FILTER DESIGN FOR POLYPHASE FILTER BANKS WITH ARBITRARY NUMBER OF SUBBAND CHANNELS

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ABSTRACT

This paper concerns the filter design problem for a recent proposed polyphase filter bank with arbitrary number of subband channels. According to the perfect reconstruction condition for 1-D case presented in [1], an analytical formula for the filter design is developed. Compared with direct numerical design algorithms, this analytical formula allows one to design the required low-pass FIR prototype filter with much less computational complexity and obtain better filter bank performance. A design example is given for illustration.

I. INTRODUCTION

Polyphase filter banks (PFB) have been successfully used to constitute a multirate signal processing system for splitting a signal into N subbands and allowing the resynthesis of the signal from the subbands. Current applications are mainly in subband coding of speech signals [2, 3], TDM-FDM transmultiplexing systems [4, 5], and short-time spectral analysis [6]. Consider the filter design for PFB, many results have been reported in the literature [7-14]. Most of these methods employ numerical algorithms directly to minimize the associated error measure in filter design process. [15] derived an analytical formula to represent the error measure. As a result, the filter coefficients can be found by a straight nonlinear optimization procedure and saving computations can be achieved. However, all of these methods have focused on the design of quadrature mirror filters (QMF).

In [1], based on nonsymmetrical frequency band allocation in constructing band-pass filter for subband channels, one of the authors has developed a one-dimensional (1-D) PFB with N arbitrary. Each constructed band-pass filter is a nonsymmetrically frequency-shifted version of a low-pass prototype filter. It has been shown in [1] that the new PFB possesses some advantages over conventional PFB's. In this paper, we concern the filter design problem for the 1-D PFB. An analytical formula for finding the coefficients of the required 1-D low-pass FIR prototype filter is derived. Using this formula, one can design the prototype filter with much less computations than direct numerical algorithms. Moreover, the simulation example shows that the resulted reconstruction error is smaller under the same number of subband channels and filter length.

II. THE PROPOSED ANALYTICAL DESIGN FORMULA

The basic structure of a 1-D filter bank with N subbands to be considered is shown in Figure 1. Its efficient structure based on polyphase network and fast Fourier transform has been developed in [1]. The band-pass filters $H_i(\omega)$ and $\hat{H}_i(\omega)$ are given as

$$H_i(\omega) = G_i(\omega) + G_{2N-1-i}(\omega) \quad \text{and}$$
$$\hat{H}_i(\omega) = G_i(\omega) - G_{2N-1-i}(\omega),$$
$$i = 0, 1, \ldots\ldots, 2N-1, \tag{1}$$

respectively, where $G_i(\omega) = G(\omega - i\pi/N)$ and $G(\omega)$ denotes the 1-D low-pass prototype filter. Assume that $G(\omega)$ is linear phase. The associated condition on the magnitude response of $G(\omega)$ for perfect reconstruction is given by [1]

$$\sum_{k=0}^{2N-1} |G(\omega - k\pi/N)|^2 = N \quad \text{for} \quad 0 \leqslant \omega \leqslant 2\pi \tag{2}$$

To design the low-pass prototype filter $G(\omega)$ which approximates the condition of (2) in some optimum sense, we define an appropriate approximation error measure E as

$$E = E_r + \alpha E_s \tag{3}$$

where E_r denotes the ripple energy over the whole frequency range and is given as

$$E_r = \int_0^{\pi} \{ \sum_{\ell=0}^{2N-1} |G(\omega - \ell\pi/N)|^2 - N \}^2 \, d\omega \tag{4}$$

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E_s denotes the stopband energy of $G(\omega)$ and is given as

$$E_s = \int_{\pi/N}^{\pi} |G(\omega)|^2 \, d\omega \qquad (5)$$

The α in (3) designates the weight between E_r and E_s. To solve the problem of minimizing E, unconstrained numerical optimization algorithms can be employed to find the filter coefficients of $G(\omega)$. However, at each iteration, we have to compute the Fourier transform of $G_i(\omega)$, $i = 0, 1, \ldots\ldots,$ $2N-1$, in order to compute E_r and E_s. This requires considerable computations and hence leads to substantial computer time requirement.

