# NBER WORKING PAPER SERIES

# FINANCIAL CAPACITY, RELIQUIFICATION, AND PRODUCTION IN AN ECONOMY WITH LONG-TERM FINANCIAL ARRANGEMENTS

Mark Gertler

Working Paper No. 2763

NATIONAL BUREAU OF ECONOMIC RESEARCH 1050 Massachusetts Avenue Cambridge, MA 02138 November 1988

This paper originated from joint work and lengthy conversations with Ben Bernanke. Any errors that crept in, however, are mine alone. Thanks also to Jon McCarthy and to the National Science Foundation. This research is part of NBER's research programs in Economic Fluctuations and Financial Markets and Monetary Economics. Any opinions expressed are those of the author not those of the National Bureau of Economic Research. NBER Working Paper #2763 November 1988

# FINANCIAL CAPACITY, RELIQUIFICATION, AND PRODUCTION IN AN ECONOMY WITH LONG-TERM FINANCIAL ARRANGEMENTS

### ABSTRACT

This paper characterizes a multi-period production economy in which borrowers and lenders enter long-term financial contracts. A key feature is that aggregate production and borrowers' capacity to absorb debt - their "financial capacity" - are jointly determined endogenous variables, in the spirit of Gurley and Shaw (1955). Expectations of future economic conditions govern financial capacity, which in turn influences current capacity utilization. Further, disturbances in the present may persist into the future by influencing borrowers' net asset positions. Finally, borrowers may substitute future for current production by preserving their assets in hard times, behavior akin to reliquification as described in Eckstein and Sinai (1986).

> Mark Gertler Department of Economics University of Wisconsin Madison, WI 53706

#### 1. Introduction.

Understanding how small disturbances can induce large output fluctuations is an ongoing quest in macroeconomics. Recent work, beginning with Bernanke (1983), resurrects the view that financial factors may play a part.<sup>1</sup> The underlying theoretical arguments evolve from extending agency theories of finance to intertemporal equilibrium settings. Two related kinds of results arise relevant to explaining business fluctuations. First, mechanisms which magnify the effects of exogenous disturbances on real activity may emerge as a consequence of the agency problems. Second, accelerator effects on investment demand may arise because the agency costs introduce a wedge between the price of internal and (uncollateralized) external finance.

To keep the problem manageable, most of these analyses abstract from settings where borrowers and lenders may enter ongoing relationships. This feature is a limitation, however. It is well understood that gains often exist from contracting over multiple periods. Further, long-term arrangements are characteristic of real-world financial markets.

This paper presents a tractable intertemporal equilibrium model of saving, investment and production, where borrowers and lenders are free to enter long-term agreements. These contracts may be an arbitrary finite number of periods in length. Part of the motivation is to demonstrate that the qualitative results developed in settings with one-shot financial contracts may extend to environments where multi-period arrangements are possible. A more important objective, however, is to demonstrate that this kind of environment enriches the description of how real-financial interactions may contribute to business volatility.

In models of single-period contracting, the net stock of financial assets

<sup>&</sup>lt;sup>1</sup>See Gertler (1988) for a survey.

the borrower begins with is typically a key determinant of the agency costs of external finance. Indeed, Bernanke and Gertler (1987, 1989) exploit this relationship to motivate an investment-accelerator mechanism.<sup>2</sup> An implication of having multi-period financial relationships is that beliefs about future economic conditions can affect current agency costs of finance, as well. One reason this possibility arises is that borrowers may offer as collateral claims on future earnings. As a net result, in the framework presented here, output in any period depends positively on the ceiling level of indebtedness that borrowers can absorb at the end of that period. The ceiling, in turn, depends on borrowers' collateralizable expected future profits, defined below. This feedback between aggregate real activity and borrowers' financial capacity is reminiscent of the arguments in Gurley and Shaw (1955).

