Field's Programme: Some Interference
Author(s): Joseph Melia
Source: *Analysis*, Vol. 58, No. 2 (Apr., 1998), pp. 63-71
Published by: Oxford University Press on behalf of The Analysis Committee
Stable URL: http://www.jstor.org/stable/3328474
Accessed: 16/06/2010 14:47

Field's programme: some interference

JOSEPH MELIA

The Quine/Putnam Indispensability Argument says that, since our best scientific theories unavoidably quantify over abstract objects, and since we have good reasons for believing our best scientific theories, we have good reasons for believing in the existence of abstract objects (Putnam 1971). Field has attacked the first premiss of this argument: he suspects that scientific theories can be formulated so that they do not quantify over abstract objects (Field 1980, 1985a). He defends this claim by providing nominalistic reformulations of two physical theories: Euclidean three-dimensional geometry and Newtonian gravitational theory. Although it does not immediately follow from this that modern physical theories can be nominalistically reformulated, Field sees no reason to suppose that the techniques used in his nominalistic reformulations could not be extended and developed to cover these theories.

Field says we should prefer his reformulations for two reasons. Firstly, there is simply the consideration of nominalism: the new formulations of scientific theories are not committed to abstract objects, the old formulations are.[1] Secondly, Field argues that, independently of the issue of nominalism, the reformulations have superior explanatory power to the old ones (Field 1980, Ch. 5).

In this paper I shall argue that (1) aside from nominalism, Field's reformulations have the same unattractive features Field pins on the platonistic ones; (2) though his reformulations are nominalist, they are nevertheless still ontologically unparsimonious; and (3) Field has failed to show that certain aspects of scientific practice can be captured in his reformulations.

[1] I too think that Field's reformulations are nominalistic, but not everybody does: Field quantifies over space-time points when reformulating his theories and this is not to every nominalist's taste. See Malament 1982 and Hale 1988. Field's formulations also require the 'full logic of Goodmanian sums' – which looks suspiciously like second order logic, a logic not usually associated with nominalism. See Shapiro 1983 for criticism and Field 1985a for rejoinders.

ANALYSIS 58.2, April 1998, pp. 63–71. © Joseph Melia

1. *Field's programme*

In *platonistic* analytic theories, physical quantities such as *mass* and *distance* are represented by functions from objects onto real numbers. So in Cartesian geometry, the distance between objects a and b is given by the familiar formula $(\Sigma(a_i - b_i)^2)^{1/2}$, where a_i (respectively, b_i) is the ith coordinate of a (respectively, b). Using this representation, one is able to express the axioms of geometry and describe the arrangements of geometric figures in the language of mathematics. Now, instead of representing distance as a function from pairs of objects onto real numbers, it is possible to introduce new relations of *Congruence* and *Betweenness* (expressed in the *synthetic* system by the predicates 'xy Cong zw' and 'x Bet yz') which hold between objects directly. Intuitively, one can think of 'ab Cong cd' as saying that the distance between a and b is the same as the distance between c and d, and 'a Bet bc' as saying that a, b and c are co-linear, and a lies between b and c – but this is of heuristic value only: the relations of Congruence and Betweenness are fundamental relations in the synthetic theory, not capable of being analysed into anything simpler.

The predicates 'xy Cong zw' and 'x Bet yz' enable nominalists to talk about the distances between objects without quantifying over *abstracta*. Suppose that c and d are the end points of the standard metre; then in order to say that the distance between objects a and b is two metres we write: '$\exists u(u \text{ Bet } ab \ \& \ au \text{ Cong } ub \ \& \ ub \text{ Cong } cd)$'. (Notice that, although this reformulation does not quantify over numbers or refer to the number 2, it does quantify over an object u midway between a and b – I shall return to this observation below). Similarly, the nominalist can say that the distance between a and b is 7/11 of a metre by a similar, if more complicated, formula. Since geometers and physicists do need to be able to express the distances between objects when practising their discipline, it is important that nominalists be able to express such relations.