Next, we derive a closed analytical formula to represent (3) and turn the optimization problem of (3) into a direct search problem of filter coefficients. Assume that $G(\omega)$ is a linear phase FIR filter with length M.
Then $G(\omega)$ can be expressed as

$$\begin{aligned} G(\omega) &= \exp(-j(M-1)\omega/2)\ \{ g((M-1)/2) + 2\sum_{n=0}^{(M-3)/2} g(n)\cos((n-(M-1)/2)\omega)\} \\ &= \exp(-j(M-1)\omega/2)\ \{\sum_{n=0}^{(M-1)/2} d(n)\cos((n-(M-1)/2)\omega)\}, \\ &\quad \text{if M is odd}, \end{aligned}$$

or

$$\begin{aligned} G(\omega) &= \exp(-j(M-1)\omega/2)\ \{2\sum_{n=0}^{(M-1)/2} g(n)\cos((n-(M-1)/2)\omega)\} \\ &= \exp(-j(M-1)\omega/2)\ \{\sum_{n=0}^{(M-1)/2} d(n)\cos((n-(M-1)/2)\omega)\}, \\ &\quad \text{if M is even}. \end{aligned} \qquad (6)$$

where

$$d((M-1)/2) = g((M-1)/2), \text{ for M odd},$$

$$d(n) = 2g(n), \text{ for } 0 \leq n \leq (M-3)/2 \text{ and M odd}, \text{ for } 0 \leq n \leq M/2-1 \text{ and M even}.$$

Using (6), we obtain

$$\begin{aligned} |G(\omega)|^2 &= \sum_{m=0}^{(M-1)/2} \sum_{n=0}^{(M-1)/2} d(m)\,d(n) \cos(\omega(m-(M-1)/2)) \cos(\omega(n-(M-1)/2)) \\ &= \frac{1}{2}\sum_{m=0}^{(M-1)/2}\sum_{n=0}^{(M-1)/2} d(m)\,d(n)\,(\cos(\omega(m-n)) + \cos(\omega(m+n-M+1))). \end{aligned} \qquad (7)$$

For the case of odd M, substituting (7) and the following relationship

$$\sum_{k=0}^{2N-1} \exp(j(\omega - 2\pi k/2N)\ell) = \begin{cases} 2N\exp(j\ell\omega), & \text{if } \ell/2N = \text{integer}, \\ 0, & \text{else}, \end{cases}$$

or equivalently,

$$\sum_{k=0}^{2N-1} \cos((\omega - k\pi/N)\ell) = \begin{cases} 2N\cos(\ell\omega), & \text{if } \ell/2N = \text{integer}, \\ 0, & \text{else}, \end{cases}$$

into (2) and performing some manipulations yields

$$\begin{aligned} \sum_{k=0}^{2N-1} |G(\omega - k\pi/N)|^2 &= N\sum_{m=0}^{(M-1)/2}\sum_{n=0}^{(M-1)/2} d(m)\,d(n)\,(A(m,n) + B(m,n)) \\ &\triangleq F(\omega) \end{aligned} \qquad (8)$$

where

$$A(m,n) = \begin{cases} \cos((m-n)\omega), & \text{if } (m-n)/2N = \text{integer}, \\ 0, & \text{else}, \end{cases}$$

$$B(m,n) = \begin{cases} \cos((m+n-M+1)\omega), & \text{if } (m+n-M+1)/2N = \text{integer}, \\ 0, & \text{else}, \end{cases}$$

for $0 \leq m, n \leq (M-1)/2$.

Consider the integral (4). We expand the square term and use the result of (8) and the following relationship

$$\int_0^{\pi} \cos(k\omega)\,d\omega = \begin{cases} \pi, & \text{if } k = 0, \\ 0, & \text{else}, \end{cases}$$

to obtain

$$E_r = \int_0^{\pi} (F^2(\omega) - 2NF(\omega) + N^2)\, d\omega$$
$$= N^2 \pi \Big(1 - \big(\sum_{n=0}^{(M-1)/2} 2d^2(n)\big) - 2d^2((M-1)/2) + \frac{1}{2} \sum_{i=0}^{(M-1)/2} \sum_{j=0} \sum_{m=0} \sum_{n=0} d(m)\, d(n)\, d(i)\, d(j)\Big) \quad (9)$$

where the four integers m, n, i, and j must satisfy one of the following relationships:

a. $m - n = 2Nk_1$ and $i - j = \pm 2Nk_1$

b. $m - n = 2Nk_2$ and $i + j - M + 1 = \pm 2Nk_2$

c. $m + n - M + 1 = 2Nk_3$ and $i + j - M + 1 = \pm 2Nk_3$

for $0 \leqslant m, n, i, j \leqslant (M-1)/2$ and integers k_1, k_2, and k_3. For the E_s of (5), using the following relationship

$$\int_{\pi/N}^{\pi} \cos(k\omega)\, d\omega = \begin{cases} \pi(1 - 1/N) & , \text{if } k = 0 \\ (-\sin(k\pi/N))/k & , \text{if } k \neq 0, \end{cases} \quad (10)$$

we obtain

$$E_s = \int_{\pi/N}^{\pi} |G(\omega)|^2\, d\omega$$
$$= \frac{1}{2} \sum_{m=0}^{(M-1)/2} \sum_{n=0}^{(M-1)/2} d(m)\, d(n)\, (C(m-n) + C(m+n-M+1)) \quad (11)$$

where $C(0) = \pi(1 - 1/N)$,

$C(k) = (-\sin(k\pi/N))/k$, if $k/N \neq$ integer

$C(k) = 0$, if $k \neq 0$ and $k/N =$ integer.

