CHAPTER 10

FIFTH ORDER GRAVITY WAVE THEORY

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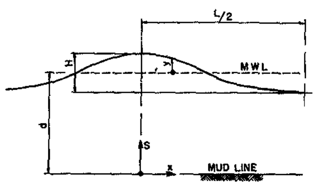
INTRODUCTION

In dealing with problems connected with gravity waves, scientists and engineers frequently find it necessary to make lengthy theoretical calculations involving such wave characteristics as wave height, wave length, period, and water depth. Several approximate theoretical expressions have been derived relating the above parameters. Airy, for instance, contributed a very valuable and complete theory for waves traveling over a horizontal bottom in any depth of water. Due to the simplicity of the Airy theory, it is frequently used by engineers. This theory, however, was developed for waves of very small heights and is inaccurate for waves of finite height. Stokes presented a similar solution for waves of finite height by use of trigonometric series. Using five terms in the series, this solution will extend the range covered by the Airy theory to waves of greater steepness. No attempt has been made in this paper to specify the range where the theory is applicable. The coefficients in these series are very complicated and for a numerical problem, the calculations become very tedious. Because of this difficulty, this theory would be very little used by engineers unless the value of the coefficient is presented in tabular form The purpose of this paper is to present the results of the fifth order theory and values of the various coefficients as a function of the parameter d/L.

The co-ordinate system and description of the wave to be considered in this paper is shown in Fig. 1.



Coordinate system
Fig. 1

The waves to be considered in this analysis are oscillatory, non-viscous, water waves of constant depth and of infinite extent in a direction normal to their propagation. Hence, the particle velocities may be obtained from a potential function as follows:

$$u = \frac{\partial \phi}{\partial x}, \qquad v = \frac{\partial \phi}{\partial s} \qquad 1.$$

provided

$$\nabla^2 \phi = 0 \qquad 2.$$

and the necessary boundary conditions on ϕ are satisfied. The boundary condition at the mud line is that the normal velocity be zero.

$$\left.\frac{\partial \phi}{\partial s}\right|_{s=0} = 0 \qquad 3.$$

One of the free surface (S = d + y) boundary conditions arises from the fact that the water particles stay on the surface. The other free surface boundary condition is that the pressure is zero. These conditions may be written as follows:

$$\left.\begin{aligned} v &= u \frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} \\ P &= 0 \end{aligned}\right\} \text{at } S = d + y \qquad 4.$$

The relationship between the pressure and the particle velocities is expressed by the Bernoulli equation.

$$\frac{2P}{\rho} + (u^2 + v^2) + 2\frac{\partial \phi}{\partial t} = -2g(K + S - d) \qquad 5.$$

where

K = a constant.

Since we are dealing with an oscillatory wave of length L, the wave profile and the potential function may be expressed in terms of a phase angle

$$\theta = \frac{2\pi}{L}(x - \bar{C}t) = \beta(x - \bar{C}t) \qquad 6.$$

where

$\bar{C}$ = the wave celerity.

Thus, combining equations 1, 4, 5, and 6, the free surface boundary conditions may be written in the form

$$\frac{\partial y}{\partial x} = \frac{-v}{\bar{C} - u} \qquad 7.$$

and $$-2u\bar{C} + (u^2 + v^2) = -2g(K + y)$$

or $$(\bar{C} - u)^2 + v^2 = C^2 - 2g(K + y) \qquad 8.$$

where

$u = \frac{\partial \phi}{\partial x}$, $v = \frac{\partial \phi}{\partial y}$ evaluated at S = d + y.

The solution to the problem is obtained by finding a solution to equation 2 that satisfies equations 3, 7, and 8. The expression for the wave profile will come out of such a solution in the process of satisfying equations 7 and 8.

Other investigators have obtained solutions correct to the third order of approximation (Ref. 1). Also, solutions for the case $d \rightarrow \infty$ have been obtained to high orders of approximation (Ref. 2). We shall proceed by assuming a similar series form of solution for the profile and the velocity potential. The unknown constants appearing in these series will then be evaluated using an iterative procedure by substitution into equations 7 and 8.

The series form for ϕ , which satisfies equations 2, 3, and symmetry requirements, will be assumed as follows:

$$\frac{\beta\phi}{\bar{C}} = \frac{2\pi\phi}{L\bar{C}} = (\lambda A_{11} + \lambda^3 A_{13} + \lambda^5 A_{15})\cosh \beta S \sin\theta$$
$$+ (\lambda^2 A_{22} + \lambda^4 A_{24})\cosh 2\beta S \sin 2\theta$$
$$+ (\lambda^3 A_{33} + \lambda^5 A_{35})\cosh 3\beta S \sin 3\theta \qquad 9.$$
$$+ \lambda^4 A_{44} \cosh 4\beta S \sin 4\theta + \lambda^5 A_{55} \cosh 5\beta S \sin 5\theta$$

The series form for the profile, y, which satisfies symmetry requirements, will be assumed to be

$$\beta y = \lambda \cos\theta + (\lambda^2 B_{22} + \lambda^4 B_{24})\cos 2\theta$$
$$+ (\lambda^3 B_{33} + \lambda^5 B_{35})\cos 3\theta + \lambda^4 B_{44} \cos 4\theta \qquad 10.$$
$$+ \lambda^5 B_{55} \cos 5\theta$$

Further, the following forms will be assumed for the constant K and the wave celerity $\bar{C}$.

$$\beta K = \lambda^2 C_3 + \lambda^4 C_4 \qquad 11.$$

$$\beta \bar{C}^2 = C_0^2 (1 + \lambda^2 C_1 + \lambda^4 C_2) \qquad 12.$$

Equations 9, 10, 11, and 12 may be substituted into equations 7 and 8 by performing the necessary expansions and retaining those terms of importance to and including the fifth order of approximations. The order of any term is represented by the power of the coefficient λ which modifies that term.

The method of expansion used in the present paper is to solve for the value of $\beta u / \bar{C}$ and $\beta v / \bar{C}$ from equations 7 and 8 and set these values equal to $\beta/c \; \partial\phi/\partial x$ and $\beta/c \; \partial\phi/\partial s$, respectively, at S = d + y. Such a procedure results in two equations involving the undetermined constants, powers of cos ϕ , and powers of the coefficient λ . These equations are grouped according to powers of λ and sub-grouped according to powers of cos ϕ . Since the equations must hold for any value of ϕ , terms in each equation involving the same order of approximation and the same power of cos ϕ are set equal, this results in twenty equations involving the twenty constants Aij, Bij, and Ci. The solution to these equations is listed below and numerical values are given in tables I, II and III. The constants involve the ratio of water depth to wave length (d/L) as a parameter. For brevity in listing the coefficients the notation is made that s = sinh(2πd/L) and c = cosh(2πd/L).

