Fiberoptic Circuit Network Design Under Reliability Constraints

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Abstract—**The network design problem with reliability constraints is a problem where, given a finite set of nodes, the objective is a cost-efficient selection of links and link capacities sufficient to satisfy the node-to-node traffic demands in normal and failed conditions. In this paper, we present a general mathematical model for this problem and a revised formulation which seems particularly appropriate for fiber-optics networks. We also describe upper and lower bounding procedures based on continuous relaxations of this modified formulation. Preliminary computational results are reported; they seem to indicate that the proposed bounds might prove suitable for a branch-and-bound approach of the problem.**

I. Introduction

GIVEN circuit requirements between origin and destination points in an area to be serviced together with the locations of switching centers and a set of candidate links, the objective of the design is to select a subset of links and to assign capacities to them so that those circuit requirements are accommodated in a cost-effective fashion.

This problem has been addressed by many researchers in the past [1], [2], [12]–[14], and a variety of mathematical formulations, exact and heuristic procedures for solving it, have been suggested. Almost all of the earlier formulations do not consider explicitly reliability constraints as part of the formulation (many of them evaluate the impact of the generated design on network reliability as part of postprocessing phase of those solutions). The papers by Gavish [4], [6], Gavish and Neuman [9], [7], and Gavish and Altinkemar [8], who examine the network design problem in packet switched networks, are an exception. The formulation developed in this paper considers reliability constraints explicitly as part of the formulation.

The following arguments are used in developing the model. Given a communication network, the network fluctuates between many states which are given by 0–1 (fail-operating) condition of links and switching centers. When failure occurs, as a first priority, the network control system attempts to recover immediately from such failures. Recovery procedures are invoked in an attempt to reconnect all the interrupted connections which passed through the system's failed components, sessions have to be rerouted, and the overall volume of traffic that a network can handle is usually reduced as a result of component failures. In parallel to the recovery process, the system initiates a restoration effort which eventually restores the network to its fully operational state (the failed components are repaired).

In our model, we account for possible link and node failures by specifying a set of *significant* system states which are defined by unique combinations of operational links and switching centers. For each of these states, we assume that we are given a traffic requirement matrix which expresses the minimal number of operational circuits that should be available for each origin-destination pair p (we use the word "commodity" to identify an origin-destination pair in the remainder of this paper) when the system is in this state. The circuit requirement matrix determines the blocking probability for customers of each O-D pair and it therefore specifies for each commodity the expected quality of service. Note that traffic requirement matrices might be identical for different states.

We also assume that we are given, for each potential link, a set of cable types which could be installed on that link. A cable type is characterized by the number of circuits that it can support.

When designing the system, the following decisions have to be taken:

1) What links should be part of the final design?

2) For each link which is a part of the final design, what combination of cable types should be installed on it?

3) What routing should be used for each commodity and for every system state which is considered in the design?

Throughout this paper, we assume that the switching center locations are given, and therefore, the objective of the model is to determine the connections between those switching centers.

In most cases, the service provider has an existing network in place, and the objective of the design is to decide

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on how to expand a network so as to accommodate additional services and future demand. We shall indicate, at the end of Section II, how the model that we propose can be modified to account for an existing network.

The paper is organized as follows. The mathematical formulation of the general model is presented in Section II. In Section III, we analyze the cost structure for "opening" the potential links. Section IV is devoted to a modified formulation which seems particularly appropriate for fiberoptics networks. In Section V, we describe a number of redundant constraints which may be added to this formulation in order to strengthen its continuous relaxation. The upper and lower bounding procedures to be used in a branch-and-bound solution approach are explained in Section VI. Computational results are reported in Section VII. Section VIII concludes the paper.