On top of enabling nominalists to talk about physical magnitudes, taking the predicates 'xy Cong zw' and 'xBet yz' as primitive serves a second purpose: it allows nominalists to axiomatize Euclidean geometry without quantifying over any mathematical objects. Thus, for example, the sentence '$\exists x \exists y \exists w(\neg x \text{ Bet } yw \ \& \ \neg y \text{ Bet } xw \ \& \ \neg z \text{ Bet } xy)$' says that space has at least three dimensions, and '$\forall x \forall y \exists z(x \text{ Bet } yz)$' says that every line is infinitely extendible (see Hilbert 1971 for a more detailed presentation). This nominalistic axiomatization of geometry enables one to prove *representation* theorems, theorems which provide the link between the nominalistic and platonistic formulations of a theory by showing how different statements in the two theories correspond to each other. The representation theorems show that, for any model of the nominalistic axiomatization of geometry, one can define a distance function $d(x, y)$

having precisely the properties one expects of a distance function. Thus we can prove that any model of the nominalistic geometry can be expanded to a model of the platonistic theory in which claims made using only the predicates of the nominalistic theory are 'equivalent' to claims of the platonistic one. So, for instance, we can prove that, in any model of the platonistic theory (i) $d(x,y) = d(z,w)$ iff $xy\,\mathrm{Cong}\,zw$, (ii) $d(x,y) + d(y,z) = d(x,z)$ iff $y\,\mathrm{Bet}\,xz$ and (iii) any function $d(x,y)$ meeting constraints (i) and (ii) is unique up to a multiplicative constant. Clearly, we would expect distance functions to satisfy these three constraints.

2. *The trouble with synthetic formulations*

Field writes:

> I believe that such 'synthetic' approaches to physical theory are advantageous not merely because they are nominalistic, but also because they are in some ways more illuminating than metric approaches: they explain what is going on without appeal to extraneous, causally irrelevant entities .... I am saying then that not only is it much likelier that we can eliminate numbers from science than electrons (since numbers, unlike electrons, do not enter causally in explanations), but also that it is more illuminating to do so. It is more illuminating because the elimination of numbers, unlike the elimination of electrons, helps us to further a plausible methodological principle: the principle that *underlying every good extrinsic explanation there is an intrinsic explanation* .... [O]ne of the things that gives plausibility to the idea that extrinsic explanations are unsatisfactory if taken as *ultimate* explanations is that the functions invoked in many extrinsic explanations are so arbitrary. (Field, 1980: 44–45)

In this passage, Field is objecting to the following: (a) the fact that numbers are extrinsic to the physical processes scientists are trying to explain; (b) the causal irrelevance of the numbers invoked in a physical explanation; (c) the arbitrariness of the numbers invoked in the physical explanation. Unfortunately for Field, the objects he quantifies over in his reformulation are open to precisely the same charge!

Consider the empty tray on Joe's desk. Joe notices that he can place two duplicate pencils top to toe in the tray, and that when he does so, there is no space between the ends of the pencils and the edge of the tray. The two pencils fit inside the tray – but only just. Had either of the pencils been the slightest bit longer, they would not have lain wholly within the tray. Had either of the pencils been the slightest bit shorter, there would have been a gap between the end of the pencils and the edge of the tray.

Why do the two duplicate pencils fit precisely into the tray? The intuitive explanation is obviously this: because the tray is twice as long as each of the two duplicate pencils. On Field's view, this explanation becomes: there is an object *u* midway between two points at the ends of the tray *a* and *b*, such that *au* Cong *cd*, where *c* and *d* are the endpoints of one of the pencils. Although there is no reference to numbers in this explanation, there is quantification over an object *u*, an object which did not appear in our intuitive explanation. And, as I shall now argue, (a) this object is extrinsic to the process to be explained; (b) this object is causally irrelevant; (c) this object is arbitrary.

(a) Field does not think that 'the distance between *a* and *b* is twice the distance between *c* and *d*' is to be *analysed* as '∃*u*(*u* Bet *ab* & *au* Cong *ub* & *ub* Cong *cd*)': the distance between *a* and *b* may be twice the distance between *c* and *d*, without *anything* (be it physical object or point of space) existing midway between *a* and *b*. For, as Field quite rightly concedes (Field 1985a: §5), the structure of space-time is an empirical matter: lines in space may very well contain gaps, and whether there is an object or point of space existing midway between *a* and *b* is independent of the distance between them. To see this, consider a line of space isomorphic to the reals, save that it is missing a single point *p*. Suppose that *a* and *b* lie on either side of the hole *p*, the same distance away from it, and let *cd* be half as long as *ab*. Although there is nothing midway between *a* and *b*, there are points *a′*, *b′*, *u′* such that (i) *a′b′* Cong *ab*, (ii) *u′* is midway between *a′* and *b′*, and (iii) *a′u′* Cong *cd*. From (ii) and (iii) it follows that *a′b′* is twice as long as *cd* and then from (i) that *ab* is twice as long as *cd*. But if the fact that *ab* is twice as long as *cd* can obtain without there being an object midway *u* between them, then the existence of such an object is not intrinsic to this fact. Accordingly, the object *u* which Field mentions in his explanation of why the two pencils fit into the tray is extrinsic to the process to be explained.

