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Field-Space Symmetry and Its Breakdown

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A unified gauge symmetry of $GL(32N, C)$ or $GL(12+2n, C)$ is proposed for combining the $SU(16N)$ or $SU(6+n)$ group of lepton-quark internal symmetry and the $SL(2, C)$ Lorentz group of space-time symmetry (where $N=1, 2, 3, \cdots$ and $n=0, 1, 2, \cdots, N$). The hierarchy of symmetries obeyed by elementary particles and the symmetry breakdown are briefly discussed. A possible origin of the Cabibbo angle is suggested in a "spinor sublepto-quark" model of leptons and quarks.

As elementary particle physics stands now, all phenomena seem to be well described by the Yang-Mills gauge theory[1] of color $SU(3)$[2] for the strong interaction of quarks and by the Weinberg-Salam gauge theory[3] of $SU(2)\times U(1)$ for the weak and electromagnetic interactions of leptons and quarks. Several unified gauge models of strong, weak and electromagnetic interactions have been proposed by Pati and Salam,[4] by Georgi and Glashow[5] and by Fritzsch and Minkowski.[6] A unified model of the Nambu-Jona-Lasinio type[7] for "all" elementary-particle forces has also been proposed by Akama, Chikashige and the present author.[8] What is left excluded by the "all" elementary-particle forces is gravity. An attempt to unify gravity with "all" other fundamental forces has been made by Akama, Chikashige, Matsuki and the present author[9] in a model of the Nambu-Jona-Lasinio type. In our model, given a set of fundamental fermions, the leptons and quarks, a single parameter, the Newtonian gravitational constant, is enough to determine not only all the other coupling strengths including the fine-structure constant and the strong, semi-weak and the Fermi coupling constants, but also the Weinberg angle and the weak-boson masses.[10] Although our model is, therefore, a very powerful working hypothesis, it is still unsatisfactory at the following two points:

1) It does not determine the lepton and quark masses, the Cabibbo angle, and the CP-violating parameters, which are left as free parameters.

2) It contains the cutoff procedure which is put in by hand. It is believable that the genuine cutoff, which is closely related to the Planck's mass,[11] is present in nature, but in a more natural mannar. To make one step further, it seems necessary to have deeper understanding of the connection between the field internal symmetry and the space-time symmetry.

The history of attempts to unify the internal symmetry of elementary particles and the Lorentz symmetry of space-time goes back to 1937 when Wigner[12] proposed

$SU(4)$ symmetry for combining $SU(2)$ isospin symmetry and $SU(2)$ (or $O(3)$) spin symmetry. Much later in 1964, Gürsey, Radicati and Sakita[13] introduced $SU(6)$ symmetry as an extension of $SU(4)$ by replacing the internal $SU(2)$ symmetry by $SU(3)$. Furthermore, Miyazawa[14] proposed in 1966 what is now called supersymmetry, a possible symmetry between fermions and bosons. On the space-time symmetry alone, one can trace the history back to the Weyl's formulation[15] of Einstein's general relativity. The Lorentz symmetry has been considered as a local gauge symmetry or alike by Utiyama and others.[16] Recently, attempts have been made by Salam and others[17] to unify gravity and other interactions in a gauge theory of groups larger than $SL(2,C)$, the Lorentz group. The way of Salam et al.[17] for unifying gravity and other interactions seems very elegant and related to ours[9] in a methodological sense, though it looks superficially different. They have proposed, as an example, $SL(6,C)$ gauge group for combining the $SU(3)$ internal flavor symmetry and the $SL(2,C)$ Lorentz symmetry. In this paper, $GL(32N,C)$ or $GL(12+2n,C)$ gauge group is proposed which contains a product of Yang-Mills gauge symmetry of color $SU(3)$, the Weinberg-Salam gauge symmetry of $SU(2)\times U(1)$, the lepton-quark symmetry and the Lorentz symmetry of $SL(2,C)$. The physical contents of this tremendously large group will be explained in what follows.

