# Fiber Optical System Performance with Avalanche Gain Detection

# FIBER OPTICAL SYSTEM PERFORMANCE WITH AVALANCHE GAIN DETECTION

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**Abstract -** The purpose of this paper is to consider optical system performance for selected digital signal formats when shot noise limited avalanche photodetection is implemented. Performance characteristics for binary on-off keyed (OOK) and pulse position modulation (PPM) are given, in addition to M-ary PPM and PAM.

## I. INTRODUCTION

Although many types of non-solid state photodetecting gain devices have been developed and well documented, new solid-state photodiodes (applicable to fiber optical systems) have gained prominence in the past decade. Such prominence is due largely to recent advances in semi-conductor fabrication. These devices are generally small, rugged, fast, and highly efficient.

Relative to optical communication implementation, the avalanche gain photodiode is the natural choice, since it far exceeds any comparable photodetection gain device in all aspects of comparison. The avalanche photodetector (APD) typically has average gains $\overline{G}$ ranging upward of two hundred and gain bandwidth products exceeding fifty gigahertz.

## II. GAIN DISTRIBUTION

Classically, the first approach in a device characterization is the determination of noise spectral density to perform SNR analysis. However, for a digital pulse-coded system a knowledge of noise spectrum alone is not sufficient to allow for prediction of bit error rates.

What is required, specifically in this case, is an avalanche gain distribution which accounts for the random generation (through the avalanche mechanism) of m carries outputted given an input of n initial carries; where the initial carries entering the avalanche region are themselves randomly created by the photon conversion process.

It has been shown, by several independent efforts [1], [6], [7], [9], that the probability distribution on m electrons generated by avalanche multiplication given the primary injection of n electrons (which are poisson distributed) is given by

$$P_m(m) = \sum_{n=1}^{\infty} \left[ \frac{n\Gamma\left(\frac{m}{1-k} + 1\right)}{m(m-n)!\ \Gamma\left(\frac{km}{1-k} + 1 + n\right)} \right] \cdot \left[ \frac{1 + k(\overline{G} - 1)}{\overline{G}} \right]^{n+km/(1-k)} \tag{1}$$

$$\cdot \left[ \frac{(1-k)(\overline{G} - 1)}{\overline{G}} \right]^{m-n} \cdot \frac{(\overline{n})^n}{n!} \exp\ (-n)$$

where $\overline{G}$ is equal to average APD gain and k is an ionization ratio coefficient which varies for different semi-conductor material. Typically, for use of a silicon APD, the nominal value of k is approximately 0.02 while for germanium it's approximately 0.500. $\overline{n}$ is the average count entering the avalanche region.

Additionally, a useful approximation of (1) which avoids computational difficulties is given by Webb [11]

$$P_m(m) = \frac{1}{(2\pi\overline{n}\ \overline{G}^2 F)^{1/2}} \cdot \frac{1}{\left(1 + \frac{(m - \overline{n}\ \overline{G})}{\sigma\lambda}\right)^{3/2}} \exp \left[ - \frac{(m - \overline{n}\ \overline{G})^2}{2\sigma^2 \left(1 + \frac{m - \overline{n}\ \overline{G}}{\sigma\lambda}\right)} \right] \tag{2}$$

where

$$F = k\overline{G} + \left(2 - \frac{1}{\overline{G}}\right)(1 - k)$$

$$\sigma^2 = \overline{n}\ \overline{G}^2 F$$

$$\lambda = (\overline{n}F)^{1/2}/(F - 1)$$

If $\overline{n}/F >> 1$, i.e. $\lambda$ large, then the approximation in (2) approaches the Gaussian distribution

$$P_m(m) = \frac{1}{(2\pi\overline{n}\ \overline{G}^2 F)^{1/2}} \exp \left[ - \frac{(m - \overline{n}\ \overline{G})^2}{2\sigma^2} \right] \tag{3}$$

Comparison of the actual distribution (1) with the approximations (2), (3) is given in Figure 1.