(b) We have just seen that, even if there were nothing midway between *a* and *b*, *ab* would still be twice as long as *cd*. Even had there been no *u* midway between *a* and *b*, the two pencils would still have fitted inside the tray. So the distance between *a* and *b* doesn't turn upon whether such a *u* exists, and in particular *u* does not cause *ab* to be twice as long as *cd*. In which case, it is hard to see how *u* can be causally relevant to the process to be explained.

(c) Given Hilbert's axiomatization of geometry, there are many sentences which are 'equivalent' to *ab* is twice as long as *cd*. '∃*u*(*u* Bet *ab* & *au* Cong *ub* & *ub* Cong *cd*)' is just one of them. '∃*u*(*c* Bet *ud* & *uc* Cong *cd* & *ud* Cong *ab*)', '∃*u*(*d* Bet *uc* & *ud* Cong *cd* & *uc* Cong *ab*)', '∃*u*∃*x*∃*y*(*xy* Cong *ab* & *u* Bet *xy* & *xu* Cong *yu* & *xu* Cong *cd*)' are all sentences which are true when *ab* is twice as long as *cd*. Any one of these could have equally well been used to express the fact that the distance between *a* and *b* is twice the distance between *c* and *d*. But different objects are quantified over in the above sentences (indeed, the number of objects quantified over is different in different sentences!). Which *u* we mention in our explanation is an arbitrary matter.[2]

Apart from the fact that Field's reformulation of physical theory is nominalistic, the advantages Field claims for these reformulations are illusory.

3. *The ontological cost of Field's programme*

Field's synthetic reformulations of Euclidean geometry and Newtonian gravitational theory commit him to an infinite number of points of space (or space-time). This may not be so bad: after all, there are a number of persuasive reasons for accepting substantivalism (see Earman 1989). But just as Field's method of replacing the analytic theory's distance function requires him to postulate a large number of points of space, so his method of replacing the mass function forces him to postulate a large number of massive bodies, and this is not so plausible.

For mass, as for distance, Field needs to introduce new predicates holding between massive bodies, and write down axioms governing their behaviour so that (1) the new predicates enable us to define nominalistically acceptable comparative mass relations such as 'the mass of *m* is 11/17ths as heavy as the mass of m^*'; (2) representation theorems are true of this theory – for any model of this axiom system, we can prove the existence of a mass-function $m(x)$ from massive bodies in the model onto real numbers which has precisely the properties one expects of a mass function.

How can the nominalist state the ratio of mass between different massive bodies? As before, he might introduce two new primitive predicates into his language: '*x* Same-mass *y*' and '*x* Bet-mass *yz*'. Informally, the first says that *x* and *y* have the same mass, the second that the mass of *x* lies between the masses of *y* and *z*. Now, note that '*a* is twice as massive as *b*' will be true if there is another object *c*, which is both wholly distinct from *b* and has the same mass as it, and the mereological sum of *b* and *c* has the same mass as *a*. Accordingly, if we take '$x \vee y$' to denote the mereological sum of *x* and *y*, and $x \mid y$ to mean that *x* and *y* share no common part,[3] then *a* will be twice as massive as *b* if '$\exists u(b \mid u$ & b Same-mass u & $(b \vee u)$ Same-mass $y)$' is true.

So far, so good. But if '$\exists u(b \mid u$ & b Same-mass u & $(b \vee u)$ Same-mass $y)$'

2 Don't say: let's choose one of these sentences as our translation of '*ab* is twice as long as *cd*'. You still have tell me why you chose one equivalent over another.

3 Both of these predicates are definable in standard mereological systems available to the nominalist.

is going to be Field's nominalistic version of 'a is twice as massive as b', then he needs it to be the case that a is twice as massive as b if *and only if* '$\exists u(b \mid u$ & b Same-mass u & $(b \vee u)$ Same-mass $y)$' is true. This is extremely dubious. Why should a's being twice as massive as b entail the existence of another body wholly disjoint from b which also has the same mass as b? Indeed, why couldn't a and b be the only two massive bodies in the universe? Certainly, Newtonian kinematics and Newtonian gravitational theory allow such possibilities; if Field's synthetic theories are meant to be nominalistic versions of these very theories, I would expect them to do the same.