$$C_o^2 = g(\tanh \beta d)$$

$$A_{11} = 1/s$$

$$A_{13} = \frac{-c^2(5c^2+1)}{8s^5}$$

$$A_{15} = \frac{-(1184c^{10}-1440c^8-1992c^6+2641c^4-249c^2+18)}{1536s^{11}}$$

$$A_{22} = \frac{3}{8s^4}$$

$$A_{24} = \frac{(192c^8-424c^6-312c^4+480c^2-17)}{768s^{10}}$$

$$A_{33} = \frac{(13-4c^2)}{64s^7}$$

$$A_{35} = \frac{(512c^{12}+4224c^{10}-6800c^8-12,808c^6+16,704c^4-3154c^2+107)}{4096s^{13}(6c^2-1)}$$

$$A_{44} = \frac{(80c^6-816c^4+1338c^2-197)}{1536s^{10}(6c^2-1)}$$

$$A_{55} = \frac{-(2880c^{10}-72.480c^8+324,000c^6-432,000c^4+163,470c^2-16,245)}{61,440s^{11}(6c^2-1)(8c^4-11c^2+3)}$$

$$B_{22} = \frac{(2c^2+1)}{4s^3}c$$

$$B_{24} = \frac{c(272c^8-504c^6-192c^4+322c^2+21)}{384s^9}$$

$$B_{33} = \frac{3(8c^6+1)}{64s^6}$$

$$B_{35} = \frac{(88,128c^{14}-208,224c^{12}+70,848c^{10}+54,000c^8-21,816c^6+6264c^4-54c^2-}{12.288s^{12}(6c^2-1)}$$

$$B_{44} = \frac{c(768c^{10}-448c^{8}-48c^{6}+48c^{4}+106c^{2}-21)}{384s^{9}(6c^{2}-1)}$$

$$B_{55} = \frac{(192,000c^{16}-262,720c^{14}+83,680c^{12}+20,160c^{10}-7280c^{8})}{12,288s^{10}(6c^{2}-1)(8c^{4}-11c^{2}+3)} + \\ + \frac{(7160c^{6}-1800c^{4}-1050c^{2}+225)}{12,288s^{10}(6c^{2}-1)(8c^{4}-11c^{2}+3)}$$

$$C_1 = \frac{(8c^{4}-8c^{2}+9)}{8s^{4}}$$

$$C_2 = \frac{(3840c^{12}-4096c^{10}+2592c^{8}-1008c^{6}+5944c^{4}-1830c^{2}+147)}{512s^{10}(6c^{2}-1)}$$

$$C_3 = -\frac{1}{4sc}$$

$$C_4 = \frac{(12c^{8}+36c^{6}-162c^{4}+141c^{2}-27)}{192cs^{9}}$$

There still remains the problem of determining the coefficients β and λ. We will assume that the wave is described by the independent parameters H, d, L (crest to trought height, water depth and wave length respectively). It is easily seen that H is related to the profile expression y by the relation

$$H = y\big|_{\theta=0} - y\big|_{\theta=\pi} \qquad 13.$$

Thus, using equation 10 and rearranging

$$\frac{\pi H}{d} = \frac{1}{(d/L)}\left\{\lambda + \lambda^{3} B_{33} + \lambda^{5}\left(B_{35} + B_{55}\right)\right\} \qquad 14.$$

Also, using equation 12 and the expression for $C_o^{\,2}$ it is easily shown that

$$\frac{d}{L_o} = \left(\frac{d}{L}\right) \operatorname{Tanh} \beta d \left\{1 + \lambda^{2} C_1 + \lambda^{4} C_2\right\} \qquad 15.$$

where

$L_o = \frac{gT^2}{2\pi}$, T = wave period and g = acceleration due to gravity.

Since the parameters H, d and L are assumed to be known for the wave and since B_{33}, B_{35}, B_{55}, C_1 and C_2 are functions of only d/L, the simultaneous solution of equations 14 and 15 yield both the values of d/L and the coefficient λ. Knowing the value of d, the value of β is easily obtained and hence the wave may be completely described in all its properties.

Unfortunately, the solution of equations 14 and 15 is rather complex and tedious. It would be advantageous to perform a computer solution to equations 14 and 15 and list the results in the manner used by Skjelbreia (Ref. 1) in a similar analysis of the third-order approximation.

The results of the fifth order theory presented in this analysis will be compared with both the third-order approximation and the first-order Airy theory for the following wave.

Given:

water depth	d = 30 ft.
wave height	H = 18 2/3 ft.
wave period	T = 7.72 sec.

Determine:

1. Wave length and wave velocity.
2. Equation for wave profile and horizontal particle velocity.

After substituting the given values for d, H, and T into equations 14 and 15, the simultaneous solutions of these equations yield the correct values for d/L and λ. The following results are obtained

$d/L = 0.120 \qquad \lambda = 0.1885$

(Note for this example a digital computer was used to solve these simultaneous equations.)

From Tables I, II, and III the following coefficients are obtained.

Wave profile:

$B_{22} = 2.5024$	$B_{33} = 5.7317$	$B_{44} = 14.034$
$B_{24} = -3.7216$	$B_{35} = -4.8893$	$B_{55} = 37.200$

Potential function:

$A_{11} = 1.2085$	$A_{22} = 0.7998$	$A_{35} = -1.5042$
$A_{13} = -5.1153$	$A_{24} = -4.9710$	$A_{44} = 0.0587$
$A_{15} = -10.6530$	$A_{33} = 0.3683$	$A_{55} = -0.0750$

