FIELD DEFORMATION IN A CURVED SINGLE-MODE FIBRE

Indexing terms: Optical fibres, Optical waveguide theory

Simple formulae are given for the field displacement caused by bending, the change in mode volume and for the mode coupling loss which arises at transitions between straight and curved sections of single-mode optical fibre. These formulae are in good agreement with other, more exact, theories.

Introduction: Experimentally, it has recently been shown[1,2] that the field distribution in a single-mode fibre is shifted considerably at a bend, with the result that a mode coupling loss is incurred at transitions between straight and curved sections of fibre. Thus any propagation study must take into account this mechanism as well as the radiation loss due to uniform curvature. These two different physical phenomena have been isolated and observed separately[3] and it has been found that the mode conversion loss, which we have called the transition loss, predominates at large bending radii. On the theoretical side, the modal fields of the curved fibre have been calculated numerically by solving an approximate wave equation via a double Fourier-Bessel expansion[4] and analytically with a perturbation method.[5] However, the results are not easy to assimilate or apply without complex numerical calculations.

However, Petermann[6] has shown that the field distribution in a curved fibre is close to Gaussian. This result has also been obtained by others[7,8] and we show here that it can form the basis for deriving simple formulae which may be used in practical design work with single-mode fibres.

Simplified theory: In the weak-guidance approximation, the electric field E of the HE_{11} mode in a curved fibre[9] is linearly polarised and is a solution of

$$\nabla^2 E + (kn)^2[1 + 2(r/R)\cos\theta]E = 0 \qquad (1)$$

where (r, θ, z) are cylindrical co-ordinates, R the radius of curvature of the fibre, k the free-space wave number, $n = n_{1,2}$ the refractive indices in the core and the cladding, and ∇ the Laplacian operator. By using a first-order perturbation method and the assumption that the radial field distribution in a straight fibre E_0 is given by

$$E_0 = A \exp[-(r/\omega_0)^2/2] \qquad (2)$$

where ω_0 is the spot size at which the power has dropped to $1/e$ of its maximum value, the electric field E can be obtained from Petermann's results[6] as

$$E = E_0[1 + (kn\omega_0)^2 (r/R)\cos\theta]\exp(-j\beta z) \qquad (3)$$

where β is the propagation constant in a straight fibre. Eqn. 3 forms the basis for our further analysis.

The spot size of the HE_{11} mode has been evaluated[7,9] by a number of techniques and in particular Marcuse has given an approximate formula

$$\omega_0 = 2^{-\frac{1}{2}}a(0{\cdot}65 + 1{\cdot}619\,V^{-1\cdot5} + 2{\cdot}879\,V^{-6}) \qquad (4)$$

where a is the radius of the fibre core and V is the normalised frequency defined by

$$V = kan_1[1-(n_2/n_1)^2]^{\frac{1}{2}} = kan_1\,\Delta^{\frac{1}{2}} \qquad (5)$$

In our simplified approach we use eqn. 4 in conjunction with eqn. 3.

Comparison with existing theories: First of all, the field distribution is calculated from the simple eqn. 3 with an ordinary pocket calculator and is then compared with the results obtained by the more exact method of Marcuse[4] which requires a digital computer. Fig. 1 shows the field distribution as a function of normalised radius in the plane of curvature for $V = 2{\cdot}4$, $n_1 = 1{\cdot}515$ and $n_2 = 1{\cdot}5$. The solid and dotted curves are derived from the simple method and Marcuse's results, respectively. It can be seen that for $R/a = 500$ the agreement is almost perfect, but there are slight differences for $R/a = 200$ which may arise from the approximate nature of the first-order perturbation method. Thus eqn. 3 gives excellent results for $R \gg a$ which is the assumption employed by Petermann. Next, a very useful parameter for the prediction of transition loss and mode spot size in a curved fibre is the beam shift d of the field maximum, which may be obtained from eqns. 3 and 4 by solving $\delta E/\delta r = 0$ to give

$$\frac{d}{a} = \frac{V^2}{4\Delta(R/a)}(0{\cdot}65 + 1{\cdot}619\,V^{-1\cdot5} + 2{\cdot}879\,V^{-6})^4 \qquad (6)$$

