FIHT2 AlGORITHM : A FAST INCREMENTAL HOUGH TRANSFORM

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ABSTRACT

FIHT2 algorithm defined by $\rho = x\cdot\cos\theta + y\cdot\sin\theta + (\pi/(2K))\cdot x\cdot\sin\theta$ *at* $0 \le \theta < \pi/2$ *and at* $\rho = x\cdot\cos\theta + y\cdot\sin\theta + (\pi/(2K))\cdot y\cdot\cos\theta$ *at* $\pi/2 \le \theta < \pi$ *is a Hough transform which requires nothing of the trigonometric and functional operations to generate the Hough distributions. It is demonstrated in this paper that the FIHT2 is a complete alternative of the usual Hough transform(HT) defined by* $\rho = x\cdot\cos\theta + y\cdot\sin\theta$ *in the sense that the both transforms could work perfectly as a line detector. It is easy to show that the Hough curves of the FIHT2 can be generated in a incremental way where addition operation is exclusively needed. It is also investigated that the difference between HT and FIHT2 could be estimated to be neglected.*

1. INTRODUCTION

It is important to reduce the computing cost of Hough transform to enforce its validity to apply to the real world of the computer vision. In order to reduce the cost, several researches have been reported on the basis of the reduction of the 'frequency cost' both of the numbers of the edge points and the divisions of the parameter space. On the other hand, it is still expectative to reduce the 'core cost' by direct reducing the Hough transform calculation. If it becomes possible to reduce the core cost, the total cost reduction of the Hough transform would be drastically improved.

This paper proposes a method, called FIHT2 algorithm, to realize the reduction of the 'core cost' of the Hough transform HT defined by eq.(1). FIHT2(Fast Incremental Hough Transform-2) algorithm is a method to generate the Hough curve by the incremental way of voting of the Hough curves to the parameter space.

$$\rho = x\cdot\cos\theta + y\cdot\sin\theta \quad 0 \le \theta < \pi \tag{1}$$

Firstly in Chapter 2, it is introduced that HT defined by eq.(1) can be approximately repalaced by the difference equations called FIHT. In chapter 3, it is proved that if a little modifications are given to eq.(1), the expression of the difference equation becomes the exact expression of the modified Hough transform. In chapter 4, from the view point of the error evaluation, the relations among FIHT2, FIHT and HT are investigated to show the validity and efficiency of FIHT2 algorithm.

2. INCREMENTAL HOUGH TRANSFORM FIHT: AN APPROXIMATED HT

When let n and K be the sequence number and the number of the division of the $\theta - axis$ of the parameter space, respectively, the HT for a point(x,y) can be expressed by eq.(2) and it is easily certified that the difference equations given by eq.(3) are approximately equivalent to the modified version of the Hough transform defined by eq.(4). A new Hough transform defined by eq.(3) is called as FIHT in this paper.

$$\begin{aligned} \rho_n &= x\cdot\cos\theta_n + y\cdot\sin\theta_n \\ n &= 0,1,2,\ldots,K-1 \\ \theta_n &= n\cdot\pi/K \end{aligned} \tag{2}$$

$$\begin{aligned} \rho_{n+1} &= \rho_n + \varepsilon\cdot\rho_n' \\ \rho_{n+1}' &= \rho_n' - \varepsilon\cdot\rho_{n+1}, \\ \text{where } \rho_0 &= x,\ \rho_0' = y \text{ and } \varepsilon = \pi/K \end{aligned} \tag{3}$$

$$\begin{aligned} \rho_n &= x\cdot\cos\theta_n + y\cdot\sin\theta_n \\ &\quad +(\pi/2K)\cdot x\cdot\sin\theta_n \\ \rho_n' &= -x\cdot\cos\theta_n + y\cdot\sin\theta_n \\ &\quad -(\pi/2K)\cdot y\cdot\sin\theta_n \\ &\quad 0 \le n < K \end{aligned} \tag{4}$$