TABLE I

FIFTH ORDER GRAVITY WAVE COEFFICIENTS

d/L	B22	B24	B33	B35	B44	B55	A11	d/L
.0500	2.5817 1	-1.0339 4	5.1727 2	-3.3425 5	9.5495 3	1.7157 5	3.1313	.0500
.0550	1.9661 1	-4.6442 3	3.0222 2	-1.1586 5	4.3146 3	6.0431 4	2.8369	.0550
.0600	1.5369 1	-2.2450 3	1.8619 2	-4.4307 4	2.1132 3	2.3733 4	2.5908	.0600
.0620	1.4016 1	-1.7082 3	1.5539 2	-3.0881 4	1.6200 3	1.6772 4	2.5032	.0620
.0640	1.2825 1	-1.3114 3	1.3057 2	-2.1782 4	1.2548 3	1.2018 4	2.4210	.0640
.0660	1.1771 1	-1.0151 3	1.1041 2	-1.5534 4	9.8140 2	8.7254 3	2.3437	.0660
.0680	1.0836 1	-7.9175 2	9.3932 1	-1.1193 4	7.7465 2	6.4136 3	2.2708	.0680
.0700	1.0004 1	-6.2195 2	8.0366 1	-8.1420 3	6.1678 2	4.7698 3	2.2020	.0700
.0720	9.2592	-4.9177 2	6.9131 1	-5.9747 3	4.9515 2	3.5868 3	2.1369	.0720
.0740	8.5916	-3.9120 2	5.9771 1	-4.4201 3	4.0062 2	2.7257 3	2.0752	.0740
.0760	7.9911	-3.1295 2	5.1931 1	-3.2945 3	3.2655 2	2.0921 3	2.0166	.0760
.0780	7.4494	-2.5164 2	4.5329 1	-2.4724 3	2.6807 2	1.6211 3	1.9610	.0780
.0800	6.9593	-2.0331 2	3.9741 1	-1.8670 3	2.2155 2	1.2675 3	1.9081	.0800
.0820	6.5149	-1.6497 2	3.4989 1	-1.4179 3	1.8429 2	9.9961 2	1.8576	.0820
.0840	6.1109	-1.3438 2	3.0930 1	-1.0823 3	1.5473 2	7.9483 2	1.8095	.0840
.0860	5.7427	-1.0986 2	2.7448 1	-8.2988 2	1.2984 2	6.3696 2	1.7636	.0860
.0880	5.4063	-9.0089 1	2.4447 1	-6.3879 2	1.0992 2	5.1428 2	1.7196	.0880
.0900	5.0985	-7.4092 1	2.1852 1	-4.9330 2	9.3566 1	4.1820 2	1.6775	.0900
.0920	4.8161	-6.1062 1	1.9599 1	-3.8193 2	8.0053 1	3.4241 2	1.6372	.0920
.0940	4.5566	-5.0429 1	1.7636 1	-2.9625 2	6.8832 1	2.8219 2	1.5986	.0940
.0960	4.3177	-4.1710 1	1.5920 1	-2.3003 2	5.9466 1	2.3403 2	1.5614	.0960
.0980	4.0973	-3.4534 1	1.4413 1	-1.7864 2	5.1609 1	1.9525 2	1.5258	.0980
.1000	3.8936	-2.8609 1	1.3088 1	-1.3860 2	4.4988 1	1.6384 2	1.4915	.1000
.1020	3.7052	-2.3700 1	1.1918 1	-1.0729 2	3.9382 1	1.3825 2	1.4584	.1020
.1040	3.5304	-1.9621 1	1.0881 1	-8.2743 1	3.4614 1	1.1728 2	1.4266	.1040
.1060	3.3682	-1.6222 1	9.9610	-6.3440 1	3.0543 1	9.9991 1	1.3959	.1060
.1080	3.2174	-1.3383 1	9.1415	-4.8229 1	2.7053 1	8.5673 1	1.3664	.1080
.1100	3.0770	-1.1005 1	8.4099	-3.6222 1	2.4048 1	7.3752 1	1.3378	.1100
.1120	2.9460	-9.0102	7.7550	-2.6732 1	2.1452 1	6.3779 1	1.3107	.1120
.1140	2.8238	-7.3328	7.1673	-1.9226 1	1.9201 1	5.5395 1	1.2835	.1140
.1160	2.7096	-5.9199	6.6388	-1.3288 1	1.7242 1	4.8316 1	1.2577	.1160
.1180	2.6026	-4.7282	6.1622	-8.5947	1.5532 1	4.2313 1	1.2327	.1180
.1200	2.5024	-3.7216	5.7317	-4.8893	1.4034 1	3.7200 1	1.2085	.1200
.1220	2.4084	-2.8704	5.3418	-1.9711	1.2717 1	3.2829 1	1.1850	.1220
.1240	2.3202	-2.1501	4.9881	3.1870 -1	1.1557 1	2.9076 1	1.1623	.1240
.1260	2.2372	-1.5402	4.6665	2.1065	1.0531 1	2.5843 1	1.1402	.1260
.1280	2.1591	-1.0235	4.3735	3.4927	9.6217	2.3047 1	1.1188	.1280
.1300	2.0855	-5.8591 -1	4.1061	4.5572	8.8134	2.0621 1	1.0980	.1300
.1320	2.0162	-2.1533 -1	3.8617	5.3640	8.0929	1.8508 1	1.0778	.1320
.1340	1.9507	9.8285 -2	3.6377	5.9645	7.4490	1.6663 1	1.0582	.1340
.1360	1.8889	3.6341 -1	3.4323	6.3998	6.8723	1.5046 1	1.0391	.1360
.1380	1.8304	5.8719 -1	3.2435	6.7030	6.3544	1.3624 1	1.0205	.1380
.1400	1.7751	7.7571 -1	3.0697	6.9009	5.8883	1.2371 1	1.0024	.1400
.1420	1.7228	9.3409 -1	2.9095	7.0150	5.4679	1.1263 1	9.8485 -1	.1420
.1440	1.6731	1.0667	2.7615	7.0626	5.0878	1.0281 1	9.6770 -1	.1440
.1460	1.6260	1.1772	2.6247	7.0579	4.7434	9.4080	9.5100 -1	.1460
.1480	1.5814	1.2689	2.4980	7.0122	4.4308	8.6300	9.3471 -1	.1480
.1500	1.5389	1.3443	2.3805	6.9346	4.1465	7.9349	9.1884 -1	.1500
.1520	1.4986	1.4059	2.2714	6.8327	3.8874	7.3124	9.0335 -1	.1520
.1540	1.4602	1.4555	2.1700	6.7124	3.6508	6.7536	8.8824 -1	.1540
.1560	1.4237	1.4950	2.0756	6.5787	3.4344	6.2508	8.7348 -1	.1560
.1580	1.3889	1.5257	1.9876	6.4353	3.2361	5.7974	8.5908 -1	.1580
.1600	1.3557	1.5489	1.9055	6.2855	3.0541	5.3878	8.4502 -1	.1600
.1620	1.3241	1.5657	1.8288	6.1317	2.8868	5.0168	8.3127 -1	.1620
.1640	1.2940	1.5770	1.7570	5.9761	2.7327	4.6802	8.1784 -1	.1640
.1660	1.2652	1.5836	1.6898	5.8201	2.5906	4.3743	8.0472 -1	.1660
.1680	1.2377	1.5863	1.6269	5.6650	2.4594	4.0956	7.9188 -1	.1680
.1700	1.2114	1.5855	1.5678	5.5118	2.3380	3.8413	7.7933 -1	.1700
.1720	1.1863	1.5819	1.5123	5.3614	2.2255	3.6089	7.6705 -1	.1720
.1740	1.1622	1.5758	1.4601	5.2141	2.1212	3.3961	7.5503 -1	.1740
.1760	1.1392	1.5677	1.4110	5.0706	2.0244	3.2009	7.4327 -1	.1760
.1780	1.1172	1.5579	1.3647	4.9310	1.9343	3.0215	7.3175 -1	.1780
.1800	1.0961	1.5467	1.3211	4.7957	1.8504	2.8565	7.2047 -1	.1800
.1820	1.0759	1.5343	1.2800	4.6647	1.7721	2.7044	7.0943 -1	1820
.1840	1.0565	1.5209	1.2411	4 5380	1 6991	2 5641	6 9861 -1	1840
.1860	1.0378	1.5068	1.2043	4 4158	1 6309	2 4344	6 8801 -1	1860
.1880	1.0200	1.4921	1.1696	4 2980	1 5671	2 3143	6 7762 -1	1880
.1900	1.0028	1.4769	1.1367	4 1846	1 5073	2 2031	6 6744 -1	1900
.1920	9.8635 -1	1.4614	1.1055	4 0754	1 4512	2 0998	6 5745 -1	1920
.1940	9.7052 -1	1.4456	1.0760	3 9704	1 3986	2 0038	6 4766 -1	1940
.1960	9.5531 -1	1.4296	1.0479	3 8694	1 3492	1 9145	6 3806 -1	1960
.1980	9.4067 -1	1.4136	1.0213	3 7725	1 3027	1 8314	6 2864 -1	1980
.2000	9.2660 -1	1.3976	9.9601 -1	3 6793	1 2590	1 7538	6 1939 -1	2000
.2100	8.6373 -1	1.3190	8.8675 -1	3 2665	1 0748	1 4353	5 7568 -1	2100
.2200	8.1146 -1	1.2457	8.0058 -1	2 9300	9 3521 -1	1 2033	5 3575 -1	2200
.2300	7.6769 -1	1.1794	7.3168 -1	2 6551	8 2747 -1	1 0304	4 9916 -1	2300
.2400	7.3081 -1	1.1204	6.7594 -1	2 4296	7 4294 -1	8 9887 -1	4 6553 -1	2400
.2500	6.9958 -1	1.0686	6.3038 -1	2 2436	6 7567 -1	7 9700 -1	4 3454 -1	2500
.2600	6.7299 -1	1.0231	5.9279 -1	2 0893	6 2148 -1	7 1686 -1	4 0591 -1	2600
.2700	6.5026 -1	9.8348 -1	5.6152 -1	1.9606	5.7733 -1	6.5294 -1	3.7942 -1	.2700
.2800	6.3076 -1	9.4892 -1	5.3534 -1	1.8527	5.4103 -1	6.0135 -1	3.5485 -1	.2800
.2900	6.1397 -1	9.1882 -1	5.1328 -1	1.7617	5.1093 -1	5.5927 -1	3.3204 -1	.2900
.3000	5.9948 -1	8.9259 -1	4.9458 -1	1.6846	4.8578 -1	5.2462 -1	3.1084 -1	.3000
.3100	5.8694 -1	8.6974 -1	4.7866 -1	1.6191	4.6464 -1	4.9585 -1	2.9110 -1	.3100
.3200	5.7607 -1	8.4981 -1	4.6506 -1	1.5631	4.4677 -1	4.7180 -1	2.7270 -1	.3200
.3300	5.6662 -1	8.3241 -1	4.5338 -1	1.5151	4.3157 -1	4.5156 -1	2.5554 -1	.3300
.3400	5.5839 -1	8.1721 -1	4.4333 -1	1.4738	4.1861 -1	4.3444 -1	2.3953 -1	.3400
.3500	5.5122 -1	8.0392 -1	4.3465 -1	1.4382	4.0749 -1	4.1988 -1	2.2456 -1	.3500
.3600	5.4495 -1	7.9230 -1	4.2714 -1	1.4075	3.9794 -1	4.0744 -1	2.1058 -1	.3600
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.3800	5.3469 -1	7.7320 -1	4.1495 -1	1.3577	3.5258 -1	3.8761 -1	1.8526 -1	.3800
.3900	5.3049 -1	7.6537 -1	4.1007 -1	1.3375	3.7640 -1	3.7970 -1	1.7380 -1	.3900
.4000	5.2681 -1	7.5851 -1	4.0572 -1	1.3200	3.7104 -1	3.7286 -1	1.6308 -1	.4000
.4200	5.2076 -1	7.4719 -1	3.9868 -1	1.2914	3.6231 -1	3.6178 -1	1.4361 -1	.4200
.4400	5.1608 -1	7.3846 -1	3.9329 -1	1.2694	3.5566 -1	3.5339 -1	1.2650 -1	.4400
.4600	5.1247 -1	7.3170 -1	3.8915 -1	1.2526	3.5057 -1	3.4699 -1	1.1147 -1	.4600
.4800	5.0968 -1	7.2647 -1	3.8596 -1	1.2397	3.4666 -1	3.4210 -1	9.8237 -2	.4800
.5000	5.0752 -1	7.2242 -1	3.8350 -1	1.2297	3.4366 -1	3.3835 -1	8.6590 -2	.5000
.5500	5.0400 -1	7.1583 -1	3.7951 -1	1.2136	3.3880 -1	3.3230 -1	6.3190 -2	.5500
.6000	5.0213 -1	7.1233 -1	3.7740 -1	1.2050	3.3624 -1	3.2912 -1	4.6133 -2	.6000

