumed to be 40 percent, i.e., $|B|=|C| \approx 0.63$ and $|A|=|D| \approx 0.78$ in (1).
wave is made orthogonal to the polarization of the input wave, the polarization of the second recirculation will again be parallel to that of the input wave. Thus it is impossible to completely eliminate the phase-induced intensity noise.
$\ln (46), C_{\delta}\left(t_{1}, t_{2}\right)$ is given by

$$
\begin{equation*}
C_{\delta}\left(t_{1}, t_{2}\right)=\sum_{M=-\infty}^{+\infty} C_{M} \delta\left[\left(t_{2}-t_{1}\right)-M \tau\right] \tag{47}
\end{equation*}
$$

$\delta(t)$ is the Dirac delta function and the intensities of the impulses are
as $\omega_{0}\left|\tau_{a}-\tau_{b}\right|$ varies. This dependence is illustrated in Fig. 32.

For the general case, when the ratio of the system delay to the coherence time $\left(\tau / \tau_{c}\right)$ can assume any value from 0 to large numbers, as the ratio $\tau / \tau_{c}$ is increased the variance of the output intensity also increases. This property was not demonstrated experimentally, but a computer plot (see Fig. 33) showing the above-mentioned behavior was generated for a coupling coefficient equal to 0.4 , for the case when there is no power loss or gain. As is seen in this figure, the

$$
C_{M}= \begin{cases}\frac{2|A C D|^{2} \gamma^{3} \exp \left(-2 \alpha_{0}\right)}{1-\gamma \exp \left(-2 \alpha_{0}\right)|B|^{2}}  \tag{48}\\ +\frac{2|A D|^{4} \gamma^{5} \exp \left(-6 \alpha_{0}\right)|B|^{2}}{\left[1-\gamma \exp \left(-2 \alpha_{0}\right)|B|^{2}\right]\left[1-\gamma^{2} \exp \left(-4 \alpha_{0}\right)|B|^{4}\right]}, & M=0 \\ -\frac{2 \gamma^{|M|+3}|A D|^{3}|C||B|^{2|M|-1} \exp \left[-2(|\mathcal{M}|+1) \alpha_{0}\right]}{1-\gamma \exp \left(-2 \alpha_{0}\right)|B|^{2}} \\ +\frac{2 \gamma^{|M|+5}|A D|^{4} \exp \left[-2(|M|+3) \alpha_{0}\right]|B|^{2|M|+1)}}{\left[1-\gamma \exp \left(-2 \alpha_{0}\right)|B|^{2}\right]\left[1-\gamma^{2} \exp \left(-4 \alpha_{0}\right)|B|^{4}\right]}, & \mathcal{M} \neq 0 .\end{cases}
$$

Since $A, B, C$ and $D$ are the elements of a unitary matrix, it follows [18] that all the $G_{M}$ with $M \neq 0$ are negative. Therefore, $C_{\delta}\left(t_{1}, t_{2}\right)$ is a symmetrical function of $t=t_{2}-t_{1}$. It is the sum of a positive impulse at $t=0$ and infinite number of equally spaced negative impulses at $t=$ $\mathcal{M} \tau(\mathcal{M} \neq 0)$. Fig. 29 shows four of these negative impulses.
The power spectral density is the product of the Fourier transforms of the two components of (46). Hence

$$
\begin{equation*}
S_{\mathrm{aa}}(f)=S_{\delta}(f) \frac{2 \tau_{c}}{1+\left(2 \pi f \tau_{c}\right)^{2}} \tag{49}
\end{equation*}
$$

with the total power spectrum given by (43b). $S_{\delta}(f)$, which is the Fourier transform of $C_{8}\left(t=t_{2}-t_{1}\right)$, is shown in Fig. 30 for various values of coupling coefficients $|B|^{2}$ and the loss $\alpha_{0}$. When the loop is lossless, infinitely deep notches appear, as seen in Fig. 30(a). But their depths decrease as the loop loss increases (see Fig. $30(\mathrm{~b})$ ). It is interesting to compare the loop behavior with the low-coherence source to its performance as a resonator with a highly coherent HeNe laser [39]. In the resonator case, a lossless loop is characterized by an infinite finesse. However, the loop transmission with the HeNe laser is extremely sensitive to micrometer-size variations in the loop length, while with our relatively incoherent source the characteristic spectrum is environmentally stable. Another interesting feature in Fig. 30 is the dependence of the form of the spectrum on the coupling ratio: the higher the coupling ratio, the flatter is the power spectral density within any given period. This dependence can be correlated with the fact that the effective number of recirculations increases with the coupling ratio. In Fig. 31(a) and (b) we have plotted curves showing the dependence of peak and notch heights of the spectrum, and the depth of the notch on the coupling coefficient for various loop transmittance factors. Also, Fig. 31(c) shows the depth of the notch as a function of the loop transmittance for various values of the coupling coefficient. Furthermore, the birefringence factor, $T$ (see (44)), changes