# III. BINARY DETECTION

## A. On-Off Keyed (OOK)

In many optical detection systems a signal is determined to be present if the output of a photodetector exceeds some pre-established decision threshold. Such is the case with the transmission of optical energy using the on-off keyed format.

The binary detection problem is to determine, in the presence of average background count rate $n_b$ the presence of $n_s$ or zero over a T second interval. This is equivalent to the detection circuit deciding between of of two hypothesis and making a declaration $H_1$ or $H_o$. Formally, the decision hypotheses are given as

$H_1$: Average signal plus noise count $\overline{n}_{s+b}$ producing m electrons given APD gain.

$H_o$: Average noise count $\overline{n}_b$ producing m electrons given APD gain.

where

$$\overline{n}_{s+b} = n_s T + n_b T$$

and

$$\overline{n}_b = n_b T.$$

The bit for bit decisions are made according to whether or not the average counts detected exceed an apriori threshold. The optimum choice of this threshold is given by the likelihood ratio test of decision theory [2]. This threshold value is optimum in the sense that it minimizes the probability of detection error.

The error probability for (OOK) can be determined once the threshold $m_T$, which is dependent on $\overline{G}$, k, $\overline{n}_{s+b}$, $\overline{n}_b$, has been established. Thus by denoting the optimum threshold by $m_T$ the minimum probability of error is given by

$$\begin{aligned} PE = {} & \frac{1}{2}\,[\text{Prob }(m) < m_T \text{ when one is sent}] \\ & + \frac{1}{2}\,[\text{Prob }(m) > m_T \text{ when zero is sent}] \\ & + \frac{1}{4}\,[\text{Prob }(m) = m_T \text{ when one is sent}] \\ & + \frac{1}{4}\,[\text{Prob }(m) = m_T \text{ when zero is sent}] \end{aligned}$$

Note that the last two terms account for the equalities in the binary test, in which case a random choice is made, with probability $\frac{1}{2}$ of being incorrect. If $m_T$ is not an integer, the last two terms will be zero.

Figure 2 gives the above performance calculation using approximations (2) and (3) with average avalanche gain $\overline{G} = 50$. In addition, actual points using (1) are indicated.

## B. Pulse Position Modulation (PPM)

A major disadvantage of using OOK transmission format is that the threshold for optimum detection requires apriori knowledge of the received signal and noise intensity plus APD characteristics. Thus for a fixed apriori threshold, performance will be suboptimal if the received field intensity changes or the APD characteristic varies. One binary signal format used to avoid this problem is pulse position modulation (PPM). This format requires sending one of two possible signals which can be specified in terms of hypothesis declaration as

$H_1$: Average signal plus noise count $\overline{n}_{s+b}$ producing m electrons given APD gain for $0 < t < T/2$ and average noise count $n_b$ producing m' electrons for $T/2 < t < T$.

$H_o$ : Average noise count producing m' electrons given APD gain for $0 < t < T/2$ and average signal plus noise count $\overline{n}_{s+b}$ producing m electrons for $T/2 < t < T$.

where

$$\overline{n}_{s+b} = n_s \cdot \frac{T}{2} + n_b \cdot \frac{T}{2}$$

and

$$\overline{n}_b = n_b \cdot \frac{T}{2}$$

For optimal performance based on a comparison test [2] the resulting PE is given by

$$PE = \sum_{m=1}^{\infty} \sum_{m'=m}^{\infty} \alpha_{m,m'} \, P_{m/H_1}(m) \, P_{m'/H_o}(m')$$

where

$$\alpha_{m,m'} = \begin{cases} \frac{1}{2} \text{ for } m = m' \\ 1 \text{ for } m \neq m' \end{cases}$$

Again using digital computation, Figure 3 reflects performance of the binary PPM signal set using the approximation (2).