TABLE II

FIFTH ORDER GRAVITY WAVE COEFFICIENTS

d/L	A_{13}	A_{15}	A_{22}	A_{24}	A_{33}	A_{35}	A_{44}	d/L
.0500	-2.6997 2	-1.5812 4	3.6054 1	-1.7843 4	3.9630 2	-2.9907 5	4.1585 3	.0500
.0550	-1.7098 2	-4.4635 3	2.4290 1	-7.3634 3	1.9646 2	-9.0711 4	1.4905 3	.0550
.0600	-1.1307 2	-1.3365 3	1.6895 1	-3.2977 3	1.0288 2	-3.0495 4	5.7854 2	.0600
.0620	-9.6847 1	-8.3832 2	1.4724 1	-2.4389 5	8.0465 1	-2.0206 4	4.0331 2	.0620
.0640	-8.3406 1	-5.3159 2	1.2863 1	-1.8219 3	6.3359 1	-1.5557 4	2.8370 2	.0640
.0660	-7.2201 1	-5.4183 2	1.1514 1	-1.5738 3	5.0202 1	-9.2007 3	2.0121 2	.0660
.0680	-6.2804 1	-2.2396 2	9.9711	-1.0450 3	4.0009 1	-6.3113 5	1.4381 2	.0680
.0700	-5.4881 1	-1.5052 2	8.8160	-8.0134 2	3.2061 1	-4.3723 5	1.0351 2	.0700
.0720	-4.8166 1	-1.0462 2	7.8186	-6.1920 2	2.5823 1	-3.0568 5	7.4987 1	.0720
.0740	-4.2446 1	-7.5850 1	6.9541	-4.8188 2	2.0899 1	-2.1553 3	5.4651 1	.0740
.0760	-3.7551 1	-5.7724 1	6.2020	-3.7752 2	1.6989 1	-1.5316 3	4.0048 1	.0760
.0780	-3.3344 1	-4.6207 1	5.5455	-2.9761 2	1.3870 1	-1.0963 3	2.9495 1	.0780
.0800	-2.9713 1	-3.8784 1	4.9706	-2.3600 2	1.1368 1	-7.8988 2	2.1872 1	.0800
.0820	-2.6566 1	-3.3883 1	4.4655	-1.8817 2	9.3519	-5.7258 2	1.6212 1	.0820
.0840	-2.5828 1	-3.0526 1	4.0205	-1.5081 2	7.7205	-4.1736 2	1.2088 1	.0840
.0860	-2.1438 1	-2.8106 1	3.6274	-1.2145 2	6.3947	-3.0572 2	9.0432	.0860
.0880	-1.9344 1	-2.6249 1	3.2791	-9.8249 1	5.3130	-2.2494 2	6.7843	.0880
.0900	-1.7504 1	-2.4728 1	2.9698	-7.9818 1	4.4272	-1.6615 2	5.1019	.0900
.0920	-1.5881 1	-2.3408 1	2.6945	-6.5102 1	3.6991	-1.2313 2	3.8442	.0920
.0940	-1.4445 1	-2.2208 1	2.4488	-5.3296 1	3.0988	-9.1501 1	2.9009	.0940
.0960	-1.3177 1	-2.1063 1	2.2291	-4.3781 1	2.6022	-6.8140 1	2.1912	.0960
.0980	-1.2039 1	-2.0010 1	2.0323	-3.6080 1	2.1902	-5.0816 1	1.6559	.0980
.1000	-1.1029 1	-1.8977 1	1.8556	-2.9822 1	1.8473	-3.7922 1	1.2511	.1000
.1020	-1.0126 1	-1.7979 1	1.6966	-2.4718 1	1.5613	-2.8296 1	9.4455 -1	.1070
.1040	-9.3160	-1.7015 1	1.5533	-2.0538 1	1.3220	-2.1089 1	7.1197 -1	.1040
.1060	-8.5884	-1.6086 1	1.4240	-1.7104 1	1.1213	-1.5682 1	5.3532 -1	.1060
.1080	-7.9352	-1.5193 1	1.3070	-1.4274 1	9.5264 -1	-1.1617 1	4.0107 -1	.1080
.1100	-7.3417	-1.4339 1	1.2011	-1.1954 1	8.1056 -1	-8.5587	2.9901 -1	.1100
.1120	-6.8067	-1.3522 1	1.1050	-9.9941	6.9064 -1	-6.2556	2.2145 -1	.1120
.1140	-6.3218	-1.2746 1	1.0177	-8.3809	5.8921 -1	-4.5215	1.6256 -1	.1140
.1160	-5.8814	-1.2009 1	9.3824 -1	-7.0360	5.0326 -1	-3.2170	1.1792 -1	.1160
.1180	-5.4807	-1.1512 1	8.6567 -1	-5.9122	4.3051 -1	-2.2375	8.4167 -2	.1180
.1200	-5.1153	-1.0653 1	7.9984 -1	-4.9710	3.6828 -1	-1.5042	5.8724 -2	.1200
.1220	-4.7816	-1.0032 1	7.3954 -1	-4.1809	3.1545 -1	-9.4751 -1	3.9635 -2	.1220
.1240	-4.4764	-9.4475	6.8439 -1	-3.5165	2.7041 -1	-5.5245 -1	2.5401 -2	.1240
.1260	-4.1966	-8.8982	6.3389 -1	-2.9567	2.3193 -1	-2.5481 -1	1.4870 -2	.1260
.1280	-3.9398	-8.5826	5.8760 -1	-2.4843	1.9903 -1	-3.8641 -2	7.1615 -3	.1280
.1300	-3.7037	-7.8989	5.4512 -1	-2.0850	1.7087 -1	1.1586 -1	1.5986 -3	.1500
.1320	-3.4863	-7.4456	5.0610 -1	-1.7470	1.4673 -1	2.2379 -1	-2.3380 -3	.1320
.1340	-3.2858	-7.0210	4.7021 -1	-1.4605	1.2601 -1	2.9664 -1	-5.0474 -3	.1340
.1360	-3.1007	-6.6234	4.3718 -1	-1.2175	1.0822 -1	3.4322 -1	-6.8355 -3	.1360
.1380	-2.9295	-6.2512	4.0675 -1	-1.0111	9.2936 -2	3.7025 -1	-7.9370 -3	.1380
.1400	-2.7709	-5.9029	3.7869 -1	-8.3577 -1	7.9784 -2	3.8284 -1	-8.5317 -3	.1400
.1420	-2.6238	-5.5769	3.5278 -1	-6.8666 -1	6.8464 -2	3.8491 -1	-8.7567 -3	.1420
.1440	-2.4873	-5.2719	3.2885 -1	-5.5983 -1	5.8714 -2	3.7943 -1	-8.7166 -3	.1440
.1460	-2.3604	-4.9864	3.0672 -1	-4.5193 -1	5.0312 -2	3.6866 -1	-8.4908 -3	.1460
.1480	-2.2422	-4.7191	2.8625 -1	-3.6013 -1	4.3068 -2	3.5430 -1	-8.1392 -3	.1480
.1500	-2.1321	-4.4689	2.6729 -1	-2.8206 -1	3.6822 -2	3.5763 -1	-7.7068 -3	.1500
.1520	-2.0294	-4.2345	2.4972 -1	-2.1571 -1	3.1434 -2	3.1965 -1	-7.2270 -3	.1520
.1540	-1.9334	-4.0150	2.3542 -1	-1.5935 -1	2.6786 -2	3.0099 -1	-6.7245 -3	.1540
.1560	-1.8457	-3.8092	2.1830 -1	-1.1154 -1	2.2777 -2	2.8224 -1	-6.2172 -3	.1560