Suppose, for definiteness, that there exist N Weinberg-Salam multiplets of leptons and quarks

$$\begin{pmatrix}\nu_j\\ l_j\end{pmatrix}_L,\quad \begin{pmatrix}u_{ji}\\ d_{ji}\end{pmatrix}_L\quad (\nu_{jR}),\ l_{jR},\ u_{jiR}\ \text{and}\ \ d_{jiR}\,, \tag{1}$$

where the $SU(3)$ color indices $i=1,\ 2,\ 3$, the unknown H-symmetry indices $j=1,\ 2,\ \cdots,\ N$ and possible Cabibbo-like rotations are ignored. The subscripts L and R denote the left- and right-handed components, $\psi_L=(1/2)(1-\gamma_5)\psi$ and $\psi_R=(1/2)(1+\gamma_5)\psi$, respectively. The total number of the fundamental fermions in this model is $8N$ (or $16N$ if the left- and right-handed components are counted seperately). The number of really existing leptons and quarks seems to be an increasing function of time. "How many leptons and quarks?" is always an intriguing question but has never been answered unambiguously. In our unified model of the Nambu-Jona-Lasinio type for "all" elementary-particle forces including gravity,[9] I have given a clue that there must exist a dozen leptons and a dozen flavors and three colors of quarks so that the fine-structure constant may be that small.[18] If this is the case, the unknown parameter N is determined to be six. In any case, a gauge symmetry of $SU(16N)$ seems to be relevant in nature since it contains the chiral $SU(8N)\times SU(8N)$ symmetry as well as the desired product of the $SU(3)\times SU(2)\times U(1)$ gauge symmetry and the lepton-quark symmetry.

If there indeed exist so many leptons and quarks, it is natural to ask why so many. An answer to this question has been given by Akama and the present author.[19),8),18)] It is the "spinor subleptoquark" model of leptons and quarks in which leptons and quarks are further made of "subleptoquarks" of spin 1/2,

$$(w_1, w_2),\ h_j,\ C_0 \quad \text{and} \quad (C_1, C_2, C_3), \tag{2}$$

where the left-handed "wakems" $(w_1, w_2)_L$ form a Weinberg-Salam $SU(2)$ doublet while the "chroms" C_0 and (C_1, C_2, C_3) form a singlet and triplet of color $SU(3)$, respectively, and the "hakams" h_j for $j=1, 2, \cdots, N$ may form a certain global symmetry such as $SU(N)$. The $8N$ leptons and quarks are expressed in terms of these $6+N$ subleptoquarks as

$$\begin{aligned} \nu_j &= (w_1\, h_j\, C_0), \quad & u_{ji} &= (w_1\, h_j\, C_i), \\ l_j &= (w_2\, h_j\, C_0), \quad & d_{ji} &= (w_2\, h_j\, C_i). \end{aligned} \tag{3}$$

An alternative expression has recently been proposed by Fujikawa.[20)] He considers the subleptoquarks $(w_1, w_2, C_0, C_1, C_2, C_3)$ as the fundamental sextet of the unified $SU(6)$ gauge symmetry of leptons and quarks, which has been proposed by Inoue, Kakuto and Nakano, by Yoshimura and by Lee and Weinberg,[21)] presenting the fifteen-plet

$$\begin{aligned} \bar{\nu}_{jL} &= (C_0 w_2 - w_2 C_0, h_j)_R\,, \quad & u_{jiL} &= (C_i w_1 - w_1 C_i, h_j)_L\,, \\ \bar{l}_{jL} &= (C_0 w_1 - w_1 C_0, h_j)_R\,, \quad & d_{jiL} &= (C_i w_2 - w_2 C_i, h_j)_L\,, \\ \bar{l}_{jR} &= (w_1 w_2 - w_2 w_1, h_j)_L\,, \quad & \bar{u}_{jiR} &= (\sum_{k,l=1}^{3} \varepsilon_{ikl} C_k C_l, h_j)_L\,, \\ & & d_{jiR} &= (C_i C_0 - C_0 C_i, h_j)_R\,. \end{aligned} \tag{4}$$

In any case, a gauge symmetry of $SU(6+n)$ for $n=0, 1, 2, \cdots, N$ seems to be relevant, containing the $SU(3)\times SU(2)\times U(1)$ gauge symmetry and the lepton-quark symmetry.

From this line of reasoning, the best candidate for the unified internal symmetry obeyed by leptons and quarks seems to be either $SU(16N)$ or $SU(6+n)$. I thus propose $GL(32N, C)$ or $GL(12+2n, C)$ as a candidate for the unified field-space gauge symmetry which contains $SU(16N)\times SL(2, C)$ or $SU(6+n)\times SL(2, C)$ and which, therefore, combines the internal symmetry of leptons and quarks and the Lorentz symmetry of space-time. The reason for not adopting a smaller $SL(32N, C)$ or $SL(12+2n, \mathrm{C})$ which also contains $SU(16N)\times SL(2, C)$ or $SU(6+n)\times SL(2, C)$ will become clear in what follows.