A second type of behavior emerging here is that borrowers may adjust production to self-insure against fluctuations in their wealth. In particular, they may cut back production by more than is essential during hard times in order to preserve financial resources for future use. The desire to smooth wealth fluctuations may arise because accumulating wealth reduces subsequent agency costs of finance. Substituting future for current production by preserving assets in bad times corresponds to "reliquification," behavior which Eckstein and Sinai (1986) describe as important in amplifying business downturns.

Section 2 presents the basic setting, a multi-period production economy with borrowing and lending. The single period version of the model is a variant of Grossman and Hart (1983) and Farmer (1985), frameworks in which asymmetric information makes underemployment possible in bad states of nature. The setting differs from Leach (1988), who also allows for multiple production

-2-

<sup>&</sup>lt;sup>2</sup>See Calomiris and Hubbard (1987) for a related theoretical analysis, and see Fazzari, Hubbard and Peterson (1988) for empirical support.

periods, because long-term contracting is permitted. The motivation and details also differ; here the concern is with financial contracts, whereas Leach studies employment contracts.

Section 3 presents a solution to the multi-period financial contracting problem posed in section 2, then constructs and analyzes equilibrium. Using insights from Green (1987), a solution is found by collapsing the multi-period contracting problem into a (backward) recursive sequence of one period problems which determine the borrower's end-of-current-period wealth as a function of the period's state realization. One difference from Green's problem is that borrowers cannot consume negatively; another is that they have finite horizons. The upshot is that the long-term contract must satisfy a "financial capacity constraint," a ceiling on the borrower's interim indebtedness that is needed to let her credibly guarantee her final wealth will be non-negative.

Overall, the environment is rigged so that in the absence of informational asymmetries production each period depends only on current economic conditions. The agency problem not only increases the sensitivity of the equilibrium to exogenous disturbances, it also introduces temporal dependence. Anticipated future economic conditions may affect current production because they determine borrowers' current financial capacity, and also because they influence borrowers' gains from smoothing wealth fluctuations. Current behavior persists into the future by affecting entrepreneurial wealth.

Sections 2 and 3 illustrate the main arguments in a setting with only two production periods. Section 4 demonstrates that the qualitative results of the previous sections extend to the N period case. It also discusses some quantitative differences. Section 5 provides concluding remarks.

-3-

#### 2. The Economic Setting

This economy lasts for three periods, denoted zero, one and two. Investment occurs in periods zero and one; production occurs in periods one and two. There are a countable infinity of people, all who live the entire time. Two kinds exist. A fraction  $\eta$  are "entrepreneurs" and a fraction  $(1-\eta)$ are "lenders." Entrepreneurs differ from lenders by having access to an investment technology described below. Both types care only about consuming in the second period and are risk neutral. Each person thus maximizes the expected value of her period two wealth.

There is one type of commodity, which is perfectly divisible. Every person begins time with  $W_0$  units of it. In each period except the last, two options are available for allocating the commodity. First, one may store it as inventory. A unit stored in period t-1 yields  $r_t$  units in period t, where the gross return  $r_t$  is exogenous. Second, one may convert the good into capital for use in a risky technology that produces output in the subsequent period. In the last period, individuals simply eat the commodity.

Each entrepreneur operates a "project" (one project per entrepreneur) which employs the risky technology. All projects are identical ex ante and work as follows: In period t-1, the entrepreneur installs capital for use in period t. Investing a unit of the endowment good at t-1 yields  $\theta_t$  units of period t capital, which fully depreciate after one period's use. Let  $i_{t-1}$  be the quantity of the endowment good invested at t-1 and  $K_t$  the amount of period t capital. Then,

$$K_{t} = \theta_{t} i_{t-1}$$
(2.1)

The technology parameter  $\theta_t$  is deterministic, though it may differ in value across time.