II. Mathematical Formulation of the General Model

In this section, we develop a mathematical programming formulation of a general version of the network design problem. To develop the mathematical model, the following notation is defined:

- N: the index set of nodes (switching centers);
- L: the index set of candidate links;
- T_l: the index set of cable types available for installation on link l, $l \in L$;
- Q_t: the number of circuits that can be supported by cable type t (cable capacity);
- n_l^t: the number of cables of type t which are selected to be installed over link l;
- $C(n_l^t)$: the cost of providing n_l^t;
- Π: the set of origin-destination pairs (commodities);
- $O(p)$: the origin node for commodity p ($p \in \Pi$);
- $D(p)$: the destination node for commodity p ($p \in \Pi$);
- R_p: the index set of all possible loopless routes which are allowed for commodity p in the network (including routes that use links which will not be part of the final design);
- δ_{rl}: the delta function which takes the value 1 when link l is part of route r and 0 otherwise;
- σ: the index set of all possible network states;
- E: an operation state of the network, $E \in \sigma$;
- 0: the state in which all network components are operational, $0 \in \sigma$;
- R_p^E: the routes in R_p which do not use links that are failed when the system is in state E;
- $\{\gamma_p^E\}$: the traffic requirement matrix expressing the minimal number of operational circuits required for commodity p when the system is in state E;
- X_{rp}^E: the fraction of the overall traffic generated by commodity p which uses route r when the system is in state E, $r \in R_p^E$;
- F_l^E: the flow on link l when the network is in state E.

Using the above notation, we obtain the flow on link l when the network is in state E as

$$F_l^E = \sum_{p\in\Pi} \sum_{r\in R_p^E} \delta_{rl} \gamma_p^E X_{rp}^E.$$

For the bifurcated flow situation, we can formulate the network design problem as the following mathematical optimization problem:

$$\text{Minimize} \sum_{l\in L} \sum_{t\in T_l} C(n_l^t) \tag{1}$$

subject to

$$\sum_{t\in T_l} n_l^t Q_t \geq F_l^E, \qquad l \in L, \qquad E \in \sigma, \tag{2}$$

$$n_l^t \geq 0 \quad \text{and integer}, \qquad t \in T_l, \qquad l \in L, \tag{3}$$

$$\sum_{r\in R_p^E} X_{rp}^E = 1, \qquad p \in \Pi, \qquad E \in \sigma, \tag{4}$$

$$X_{rp}^E \geq 0, \qquad r \in R_p^E, \qquad p \in \Pi, \qquad E \in \sigma. \tag{5}$$

The constraints in (2) assure that the capacity assigned to link l is high enough to accommodate the traffic on the link for all possible system states. Constraints (3) ensure that the number of cables type t selected for link l is integer. Constraints (4) and (5) assure that the demand γ_p^E is satisfied for each commodity in every possible system state. For a nonbifurcated flow formulation, one only has to replace the constraint (5) by (5′) $X_{rp}^E \in \{0, 1\}$, $r \in R_p^E$, $p \in \Pi$, $E \in \sigma$.

Note that one may easily take into account an already existing network in the previous formulation by using incremental costs for the cost functions $C(n_l^t)$, i.e., $C(n_l^t)$ would now be the additional cost to be incurred if the number of cables of type t on link l were changed from its current value (which may be 0) to n_l^t. With such a formulation, if there are already $\bar{n}_l^t$ cables of type t on link l, then by setting $C(\bar{n}_l^t)$ to 0 we account only for the incremental cost of installing additional cable types over that line. Obviously there are nonzero maintenance and operational costs to existing links.

For the remainder of the paper, we will assume that the network has to be designed from scratch.

III. Cost Structure of the Problem

Past investigations have assumed a fairly simple cost structure for laying down a cable between two points. Careful analysis reveals that it is more complicated than previously assumed. In order to be able to lay down a cable (or cables) over link l, the company has to obtain the right of way over that path which is typically independent of the number of cables that are laid down over this path (actually in most instances the land owner does not know how many cables are strung under his property). This portion of the cost can be looked upon as a fixed set up cost for having one or more cables passing over the link. In order to be able to lay down the links, a ditch has to be dug and appropriate pipes (conduits) have to be installed over that path. The digging and conduit installation is in most cases almost independent of the number of cables that are later pooled through those conduits.