Following Salam,[17)] let us take the Lagrangian

$$L = i\,\mathrm{tr}[L^\mu, L^\nu]B_{\mu\nu} + i\bar{\psi}L^\mu(\partial_\mu + iB_\mu)\psi \tag{5}$$

with

$$\begin{aligned} L^\mu &= L^{\mu\alpha a}\gamma_\alpha \tfrac{1}{2}\lambda^a + L^{5\mu\alpha a}\gamma_\alpha\gamma_5\tfrac{1}{2}\lambda^a\,, \\ B_\mu &= B_\mu{}^{\alpha\beta a}\tfrac{1}{4}\sigma_{\alpha\beta}\lambda^a + B_\mu{}^a\tfrac{1}{2}\lambda^a + B_\mu{}^{5a}\gamma_5\tfrac{1}{2}\lambda^a \end{aligned} \tag{6}$$

and

$$B_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu+i[B_\mu, B_\nu],$$

where $L^{\mu\alpha 0}$ are the usual vierbein tensor related to the space-time metric tensor $g_{\mu\nu}$ by $g^{\mu\nu}=\eta_{\alpha\beta}L^{\mu\alpha 0}L^{\nu\beta 0}$ (where $\eta_{\alpha\beta}=+1, -1, -1, -1$ for $\alpha=\beta=0, 1, 2, 3$ and $\eta_{\alpha\beta}=0$ for $\alpha\neq\beta$), B_μ are the "gauge connections", $\lambda^a (a=0, 1, 2, \cdots, (16N)^2-1$ or $(6+n)^2-1)$ are the generalized Gell-Mann's matrices of $U(16N)$ or $U(6+n)$, and ψ denotes a fundamental multiplet of fermion fields, the 16N chiral leptons and quarks or the $6+n$ subleptoquarks. The natural unit convention of $\hbar=c=16\pi G=1$ where G is the Newtonian gravitational constant should be understood. This Lagrangian is invariant under the local gauge transformation of $GL(32N, C)$ or $GL(12+n, C)$

$$\psi\rightarrow\Omega\psi\,, \quad L^\mu\rightarrow\Omega L^\mu\Omega^{-1} \quad \text{and} \quad B_\mu\rightarrow\Omega B_\mu\Omega^{-1}-i\Omega\partial_\mu\Omega^{-1} \tag{7}$$

with

$$\Omega(x)=\exp i[\varepsilon^{\alpha\beta a}(x)\tfrac{1}{4}\sigma_{\alpha\beta}\lambda^a+\varepsilon^a(x)\tfrac{1}{2}\lambda^a+\varepsilon^{5a}(x)\gamma_5\tfrac{1}{2}\lambda^a],$$

where ε's are arbitrary functions and $(\frac{1}{4}\sigma_{\alpha\beta}\lambda^a, \frac{1}{2}\lambda^a, \gamma_5\frac{1}{2}\lambda^a)$ form $2(32N)^2$ or $2(12+2n)^2$ generators of $GL(32N, C)$ or $GL(12+2n, C)$ group. It can be shown[17] that the first term in the Lagrangian (5) produces the Ricci's scalar curvature and that this field-space gauge model, therefore, correctly describes gravity in Einstein's general relativity. The equations of motion derived from this Lagrangian are the Einstein's "curvature equation":

$$i[L^\mu, B_{\mu\nu}]=T_\nu \tag{8}$$

and the Cartan's "torsion equation"

$$\partial_\mu[L^\mu, L^\nu]+i[B_\mu[L^\mu, L^\nu]]=S^\nu\,, \tag{9}$$

where T_ν and S^ν are the matter stress-energy and the matter spin, respectively.