Output in period t depends on a productivity disturbance  $\widetilde{\psi}_{_{+}}$  and on the

-4-

quantity of capital employed in production. The random variable  $\tilde{\psi}_t$  is independent and identically distributed over time and has the following two-point distribution:

$$\widetilde{\psi}_{t} = \begin{cases} \psi^{g} = 1 + \Delta & \text{with probability } \pi^{g} \\ \psi^{b} = 1 & " & " & \pi^{b} \end{cases}$$
(2.2)

with  $\Delta > 0$ ,  $0 < \pi^{g} < 1$ , and  $\pi^{g} + \pi^{b} = 1$ . Note  $\psi^{g} - \psi^{b} = \Delta > 0$ . The realizations of  $\tilde{\psi}_{t}$  across projects are mutually independent, implying there is no aggregate risk (per project).

Let  $q^j$  be project output in period one given that productivity state j arises (j = g,b);  $q^{jk}$  period two output given that state j occurs in period one and state k occurs in period two (k = g,b);  $x^j$  and  $y^{jk}$  the state-contingent amounts of capital used as input in periods one and two, respectively; and  $K_2^j$  the period two capital stock selected after state j arises in period one. Then the production technology is given by

$$q^{j} = \psi^{j} x^{j} - c(x^{j})$$
(2.3a)

$$K_1 \ge x^J$$
 (2.3b)

$$q^{jk} = \psi^k y^{jk} - c(y^{jk})$$
 (2.4a)

$$K_2^j \ge y^{jk}$$
 (2.4b)

The function  $c(\cdot)$  is twice continuously differentiable, strictly increasing and strictly convex, with c(0) = 0, c'(0) = 0, and  $c'(z) \rightarrow \infty$  as  $z \rightarrow \infty$ .

Output in each period is a strictly increasing and strictly concave function of the quantity of capital input. While the entrepreneur must select the capacity level of capital before the productivity shock is realized - it takes time to install capital - she is able to determine capacity utilization after this event. Employing a unit of installed capital costs  $c'(\cdot)$  units of endowment at the margin, where  $c'(\cdot)$  is increasing in the total quantity of capital employed. For this reason, the entrepreneur may choose to operate at less than full capacity when the bad productivity state arises (see below).<sup>3</sup> Assume further that

~

$$\pi^{9}\Delta > r_{t}^{\prime}\theta_{t}$$
 (2.5a)  
 $\pi^{9}\Delta > \pi^{b}$  (2.5b)

Condition (2.5a) ensures that capital input is always higher in the good state than in the bad, while (2.5b) makes the incentive problem connected with investment finance interesting (again, see below).

Information is structured as follows: The distribution of the project specific productivity shock is common knowledge. However, the realizations of both  $\tilde{\psi}_t$  and project output each period are the private knowledge of the respective entrepreneur. On the other hand, investment and capacity utilization each period are publicly observable.

Lenders' behavior is simple to characterize. They allocate their period zero wealth between loans to entrepreneurs and inventory storage. They repeat the process in period one and consume in the final period.

The behavior of entrepreneurs is more complex. Let the random variable  $\tilde{W}_2$  be the period two wealth of a representative entrepreneur and  $W^{jk}$  be her period two wealth contingent on states j and k arising in periods one and two respectively. The entrepreneur's objective is to maximize

<sup>&</sup>lt;sup>3</sup>The production costs could reflect the use of intermediate inputs or a variable factor such as labor. The key feature of the technology is that it is always optimal to use less input in the bad productivity state than in the good one.

$$E\{\widetilde{W}_{2}\} = \sum_{k} \sum_{j} \pi^{jk} W^{jk}$$
(2.6)

where E( ) is the expectation operator and where  $\pi^{j\,k}$  equals  $\pi^j\cdot\pi^k.$ 