# IV. M-ARY DETECTION

## A. M-ary PPM

An important application employing a block encoded orthogonal signal set is that of pulse position modulation (PPM). In a pulse position format the T sec interval is divided into M time slots, and an optical pulse is placed in one of the $i^{th}$ time slots to represent each block word. More formally, we would expect for shot noise limited operation to receive one of M signals, in an average background noise count, given by (4).

$$M_i = \begin{cases} \bar{n}_b & 0 \le t < \frac{(i-1)}{M} T \\ \bar{n}_{s+b} & \frac{(i-1)T}{M} \le t < \frac{iT}{M} \\ \bar{n}_b & \frac{iT}{M} \le t < T \end{cases} \tag{4}$$

In terms of performance, it can be shown [2] that the probability of a word error (PWE) given the M-ary PPM signal format and shot noise limited operation with avalanche detection is

$$PWE = 1 - \sum_{m=1}^{\infty} P_{m/H_1}(m) \left[ \sum_{m'=1}^{m-1} P_{m'/H_0}(m') \right]^{M-1}$$

Figure 4 represents the PWE using equation (2) for a value of gain $\overline{G} = 50$ and M = 2, 4, 8, 16.

## B. M-ary PAM

Although PPM optical signaling is favorable for most block coded operations, it does have certain disadvantages. For example, it requires relatively wide intensity bandwidths which implies extremely accurate synchronization and hardware implementation. To circumvent some of the disadvantages inherent with PPM other forms of block encoding may be more applicable. One such possibility is multilevel amplitude modulation (MPAM).

In MPAM, each block word is transmitted as an optical pulse with a different intensity level $\bar{n}_{s_i}$ . After counting over each block interval a decision is made as to which level is being received. The decision is made according to a computed likelihood function based on apriori knowledge of the selected intensity levels $\bar{n}_{s_i}$ and characteristics of the detector. That is to say, if we use the approximation (2), the receiver must compute the likelihood function

$$\bar{\Lambda}_i = \bar{G}\ \ln\ (\bar{n}_{s_i+b}) + 3\bar{G}\ \ln\left(\frac{m(F-1)}{\bar{n}_{s_i+b}}\right) + \frac{(m - \bar{n}_{s_i+b}\bar{G})^2}{m(F-1) + \bar{n}_{s_i+b}\bar{G}} \tag{5}$$

and the j[th] word in the block code is selected if

$$\bar{\Lambda}_j = \min\ (\bar{\Lambda}_i) \qquad i = 1,\ M$$

Once the optimum M-1 thresholds have been selected the probability of a word error, assuming non-integer thresholds, is given by

$$PWE = \frac{1}{M} \sum_{i=1}^{M} \Big[ \{\text{Prob}\ (m \leq m_{T_i-1}) | \text{intensity } i \text{ sent}\} + \sum_{i=1}^{M} \{\text{Prob}\ (m > m_{T_i}) | \text{intensity } i \text{ sent}\} \Big] \tag{6}$$

where $m_{T_i-1}$ and $m_{T_i}$ are the lower and upper thresholds with $m_{T_0} = 0$ and $m_{T_M} = \infty$.

The intensity levels can be either spaced uniformly or non-uniformly That is, if $\bar{n}_s$ is the upper intensity level based upon energy restrictions, and the remaining levels are selected on a uniform spaced basis, then

$$\bar{n}_{s_i} = \left(\frac{i}{M}\right) \bar{n}_s \tag{7}$$

Word error probabilities given by (6) and using (7) have been determined for gains $\bar{G}$ = 25, 50 and M = 2, 3, 4, 8 [Ref. Figure 5].

Additionally, Figure 5 contains multimode poisson for comparison when M = 3.

## CONCLUSION

It has been the purpose of this paper to consider selected digital signal performance when shot noise limited avalanche photodetection is implemented. The most notable results are performance degradation with increased avalanche gain. That is to say, with increased gains, the system designer is sacrificing performance compared to unity gain shot noise limited poisson characteristics.