A second cost component is related to each cable which is pulled over this link. There is a cost for pulling the cable through the conduit (mainly labor costs), the cost of the cable itself (mainly material costs), and the cost of the amplifiers that have to be installed at regular intervals over the link. Finally, different technologies have different operational and maintenance costs. Different cable types have different cable costs, pulling costs, and amplification costs over the link. A typical cost structure for laying a cable is thus

$$S_l + s(n_l^t) + v(n_l^t) + M(n_l^t)$$

where

- S_l: is the cost of obtaining the right of way and the cost of laying down the conduit over link l;
- $s(n_l^t)$: is the set up cost for laying down and pulling n_l^t units of cable type t over the link l;
- $v(n_l^t)$: is the variable cost for laying down and installing n_l^t units of cable type t over the link l ($v(n_l^t)$ typically increases proportionally with n_l^t, i.e., one has $v(n_l^t) = \alpha_l^t n_l^t$ for some constant α_l^t);
- $M(n_l^t)$: present discounted cost for operating and maintaining cable type t over link l.

Note that the resulting cost function has a staircase shape as a function of the number of circuits.

IV. A Modified Formulation

As shown in Section III, the cost structure of a cable can be quite complicated. In order to simplify the analysis and better cope with the very general cost structure, we take advantage of the following observation. A fiberoptics cable can carry a very large volume of traffic. When dealing with fiberoptics systems, it is highly likely that a few cables will be enough to carry all of the traffic that has to be carried over a link. In order to cope with the cost complexity and to formulate the problem, we redefine the set of cable types T_l over a link l as follows: instead of using two parameters (the cable type t and the number n_l^t of such cables), we will use only one, still referring to it as a cable type, but that will now imply a specific number of cables. The capacity and cost of these new cable types will be directly given by the new parameters C_{lt} and Q_{lt}. T_l will be represented by the set of indexes $\{1, 2, \cdots, |T_l|\}$.

Let Z_{lt} be a binary variable which is equal to 1 if we are laying cable type t over link l, and 0 otherwise.

C_{lt} is the cost of using cable type t on link l.

Q_{lt} is the capacity (in circuits) of cable type t over link l ($Q_{l1} < \cdots < Q_{l|T_l|}$).

The problem can be formulated as that of finding the values Z_{lt} which minimize the objective expressed below by (6) subject to constraints (7)–(11). Denote this problem formulation by P1.

$$\text{(P1)} \qquad \text{Minimize} \sum_{l \in L} \sum_{t \in T_l} C_{lt} Z_{lt} \tag{6}$$

subject to

$$\sum_{p \in \Pi} \sum_{r \in R_p^E} \delta_{rl} \gamma_p^E X_{rp}^E \le \sum_{t \in T_l} Q_{lt} Z_{lt}, \quad l \in L, \quad E \in \sigma, \tag{7}$$

$$\sum_{t \in T_l} Z_{lt} \le 1, \quad l \in L, \tag{8}$$

$$\sum_{r \in R_p^E} X_{rp}^E = 1, \quad p \in \Pi, \quad E \in \sigma, \tag{9}$$

$$X_{rp}^E \ge 0, \quad r \in R_p^E, \quad p \in \Pi, \quad E \in \sigma, \tag{10}$$

$$Z_{lt} \in \{0, 1\}, \quad l \in L, \quad t \in T_l. \tag{11}$$

The constraints in (7) ensure that the capacity allocated to link l is high enough to carry all the traffic allocated to it for every system state considered. The constraints in (8) select at most one cable type for link l. Again, the above formulation is stated in terms of bifurcated flow. In order to account for nonbifurcation, constraints (10) are modified to (10′) $X_{rp}^E \in \{0, 1\}$, for all $r \in R_p^E$, $p \in \Pi$, $E \in \sigma$.

This is an NP-complete problem (see the network design problem in [3]), we therefore develop in the following sections lower bounding procedures for the problem and heuristics for generating feasible solutions. The difference between the upper and lower bounds provides an estimate on the quality of the solution generated by the heuristic.

V. Additional Constraints

Relaxing the integrality requirements in (11) and solving for it shows very limited promise. Obviously, nothing prevents the Z_{lt}'s from being fractional, such that often the right-hand side of (7) is equal to its left-hand side resulting in a significant portion of the fixed cost being avoided. Additional constraints have to be added to the relaxation of (P1) in order to obtain a better lower bound. Three classes of constraints were developed for this purpose. Their combined effect produced on our test problem a drastic improvement over the simple relaxation of (P1), as will be seen in Section VII. These classes of constraints are now described and motivated.

