

# Fiber Orientation Factor on Rectangular Cross-Section in Concrete Members

Seong-Cheol Lee, Jeong-Hwan Oh, and Jae-Yeol Cho

**Abstract**—In order to predict the post-cracking tensile behavior of fiber reinforced concrete, it is necessary to evaluate the fiber orientation factor which indicates the number of fibers bridging a crack. For investigation of fiber orientation factor on a rectangular section, in this paper, dog-bone fiber reinforced concrete specimens were prepared with the variables of concrete compressive strength, rectangular cross-section size, fiber type, and fiber volumetric ratio. After direct tension tests, the fiber orientation factor could be evaluated through counting the number of fibers on a crack. From the test results, it was investigated that the fiber orientation factor was larger than 0.5 which is generally adopted for large members, as fibers distribution is affected by the specimen size. For rational prediction of the fiber orientation factor on a rectangular concrete section, a simple model was derived from the Diverse Embedment Model (DEM), which is a rigorous model to predict the tensile behavior of steel fiber reinforced concrete. From the comparison of the measured data and the predicted values, it was found that the actual fiber orientation factor was well predicted by the proposed model.

**Index Terms**—Fiber orientation factor, fiber reinforced concrete, rectangular cross-section, steel fiber.

## I. INTRODUCTION

For last a couple of decades, a number of researches [1]-[4] have been conducted to use steel fiber reinforced concrete as a structural member since steel fiber reinforced concrete exhibits ductile tensile behavior even after cracking. Meanwhile, several models [5]-[11] have been proposed in order to quantitatively represent the ductile tensile behavior of steel fiber reinforced concrete. From the models for the tensile behavior of steel fiber reinforced concrete, the tensile stress attained by steel fibers bridging a crack is generally evaluated by the followings;

$$f_f = \alpha_f V_f \sigma_{f,avg} \quad (1)$$

where,  $f_f$  is tensile stress attained by steel fibers,  $\alpha_f$  is fiber orientation factor,  $V_f$  is fiber volumetric ratio, and  $\sigma_{f,avg}$  is average tensile stress of fibers at a crack.

As presented in the equation, the tensile stress of steel fiber reinforced concrete is directly affected by the fiber orientation factor. Generally, the fiber orientation factor is

0.5, which is theoretically derived for randomly distributed fibers in large concrete members. However, the fiber orientation factor can be significantly affected by the concrete member size; it reaches up to 1.0 when the concrete cross-section is the same with the cross-section of a fiber. Since experimental programs with steel fiber reinforced concrete are generally conducted with relatively small specimens in a laboratory, it is important to more rationally evaluate the fiber orientation factor with the consideration of concrete member size as well as fiber length.

In this paper, therefore, the fiber orientation factor in small concrete members will be experimentally evaluated, and a simple model will be derived for the fiber orientation factor considering concrete member size and fiber length.

## II. TEST PROGRAM FOR FIBER ORIENTATION FACTOR

### A. Test Variables, Material Properties, and Test Procedure

In order to investigate fiber orientation factor on rectangular concrete section, dog-bone fiber reinforced concrete specimens with rectangular cross-section of 70×100 mm or 50×50 mm were prepared as illustrated in Fig. 1 and 2. Test variables were concrete compressive strength, fiber type, and concrete cross-section size. Notations for test variables are presented in Fig. 3, and the concrete mixture design and fiber details are presented in Table I and Table II, respectively.