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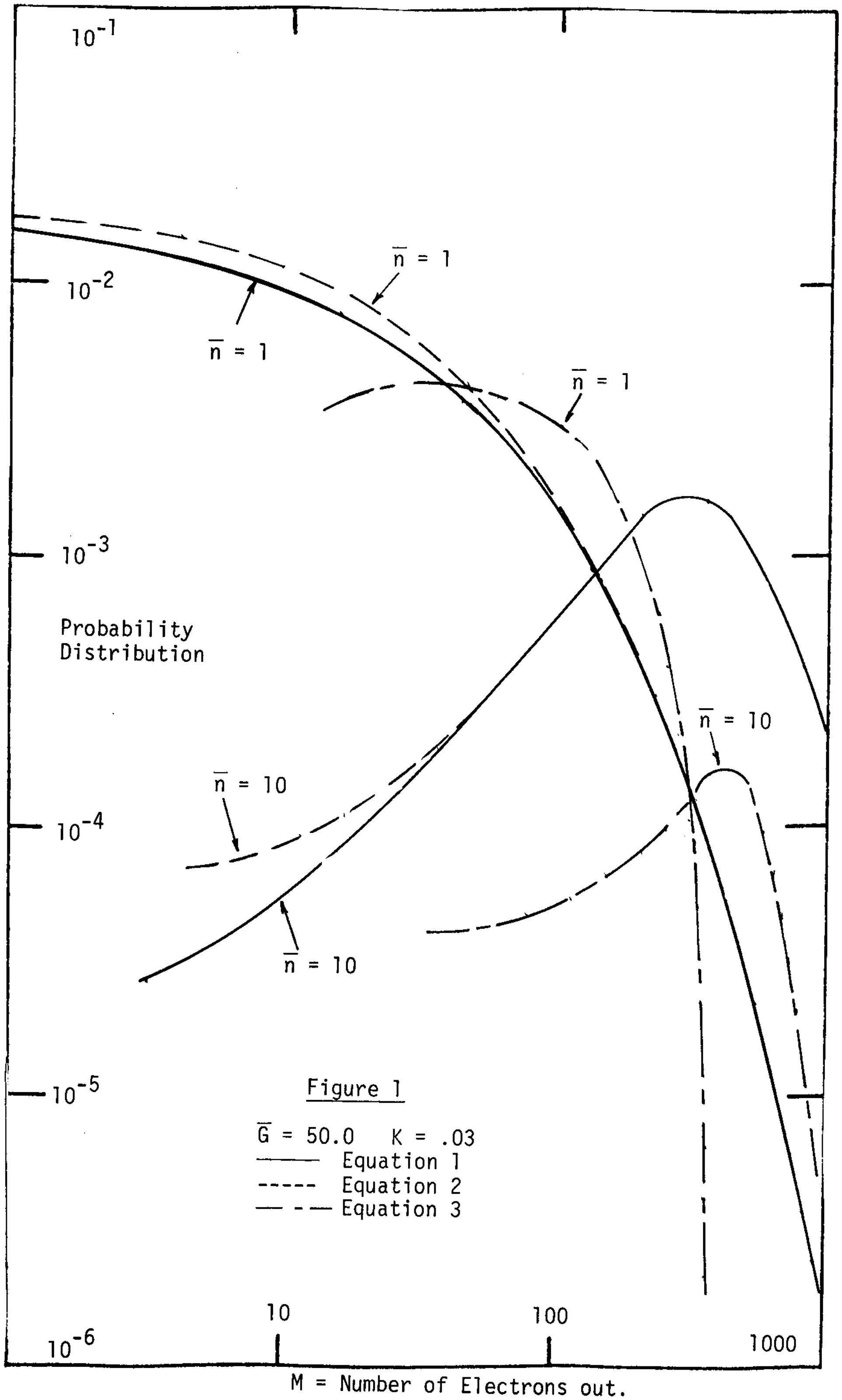