A. First Class of Additional Constraints

Whenever a node n belongs to at least one commodity (i.e., is the origin or destination of a commodity) and whenever the set of states σ is such that for each incident link to n there exists at least one state such that the link cannot be used (either because it has failed, or because every route including it and being allowed for the commodity cannot be used since another link of the route has failed), then we can require the degree of n to be at least two. As a matter of fact, since n belongs to a commodity, each state of σ induces at least one route beginning (or ending) at n and therefore using a link incident to n. Under the above hypothesis, there is at least one state where this link cannot be used, and therefore a route using a different link incident to n must be used. All in all, the network must contain at least two links incident to n.

We can thus require that

$$\sum_{l \in A(n)} \sum_{t \in T_l} Z_{lt} \geq 2, \qquad n \in M, \tag{12}$$

where

$$A(n) = \{l \in L \mid l \text{ is incident to } n\}, \qquad n \in N;$$

$$M = \Big\{ n \in N \mid \big(\exists p \in \Pi \mid \big((O(p) = n \text{ or } D(p) = n) \text{ and } (\forall l \in A(n)\ \exists E \in \sigma \mid \forall r \in R_p^E \quad \delta_{rl} = 0) \big) \big) \Big\}.$$

B. Second Class of Additional Constraints

Each link of the network must have a positive capacity in order to be used, i.e., whenever a route that comprises a link l is used, one of the Z_{lt} ($t = 1, \cdots, |T_l|$) variables must be equal to one. The X_{rp}^E variables indicate whether or not a route is used, and the Z_{lt}'s indicate the selected capacity of the links. By linking the two sets of variables, we aim at pushing up the values of the Z_{lt}'s whenever link l is used in a certain state. This is expressed as follows:

$$\sum_{t \in T_l} Z_{lt} \geq \sum_{r \in R_p^E} \delta_{rl} X_{rp}^E, \qquad l \in L, \qquad E \in \sigma, \qquad p \in \Pi. \tag{13}$$

The right-hand side expresses the total percentage of the traffic requirement for commodity p in state E that uses link l. This fraction is limited to one by constraint (9). On the other hand, we would like the left-hand side to be equal to one whenever the right-hand side is greater than zero. In all cases, therefore, constraints (13) are valid.

C. Third Class of Additional Constraints

In order to express this class of constraints, we need to redefine the Z_{lt} variables and, as a consequence, the C_{lt} and Q_{lt} parameters also, as follows:

$Q'_{lt} = Q_{lt} - Q_{lt-1}$ is the added capacity (in circuits) on link l of cable type t over cable type $t - 1$, where Q_{l0} is defined to be zero;

$C'_{lt} = C_{lt} - C_{lt-1}$, where C_{l0} is defined to be zero;

$$Z'_{lt} = \begin{cases} 1 & \text{if we are laying over link } l \text{ a cable type of a capacity of at least } Q_{lt}; \\ 0 & \text{otherwise.} \end{cases}$$

Rewriting (P1) using the new parameters and variables, we obtain

$$\text{(P2)} \qquad \text{Minimize} \sum_{l \in L} \sum_{t \in T_l} C'_{lt} Z'_{lt} \tag{14}$$

subject to

$$\sum_{p \in \Pi} \sum_{r \in R_p^E} \delta_{rl} \gamma_p^E X_{rp}^E \leq \sum_{t \in T_l} Q'_{lt} Z'_{lt}, \qquad l \in L, \quad E \in \sigma, \tag{15}$$

$$Z'_{lt} \geq Z'_{lt+1}, \qquad l \in L, \quad t = 1, \cdots, |T_l| - 1, \tag{16}$$

$$\sum_{r \in R_p^E} X_{rp}^E = 1, \qquad p \in \Pi, \qquad E \in \sigma, \tag{17}$$

$$X_{rp}^E \geq 0, \qquad r \in R_p^E, \qquad p \in \Pi, \qquad E \in \sigma, \tag{18}$$

$$Z'_{lt} \in \{0, 1\}, \qquad l \in L, \qquad t \in T_l. \tag{19}$$

Constraints (16) now replace constraints (8). Together with (19) they assure that the Z'_{lt} variables take the value 1 in sequence so as to simulate the Z_{lt} variables in (P1). We now indeed have that

$$Z'_{lt_0} = 1 \quad \text{only if } Z'_{lt} = 1 \quad \text{for } t = 1, \cdots, t_0 - 1.$$