.1560	-1.7596	-3.6164	2.0425 -1	-7.1051 -2	1.9318 -2	2.6575 -1	-5.7178 -3	.1580
.1600	-1.6809	-3.4354	1.9120 -1	-3.6825 -2	1.6335 -2	2.4579 -1	-5.2352 -3	.1600
.1620	-1.6071	-3.2657	1.7906 -1	-7.9688 -3	1.3765 -2	2.2853 -1	-4.7752 -3	.1620
.1640	-1.5377	-3.1065	1.6777 -1	1.6285 -2	1.1547 -2	2.1209 -1	-4.3415 -3	.1640
.1660	-1.4726	-2.9566	1.5725 -1	3.6590 -2	9.6392 -3	1.9654 -1	-3.9360 -3	.1660
.1680	-1.4112	-2.8160	1.4746 -1	5.3507 -2	7.9972 -3	1.8191 -1	-3.5596 -3	.1680
.1700	-1.3535	-2.6838	1.3833 -1	6.7518 -2	6.5857 -3	1.6820 -1	-3.2122 -3	.1700
.1720	-1.2991	-2.5594	1.2981 -1	7.9035 -2	5.3737 -3	1.5541 -1	-2.8932 -3	.1720
.1740	-1.2477	-2.4425	1.2187 -1	8.8413 -2	4.5346 -3	1.4350 -1	-2.6015 -3	.1740
.1760	-1.1993	-2.3320	1.1445 -1	9.5957 -2	3.4451 -3	1.3244 -1	-2.3552 -3	.1760
.1780	-1.1535	-2.2281	1.0752 -1	1.0193 -1	2.6853 -3	1.2220 -1	-2.0933 -3	.1780
.1800	-1.1101	-2.1302	1.0104 -1	1.0655 -1	2.0576 -3	1.1272 -1	-1.8740 -3	.1800
.1820	-1.0691	-2.0378	9.4988 -2	1.1003 -1	1.4871 -3	1.0397 -1	-1.6757 -3	.1820
.1840	-1.0305	-1.9506	8.9325 -2	1.1252 -1	1.0206 -3	9.5867 -2	-1.4967 -3	.1640
.1860	-9.9345 -1	-1.8682	8.4025 -2	1.1417 -1	6.2681 -4	8.8436 -2	-1.3353 -3	.1660
.1880	-9.5848 -1	-1.7904	7.9064 -2	1.1511 -1	2.9583 -4	8.1570 -2	-1.1902 -3	.1880
.1900	-9.2530 -1	-1.7168	7.4416 -2	1.1545 -1	1.9110 -5	7.5247 -2	-1.0598 -3	.1900
.1920	-8.9378 -1	-1.6472	7.0062 -2	1.1528 -1	-2.1079 -4	6.9426 -2	-9.4284 -4	.1920
.1940	-8.6380 -1	-1.5814	6.5981 -2	1.1467 -1	-4.0053 -4	6.4069 -2	-8.5800 -4	.1940
.1960	-8.3527 -1	-1.5190	6.2154 -2	1.1370 -1	-5.5514 -4	5.9141 -2	-7.4416 -4	.1960
.1980	-8.0811 -1	-1.4598	5.6563 -2	1.1244 -1	-6.8010 -4	5.4606 -2	-6.6024 -4	.1980
.2000	-7.8225 -1	-1.4058	5.5194 -2	1.1092 -1	-7.7945 -4	5.0435 -2	-5.8526 -4	.2000
.2100	-6.6955 -1	-1.1633	4.1187 -2	1.0097 -1	-1.0051 -3	3.4077 -2	-3.1592 -4	.2100
.2200	-5.7940 -1	-9.7612 -1	3.0896 -2	8.9427 -2	-9.7708 -4	2.5256 -2	-1.6607 -4	.2200
.2300	-5.0624 -1	-8.2840 -1	2.3281 -2	7.8045 -2	-8.5102 -4	1.6046 -2	-8.4257 -5	.2300
.2400	-4.4608 -1	-7.1039 -1	1.7613 -2	6.7580 -2	-7.0020 -4	1.1197 -2	-4.0458 -5	.2400
.2500	-5.9605 -1	-6.1498 -1	1.3570 -2	5.8291 -2	-5.5693 -4	7.9020 -5	-1.7649 -5	.2500
.2600	-3.5392 -1	-5.5697 -1	1.0180 -2	5.0200 -2	-4.3339 -4	5.8585 -5	-6.2124 -6	.2600
.2700	-3.1815 -1	-4.7252 -1	7.7713 -3	4.5224 -2	-3.3225 -4	4.0667 -3	-8.1765 -7	.2700
.2800	-2.8744 -1	-4.1873 -1	5.9460 -3	3.7244 -2	-2.5202 -4	2.9654 -3	1.4553 -6	.2800
.2900	-2.6089 -1	-5.7341 -1	4.5584 -3	3.2128 -2	-1.8969 -4	2.1808 -3	2.1816 -6	.2900
.3000	-2.3775 -1	-3.3489 -1	3.5008 -3	2.7755 -2	-1.4194 -4	1.6200 -3	2.1927 -6	.3000
.3100	-2.1742 -1	-3.0187 -1	2.6927 -3	2.4014 -2	-1.0573 -4	1.2142 -3	1.9148 -6	.5100
.3200	-1.9945 -1	-2.7336 -1	2.0739 -3	2.0811 -2	-7.8487 -5	9.1765 -4	1.5562 -8	.3200
.3300	-1.8548 -1	-2.4855 -1	1.5992 -3	1.8064 -2	-5.8102 -5	6.9903 -4	1.2113 -6	.3300
.3400	-1.6919 -1	-2.2681 -1	1.2544 -3	1.5702 -2	-4.2917 -5	5.3645 -4	9.1628 -7	.5400
.3500	-1.5635 -1	-2.0765 -1	9.5567 -4	1.3669 -2	-3.1644 -5	4.1453 -4	8.7931 -7	.5500
.5600	-1.4476 -1	-1.9066 -1	7.3759 -4	1.1914 -2	-2.5298 -5	3.2239 -4	4.9624 -7	.3600
.3700	-1.3426 -1	-1.7551 -1	5.7056 -4	1.0398 -2	-1.7135 -5	2.5222 -4	3.5846 -7	.3700
.3800	-1.2469 -1	-1.6193 -1	4.4174 -4	9.0841 -3	-1.2586 -5	1.9841 -4	2.5668 -7	.5800
.3900	-1.1597 -1	-1.4971 -1	3.4219 -4	7.9445 -3	-9.2388 -6	1.5688 -4	1.8253 -7	.3900
.4000	-1.0797 -1	-1.3866 -1	2.6521 -4	6.9542 -3	-6.7768 -6	1.2461 -4	1.2908 -7	.4000
.4200	-9.3873 -2	-1.1950 -1	1.5949 -4	5.3412 -3	-3.6402 -6	7.9605 -5	6.3720 -8	.4200
.4400	-8.1874 -2	-1.0351 -1	9.6042 -5	4.1132 -3	-1.9520 -6	5.1576 -5	3.1059 -8	.4400
.4600	-7.1584 -2	-9.0008 -2	5.7892 -5	5.1743 -3	-1.0455 -6	3.5805 -5	1.5004 -8	.4600
.4800	-6.2708 -2	-7.8514 -2	3.4924 -5	2.4538 -3	-5.5939 -7	2.2366 -5	7.2013 -9	.4600
.5000	-5.5015 -2	-8.8655 -2	2.1081 -5	1.8994 -3	-2.9911 -7	1.4910 -5	3.4397 -9	.5000
.5500	-3.9842 -2	-4.9448 -2	5.9790 -6	1.0054 -3	-6.2404 -8	5.5409 -6	5.3497 -10	.5500
.6000	-2.6968 -2	-5.5848 -2	1.6985 -6	5.3408 -4	-1.2997 -8	2.1042 -6	8.2250 -11	.6000