The expected final-period payoff must satisfy the following intertemporal budget constraint:

$$E\{\tilde{W}_{2}\} = W_{0} + (E\{\tilde{q}_{1}\}/r_{1} + E\{\tilde{q}_{2}\}/r_{1}r_{2}) - (I_{0} + E\{\tilde{I}_{1}\}/r_{1})$$
(2.7)

with

$$\mathsf{E}\{\widetilde{\mathsf{q}}_1\} = \sum_{j} \pi^j \mathsf{q}^j, \qquad \mathsf{E}\{\widetilde{\mathsf{q}}_2\} = \sum_{k} \sum_{j} \pi^{jk} \mathsf{q}^{jk}, \qquad \mathsf{E}\{\widetilde{\mathsf{1}}_1\} = \sum_{j} \pi^j \mathsf{1}_1^j$$

where the random variables  $\tilde{q}_t$  and  $\tilde{l}_t$  are period t project output and investment, respectively, and where  $i_1^j$  is investment during the first period contingent on state j. Eq. (2.7) requires that  $E\{\tilde{W}_2\}$  equal the sum of the entrepreneur's initial endowment  $W_0$  and the expected present discounted value of project output (the second term) minus the expected present discounted value of the cost of investing (the third term). The return on storage  $r_t$  is the appropriate discount rate if there is inventory accumulation in equilibrium (so that  $r_t$  is the competitive equilibrium interest rate - see below).

If the entrepreneur needs to borrow, she will enter into a financial contract with lenders in period zero. In general, the contract will specify the state-contingent values of all the publicly observable variables over the project's lifetime and the state-contingent payoffs for each party. Further, an optimal contract will account for the possibility that the entrepreneur may want to misreport the sequence of productivity states. The customary way to address this issue is to restrict attention to the class of contracts where the entrepreneur has no incentive to lie. Let  $s_t(z)$  be the reported

productivity state for period t, given that state z was the true realization. To induce truth-telling, the entrepreneur's payoff  $W^{s_1(j)s_2(k)}$  from misrepresenting any sequence of productivity outcomes (j,k) cannot exceed her gain from honest reporting  $W^{jk}$ . Thus, a constraint on the contract is that

$$W^{jk} \ge W_1^{s_1(j)s_2(k)} \tag{2.8}$$

Another restriction is that the entrepreneur's final wealth in any state cannot be negative:

$$W^{jk} \ge 0$$
 (2.9)

The entrepreneur may be a net debtor in the interim time. This differs from single period contracting, where accounts are settled immediately at the end of each period. Debts may be rolled over under long-term contracting. The maximum amount will depend on beliefs about the entrepreneur's future project returns, as discussed below.

All individuals act competitively. Think of each entrepreneur as picking a contract in period zero to offer lenders.<sup>4</sup> Formally, the contract maximizes the entrepreneur's expected wealth (eq. (2.6)) subject to eqs. (2.1) - (2.5) and (2.7) - (2.9), and to the feasibility requirement that physical quantities such as output be non-negative. The decision variables are  $W^{jk}$ ,  $K_1$ ,  $i_0$ ,  $K_2^j$ ,  $i_1^j$ ,  $q^j$ ,  $q^{jk}$ ,  $x^j$ , and  $y^{jk}$ , for j = g, b and k = g, b.

Once a solution to the contracting problem is found, it is easy to characterize the intertemporal competitive equilibrium. Since each entrepreneur is identical ex ante, each chooses the same contract. All

<sup>&</sup>lt;sup>4</sup>One could introduce the fiction of competitive intermediaries facilitating loans between entrepreneurs and lenders. Since these institutions would earn zero profits and would not use any resources, the analysis that follows can still proceed safely without reference to them.

contracts therefore assign the same state-contingent values for project output and investment. Thus, if  $\hat{q}_t$  denotes per capita output in period t then the weak law of large numbers implies,

$$\hat{q}_{1} = \eta \sum_{i} \pi^{j} q_{i}^{j} + r_{1} (W_{0} - \eta i_{0})$$
 (2.10a)

$$\hat{q}_{2} = \eta \sum_{k} \sum_{j} \pi^{jk} q^{jk} + r_{2} (\hat{q}_{1} - \eta \sum_{j} \pi^{j} i_{1}^{j})$$
(2.10b)

In both eqs. (2.10a) and (2.10b), the two terms on the right are per capita output from project investment and inventory storage respectively.