Figure 1

$\bar{G} = 50.0$   K = .03
——— Equation 1
----- Equation 2
— — — Equation 3

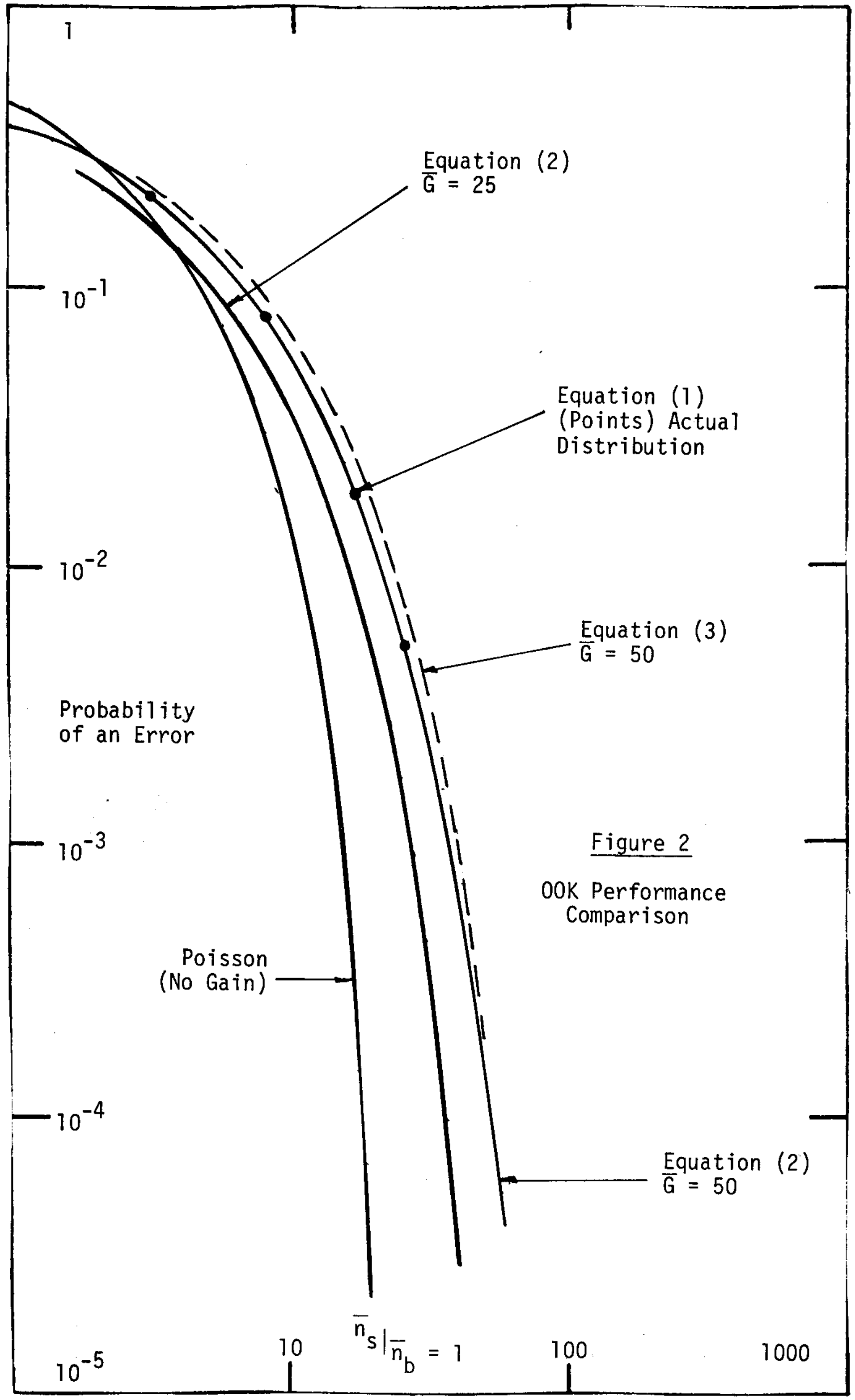