Fig. 30. The power spectral density of the autocovariance function, $C_{\delta}\left(t_{1}, t_{2}\right)$, see (47), (48). (a) A lossless loop ( $\gamma=$ $1, \alpha_{0}=0$ ). (b) $\gamma=1, \alpha_{0}=0.1$. 1) $|B|=\mid C=0.32$ and $|A|=$
$|D|=0.95$ in (1). 2) $|B|=|C|=0.63$ and $|A|=|D|=0.77$. 3)
$|B|=|C|=0.84$ and $|A|=|D|=0.55$. 4) $|B|=|C|=0.95$ and
$|A|=|D|=0.32$.
variance reaches its maximum saturation value for $\tau / \tau_{c}>$ - 4. This ratio was much higher in all our experiments.

The results obtained so far correctly predict, at least qualitatively, all the observed data, including:
a) the periodic form of the spectrum with a characteristic notch at the origin;


Fig. 31. Dependence of the peaks and notches and the depth of the notch of the output spectrum for the case when $\tau / \tau_{c} \gg 1$. (a) Peak and notch heights as a function of the coupling coefficient for various values of the loop transmittance factor. (b) Depth of the notch as a function of the coupling coefficient for various values of the loop transmittance factor. (c) Depth of the notch as a function of the loop transmittance factor for various values of the coupling coefficient.
b) the shape of the basic period as a function of the coupling coefficient;
c) the dependence of the spectrum on the setting of the polarization controller;
d) the insensitivity of the effect to environmental conditions (unlike the fiber-optic resonator).


Fig. 32. The birefringence factor (see (44)), $T$, as a function of polarization phase difference $\omega_{0}\left|\tau_{a}-\tau_{b}\right|$.


Fig. 33. The variance of the output intensity as a function of $\tau / \tau_{c}$. This curve is for the case when the coupling coefficient is equal to 0.4 , with no power loss or gain in the system. It is also assumed that the loop length is an integer multiple of the source wavelength.

Further investigation is required on the following issues:
a) Extension of the theory to a multimode laser.
b) More general theoretical treatment that will also include the laser intensity noise.
c) Quantitative experimental confirmations of the theoretical predictions.
d) Measurements of the phase-induced noise with sources with various values of coherence time $\tau_{c}$.
e) The study of the implications of the laser phase noise on the performance of single-mode fiber-optic systems.

## APPENDIX

In the following some of the related theorems of the positive systems theory [15], [27] are stated and then used to prove the relation given in (24).

## Theorem 1. The Frobenius-Perron Theorem

If $A>0$ (strictly positive), then there exists $\lambda_{0}>0$ and $x_{0}>0$ such that
a) $A x_{0}=\lambda_{0} x_{0}$;
b) if $\lambda \neq \lambda_{0}$ is any other eigenvalue of $A$, then $|\lambda|<\lambda_{0}$;
c) $\lambda_{0}$ is an eigenvalue of geometric and algebraic multiplicity 1 .

## Theorem 2

If $A \geqslant 0$ (positive or strictly nonnegative) and $A^{q}>0$ for some positive integer $q$, then all the conclusions of Theorem 1 apply to $\boldsymbol{A}$.

## Theorem 3

If $A \geqq 0$ (nonnegative), then there exists $\lambda_{0} \geqslant 0$ and $x_{0} \geqslant 0$ such that
a) $A x_{0}=\lambda_{0} x_{0} ;$
b) if $\lambda \neq \lambda_{0}$ is any other eigenvalue of $\boldsymbol{A}$, then $|\lambda| \leqslant \lambda_{0}$.

Lemma 1 (Series Expansion of Inverse): If $\boldsymbol{A}$ is a matrix with all eigenvalues strictly inside the unit circle $(|\lambda(A)|<$ 1), then

$$
\begin{equation*}
[I-A]^{-1}=I+A+A^{2}+A^{3}+\cdots \tag{A1}
\end{equation*}
$$

where $I$ is the identity matrix.