FIFTH ORDER GRAVITY WAVE THEORY

TABLE III

FIFTH ORDER GRAVITY WAVE COEFFICIENTS

d/L	A_{55}	C_1	C_2	C_3	C_4	SINH($\frac{2\pi d}{L}$)	COSH($\frac{2\pi d}{L}$)	d/L
.0500	4.0709 4	1.1897 2	2.5369 5	-7.4573 -1	-3.5557 2	3.1935 -1	1.0498	.0500
.0550	1.0458 4	8.1917 1	9.9955 4	-6.6889 -1	-1.7030 2	3.3249 -1	1.0603	.0550
.0600	2.9402 3	5.8396 1	4.7948 4	-6.0425 -1	-8.5328 1	3.8598 -1	1.0719	.0600
.0620	1.8067 3	5.1438 1	3.1292 4	-5.8115 -1	-6.5353 1	3.9949 -1	1.0768	.0620
.0640	1.1213 3	4.5511 1	2.3054 4	-5.5941 -1	-5.0280 1	4.1305 -1	1.0619	.0640
.0660	7.0206 2	4.0436 1	1.7164 4	-5.3692 -1	-3.8826 1	4.2668 -1	1.0872	.0660
.0680	4.4301 2	3.6070 1	1.2906 4	-5.1955 -1	-3.0071 1	4.4037 -1	1.0927	.0680
.0700	2.8141 2	3.2297 1	9.7969 3	-5.0122 -1	-2.3341 1	4.5414 -1	1.0983	.0700
.0720	1.7975 2	2.9022 1	7.5036 3	-4.8385 -1	-1.8140 1	4.6796 -1	1.1041	.0720
.0740	1.1532 2	2.6169 1	5.7966 3	-4.6736 -1	-1.4105 1	4.8169 -1	1.1101	.0740
.0760	7.4212 1	2.3673 1	4.5147 3	-4.5167 -1	-1.0960 1	4.9588 -1	1.1162	.0760
.0780	4.7841 1	2.1482 1	3.5439 3	-4.3674 -1	-8.5003	5.0994 -1	1.1225	.0780
.0800	3.0845 1	1.9557 1	2.8028 3	-4.2251 -1	-6.5712	5.2409 -1	1.1290	.0800
.0820	1.9851 1	1.7847 1	2.2328 3	-4.0892 -1	-5.0542	5.3832 -1	1.1557	.0820
.0840	1.2724 1	1.6336 1	1.7911 5	-3.9594 -1	-3.8589	5.5265 -1	1.1425	.0840
.0860	8.0967	1.4992 1	1.4464 5	-3.8352 -1	-2.9157	5.6704 -1	1.1496	.0860
.0880	5.0942	1.3794 1	1.1756 5	-3.7163 -1	-2.1706	5.8153 -1	1.1568	.0880
.0900	3.1500	1.2724 1	9.6148 2	-3.6024 -1	-1.5820	5.9611 -1	1.1642	.0900
.0920	1.8963	1.1764 1	7.9112 2	-3.4931 -1	-1.1173	6.1079 -1	1.1718	.0920
.0940	1.0938	1.0902 1	6.5475 2	-3.3881 -1	-7.5073 -1	6.2536 -1	1.1795	.0940
.0960	5.8568 -1	1.0126 1	5.4497 2	-3.2873 -1	-4.6235 -1	6.4043 -1	1.1873	.0960
.0960	2.6930 -1	9.4249	4.5609 2	-3.1903 -1	-2.3623 -1	6.5341 -1	1.1936	.0980
.1000	7.7244 -2	8.7912	3.8374 2	-3.0970 -1	-5.9806 -2	6.7048 -1	1.2040	.1000
.1020	-3.4780 -2	8.2168	3.2454 2	-3.0071 -1	7.6924 -2	6.8567 -1	1.2125	.1020
.1040	-9.5826 -2	7.6952	2.7586 2	-2.9205 -1	1.8191 -1	7.0096 -1	1.2212	.1040
.1060	-1.2491 -1	7.2206	2.3562 2	-2.8370 -1	2.6153 -1	7.1636 -1	1.2301	.1060
.1080	-1.3443 -1	6.7880	2.0221 2	-2.7565 -1	3.2084 -1	7.3187 -1	1.2392	.1080
.1100	-1.3241 -1	6.3929	1.7435 2	-2.6788 -1	3.6396 -1	7.4750 -1	1.2485	.1100
.1120	-1.2398 -1	6.0315	1.5099 2	-2.6037 -1	3.9415 -1	7.6325 -1	1.2580	.1120
.1140	-1.1240 -1	5.7004	1.3134 2	-2.5312 -1	4.1406 -1	7.7912 -1	1.2677	.1140
.1160	-9.9660 -2	5.3965	1.1473 2	-2.4611 -1	4.2582 -1	7.9511 -1	1.2776	.1160
.1180	-8.6963 -2	5.1171	1.0063 2	-2.3933 -1	4.3114 -1	8.1123 -1	1.2677	.1180
.1200	-7.4980 -2	4.8600	8.8625 1	-2.3277 -1	4.5142 -1	8.2746 -1	1.2980	.1200
.1220	-6.4054 -2	4.6229	7.8354 1	-2.2642 -1	4.2776 -1	8.4365 -1	1.3085	.1220
.1240	-5.4324 -2	4.4041	6.9535 1	-2.2027 -1	4.2108 -1	8.6036 -1	1.3192	.1240
.1260	-4.5805 -2	4.2018	6.1936 1	-2.1432 -1	4.1211 -1	8.7701 -1	1.3501	.1260
.1260	-3.8433 -2	4.0146	5.5365 1	-2.0855 -1	4.0143 -1	8.9379 -1	1.3412	.1280
.1300	-3.2119 -2	3.8410	4.9663 1	-2.0296 -1	3.8951 -1	9.1072 -1	1.3526	.1300
.1320	-2.6750 -2	3.6800	4.4697 1	-1.9753 -1	3.7674 -1	9.2779 -1	1.3641	.1320
.1540	-2.2213 -2	3.5304	4.0360 1	-1.9228 -1	3.6341 -1	9.4500 -1	1.3759	.1340
.1360	-1.8396 -2	5.3913	3.6559 1	-1.8718 -1	3.4977 -1	9.6237 -1	1.3879	.1360
.1380	-1.5199 -2	3.2617	3.3217 1	-1.8223 -1	3.3601 -1	9.7989 -1	1.4001	.1380
.1400	-1.2329 -2	3.1410	3.0271 1	-1.7743 -1	3.2229 -1	9.9756 -1	1.4125	.1400
.1420	-1.0307 -2	3.0283	2.7665 1	-1.7276 -1	3.0871 -1	1.0134	1.4231	.1420
.1440	-8.4608 -3	2.9230	2.5354 1	-1.6824 -1	2.9537 -1	1.0334	1.4380	.1440
.1460	-6.9315 -3	2.8246	2.3299 1	-1.6384 -1	2.8235 -1	1.0515	1.4511	.1460
.1480	-5.6667 -3	2.7324	2.1466 1	-1.5957 -1	2.6968 -1	1.0698	1.4644	.1480
.1500	-4.6227 -3	2.6461	1.9828 1	-1.5542 -1	2.5741 -1	1.0883	1.4780	.1500