Figure 2

OOK Performance Comparison

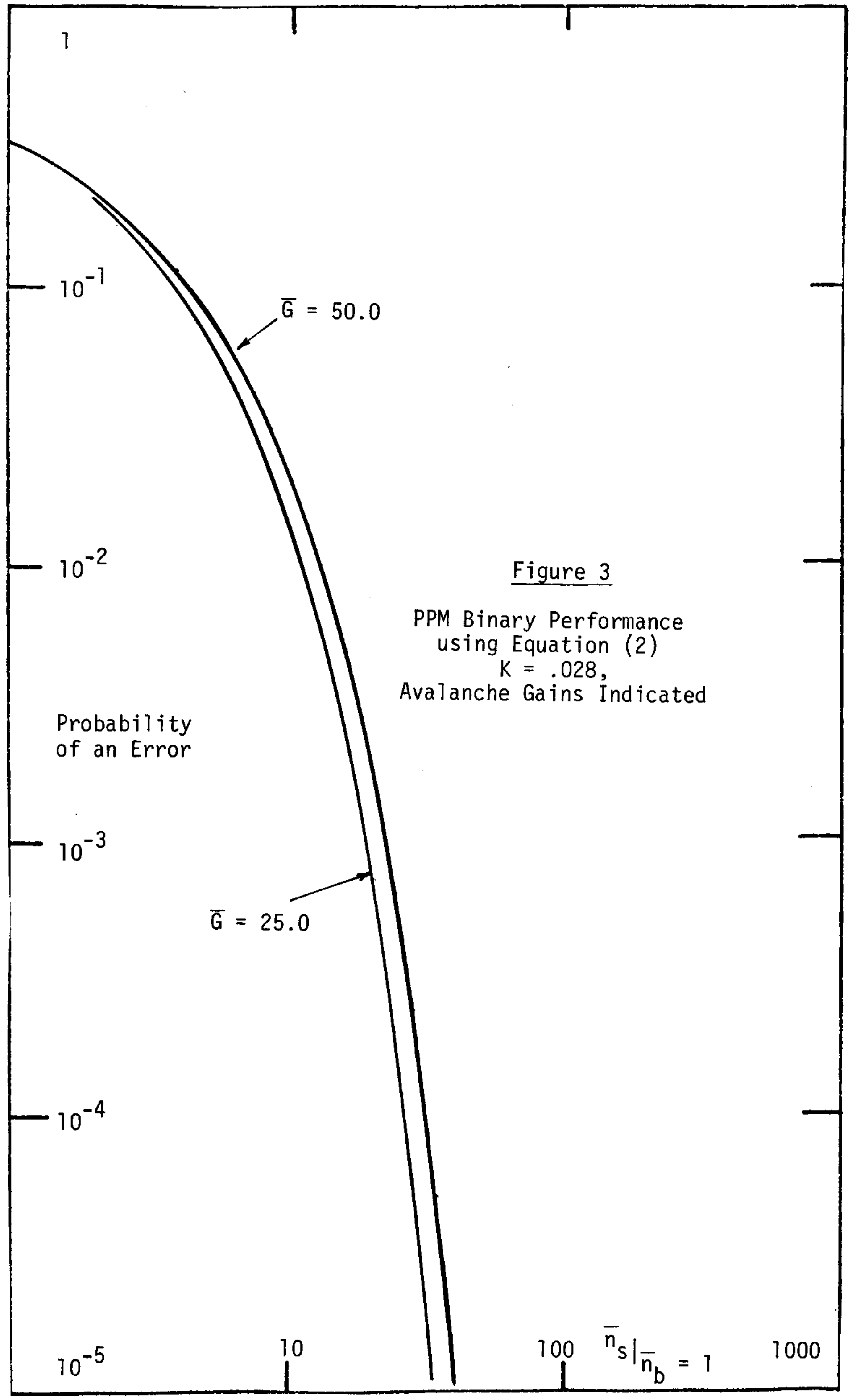


Figure 3

PPM Binary Performance
using Equation (2)