## Theorem 4

If $\boldsymbol{A} \geqq 0$ (nonnegative) with associated Frobenius-Perron eigenvalue $\lambda_{0}$, then the matrix $[\lambda I-A]^{-1}$ exists and is positive if and only if $\lambda>\lambda_{0}$.

## Some Matrix Inequalities

i) The Cauchy-Schwarz Inequality

$$
\begin{equation*}
[A \cdot B]^{+} \leqq A^{+} \cdot B^{+} \tag{A2}
\end{equation*}
$$

where the " + " sign (the matrix modulus) and the inequality signs are as defined earlier in Section IV-D. The equality sign in (A2) holds for the $i-j$ th elements if $a_{i k}$ and $b_{k j}$ are real and have like signs for all possible values of $k$. For the special case when $\boldsymbol{A}$ and $\boldsymbol{B}$ are both nonnegative matrices $[\boldsymbol{A} \cdot \boldsymbol{B}]^{+}=\boldsymbol{A} \cdot \boldsymbol{B}$.
ii) The Triangle Inequality

$$
\begin{equation*}
[A+B]^{+} \leqq A^{+}+B^{+} \tag{A3}
\end{equation*}
$$

Here the equality sign holds for the $i-j$ th elements when $a_{i j}$ and $b_{i j}$ are real and have like signs.

Using the theoretical background described so far, in the following section we will prove that the modulus of the transfer function (or frequency response) of a linear shiftinvariant positive system takes its maximum value at the origin (zero frequency).

## Proof that $\left[\boldsymbol{H}\left(\mathrm{e}^{j \omega T}\right)\right]^{+} \leqslant \boldsymbol{H}(1)$

Using $Z$-transform methods, the transfer function of a linear shift-invariant system, described by the state-space form of (22a), is given by

$$
\begin{equation*}
H(z)=D+C(z I-A)^{-1} B \tag{A4}
\end{equation*}
$$

where $\boldsymbol{H}$, the $(p \times m)$ transfer function matrix whose $i-j$ th element $H_{i j}$ is the transfer function from the $j$ th input to the $i$ th output, is shown in its expanded form in the following:

$$
H=\left(\begin{array}{cccc}
H_{11} & H_{12} & \cdots & H_{1 m}  \tag{A5}\\
H_{21} & H_{22} & \cdots & H_{2 m} \\
\cdot & \cdot & \cdots & \cdot \\
H_{p 1} & H_{p 2} & \cdots & H_{p m}
\end{array}\right)
$$

The frequency response of the system is, in general, a complex-valued function which is obtained by evaluating (A4) on the unit circle, $z=e^{j \omega T}$, in the complex $Z$-plane ( $\omega$ is the angular modulation frequency, and $T$ is the unit time delay of the system). Now taking the modulus of both sides of (A4), we will obtain the following expression for the magnitude of the frequency response of the system:

$$
\begin{align*}
{\left[\boldsymbol{H}\left(e^{j \omega T}\right)\right]^{+} } & =\left[\boldsymbol{D}+\boldsymbol{C}\left(e^{j \omega T} \boldsymbol{I}-\boldsymbol{A}\right)^{-1} \boldsymbol{B}\right]^{+} \\
& =\left[\boldsymbol{D}+\boldsymbol{C} e^{-j \omega T}\left(\boldsymbol{I}-e^{-j \omega T} \boldsymbol{A}\right)^{-1} \boldsymbol{B}\right]^{+} \tag{A6}
\end{align*}
$$

Since the transfer function is periodic (with period $\omega T=2 \pi$ ) and also symmetric (for real impulse response) about $\omega T=$ $\pi$, we can limit our discussion to the range $0 \leqslant \omega T \leqslant \pi$. Assuming a strictly stable system $(|\lambda(A)|<1)$ it is obvious that the matrix $e^{-j \omega T} A$ is stable too $\left(\left|e^{-j \omega T}\right|=1\right.$ ); therefore using Lemma I we can expand (A6) as

$$
\begin{align*}
& {\left[\boldsymbol{H}\left(e^{j \omega T}\right)\right]^{+}} \\
& \quad=\left[\boldsymbol{D}+\boldsymbol{C} e^{-j \omega T}\left(1+\boldsymbol{e}^{-j \omega T} \boldsymbol{A}+\boldsymbol{e}^{-2 j \omega T} \boldsymbol{A}^{2}+\cdots\right) \boldsymbol{B}\right]^{+} \tag{A7}
\end{align*}
$$