.1520	-3.7625 -3	2.5652	1.8359 1	-1.5139 -1	2.4557 -1	1.1070	1.4918	.1520
.1540	-5.0548 -3	2.4892	1.7039 1	-1.4747 -1	2.3416 -1	1.1258	1.5058	.1540
.1560	-2.4735 -3	2.4179	1.5850 1	-1.4366 -1	2.2320 -1	1.1448	1.5201	.1580
.1580	-1.9970 -3	2.3508	1.4777 1	-1.3995 -1	2.1268 -1	1.1640	1.5346	.1580
.1600	-1.6069 -3	2.2877	1.5805 1	-1.3655 -1	2.0262 -1	1.1834	1.5493	.1600
.1620	-1.2881 -3	2.2282	1.2924 1	-1.3285 -1	1.9299 -1	1.2030	1.5645	.1620
.1640	-1.0281 -3	2.1722	1.2124 1	-1.2944 -1	1.8380 -1	1.2227	1.5796	.1640
.1660	-8.1654 -4	2.1193	1.1395 1	-1.2613 -1	1.7503 -1	1.2427	1.5951	.1660
.1680	-6.4471 -4	2.0695	1.0729 1	-1.2290 -1	1.6667 -1	1.2628	1.6108	.1680
.1700	-5.0550 -4	2.0223	1.0121 1	-1.1976 -1	1.5871 -1	1.2832	1.6268	.1700
.1720	-3.9305 -4	1.9778	9.5639	-1.1671 -1	1.5113 -1	1.3057	1.6431	.1720
.1740	-3.0247 -4	1.9357	9.0525	-1.1374 -1	1.4591 -1	1.5245	1.6596	.1740
.1760	-2.2978 -4	1.8938	8.5824	-1.1085 -1	1.3705 -1	1.3454	1.6763	.1760
.1780	-1.7168 -4	1.8380	8.1494	-1.0803 -1	1.3052 -1	1.3666	1.6934	.1780
.1800	-1.2546 -4	1.8222	7.7500	-1.0529 -1	1.2432 -1	1.3880	1.7107	.1800
.1820	-8.8893 -5	1.7863	7.3610	-1.0262 -1	1.1843 -1	1.4096	1.7283	.1820
.1840	-6.0149 -5	1.7560	7.0394	-1.0002 -1	1.1283 -1	1.4314	1.7461	.1840
.1860	-3.7734 -5	1.7254	6.7229	-9.7493 -2	1.0751 -1	1.4555	1.7642	.1860
.1880	-2.0420 -5	1.6964	6.4291	-9.5030 -2	1.0246 -1	1.4738	1.7627	.1880
.1900	-7.2076 -6	1.6687	6.1561	-9.2631 -2	9.7637 -2	1.4983	1.8013	.1900
.1920	2.7225 -6	1.6424	5.9019	-9.0294 -2	9.3098 -2	1.5210	1.6203	.1920
.1940	1.0036 -5	1.6174	5.6650	-8.8018 -2	6.8766 -2	1.5440	1.8396	.1940
.1960	1.5275 -5	1.5936	5.4439	-6.5801 -2	8.4656 -2	1.5673	1.8591	.1960
.1980	1.8879 -5	1.5709	5.2374	-8.3642 -2	8.0749 -2	1.5907	1.8790	.1980
.2000	2.1204 -5	1.5492	5.0441	-8.1538 -2	7.7038 -2	1.6145	1.8991	.2000
.2100	2.1944 -5	1.4550	4.2434	-7.1804 -2	6.1082 -2	1.7371	2.0044	.2100
.2200	1.6255 -5	1.3797	3.6510	-6.3252 -2	4.8698 -2	1.8665	2.1175	.2200
.2300	1.0620 -5	1.3190	3.2026	-5.5734 -2	3.9048 -2	2.0033	2.2391	.2500
.2400	6.4895 -6	1.2696	2.8561	-4.9118 -2	3.1492 -2	2.1481	2.3694	.2400
.2500	5.7992 -6	1.2289	2.5840	-4.3295 -2	2.5545 -2	2.3015	2.5092	.2500
.2600	2.1545 -6	1.1953	2.3670	-3.8166 -2	2.0839 -2	2.4636	2.6588	.2600
.2700	1.1891 -6	1.1673	2.1919	-3.3648 -2	1.7094 -2	2.6556	2.8190	.2700
.2800	6.3915 -7	1.1436	2.0489	-2.9668 -2	1.4098 -2	2.8181	2.9902	.2800
.2900	3.3367 -7	1.1259	1.9511	-2.6159 -2	1.1687 -2	3.0116	3.1795	.2900
.5000	1.6799 -7	1.1071	1.8537	-2.5066 -2	9.7559 -5	5.2171	3.5689	.3000
.5100	8.0589 -8	1.0928	1.7515	-2.0540 -2	8.1491 -5	5.4355	3.5779	.5100
.3200	3.5471 -8	1.0806	1.6622	-1.7957 -2	6.8515 -5	3.6670	3.8009	.5200
.3500	1.3536 -6	1.0701	1.6238	-1.5817 -2	5.7647 -5	5.9132	4.0590	.5500
.3400	3.1093 -9	1.0611	1.5740	-1.3949 -2	4.9034 -5	4.1749	4.2930	.5400
.3500	-1.1659 -9	1.0335	1.5315	-1.2501 -2	4.1717 -3	4.4531	4.5640	.3500
.3600	-2.5696 -9	1.0466	1.4950	-1.0848 -2	3.5615 -3	4.7488	4.8529	.5600
.3700	-2.6985 -9	1.0407	1.4635	-9.5667 -5	3.0499 -3	5.0635	5.1611	.3700
.3600	-2.3355 -9	1.0356	1.4364	-8.4369 -3	2.6195 -3	5.3978	5.4896	.3800
.3900	-1.8455 -9	1.0312	1.4129	-7.4404 -3	2.2559 -3	5.7536	5.8399	.3900
.4000	-1.5835 -9	1.0274	1.3926	-6.5617 -3	1.9475 -3	6.1321	6.2131	.4000
.4200	-7.0894 -10	1.0211	1.3594	-5.1034 -3	1.4608 -3	6.9654	7.0349	.4200
.4400	-3.5793 -10	1.0165	1.5345	-3.9692 -3	1.1038 -3	7.9048	7.9678	.4400
.4600	-1.5439 -10	1.0126	1.3150	-3.0871 -3	8.3908 -4	8.9712	9.0268	.4600
.4800	-6.8651 -11	1.0098	1.3003	-2.4011 -3	6.4090 -4	1.0179 1	1.0226 1	.4800
.5000	-2.9969 -11	1.0076	1.2889	-1.8675 -3	4.9141 -4	1.1549 1	1.1592 1	.5000
.5500	-3.5928 -12	1.0040	1.2706	-9.9627 -4	2.5605 -4	1.5825 1	1.5857 1	.3500
.6000	-4.1440 -13	1.0021	1.2609	-5.3150 -4	1.3483 -4	2.1677 1	2.1700 1	.6000