When we turn to what axioms we must add to our nominalist theory so that Hilbert-style representation theorems are provable, matters are worse: Field must postulate an *infinite* number of massive bodies. In the case of mass, we expect our representation theorems to guarantee that, for any model M of the nominalistic theory, there is a function $m(x)$ from the domain of M into the real numbers such that, (i) $m(x) = m(y)$ iff x Same-mass y; (ii) $m(y) < m(x) < m(z)$ iff x Mass-bet yz; (iii) $m(x \vee y) = m(x) + m(y)$ where x and y are disjoint, and finally (iv) any mass function satisfying (i) to (iii) is unique up to a multiplicative constant. But such mass functions on M will not satisfy (iv) unless the massive bodies in M themselves instantiate the right kind of structure. For instance, suppose there were a model M* containing just three massive bodies, a, b and $a \vee b$, where a is more massive than b.[4] Though M* seems sensible enough, it is not a model Field can allow, for there many mass functions definable on M* which do not differ by a mere multiplicative constant (see Field 1985b). If $m(a) = 1$ then, let $m(b)$ be any number you like, and let $m(a \vee b) = 1 + m(b)$. Any such $m(x)$ satisfies (i) to (iii). Accordingly, if T is a theory which has M* as a model, then the representation theorems are not true of T.

This problem didn't arise for the distance function since Hilbert's axioms for synthetic geometry guarantees that the points along a line are isomorphic to the reals. If lines are like that in every model, then any two distance functions satisfying the corresponding constraints will be unique up to a multiplicative constant. But do we really want to add axioms to our nominalist theories guaranteeing that the massive bodies are isomorphic to the reals? This seems absurd. A commitment to substantivalism is one thing, but a commitment to an infinite number of masses is another. True, massive bodies may be concrete objects, but such a commitment is as ontologically unparsimonious as a commitment to an infinite number of abstract numbers. Yet without such a commitment, how can Field guarantee that the representation theorems are true of his reformulations?

4 Thus, in M*, the extension of 'x Same-mass y' is empty, and $\langle a, b, a \vee b \rangle$ is the sole member of 'x Mass-bet yz'.

4. *An incompleteness in Field's programme*

We have seen how, for an ontological cost, we can say nominalistically that the distance between two objects is m/n of a metre, for any choice of m and n. But we have not been taught how to say nominalistically that the distance between two objects is, say, e of a metre.[5] The systematic treatment of nominalistic predicates outlined above is limited to rational ratios; if the ratio is irrational, then it is not at all clear that a nominalistic predicate can be constructed in the synthetic system capable of expressing this ratio.

Hilbert showed how to formulate the fundamental axioms of geometry synthetically. But there is more to doing geometry than giving the fundamental axioms and drawing conclusions from these axioms. For instance, we may become interested in the geometric properties of a Euclidean right angled triangle, whose base is one unit long and whose height is e units high. But how can we even begin to study such triangles using only the linguistic resources found in Hilbert's synthetic formulation of Euclidean geometry? Field has provided no guarantee that his system is capable of describing such triangles.

Field showed how to formulate the basic laws of Newtonian gravitational theory synthetically. But there is more to Newtonian gravitational theory than the statement of the fundamental laws and the consequences of these laws. We cannot predict the behaviour of a particular massive body from just the fundamental laws alone – the behaviour can be derived only from the fundamental laws plus the initial conditions. But there is no guarantee that the conceptual and linguistic resources capable of stating the fundamental laws are also capable of stating the initial conditions. For instance, we may be interested in studying a Newtonian system consisting of two massive bodies, one of which is e times as heavy as the other, and which are π metres apart. But how can we even begin to study such systems using only the linguistic resources found in Field's synthetic formulation of Newtonian Gravitational theory? Field has provided no guarantee that his synthetic theory is capable of describing such systems.

We might also need to mention irrational numbers in the statement of the fundamental laws. For instance, Coulomb's law states that the force

[5] e, the base of natural logarithms, is the limit of $(1 + 1/n)^n$ as n tends towards infinity. Why didn't I use a more familiar irrational number, such as $\sqrt{2}$, as my example? Because, as a matter of fact, $\sqrt{2}$ is definable in Field's system – though by a formula quite different from those that work for rational ratios (in Field 1980, ch. 8, sec B, the formula Field suggests which would enable him to define such a ratio takes a cardinality quantifier as primitive). However, I see no way of representing e in Field's system, nor has Field given us any guarantee that any ratio between concrete objects which is expressible in the analytic system is also expressible in the synthetic system.

along a straight line from one charge q_1 to another q_2 is $(1/4\pi e)q_1q_2$. But Field has given us no way of stating this law nominalistically. The methods Field has used to deal with rational numbers do not extend to irrational numbers. Of course, as a matter of fact, Coulomb's law is not a fundamental law, but follows from Maxwell's equations. But I can see no a priori reason to rule out the possibility that we may have to use irrational numbers to state the most fundamental physical laws.