Let us proceed to brief discussion of the hierarchy of symmetries obeyed by leptons and quarks on the top of which sits the highest symmetry of $GL(32N, C)$ or $GL(12+2n, C)$. Detailed discussion of the field-space symmetry and its breakdown will be given elsewhere. Let us classify the energy regions in particle physics into the following four: the high energy region with energies between about 1 and 10^2 GeV, the very high energy region with energies between about 10^2 and $10^{15}-10^{18}$ GeV, the superhigh energy region with energies between about $10^{15}-10^{18}$ and 10^{19} GeV and the ultrahigh energy region with energies larger than 10^{19} GeV. The field-space gauge symmetry is supposed to be broken by the following three steps 1) $\sim$3):

0) The field-space gauge symmetry of $GL(32N, C)$ or $GL(12+2n, C)$ holds in the ultrahigh energy region where energy is larger than the Planck's mass ($G^{-1/2}\sim 10^{19}$ GeV) and, therefore, gravity competes with strong, weak, and electromagnetic forces in particle phenomena.

1) The field-space gauge symmetry of $GL(32N, C)$ or $GL(12+2n, C)$ is effectively broken down to the gauge symmetry of $SU(16N)\times SU(2, C)$ or $SU(6+n)\times SL(2, C)$ in the superhigh energy region. The gravitational interaction becomes negligible but the weak and electromagnetic interactions still compete with the strong interaction. The lower limit of energy for this region is estimated[22)] to be order $10^{17}-10^{18}$ GeV by a renormalization group consideration of where the asymptotically not free, weak and electromagnetic interactions compete with the asymptotically free strong interaction. The energy of order 10^{15} GeV is obtained,[5)] on the other hand, by estimating, from the stability of the proton, the masses of the superheavy "leptoquark" gauge bosons which breaks the unified gauge symmetry of strong, weak and electromagnetic interactions down to a lower symmetry.

2) The gauge symmetry of $SU(16N)\times SL(2, C)$ or $SU(6+n)\times SL(2, C)$ is broken down to that of $SU(3)\times SU(2)\times U(1)\times SL(2, C)$ in the very high energy region. The weak and electromagnetic interactions are much weaker than the strong interaction but the weak interaction still competes with the electromagnetic one. The lower limit of energy for this region lies around the masses of the weak bosons, which are predicted in the Weinberg-Salam model to be of order 10^2 GeV. Notice that an approximate global H-symmetry[19)] of $U(N)\times U(N)$ for leptons and quarks appears automatically as an accidental symmetry in this region where the masses of leptons and quarks are much smaller than the energies involved.

3) The gauge symmetry of $SU(3)\times SU(2)\times U(1)\times SL(2, C)$ is broken further down to that of $SU(3)\times U(1)\times SL(2, C)$ where the latter $U(1)$ is the electromagnetic gauge symmetry. This breakdown is supposed to occur spontaneously and to generate the weak boson masses as well as the lepton and quark masses. The approximate H-symmetry of $U(N)\times U(N)$ is also reduced to a smaller one of $U(1)\times U(1)$, which simply denotes the conservation of lepton number and quark number.

In conclusion, let us return to the problems to be solved in a unified model of "all" elementary-particle forces. If CP-violation occurs due to the non-trivial phases in a unitary matrix diagonalizing the quark mass matrix, as suggested by Kobayashi and Maskawa,[23)] all the problems on the lepton and quark masses, on the Cabibbo angle and on the CP-violating parameters can be reduced to a single problem on the lepton-quark mass matrix. Many attemps have already been made to determine the Cabibbo angle in a gauge model of weak and electromagnetic interactions but none of them seems to be both successful and convincing. Here another possible origin of the Cabibbo angle is suggested. If quarks are indeed made of subleptoquarks,[19),8),20)] the weak currents are more fundamentally written in terms of the wakems, w_1 and w_2, as $\boldsymbol{J}_\mu=\bar{w}_L\gamma_\mu\boldsymbol{\tau}w_L$, etc. The Cabibbo angle θ_C then appears as the ratio of the transition matrix element of the charged current between the proton quark and the lamda quark to that between the proton quark and the neutron quark:

$$\tan\theta_C = \frac{\langle p|\bar{w}_{1L}\gamma_\mu w_{2L}|\lambda\rangle}{\langle p|\bar{w}_{1L}\gamma_\mu w_{2L}|n\rangle}. \tag{10}$$

Although this picture does not provide an easy solution of the Cabibbo angle because of the ambiguous subleptoquark dynamics, it strongly suggests that the Cabibbo angle would vary at higher momentum transfers where the subleptoquark structure of quarks may become relevant. This effect would be observed in future high energy neutrino or lepton reactions. Notice that the possibility of vanishing Cabibbo angle in strong electromagnetic field has recently been emphasized by Salam[24)] in a spontaneously broken gauge model of the Cabibbo rotation.

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