Now, applying the matrix inequalities (A2) and (A3) to (A7) we get

$$
\begin{align*}
{\left[\boldsymbol{H}\left(e^{j \omega T}\right)\right]^{+} \leqslant } & \boldsymbol{D}^{+}+\boldsymbol{C}^{+}\left|\boldsymbol{e}^{-j \omega T}\right| \\
& \cdot\left(\boldsymbol{I}+e^{-j \omega T} \boldsymbol{A}+e^{-2 j \omega T} \boldsymbol{A}^{2}+\cdots\right)^{+} \boldsymbol{B}^{+} \\
\leqslant & \boldsymbol{D}^{+}+\boldsymbol{C}^{+}\left(\boldsymbol{I}+\boldsymbol{A}^{+}+\left(\boldsymbol{A}^{2}\right)^{+}+\cdots\right) \boldsymbol{B}^{+} \tag{A8}
\end{align*}
$$

But from the positivity constraint on the system we have $A, B, C, D \geqslant 0$, which means that $A^{+}=A, B^{+}=B, C^{+}=C$, and $\boldsymbol{D}^{+}=\boldsymbol{C}$, therefore (A8) can be rewritten as

$$
\begin{equation*}
\left[H\left(e^{j \omega T}\right)\right]^{+} \leqslant D+C\left(I+A+A^{2}+\cdots\right) B \tag{A9}
\end{equation*}
$$

Moreover, since we have assumed a strictly stable system, from Lemma I we have

$$
\begin{equation*}
I+A+A^{2}+\cdots=(I-A)^{-1} \tag{A10}
\end{equation*}
$$

Plugging (A10) into (A9), we get

$$
\begin{equation*}
\left[\boldsymbol{H}\left(\mathrm{e}^{j \omega T}\right)\right]^{+} \leqslant \boldsymbol{D}+\boldsymbol{C}(I-A)^{-1} B \tag{A11}
\end{equation*}
$$

But

$$
\begin{equation*}
H(1)=D+C(I-A)^{-1} B \tag{A12}
\end{equation*}
$$

Therefore, comparing (A11) and (A12), we will finally obtain

$$
\begin{equation*}
\left[\boldsymbol{H}\left(e^{j \omega T}\right)\right]^{+} \leqslant \boldsymbol{H}(1) . \tag{A13}
\end{equation*}
$$

We also conclude that $\boldsymbol{H}(1)$ is real and positive, which also means that the phase of the transfer function is zero at $\omega=0$.

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OUTPUT PULSES $\omega+$
(a)

## $\left[\begin{array}{l}1.5 \\ 1.5\end{array}\right]=\left[\begin{array}{ll}1 & 1 \\ 1 & 1\end{array}\right]\left[\begin{array}{c}1 \\ 0.5\end{array}\right]$



OUTPUT PULSES $+\rightarrow$
(b)

Fig. 14. Experimental matrix-vector products in two different cases (arbitrary vertical scales). The upper traces show the electronic input pulses which are of negative polarity. The bias point of the laser source was set such that negative pulses were required for positive modulation of the laser light. The undershoots that follow the pulses result from the high-pass characteristics of the electronic circuitry. Note that the first and the fourth pulses are proportional to $a_{12} x_{1}$ and $a_{21} x_{2}$, respectively.


Fig. 19. Frequency responses of the recursive lattice structure, used for matrix-vector multiplication. (a) Log-magnitude response in the frequency range $0-1200 \mathrm{MHz}$, for the case when the first three output pulses of the impulse response are equal. The upper trace shows the frequency response of the electronics. (b) Phase and log-magnitude responses, illustrated in one cycle ( $0-100 \mathrm{MHz}$ ). (c) Log-magnitude response in the frequency range $0-600 \mathrm{MHz}$, for the case when the coupling coefficient of the middle coupler is maximum ( $a_{1} \approx 1$ ).


Fig. 25. Output pulses of the fiber-optic systolic array when inflicted by the phase noise.

(a)

(b)

Fig. 28. A recirculating delay line with a $10-\mathrm{m}$ loop. (a) Spectrum of the output intensity noise. The loop delay is 50 ns , and the frequency spacing between successive notches is 20 MHz . (b) Intensity impulse response of the $10-\mathrm{m}$ loop as determined by pulsing the laser with a $35-\mathrm{ns}$ pulse. The input pulse is partially coupled out (the first pulse in the picture); after one circulation it is split again, and the second pulse in the picture represents its uncoupled part. The process repeats itself, and this time-domain display clearly shows several recirculations.


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