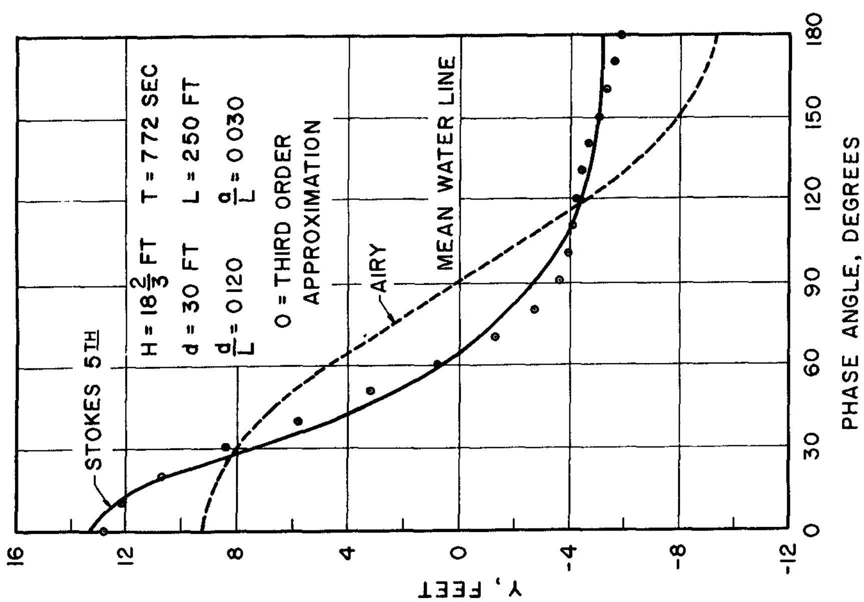

Fig. 2 - Wave profile

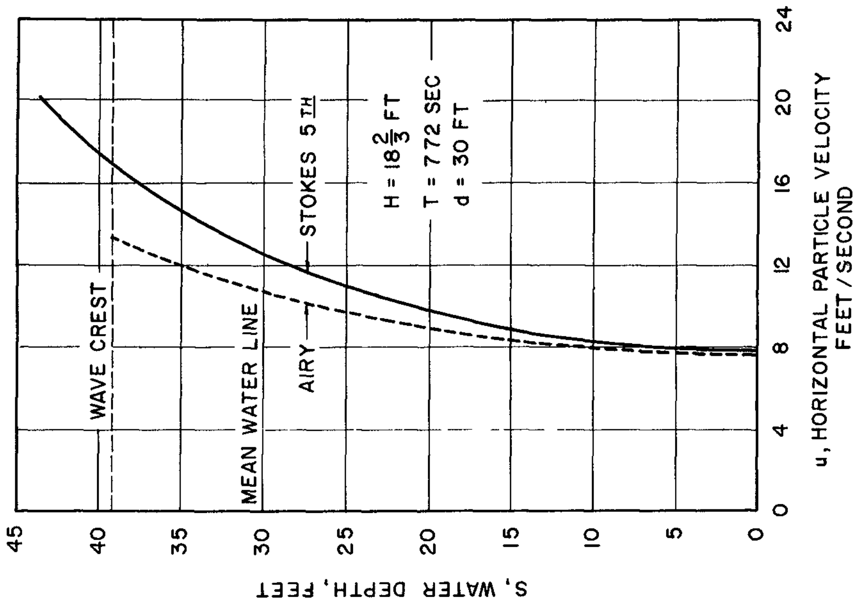

Fig. 3 - Horizontal velocity distribution on vertical section through crest

Other constants:

$$C_1 = 4.8600 \qquad C_3 = 0.2328 \qquad \sinh \beta d = 0.8275$$

$$C_2 = 88.6250 \qquad C_4 = 0.4314 \qquad \cosh \beta d = 1.2980$$

Substituting these constants into equations 12, 10, 9, and 1 the following results are obtained.

Wave velocity: $\overline{C} = 32.39$ ft/sec
Wave length: $L = \overline{C}T = 250$ ft.

Wave profile:

$$y = 7.50 \cos \theta + 3.35 \cos 2\theta + 1.48 \cos 3\theta + 0.70 \cos 4\theta + 0.35 \cos 5\theta$$

Horizontal particle velocity:

$$u = 6.186 \cosh \beta s \cos \theta + 1.434 \cosh 2\beta s \cos 2\theta + 0.205 \cosh 3\beta s \cos 3\theta + 0.021 \cosh 4\beta s \cos 4\theta - 0.003 \cosh 5\beta s \cos 5\theta$$

The wave profile is plotted in Fig. 2 together with profile obtained by the Airy theory-points obtained from the Third Order Approximation as also shown on the same graph.

The horizontal particle velocity distribution at the crest station ($\theta = 0$) is plotted in Fig. 3 together with the velocity distribution obtained by the Airy theory.

NOMENCLATURE

$\overline{C}$	=	Wave celerity
θ	=	$\beta (X - \overline{C}r)$ = phase angle
β	=	$2\pi/L$
L	=	Wave length
d	=	Mean water depth
H	=	Trough to crest wave height
T	=	Wave period = $L/\overline{C}$
P	=	Absolute pressure
X	=	Horizontal coordinate distance, measured from crest

t	=	Time
S	=	Vertical coordinate distance, measured positively upwards from mud line
y	=	Profile coordinate, measured positively upward from mean water line
ϕ	=	Velocity potential function
∇^2	=	Laplacian operator = $\partial^2/\partial x^2 + \partial^2/\partial y^2$
c	=	cosh β d
s	=	sinh β d
v	=	Vertical particle velocity
u	=	Horizontal particle velocity
λ	=	β a
a	=	A constant to be determined for each wave
Lo	=	$\frac{gT^2}{2\pi}$ = wave length for "deep-water" wave

REFERENCES

1. Skjelbreia, Lars, Gravity Waves Stokes' Third Order Approximation Tables of Functions, (June 11, 1958).

2. Wilton, J. R., On Deep Water Waves, Phil. Mag. S. 6 Vol. 27, No. 158, (Feb. 1914).