TABLE I: FIBER REINFORCED CONCRETE MIX PROPORTION

	W/B	Unit weight (kgf/m <sup>3</sup> )					AD
		W	C	SF	S	G	
N	0.35	200	572	-	798	627	1.43
H	0.25	200	737	64	667	569	6.01

TABLE II: PROPERTIES FOR END-HOOKED STEEL FIBERS

Fiber type	Length (mm)	Diameter (mm)	Tensile strength (MPa)	Aspect Ratio
$F_1$	50	1.05	1000	47.6
$F_2$	35	0.55	1100	63.6
$F_3$	30	0.38	2300	78.9

All specimens had been subjected to uniaxial tension until they were entirely separated due to a crack, then the number of fibers on crack surfaces were counted. From the number of fibers ( $N_f$ ), the fiber orientation factor can be evaluated from the cross-sectional area of fiber ( $A_f$ ), fiber volumetric ratio ( $V_f$ ), and cross-sectional area of concrete member ( $A_c$ ), as following equation [12], [13].

$$\alpha_f = (N_f A_f) / (V_f A_c) \quad (2)$$

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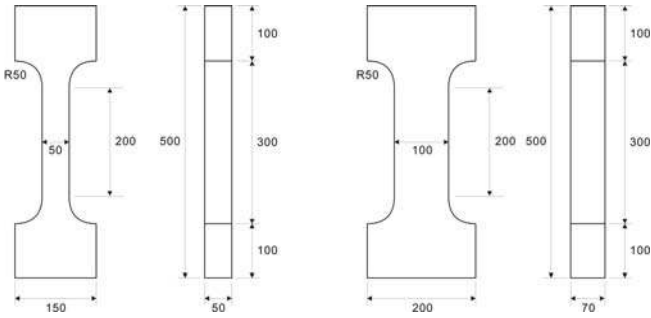


Fig. 1. Details of the specimens.

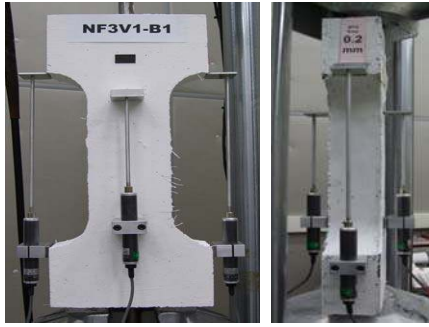


Fig. 2. Test set-up.

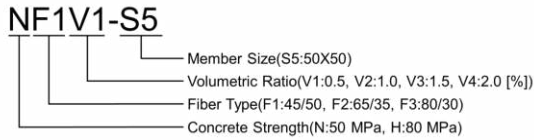
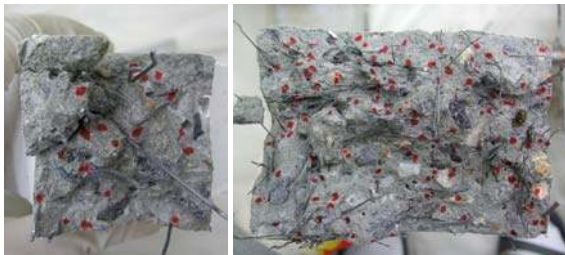


Fig. 3. Notations for the test variables.



(a) NF1V3-S5 (b) NF3V3  
Fig. 4. Counting the number of fibers.

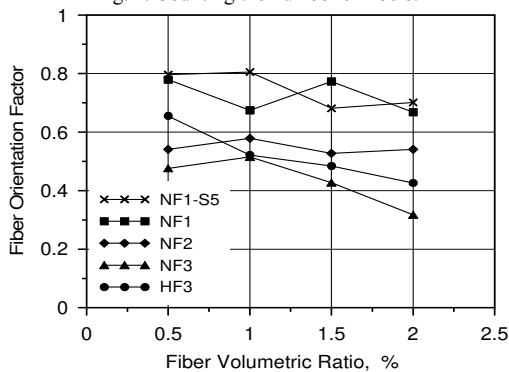


Fig. 5. Test results for fiber orientation factor.

### B. Test Results

The fiber orientation factors observed through the number of fibers on a crack (see Fig. 4) are presented in Fig. 5 and Table III. It is noted that each point in the graph represents the fiber orientation factor averaged through four specimens. As presented in the figure, the fiber orientation factor with F1 on the small section was high because fiber distribution is

significantly affected by the boundary surfaces of the rectangular cross-section. (Compare NF1 and NF1-S5.) In the case of NF3 and HF3 specimens, it was investigated that the fiber orientation factor generally decreases as the fiber volumetric ratio increases, since fibers were not evenly distributed because of fiber bundle easily found with small fibers.