It is known by the reference [6] that the resolution of the difference equation shown in eq.(3) is given by eq.(5). If $d = \pi/K$, as shown pictorially in Fig.1 then it is obvious from eq.(2) that $\theta_n = n\cdot d$. At the same time, basing on the Maclaurin's Theorem given by eq.(6), it is known that $\varepsilon = 2\cdot\sin(d/2) = d$ and $\cos(d/2) = 1$, because $d << 1$ and therefore $d/2 = 0$ when K is selected to be large enough. Replacing $\cos(d/2)$ and $\sin(d/2)$ by 1 and $\pi/(2K)$, respectively in eq.(5), eq.(4) can be automatically introduced.

$$\begin{aligned} \rho_n &= \frac{1}{\cos(d/2)}(x\cdot\cos(n\cdot d - d/2) \\ &\quad +y\cdot\sin(n\cdot d)) \\ \rho_n' &= \frac{1}{\cos(d/2)}(-x\cdot\sin(n\cdot d) \\ &\quad +y\cdot\cos(n\cdot d + d/s)) \\ \varepsilon &= 2\cdot\sin(d/2) \text{ or } d = 2\cdot\arcsin(\varepsilon/2) \end{aligned} \tag{5}$$

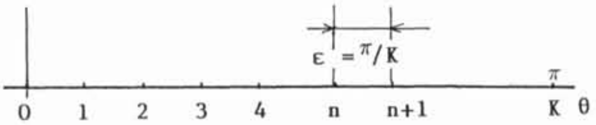

Fig.1 How to digitize θ-axis in FIHT and FIHT2

$$2\sin\cdot(d/2) \;=\; d-(d^2/4)\cdot\sin(z\cdot d/2) \qquad (6)$$
$$\cos(d/2) \;=\; 1-(d^2/8)\cdot\cos(z\cdot d/2)\;;\; 0<z<1$$

It is clarified that as K is ordinally large enough, eq.(3) can generate approximately the same Hough distribution as eq.(4). On the contrary to eq.(4), it is promising that eq.(3) can be executed incrementaly. As a result, FIHT algorithm defined by eq.(3) was established, and especially when the value of ε is selected carefully so that $\varepsilon = 1/(2^m)$ or $K = \pi\cdot 2^m$ where m is a natural number, the multiplications in eq.(3) could be replaced by the shift operations in the data register. In this case, FIHT algorithm can be executed without any trigonometric function and multiplication operations. Therefore, the procedure to execute FIHT algorithm is firstly to the Hough distribution by eq.(3), and secondarily to inversely transform by eq.(4) using some peaks in the parameter space.

3. FIHT2:AN ALTERNATIVE OF HOUGH TRANSFORM

FIHT algorithm introduced in Eq.'s (3) and (4) can be an alternative of HT in the sense that eq.(4) can be regarded as a complete transform for the substitute of the usual HT function given by eq.(2).

3.1 FIHT2 Is Not An Approximated, But A New Hough Transform

When let ρ_n' and $\theta_{n+K/2}$ be repalced by $\rho_{n-K/2}$ and $\theta_n - \pi/2$, respectively in eq.(4), eq.(4) can be easily substituted by eq.(7), and the new formalism defined by eq.(7) is indicated as FIHT2 in this paper. It is apparent just in the same way of FIHT that the generation of ρ_n in eq.(7) can be executed directly by the difference equation given by eq.(8), as shown in Fig.2.

$$\begin{aligned}\rho_n &= x\cdot\cos\theta_n + y\cdot\sin\theta_n \qquad (7)\\ &\quad +(\pi/(2K))\cdot x\cdot\sin\theta_n\\ &\quad \text{for}\quad 0\le n< K/2-1 \text{ or } 0\le\theta_n<\pi/2\\ \rho_n &= x\cdot\cos\theta_n + y\cdot\sin\theta_n + (\pi/(2K))\cdot y\cdot\cos\theta_n\\ &\quad \text{for}\quad K/2\le n<K-1 \text{ or } \pi/2\le\theta_n<\pi\end{aligned}$$

$$\begin{aligned}\rho_0 &= x\;,\; \rho_0' = y\;,\; \varepsilon = \pi/K \qquad (8)\\ \rho_{n+1} &= \rho_n + \varepsilon\cdot\rho_n' \text{ for } 0\le n<K/2-1\\ \rho_{n+1}' &= \rho_n' - \varepsilon\cdot\rho_{n+1} \text{ for } K/2\le n<K\end{aligned}$$