From (9) and (11), we note that the approximation error measure E can be expressed as a closed analytical formula in terms of the filter coefficients, $g(0), g(1), \ldots\ldots, g(M-1)$. On the other hand, for the case of even M, following the similar procedure, we can obtain the similar result. Therefore, computing the filter coefficients by minimizing E can be performed by utilizing a simple direct search method.

III. EXPERIMENTAL RESULTS

In this section, we present an example for illustration. The filter length M and the weight α were set to 28 and 1, respectively. The number N of subbands was 3. Based on the proposed analytical formula, we employed a simple gradient method to iteratively search the filter coefficients, $g(0), g(1), \ldots\ldots, g(27)$. The number of iterations was 200. For comparison, the direct numerical optimization of (3) was also performed. Figure 2 shows the frequency responses of $G(\omega)$ using both methods. The corresponding reconstruction errors are shown in Figure 3. Table 1 compares the computational complexity in terms of the number of operations required at each iteration step. Table 2 lists the computed filter coefficients, $g(0), g(1), \ldots\ldots, g(13)$, for the simulation of linear phase FIR filter design. From these results, we observe that the proposed analytical design formula is superior to the direct numerical optimization algorithm.

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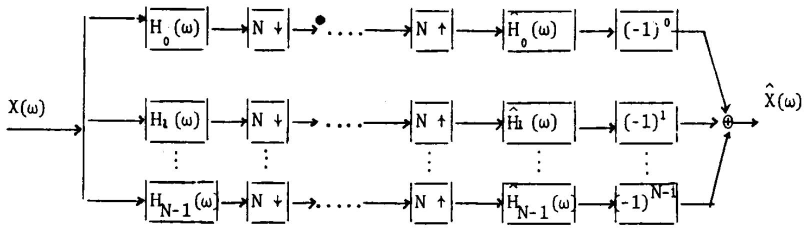

Analysis Filter Bank Synthesis Filter Bank

Figure 1

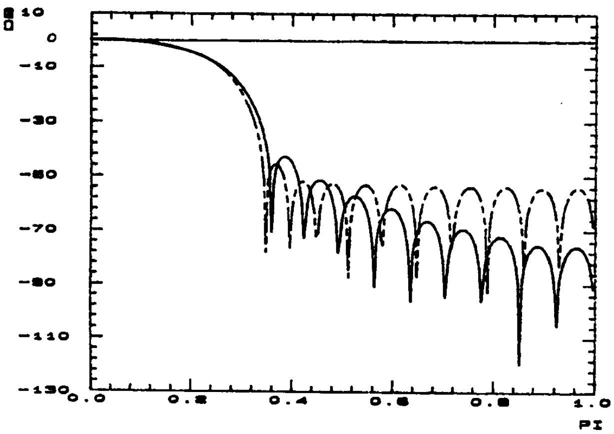

Figure 2. The Magnitude Response of G(ω)
—— Using The Analytic Design Formula
----- Using A Conventional Numerical Algorithm

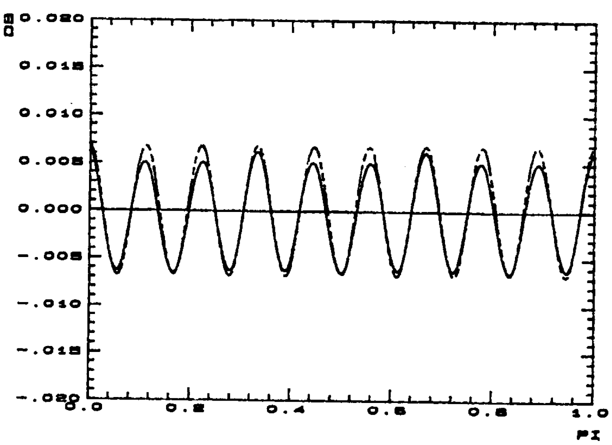

Figure 3. Reconstruction Error Between The Output and Input of The System of Fig. 1
—— Using The Analytic Design Formula
----- Using A Conventional Numerical Algorithm

Table 1 The Computational Complexity

Design Method	Real Additions	Real Multiplications
Numerical Algorithm	4521	8482
Analytical Formula	1514	3723

Table 2 The Filter Coefficients

g(0) = -0.301554350E-02
g(1) = -0.444925447E-02
g(2) = -0.314682306E-02
g(3) = 0.260439676E-02
g(4) = 0.915403406E-02
g(5) = 0.899816595E-02
g(6) = -0.255303234E-02
g(7) = -0.198699317E-01
g(8) = -0.253663825E-01
g(9) = 0.255687792E-02
g(10)= 0.750467467E-01
g(11)= 0.181366878E 00
g(12)= 0.288648657E 00
g(13)= 0.356221275E 00