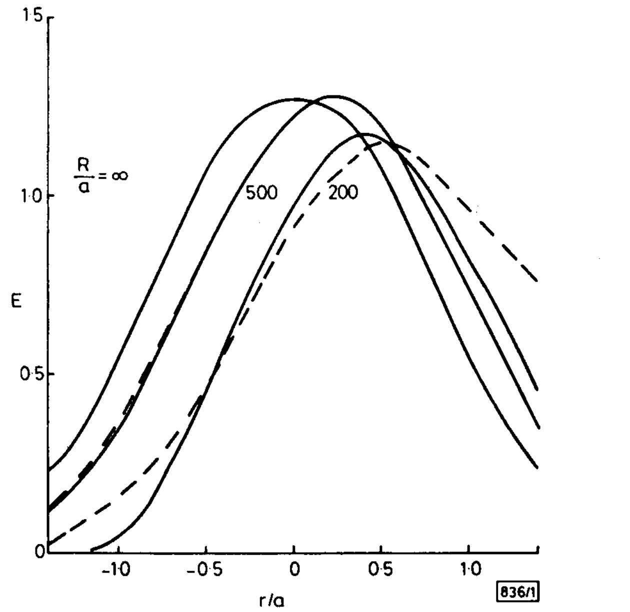

Fig. 1 *Field distribution as a function of normalised radius in the plane of curvature for various values of R/a*
The solid curves are derived from eqn. 3 and the dotted lines are the results of Marcuse[4]
$V = 2{\cdot}4$, $n_1 = 1{\cdot}515$, $n_2 = 1{\cdot}5$

This normalised beam shift is shown in Fig. 2 in the form of a universal curve which can be used for any value of R, d, Δ and over the range of V given. Miyagi and Yip[5] have calculated the beam shift by using the complex modal field and a comparison with their results for the case of $V = 2{\cdot}38$, $a = 4{\cdot}05\ \mu m$ and $\Delta = 0{\cdot}0016$ is given in Fig. 3. The solid curve is obtained from eqn. 6 and the dotted curve is the result of

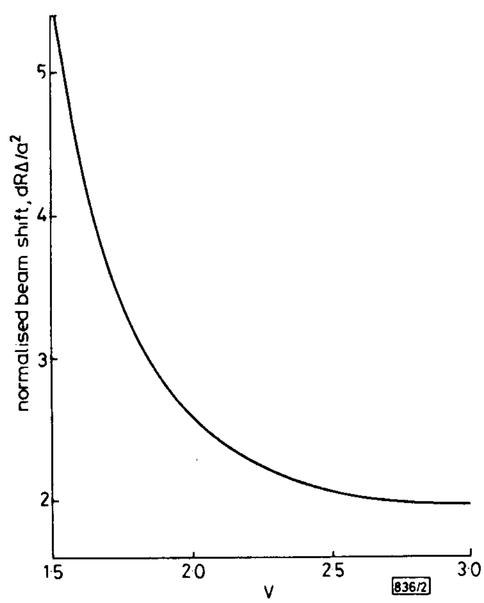

Fig. 2 *Normalised beam shift as a function of normalised frequency of fibre*

Miyagi and Yip. It is seen that the agreement is excellent except for a slight departure at small bend radii.

It is apparent from the results quoted here that the mode field not only shifts away from the guide axis with decreasing radius of curvature but may also change substantially in width while maintaining a quasiGaussian transverse distribution. Now the mode spot size, in curved as well as straight fibres, is an important parameter because the width of the field distribution has a significant influence on the bending (including microbending) loss[4,6] The mode spot size in a curved fibre may be obtained by substituting eqn. 3 into Petermann's definition[6] to give the remarkably simple result

$$\delta\omega = (\omega - \omega_0)/\omega_0 = \tfrac{1}{2}\left(\frac{d}{\omega_0}\right)^2 \qquad (7)$$

Eqn. 7 shows that the change of relative mode spot size due to fibre curvature depends on the beam shift while the actual spot size ω increases with decreasing radius of curvature. This implies that the actual curvature loss in a single-mode fibre is larger than that predicted by conventional calculations[4] which assume that the spot size is unchanged at a bend. For example, for $V = 2{\cdot}4$, $a = 5\ \mu m$ and $\Delta = 0{\cdot}001$, the change of spot size is 0·9% for $R = 10$ cm and 3·7% for $R = 5$ cm. In contrast a reduction in spot size with increasing curvature is predicted in multimode fibres.