It is essential in FIHT2 that the transform function given by eq.(7) is not an approximated function of HT, but a new transform function which is perfectly applicable to the line detector just in the same way as the usual HT. In other words, as precisely demonstrated in [5] and [8], a point in the $x-y$ space can be uniquely

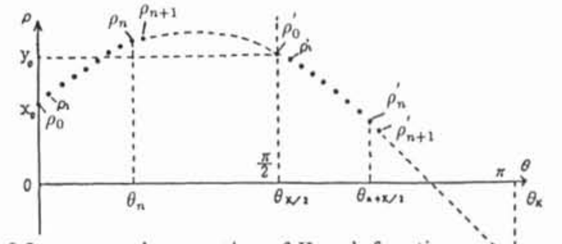

Fig.2 Incremental generation of Hough function

transformed to a piece of Hough curve in the $\theta-\rho$ space by FIHT2 function given by eq.(7), an instance of the EHT(Extended Hough Transform) functions, and vice varsa. An example of the Hough curve of FIHT2 for a point (x,y) is shown in Fig.3. This fact asserts that a parameter pair (θ^*,ρ^*) extracted from the parameter space of FIHT2 would provide a line in the $x-y$ space by means of eq.(9).

$$\begin{aligned}\rho^* &= x\cdot(\cos\theta^* + (\pi/(2K))\cdot\sin\theta^*) + y\cdot\sin\theta^* \;(9)\\ &\quad \text{for } 0\le\theta^*<\pi/2\\ \rho^* &= x\cdot\cos\theta^* + y\cdot(\sin\theta^* + (\pi/(2K))\cdot\cos\theta^*)\\ &\quad \text{for } \pi/2\le\theta^*<\pi\end{aligned}$$

3.2 Further Reduction Of Cost

When the value of ε be selected carefully as $1/2^m$ where m is a large natural number, eq.(8) can be replaced by eq.(10). In eq.(10), as the division by 2^m can be executed by the shift operations which is less expensive than the division, the total cost of FIHT2 can be reduced as shown in Table 1. If m be 6, for example, $\varepsilon = 1/64$ and therefore $K = 201$.

$$\begin{aligned}\rho_0 &= x,\; \rho_0' = y \qquad (10)\\ \rho_{n+1} &= \rho_n + \rho_n'/(2^m)\\ \rho_{n+1}' &= \rho_n' - \rho_{n+1}/(2^m)\end{aligned}$$

4. RELATIONS AMONG HT, FIHT AND FIHT2:ERROR EVALUATION

It was theoretically presented that FIHT is an ap-

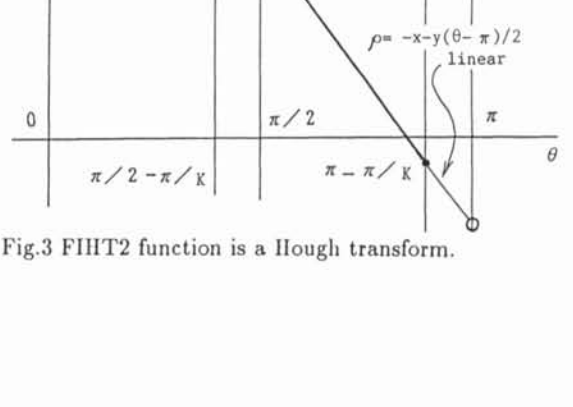

Fig.3 FIHT2 function is a Hough transform.

Table 1 Theoretical evaluation of the cost of FIHT2

	RAM access	Add/Sub	Mult	Shift
HT	2NK	NK	2NK	0
FIHT 2	0	N(K−2)	0	N(K−2)

proximation of the usual HT, and that FIHT2 is a new formalism for a substitute of the usual HT. It is investigated here to evaluate the error of FIHT against HT, and to clarify the difference between FIHT2 and the usual HT.

Using the definitions of eq.'s (11), (12) and (13), En and Dn is introduced by eq.(14) for the measures to visualize the relations among them, and an example of the differences En and Dn among the Hough curves generated by HT, FIHT and FIHT2 is shown in Fig.4, where $m = 6$ and $(x, y) = (70, 70)$. The measure Dn represents the difference between the definitions of the transform function of HT and FIHT2, and En represents the error caused by the numerical approximation of the FIHT2 transform function by the difference equation.

