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978-0-521-87512-7 - Filtering and System Identification: A Least Squares Approach

Michel Verhaegen and Vincent Verdult

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## FILTERING AND SYSTEM IDENTIFICATION

Filtering and system identification are powerful techniques for building models of complex systems in communications, signal processing, control, and other engineering disciplines. This book discusses the design of reliable numerical methods to retrieve missing information in models derived using these techniques. Particular focus is placed on the least squares approach as applied to estimation problems of increasing complexity to retrieve missing information about a linear state-space model.

The authors start with key background topics including linear matrix algebra, signal transforms, linear system theory, and random variables. They then cover various estimation and identification methods in the state-space model. A broad range of filtering and system-identification problems are analyzed, starting with the Kalman filter and concluding with the estimation of a full model, noise statistics, and state estimator directly from the data. The final chapter on the system-identification cycle prepares the reader for tackling real-world problems.

With end-of-chapter exercises, MATLAB simulations and numerous illustrations, this book will appeal to graduate students and researchers in electrical, mechanical, and aerospace engineering. It is also a useful reference for practitioners. Additional resources for this title, including solutions for instructors, are available online at [www.cambridge.org/9780521875127](http://www.cambridge.org/9780521875127).

MICHEL VERHAEGEN is professor and co-director of the Delft Center for Systems and Control at the Delft University of Technology in the Netherlands. His current research involves applying new identification and controller design methodologies to industrial benchmarks, with particular focus on areas such as adaptive optics, active vibration control, and global chassis control.

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# Filtering and System Identification

A Least Squares Approach

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## Preface

This book is intended as a first-year graduate course for engineering students. It stresses the role of linear algebra and the least-squares problem in the field of filtering and system identification. The experience gained with this course at the Delft University of Technology and the University of Twente in the Netherlands has shown that the review of undergraduate study material from linear algebra, statistics, and system theory makes this course an ideal start to the graduate course program. More importantly, the geometric concepts from linear algebra and the central role of the least-squares problem stimulate students to understand how filtering and identification algorithms arise and also to start developing new ones. The course gives students the opportunity to see mathematics at work in solving engineering problems of practical relevance.

The course material can be covered in seven lectures:

- (i) Lecture 1: Introduction and review of linear algebra (Chapters 1 and 2)
- (ii) Lecture 2: Review of system theory and probability theory (Chapters 3 and 4)
- (iii) Lecture 3: Kalman filtering (Chapter 5)
- (iv) Lecture 4: Estimation of frequency-response functions (Chapter 6)
- (v) Lecture 5: Estimation of the parameters in a state-space model (Chapters 7 and 8)
- (vi) Lecture 6: Subspace model identification (Chapter 9)
- (vii) Lecture 7: From theory to practice: the system-identification cycle (Chapter 10).

The authors are of the opinion that the transfer of knowledge is greatly improved when each lecture is followed by working classes in which the

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students do the exercises of the corresponding classes under the supervision of a tutor. During such working classes each student has the opportunity to ask individual questions about the course material covered. At the Delft University of Technology the course is concluded by a real-life case study in which the material covered in this book has to be applied to identify a mathematical model from measured input and output data.

The authors have used this book for teaching MSc students at Delft University of Technology and the University of Twente in the Netherlands. Students attending the course were from the departments of electrical, mechanical, and aerospace engineering, and also applied physics. Currently, this book is being used for an introductory course on filtering and identification that is part of the core of the MSc program Systems and Control offered by the Delft Center for Systems and Control (<http://www.dcsc.tudelft.nl>). Parts of this book have been used in the graduate teaching program of the Dutch Institute of Systems and Control (DISC). Parts of this book have also been used by Bernard Hanzon when he was a guest lecturer at the Technische Universität Wien in Austria, and by Jonas Sjöberg for undergraduate teaching at Chalmers University of Technology in Sweden.