We therefore have a one-to-one relationship between (P1) and (P2). To each solution $\{Z_{lt}\}$ in (P1) corresponds a $\{Z'_{lt}\}$ solution (P2) and conversely. The relation is as follows:

$$\left.\begin{array}{l} Z_{lt_0} = 1, \\ Z_{lt} = 0, \quad t \neq t_0, t \in T_l, \end{array}\right\} \leftrightarrow \begin{cases} Z'_{lt_0} = 1, & t = 1, \cdots, t_0, \\ Z'_{lt} = 0, & t > t_0. \end{cases}$$

And for each such pair of solutions, we have that

$$\sum_{t \in T_l} C_{lt} Z_{lt} = \sum_{t \in T_l} C'_{lt} Z'_{lt},$$

$$\sum_{t \in T_l} Q_{lt} Z_{lt} = \sum_{t \in T_l} Q'_{lt} Z'_{lt}.$$

The first two classes of additional constraints can also be expressed in terms of the Z'_{lt}, Q'_{lt}, C'_{lt}'s: (12) becomes (20)

$$\sum_{l \in A(n)} Z'_{l1} \geq 2, \qquad n \in M, \tag{20}$$

and (13) becomes (21)

$$Z'_{l1} \geq \sum_{r \in R_p^E} \delta_{rl} X_{rp}^E, \qquad l \in L, \qquad E \in \sigma, \qquad p \in \Pi. \tag{21}$$

Before presenting the new class of constraints, we need to make the assumption that the number of capacity types is the same for every link, and that the "capacity levels" are equal, i.e.,

$$|T_l| = T, \qquad \forall l \in L,$$

$$Q'_{lt} = Q, \qquad \forall l \in L, \qquad \forall t \in T_l.$$

The third class of additional constraints is expressed by two sets of inequalities:

$$\sum_{l \in A(n)} \Delta(R_n^E(l)) \sum_{t=1}^{T} Z'_{lt} \ge NL(D_{nE}), \quad n \in N, \quad E \in \sigma, \tag{22}$$

$$\sum_{l \in A(n)} \Delta(R_n^E(l)) Z'_{l1} \ge \left\lceil \frac{NL(D_{nE})}{T} \right\rceil, \quad n \in N, \quad E \in \sigma, \tag{23}$$

where

$$P(n) = \{p \in \Pi \mid O(p) = n \text{ or } D(p) = n\},$$

$$D_{nE} = \sum_{p \in P(n)} \gamma_p^E, \qquad n \in N, \quad E \in \sigma,$$

$$NL(D_{nE}) = \left\lceil \frac{D_{nE}}{Q} \right\rceil,$$

$$R_n^E(l) = \left\{ r \in \bigcup_{p \in P(n)} R_p^E \mid \delta_{rl} = 1 \right\}, \qquad n \in N, \quad E \in \sigma, \quad l \in L$$

$$\Delta(R_n^E(l)) = \begin{cases} 1, & \text{if } R_n^E(l) \ne \varnothing, \\ 0 & \text{otherwise.} \end{cases}$$

D_{nE} is the total demand that originates or terminates at node n in state E. To reach node n, that demand must use its incident links. The right-hand side of (22) expresses the minimal (integer) number of "capacity levels" globally required amongst the incident links in order to accommodate D_{nE}, whereas the left-hand side evaluates the number of such selected levels.

Knowing the minimal number of capacity levels required, and since each link can support at most T capacity levels, we can now have an expression for the minimal number of links incident to node n. This is just what the right-hand side of (23) expresses, and the left-hand side counts the number of selected links incident to n, using only the first capacity level (i.e., $t = 1$).

We will now examine how those three classes of additional constraints can be incorporated into bounding procedures in order to get a lower and an upper bound on the optimal solution.

VI. Bounding Procedures

A. Lower Bounding Procedure

In order to get a lower bound, we solve a sequence of linear programs. The sequential aspect is due to the fact that we chose not to include all the constraints (21) at once because of their large number, but to add them only upon violation. The procedure stops when such constraints are no longer violated. This is similar to the augmented Lagrangean procedure presented in [10].