The deviations given by En between the curves in Fig.4 shows that the incremental generation of the Houg h curves of FIHT2 function is exactly the same one of the direct generation by eq.(8) or (10). As the magnitude of the deviation 10 in Fig.4, for example, provides only 1/500 deviation in the real dimension, and therefore as this deviation is always below 1 pixel resolution in the digitized Hough parameter space, it was certified again that FIHT2 can be perfectly implemented provided that the value of the parameter K be selected large enough.

$$\rho_n^{(HT)} = x \cdot \cos\theta_n + y \cdot \sin\theta_n \quad 0 \le \theta_n < \pi \qquad (11)$$

$$\begin{aligned} \rho_n^{(FIHT)} &= x \cdot (\cos\theta_n + (\pi/(2K) \cdot \sin\theta_n) \qquad (12) \\ &\quad + y \cdot \sin\theta_n \text{ for } 0 \le \theta < \pi/2 \\ \rho_n^{(FIHT)'} &= x \cdot \cos\theta_n + y \cdot (\sin\theta_n + (\pi/(2K) \cdot \sin\theta_n) \\ &\quad \text{for } \pi/2 \le \theta < \pi \\ \rho_0^{(FIHT2)} &= x,\ \rho_0^{(FIHT2)'} = y,\ \varepsilon = \pi/K \end{aligned}$$

$$\begin{aligned} \rho_{n+1}^{(FIHT2)} &= \rho_n^{(FIHT2)} + \varepsilon \cdot \rho_n^{(FIHT2)'} \qquad (13) \\ \rho_{n+1}^{(FIHT2)'} &= \rho_n^{(FIHT2)'} - \varepsilon \cdot \rho_{n+1}^{(FIHT2)} \end{aligned}$$

$$\begin{aligned} E_n &= \rho_n^{(FIHT2)} \qquad (14) \\ &\quad +5000 \cdot (\rho_n^{(FIHT)} - \rho_n^{(FIHT2)}) \\ D_n &= \rho_n^{(HT)} + 50 \cdot (\rho_n^{(FIHT)} - \rho_n^{(HT)}) \end{aligned}$$

5. EXPERIMENTAL DEMONSTRATIONS AND CONSIDERATIONS

It is demonstrated here that FIHT2 can be executed about 4 times faster than the usual HT. In the experiments, the values of K and m were 201 and 6, respectively. The tables of sin and cos functions were prepared for the calculation of the usual HT.

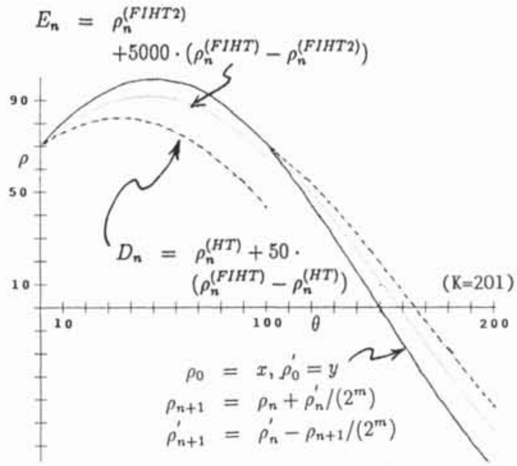

Fig.4 Relations among HT,FIHT and FIHT2

5.1 Simulations

Table 2 shows a result of a simulation where the number N of the points is 100,and BASIC program on PC and Assembly program on MC68000 are utilized as the environments of the experiments. It was known that the costs of the BASIC and Assembler programs of FIHT2 algorithm are reduced to about 1/2 and 1/4, respectively.

5.2 Practical Application

A grey image(256x256, 256 grey levels) of the mechanical parts shown in Photo.1(a) was used here and its edge enhanced image shown in Photo.1(b) was provided to the experiment of the edge extraction by FIHT 2. The number N of Photo.1(b) is 2968, and the parameters K and L(the parameter for $\rho - axis$ of the Hough parameter space) are 201 and 240, respectively. Photo.2 (a) and (b) show the Hough distributions generated by HT and FIHT2, respectively. As theoretically demonstrated in 4, it is also obvious experimentally that the distribution of FIHT2 is just the same distribution of HT. The two highest peaks which are located around the center of the respective Photo.2(a) and (b) represent a pair of straight edges appear in Photo.1. Photo.3(a) and (b) shows the extracted lines which are corresponding to the most highest peaks in Photo.2(a) and (b) respectively generated by HT and FIHT2.