5. Moral

Aside from their nominalism, synthetic reformulations have nothing to recommend them. Moreover, although the ontology of the synthetic formulations may be nominalistic, it can scarcely be described as attractive. Finally, though there may exist synthetic formulations of the basic laws of Newtonian gravitational theory, still there is much which physicists would count as Newtonian gravitational theory which has not been captured. The attempt to reformulate physical theory in the interests of a safe and sane ontology is looking shaky. So should we bite the bullet and accept abstracta into our ontology?

Forget the fact that many of us have a natural prejudice against abstract objects. Forget the fact that we find ontologies containing an infinity of space-time points, or an infinity of different massive particles or an infinity of different charged particles unattractive. Let us ask why we are told we ought to believe that such things exist. The short answer Quine/Putnam gives is: because we have to quantify over these objects in order to do physics. But can this be taken as a reason for believing in such things? We have no problems accepting atoms, electrons and quarks into our ontology not because these are concrete objects, nor because we quantify over these objects in our best physical theories, but because these entities play a role in explaining the observable phenomena. In Cartesian geometry, numbers give us the wherewithal to, amongst other things, pick out various possible distance relations between concrete objects or describe situations involving concrete objects – situations which it is not clear how to describe without referring to numbers. But nobody thinks that the numbers explain why certain bodies stand in the distance relations they do. When we say 'x is root two of a metre away from y', we use the number $\sqrt{2}$ to pick out a particular distance relation – but nobody thinks that the number $\sqrt{2}$ plays any part in explaining why these objects stand in this relation. I agree with Field that the utility of mathematical objects is different from the utility of theoretical objects (see Field 1980, chs 1 and 5) – but their utility shows up not in the conservativeness of mathematics, but in the fact that the mathematics is used simply in order to make more things sayable about concrete objects. And it scarcely seems like a good reason to accept objects into our ontology simply because quantifying over such objects allows us to express more things. Notice that this qualm has nothing to do with abstractness. As we have seen, in order to say that the mass of m is twice that of m^* nominalistically, Field must quantify over body m', distinct from m^*, having the same mass as m^*. Well, postulating bodies such as m' can scarcely be said to offend any nominalistic scruples – but would anyone own up to believing in body m' just because it enabled them to say that m and m^* stand in a certain relation?

Field accepts the argument in the Quine/Putnam argument, but tries to show that one of its premisses is incorrect: namely, the premiss that scientific theories can be formulated without reference to abstracta. We have seen some of the difficulties besetting this programme above. I suggest that it may be time to reassess the argument: the bare fact that our best theories quantify over a certain kind of object may be no reason to believe in that kind of object.[6]

University of York
York, Y01 5DD, UK
jwm9@york.ac.uk

References

Earman, J. 1989. *World Enough and Space-time*. Cambridge Mass: MIT Press.

Field, H. 1980. *Science Without Numbers*. Oxford: Basil Blackwell.

Field, H. 1985a. On conservativeness and incompleteness. *Journal of Philosophy* 81: 239–60. Reprinted in Field 1989.

Field, H. 1985b. Can we dispense with space-time? In *PSA 1984: Proceedings of the 1984 Biennial Meeting of the Philosophy of Science Association* vol. 2, ed. P. Asquith and P. Kitcher, 33–90. Reprinted in Field 1989.

Field, H. 1989. *Realism, Mathematics and Modality*. Oxford: Basil Blackwell.

Hale, S. 1988. Space-time and the abstract/concrete distinction. *Philosophical Studies* 53: 85–120.

Hilbert, D. 1971 *Foundations of Geometry*. LaSalle: Open Court.

Malament, D. 1982. Review of Field's *Science Without Numbers*. *Journal of Philosophy* 79: 523–34.

Melia, J. 1998. On the indispensability argument. forthcoming.

Putnam, H. 1971. *Philosophy of Logic*. New York: Harper.

Shapiro, S. 1983. Conservativeness and incompleteness. *Journal of Philosophy*, 80: 521–31.

[6] See Melia 1998 for further elaboration on these ideas. I would like to thank Rosanna Keefe and the Editor of *Analysis* for many helpful comments and discussions.