TABLE III: MEASURED FIBER ORIENTATION FACTOR

Fiber type	Fiber volumetric ratio (%)	Number of fibers on a crack	Fiber orientation factor
NF1V1-S5	0.5	12	0.797
NF1V2-S5	1.0	23	0.805
NF1V3-S5	1.5	30	0.681
NF1V4-S5	2.0	41	0.701
NF1V1	0.5	32	0.779
NF1V2	1.0	55	0.674
NF1V3	1.5	94	0.773
NF1V4	2.0	108	0.668
NF2V1	0.5	80	0.541
NF2V2	1.0	171	0.579
NF2V3	1.5	233	0.527
NF2V4	2.0	319	0.541
NF3V1	0.5	147	0.476
NF3V2	1.0	318	0.515
NF3V3	1.5	396	0.427
NF3V4	2.0	392	0.318
NF3V1	0.5	202	0.655
NF3V2	1.0	322	0.522
NF3V3	1.5	448	0.484
NF3V4	2.0	526	0.426

### III. SIMPLE MODEL FOR FIBER ORIENTATION FACTOR

#### A. Derivation of a Simple Model

In the Diverse Embedment Model (DEM) [9], [10], the fiber orientation factor on rectangular concrete cross-section is evaluated through double numerical integration, so it is quite complicating. In this paper, through regression procedure to remove the double numerical integration, a simple fiber orientation factor has been derived as followings.

$$\alpha_f = -0.1 \left( \left( \frac{h}{l_f} \right)^{2.8} + \left( \frac{b}{l_f} \right)^{2.8} \right) + 1 \quad (3)$$

$$\text{for } \frac{h}{l_f} \leq 1.0, \frac{b}{l_f} \leq 1.0$$

$$\alpha_f = - \frac{(0.05(b/l_f)^{2.8} ((h/l_f)^{1.12} + 1) - 0.26)}{(h/l_f)^{1.12}} + 0.64 \quad (4)$$

$$\text{for } \frac{h}{l_f} \leq 1.0, \frac{b}{l_f} > 1.0$$

$$\alpha_f = \frac{0.13}{(bh/l_f^2)^{1.12}} + 0.087 \left( \left( \frac{l_f}{b} \right)^{1.12} + \left( \frac{l_f}{h} \right)^{1.12} \right) + 0.5 \quad (5)$$

$$\text{for } \frac{h}{l_f} > 1.0, \frac{b}{l_f} > 1.0$$

In Fig. 6, the predictions of the simple fiber orientation factor model are compared with the results of the DEM. As

presented in the figure, the both DEM and the proposed model predicts that the fiber orientation factor for infinite concrete cross-section is 0.5, and it increases up to 1.0. In addition, the fiber orientation factor increases more quickly when width or height of concrete cross-section is less than two times of the fiber length, since fiber orientation is significantly affected by more than one boundary surface together. As compared in the figure, it can be concluded that the simple fiber orientation factor model shows good agreement with the DEM.

**B. Verification of the Simple Model**

Fiber orientation factors for the test specimens have been evaluated with the proposed simple model, and compared with the test results as presented in Table IV and Fig. 7. As presented in the figure, the actual fiber orientation factors measured through the test program are well predicted by the proposed simple model; average and standard deviation are 0.938 and 0.197, respectively, for the ratio of the test result to the prediction.

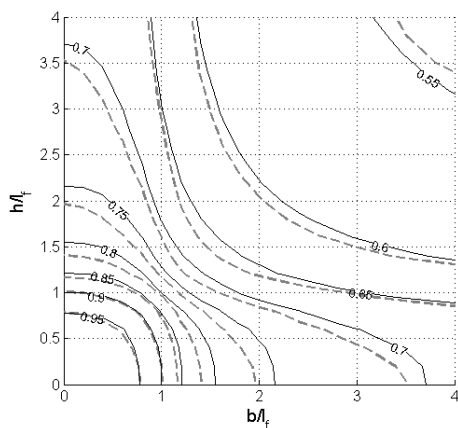


Fig. 6. Comparison of the simple model and the DEM.