It is important, at first, to know that FIHT2 is really able to extract a line from the practical grey images just in the same way of the usual HT. Secondarily, it is important that the cost of FIHT2 is extremely reduced to 1/4 of the cost of the usual HT as shown in Table 3.

6. CONCLUSION

FIHT2, a new Hough transform algorithm, is proposed in this paper in order to reduce the computing cost. FIHT2 does not require any operations except for the incremental addition operations. As a result, it was clarified from the theoretical and experimental

Table 2 Computing cost(Simulation)

	BASIC(PC)	Assemly(MC6800)
HT	50 sec	0.20 sec
FIHT2	25	0.052
cost	1/2	1/4

K=201, N=100

view points that the reduction of the cost of FIHT2 can be remarkably reduced to about 1/4 of the cost of the usual Hough transform.

As FIHT2 can reduce the computing cost, especially the 'core cost', it is promising to use FIHT2 algorithm together with some algorithm such as [2] for the reduction of the 'frequency cost' in order to the further reduction of computing cost. It is interesting to investigate FIHT2 from the view point of the more extended scope of the Hough-type transform function proposed in [9].

References

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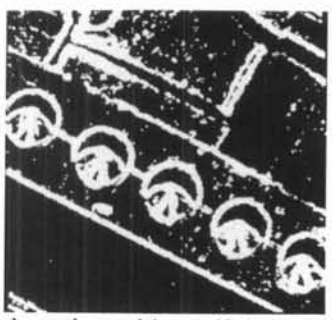
Table 3 Computing cost(Experiment)

	transform	RAM access	Total
HT	6.0 sec	1.2 sec	7.2 sec
FIHT2	1.5	1.2	2.7

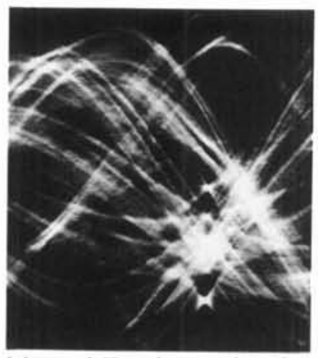
K=201, L=240, N=2968(Photo.1(b))
m=6(FIHT2)

(a) a grey image(mechanical parts)

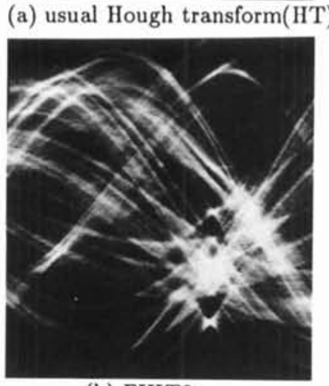


(b) edge enhanced image(Sobel operator)

Photo.1 A grey image and its edge enhanced image

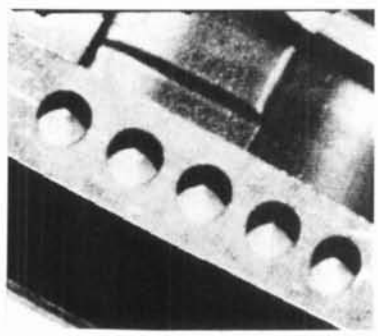


(a) usual Hough transform(HT)



(b) FIHT2

Photo.2 Hough distributions in the parameter space



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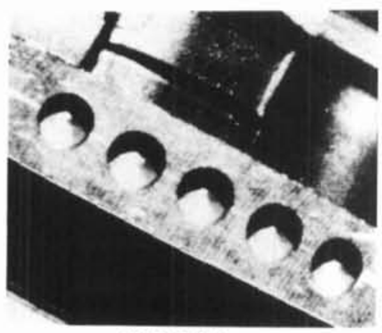

(b) FIHT2

Photo.3 Extracted edge lines