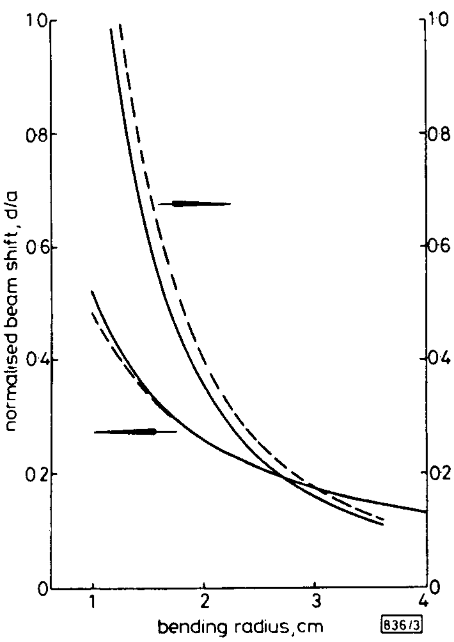

Fig. 3 *Comparison between simple method, eqns. 6 and 9, of determining d/a and P_R (solid curves) and the results obtained by Miyagi and Yip[5,10] (dotted curves)*

Another important consideration is the prediction of transition loss P_R between straight and curved fibre sections[1,3] which is caused by the field deformation in a curved fibre and is given by

$$P_R = \frac{\int_A (E - E_0)^2\, dA}{\int_A E_0^2\, dA} \qquad (8)$$

Again substitution of eqn. 3 gives

$$P_R = \tfrac{1}{2}\left(\frac{d}{\omega_0}\right)^2 \qquad (9)$$

It is interesting to note that the transition loss is also expressed simply in terms of the beam shift and the mode spot size. Eqn. 9 gives the transition loss between straight and curved sections of fibre, but it can be easily extended to the general case, so that the loss between fibres with curvatures R_1 and R_2 can be written as

$$P_R = \tfrac{1}{2}[(d_1 - d_2)/\omega_0]^2$$

where d_1, d_2 are the beam shifts for R_1 and R_2. Thus the loss for a change of curvature from R to $-R$ is 4 times larger than that[3] from R to ∞, because the total beam shift is double. It is also worth mentioning that the splice loss due to fibre off-set d is given by[7]

$$2\alpha = 1 - \exp\,[-(d/\omega_0)^2/2] = \tfrac{1}{2}\left(\frac{d}{\omega_0}\right)^2 \qquad (10)$$

assuming that the two jointing fibres have the same ω_0. The fact that eqns. 9 and 10 give the same result is not surprising since in each case the loss is caused by mismatched Gaussian field distributions (due to displacement). Finally the transition loss given by eqn. 8 is compared with the results of Miyagi and Yip[10] in Fig. 3 for $V = 2{\cdot}43$, $a = 3{\cdot}9\ \mu m$ and $\Delta = 0{\cdot}0019$. The two curves are very similar; their maximum difference being only about 11%.

Conclusions: The field distribution in a step-index single-mode fibre is very similar to Gaussian in shape. Using this assumption Petermann[6] has obtained a simple expression for the field distribution in a curved fibre. We have extended his work and derive simple formulae for the effect of curvature on (*a*) beam shift, (*b*) spot size, and (*c*) transition loss, which are in good agreement with other, more complex, calculations. Our simplified technique can be directly applied to practical design work on single-mode fibres using only an ordinary pocket calculator whereas the conventional techniques require a computer.

Acknowledgments: We wish to thank D. N. Payne for stimulating discussions; the Pirelli General Cable Company for the award of a research fellowship and the UK Science Research Council for its support.

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13th January 1978

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0013-5194/78/0836-0130 $1.50/0