K = .028,
Avalanche Gains Indicated

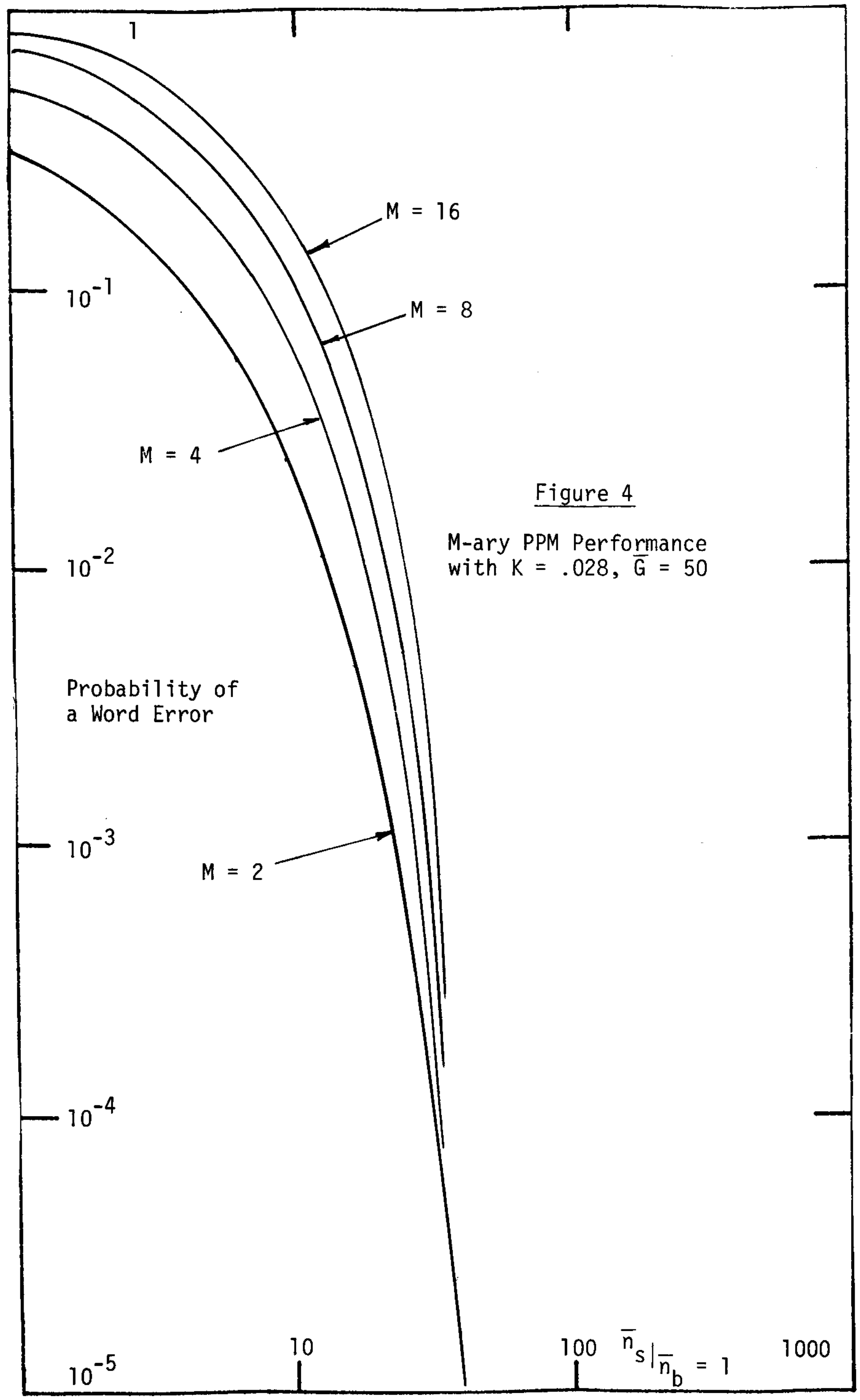


Figure 4

M-ary PPM Performance with