Field Inhomogeneity Correction based on Gridding Reconstruction

H. Eggers[1], T. Knopp[2], D. Potts[3]

[1]Sector Technical Systems, Philips Research, Hamburg, Germany, [2]Institute of Mathematics, University of Luebeck, Luebeck, Germany, [3]Department of Mathematics, Chemnitz University of Technology, Chemnitz, Germany

Introduction

The spatial variation of the main magnetic field distorts the desired Fourier encoding and gives rise to image artifacts if neglected in reconstruction. Several algorithms have been developed over the years to correct these off-resonance effects, mainly for echo planar and spiral imaging [1-3]. Especially those for spiral imaging generally introduce an interpolation to reduce the computational complexity. So far, this has been done independent of the approximation that the reconstruction of non-Cartesian acquisitions usually relies on. The present work proposes a new algorithm for field inhomogeneity correction, which is based on the same concept as standard gridding reconstruction and which is demonstrated to possess several attractive properties in simulations and phantom experiments.

Methods

Most gridding and non-equispaced Fast Fourier Transform (NFFT) [4] algorithms make use of an approximation of the form

$$e^{\mathrm{i}kx} \approx \frac{1}{\alpha N \hat{w}(x)} \sum_{l=-\alpha N/2}^{\alpha N/2-1} w'(k - l/\alpha N)\, e^{\mathrm{i}xl/\alpha N}, \quad k \in [-\pi, \pi],\ x = -N/2,\dots,N/2-1,$$

where α denotes an oversampling factor, N the number of samples in the image domain, w' a window function of kernel size $2m$, w the periodization of w', and $\hat{w}$ the Fourier transform of w. It can be shown that this approximation remains valid for real $x \in [-N/2, N/2]$ if $k \in [-\pi+2\pi m/\alpha N, \pi-2\pi m/\alpha N]$.

Most field inhomogeneity correction algorithms for non-Cartesian acquisitions, including both direct and iterative ones, involve an evaluation of

$$s_\kappa = \sum_{\rho=0}^{N_1N_2-1} m_\rho e^{\mathrm{i}\omega_\rho t_\kappa} e^{\mathrm{i}\mathbf{k}_\kappa \mathbf{r}_\rho} \quad \text{or} \quad m_\rho \approx \sum_{\kappa=0}^{M-1} d_\kappa s_\kappa e^{-\mathrm{i}\omega_\rho t_\kappa} e^{-\mathrm{i}\mathbf{k}_\kappa \mathbf{r}_\rho},$$

where s_κ denotes the estimated signal at position $\mathbf{k}_\kappa$ at time t_κ, m_ρ the magnetization and ω_ρ the angular off-resonance frequency at position $\mathbf{r}_\rho$, N_1N_2 the number of pixels, M the number of samples in the k-space domain, and d_κ an optional sampling density compensation. In principle, ω and t may be considered as extra dimensions in image and k-space, respectively. Evaluating one of the sums then amounts to calculating a trivariate Fourier transform with non-equispaced sampling in both domains. To improve accuracy and efficiency, we suggest to directly apply the above approximation to the field inhomogeneity-induced exponential. For this purpose, we define

$$N_3 \geq 4(\max(|\omega_\rho t_\kappa/2\pi|) + m/2\alpha) \quad \text{and} \quad T = \max(|t_\kappa|)/(\pi(1-2m/\alpha N_3)).$$

The first of the two sums may then be rewritten as

$$s_\kappa \approx \sum_{l=-\alpha N_3/2}^{\alpha N_3/2-1} w'(t_\kappa/T - l/\alpha N_3) \sum_{\rho=0}^{N_1N_2-1} \left(\frac{m_\rho}{\alpha N_3 \hat{w}(\omega_\rho T)} e^{\mathrm{i}\omega_\rho Tl/\alpha N_3}\right) e^{\mathrm{i}\mathbf{k}_\kappa \mathbf{r}_\rho}.$$

Thus, its evaluation involves αN_3 times a weighting of the magnetization m_ρ and the calculation of a 2D NFFT, and once a local convolution along the t axis. The second sum may be expressed similarly.

To assess this approach, we integrated it into a conjugate phase and an iterative reconstruction, and applied it to simulated and measured spiral k-space data. We used the same Kaiser-Bessel window for $\hat{w}$ [5] and identical settings for α and m for the NFFT and the field inhomogeneity correction.

Results

The accuracy of the new algorithm is compared to that of others for $\alpha = 1.25$ and $m = 2$ in Fig. 1. In this example, αN_3 equals 14. If more than 14 segments are used, the accuracy of the least squares and gridding-based approaches is obviously dominated by the NFFT. The computation times per iteration are listed in Tab. 1 for a resolution of 256 x 256. Differences mainly result from the varying locality of the convolution along the t axis. Finally, the new algorithm is demonstrated on phantom data in Fig. 2. The resonance frequency deviated by ±95Hz in this case.

Conclusions

For standard oversampling factors α and kernel sizes m, our gridding-based approach achieves a similar accuracy as a least squares approach. It provides a rule for choosing the number of segments in the interpolation and allows balancing the accuracy of the reconstruction and of the correction. Moreover, the interpolation coefficients are simply given by the window function w', and the convolution along the t axis is local, thus reducing the computational complexity substantially.

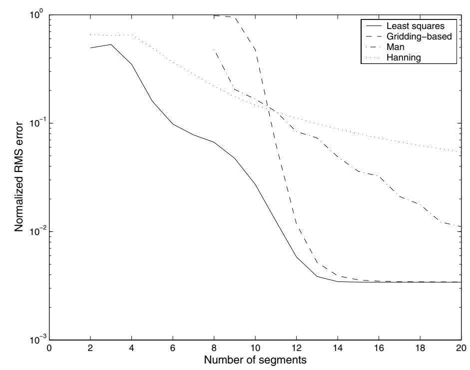

Fig. 1. Error as a function of the number of segments in the interpolation, determined in simulations using an iterative reconstruction. Gridding-based denotes the proposed new algorithm, Least squares [3], Man [2], and Hanning [1] other existing ones.

Method	Running time
Least squares	1530 ms
Gridding-based	840 ms
Man	1530 ms
Hanning	710 ms

Tab. 1. Computation times per iteration using 14 segments, excluding the initial calculation of interpolation coefficients.

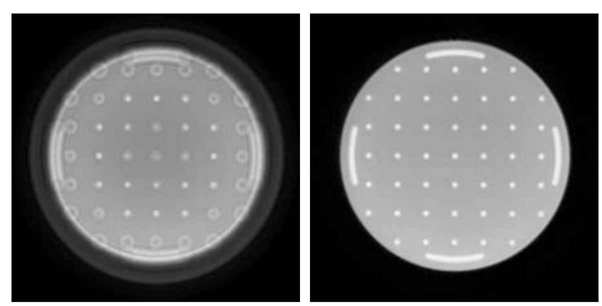

Fig. 2. Phantom image obtained with standard gridding reconstruction (left) and the proposed field inhomogeneity correction (right) from data acquired with a segmented spiral sequence (8 interleaves, 42.5 ms sampling window).

References

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