Algorithm:

Step 0: Solve (P2), with the integrality constraint relaxed, to which constraints (20), (22), and (23) are added.

Step 1: List all instances of constraints (21) that are violated. If the list is empty, then STOP. If not, add the violated constraints to the dual form and solve again. Repeat step 1.

B. Upper Bounding Procedure

We now describe a simple and direct heuristic solution scheme. Such a scheme generates feasible solutions to the problem which could be incorporated into a branch-and-bound algorithm.

In this heuristic procedure, we are also solving a sequence of linear programming problems. Initially we obtain a lower bound using the algorithm described in Section VI-A. The Z'_{lt} variables which are equal to 1 in the lower bound solution are fixed at 1 in all subsequent problems. In each successive problem solved, we substitute the largest fractional Z'_{lt} variable by a variable set to 1. In such a procedure, the maximal number of LP's solved is bounded by the cardinality of L times T. We add the additional constraints to the dual form in order to continue with a feasible basis at each setting of a Z'_{lt} variable to 1.

Algorithm:

Step 0 (Initialization): Obtain a lower bound using the algorithm described in Section VI-A. Fix to one (now and for the remainder of the algorithm) all Z'_{lt} variables with the value 1 in the lower bound solution. If all Z'_{lt} are 1 or 0, STOP.

Step 1: List all instances of fractional Z'_{lt} variables. If the list is empty, then STOP. If not, select the fractional Z'_{lt} variable closest to 1 in the optimal LP solution. Fix this value to 1 (now and for the remainder of the algorithm). Repeat Step 1.

Note that when there is only one fractional Z'_{lt} variable, we still have to solve an LP after fixing its value to 1. The new LP solution determines the X_{rp}^E variables which correspond to the routing solutions for the commodities in case of bifurcated flow.

VII. Computational Experiments

In this section, we discuss in detail the results of computational testing for a simple 8 node, 13 link problem described in [11] and [12].

Network: The network (N, L) is described in Fig. 1. It shows the potential links, i.e., those that can be included in the final design. Π, the set of commodities, is defined from L, i.e., each potential link $\{i, j\}$ (a nonoriented pair) defines two commodities: (i, j) and (j, i). With 13 potential links, we end up with 26 commodities.

Capacities: It is assumed that the Q'_{lt}'s are all equal to 60 and that all the $|T_l|$'s are equal to 5. We therefore can apply the constraints (22) and (23).

Cost Structure: It is assumed that C'_{lt} is defined as follows:

$$C'_{lt} = \begin{cases} a_l + b_l & \text{for } t = 1, \\ b_l & \text{for } t = 2, \cdots, 5, \end{cases}$$

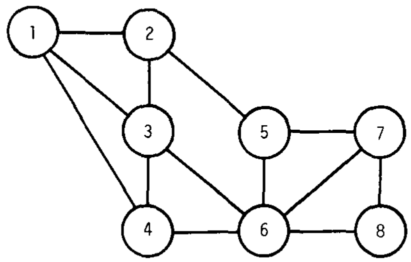

Fig. 1. Test network.

where

$$a_l = 30, \quad \text{for all } l \in L;$$

and

$$b_l = 6 * \text{length of link } l, \quad \text{for all } l \in L.$$

The link lengths are obtained using the Euclidean metric assuming neighboring rows and columns are one unit apart in Fig. 1. The link variable costs (b_l's) appear in Table I in the following format: if $l = \{i, j\}$ with $i \leq j$, then b_l will be the (i, j)th entry.

States:

$$\sigma = \{0\} \cup \left[\bigcup_{l \in L} \{\text{state } E \mid \text{link } l \text{ is the only link in failure in } E\} \right].$$

Demands: The demands for each state and each commodity are defined as follows:

$$\gamma_p^E = \begin{cases} 0.99 \text{ dem}_{ij}, & \text{if } E = 0; \\ 0.90 \text{ dem}_{ij}, & \text{otherwise,} \end{cases}$$

where $i = O(p)$ and $j = D(p)$. The coefficients $\{\text{dem}_{ij}\}$ are given in Table II.