K = .028, $\bar{G}$ = 50

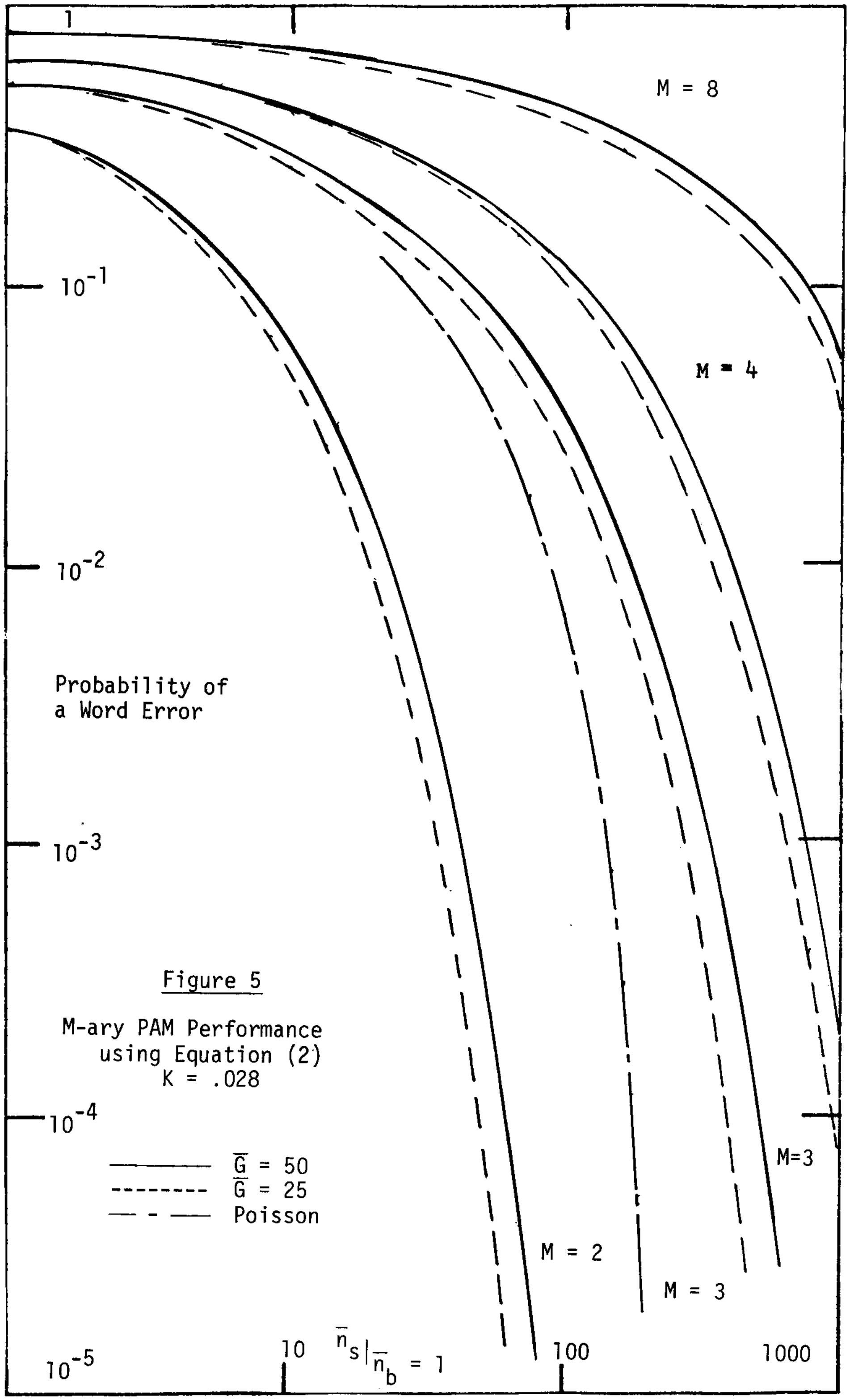


Figure 5

M-ary PAM Performance
using Equation (2)
K = .028