The writing of this book stems from the attempt of the authors to make their students as enthusiastic about the field of filtering and system identification as they themselves are. Though these students have played a stimulating and central role in the creation of this book, its final format and quality has been achieved only through close interaction with scientist colleagues. The authors would like to acknowledge the following persons for their constructive and helpful comments on this book or parts thereof: Dietmar Bauer (Technische Universität Wien, Austria), Bernard Hanzon (University College Cork, Ireland), Gjerrit Meinsma (University of Twente, the Netherlands), Petko Petkov (Technical University of Sofia, Bulgaria), Phillip Regalia (Institut National des Télécommunications, France), Ali Sayed (University of California, Los Angeles, USA), Johan Schoukens (Free University of Brussels, Belgium), Jonas Sjöberg (Chalmers University of Technology, Sweden), and Rufus Fraanje (TU Delft).

Special thanks go to Niek Bergboer (Maastricht University, the Netherlands) for his major contributions in developing the Matlab software and guide for the identification methods described in the book. We finally would like to thank the PhD students Paolo Massioni and Justin Rice for help in proof reading and with the solution manual.

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## Notation and symbols

$\mathbb{Z}$	the set of integers
$\mathbb{N}$	the set of positive integers
$\mathbb{C}$	the set of complex numbers
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^n$	the set of real-valued $n$ -dimensional vectors
$\mathbb{R}^{m \times n}$	the set of real-valued $m$ by $n$ matrices
$\infty$	infinity
Re	real part
Im	imaginary part
$\in$	belongs to
$=$	equal
$\approx$	approximately equal
$\square$	end of proof
$\otimes$	Kronecker product
$I_n$	the $n \times n$ identity matrix
$[A]_{i,j}$	the $(i, j)$ th entry of the matrix $A$
$A(i, :)$	the $i$ th row of the matrix $A$
$A(:, i)$	the $i$ th column of the matrix $A$
$A^T$	the transpose of the matrix $A$
$A^{-1}$	the inverse of the matrix $A$
$A^{1/2}$	the symmetric positive-definite square root of the matrix $A$
$\text{diag}(a_1, a_2, \dots, a_n)$	an $n \times n$ diagonal matrix whose $(i, i)$ th entry is $a_i$
$\det(A)$	the determinant of the matrix $A$
$\text{range}(A)$	the column space of the matrix $A$
$\text{rank}(A)$	the rank of the matrix $A$
$\text{trace}(A)$	the trace of the matrix $A$

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*Notation and symbols* $\text{vec}(A)$ a vector constructed by stacking the columns of the matrix  $A$  on top of each other $\|A\|_2$ the 2-norm of the matrix  $A$  $\|A\|_F$ the Frobenius norm of the matrix  $A$  $[x]_i$ the  $i$ th entry of the vector  $x$  $\|x\|_2$ the 2-norm of the vector  $x$  $\lim$ 

limit

 $\min$ 

minimum

 $\max$ 

maximum

 $\sup$ 

supremum (least upper bound)

 $E[\cdot]$ 

statistical expected value

 $\delta(t)$ 

Dirac delta function (Definition 3.8 on page 53)

 $\Delta(k)$ 

unit pulse function (Definition 3.3 on page 44)

 $s(k)$ 

unit step function (Definition 3.4 on page 44)

 $X \sim (m, \sigma^2)$ Gaussian random variable  $X$  with mean  $m$  and variance  $\sigma^2$

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## List of abbreviations

ARX	Auto-Regressive with eXogeneous input
ARMAX	Auto-Regressive Moving Average with eXogeneous input
BIBO	Bounded Input, Bounded Output
BJ	Box-Jenkins
CDF	Cumulative Distribution Function
DARE	Discrete Algebraic Ricatti Equation
DFT	Discrete Fourier Transform
DTFT	Discrete-Time Fourier Transform
ETFE	Empirical Transfer-Function Estimate
FFT	Fast Fourier Transform
FIR	Finite Impulse Response
FRF	Frequency-Response Function
IID	Independent, Identically Distributed
IIR	Infinite Impulse Response
LTI	Linear Time-Invariant
LTV	Linear Time-Varying
MIMO	Multiple Input, Multiple Output
MOESP	Multivariable Output-Error State-sPace
N4SID	Numerical algorithm for Subspace IDentification
PDF	Probability Density Function
PEM	Prediction-Error Method
PI	Past Inputs
PO	Past Outputs
OE	Output-Error
RMS	Root Mean Square
SISO	Single Input, Single Output
SRCF	Square-Root Covariance Filter
SVD	Singular-Value Decomposition
WSS	Wide-Sense Stationary