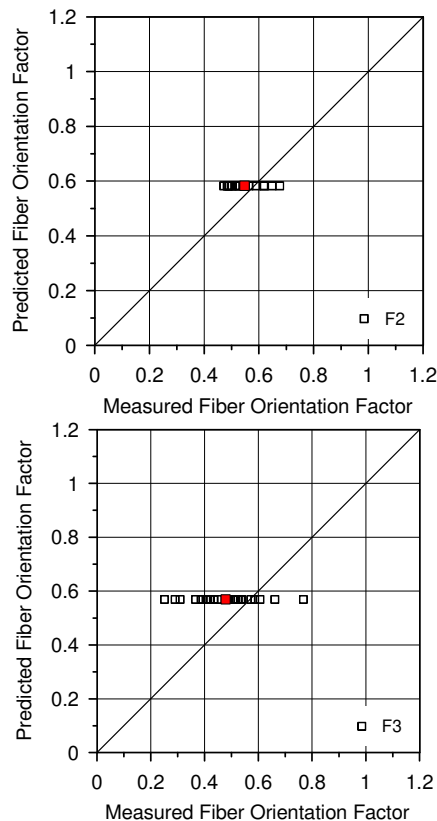
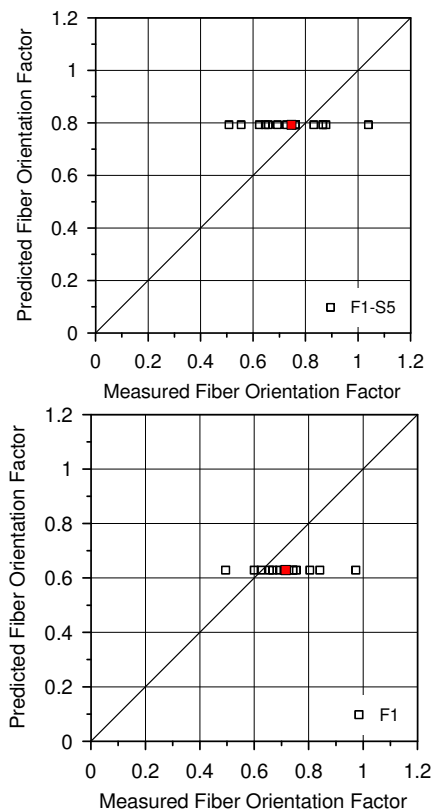


Fig. 7. Comparison of the prediction and the test results on fiber orientation factor.

TABLE IV: COMPARISON OF THE PREDICTION WITH THE TEST RESULTS ON FIBER ORIENTATION FACTOR

Fiber type	Fiber orientation factor		Measured / Prediction	Standard deviation
	Measured	Predicted		
F1-S5	0.746	0.793	0.941	0.174
F1	0.716	0.629	1.138	0.175
F2	0.547	0.583	0.939	0.108
F3	0.547	0.569	0.840	0.188
Total			0.938	0.197

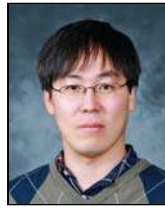
**IV. CONCLUSIONS**

In this paper, fiber orientation factor on rectangular cross-section in concrete members were investigated through test program with dog-bone specimens. From the test results, it was investigated that fiber orientation factor was significantly affected by the concrete member cross-section size and the fiber length; fiber orientation factor increases as the cross-section size-to-fiber length ratio decreases. In order to predict fiber orientation factor considering the size effect, a simple model has been derived. The predictions of the proposed simple model showed good agreement with the test results as well as the predictions of the previous rigorous model. The results presented in this paper can be useful for researches on structural behavior of concrete members and structures with steel fibers.

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