Routes: R_p^0, the index set of routes that support commodity p when there is no failure, is not the set of all possible routes on (N, L) between nodes $O(p)$ and $D(p)$. We limited it, as in [11], to exactly five routes per commodity. Note that the direct link is always one of the five routes, but that when $O(p) = D(p')$ and $D(p) = O(p')$, $R_{p'}$ is not necessarily obtained by reversing each route of R_p.

In general,

$$R_p^E = R_p^0 - \{r \in R_p^0 \mid \exists \text{ link } l \text{ in } r \text{ that is in failure in state } E\}.$$

Even for this simple problem of 8 nodes and a maximum of 13 links, the number of potential configurations is very high. Since each link can have 6 different link capacities (including 0) and since at least 8 links need to have a positive capacity (because the network has to be connected, but not a tree), the number of potential configurations is $\binom{13}{8} 5^8 6^{13-8} \simeq 3 \times 10^{13}$.

The computational results are summarized in Table III.

TABLE I
LINK VARIABLE COSTS

i \ j	1	2	3	4	5	6	7	8
1	—	6.0	8.5	13.4	—	—	—	—
2		—	6.0	—	8.5	—	—	—
3			—	6.0	—	8.5	—	—
4				—	—	6.0	—	—
5					—	6.0	6.0	—
6						—	8.5	6.0
7							—	6.0
8								—

TABLE II
CIRCUIT DEMAND COEFFICIENTS dem_{ij}

i \ j	1	2	3	4	5	6	7	8
1	0	59	45	125	0	0	0	0
2	31	0	32	0	43	0	0	0
3	50	71	0	23	0	84	0	0
4	39	0	65	0	0	57	0	0
5	0	22	0	0	0	16	50	0
6	0	0	58	50	50	0	32	50
7	0	0	0	0	50	50	0	50
8	0	0	0	0	0	67	50	0

In the first column, we describe the specific formulation used to obtain a lower bound. They vary according to the set of additional constraints used. On the first line, none of them was used (thus, only a simple relaxation of P2 was performed), whereas on the third line, for instance, we included constraints (20) and (21), the first two classes of additional constraints. The next three columns refer to the results obtained for the lower bound, respectively, its value, the CPU time (using a CDC CYBER 173 and APEX III for the LP), and the number of iterations required to obtain it. The next three columns give the same information about the upper bound. Note that here CPU refers to the additional time required once the lower bound is obtained. Finally, the last column gives the importance of the gap between the lower and upper bounds.

As expected, the simple relaxation of P2 generated a large gap (53 percent). Every successive line then shows the cumulative impact of adding each set of additional constraints. Using all three sets reduces the gap to 8 percent, a figure that makes it realistic to apply a branch-and-bound procedure to get an optimal solution to the problem (at least for a problem of similar size). We can also note that the last class of constraints is generative of integrality, many variables being already integral when the lower bound is obtained, thus reducing the number of iterations required to get the upper bound.

VIII. CONCLUSIONS

The conclusions reached in this paper are in terms of modeling, algorithm development, and performance expectations. In terms of modeling, this circuit network de-

TABLE III
COMPUTATIONAL RESULTS

Formulation	L.B.	CPU	No. of iter.	U.B.	CPU	No. of iter.	(UB − LB)/LB
(P2)	432.3	156	1	661.6	1,147	38	.53
(P2) + (20)	486.7	169	1	665.1	1,067	36	.37
(P2) + (20) − (21)	547.8	790	8	665.1	1,487	35	.21
(P2) + (20) − (23)	594.4	582	8	642.2	804	19	.08

sign problem under reliability constraints can be formulated mathematically as a mixed integer linear programming problem. The size of this problem for a given network is highly dependent on the number of possible states for the system. The maximal size of such problems that can be solved with the help of the methods outlined in this paper remains an open question for the computational testing stage. In terms of the algorithmic development, we have presented one solution approach for this problem: relaxing the mixed integer programming formulation and solving instead the corresponding linear programming problem augmented by three classes of cuts. The resulting fractional values are cut off with a myopically directed adjustment method which requires dual manipulation of the linear programming solution.

Limited computational results indicate a good performance of the algorithm, producing a gap between lower and upper bounds that is sufficiently small for a branch-and-bound procedure to be applicable.

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