When informational asymmetries are absent, the only connection between  $\hat{q}_1$ and  $\hat{q}_2$  is that output in the first period affects the quantity of storage in the second. There is no interdependency between period one and period two output from investment. This is because capital depreciates after one period's use and because productivity shocks are i.i.d.. The presence of informational asymmetries changes the situation. At the aggregate level, first period output depends on the perceived state of the economy in period two. In turn, second period output (including output from project investment) depends on the performance of the economy in period one. The sections that follow derive and elaborate these results.

# 3. Equilibrium Under The Optimal Financial Contract

The way to solve the long-term contracting problem posed in section 2 is to apply the logic of dynamic programming. Following Green (1987), imagine setting up an account balance which records the entrepreneur's asset position as it evolves over time. This account balance will be the state variable in the programming problem. The period zero entry is the initial endowment  $W_0$ . Let  $W^J$  denote the entry at the end of period one contingent on state j (i.e., the contract adds  $W^J - W_0$  to the entrepreneur's account in period one if state j arises). The period two entry is the entrepreneur's final state-contingent payoff  $W^{jk}$  (so that  $W^{jk} - W^{j}$  is the amount the contract adds to her account in period two contingent on state k occurring).

The optimal long-term contract is found by working backwards and first deriving the optimal contractual arrangement for period two from the vantage of period one, given state j having occurred in period one. This amounts to solving a one period contracting problem, assuming the entrepreneur has an initial asset position of  $W^{j}$ . The solution yields a value function  $V(W^{j})$  which expresses the entrepreneur's expected period two payoff as a function of  $W^{j}$ . Once  $V(W^{j})$  is obtained it is then possible to find the complete solution by moving back to period zero and solving a contracting problem which picks  $W^{j}$  for j = g, b to maximize the entrepreneur's expected final wealth.

This section first derives the value function and then subsequently solves the complete programming problem. It concludes by analyzing the optimal financial contract and the associated real equilibrium.

3a. <u>Construction of the value function</u>. Suppose an entrepreneur is about to enter period two with an account balance of  $W^{j}$  resulting from the realization of productivity state j in period one. Her expected final period wealth conditioned on this event is by definition

$$\mathbb{E}\{\widetilde{W}_{2} \mid \widetilde{\psi}_{1} = \psi^{j}\} = \sum_{k} \pi^{k} \psi^{jk}$$
(3.1)

The problem here is to find a contract which maximizes (3.1). The contract must offer lenders a competitive return, implying

$$\sum_{k} \pi^{k} (\psi^{k} y^{jk} - c(y^{jk}) - W^{jk}) = r_{2} (K_{2}^{j} \theta_{2} - W^{j})$$
(3.2)

The left side of eq. (3.2) is the expected payment the contract offers lenders (after using eq. (2.4a) to eliminate  $q^{jk}$ ). The right side is the opportunity

-10-

cost of the funds borrowed (after using eq. (2.1) to eliminate  $i_1^{j}$ ).

Only contracts which induce truthful reporting receive consideration, following the discussion in section two. The issue arises here because the entrepreneur may want to pretend times are bad when actually the good productivity state is realized. By doing so, she may be able to substantially lower her obligation to outside lenders. The relevant incentive constraint is

$$W^{jg} \ge W^{jb} + \Delta y^{jb} \tag{3.3}$$

An entrepreneur who falsely pleads hard times must set capacity utilization at  $y^{jb}$  in order to mimic the bad state. Her gain is the unreported income  $\Delta y^{jb}$  (the difference between  $\psi^{g}y^{jb}$  and  $\psi^{b}y^{jb}$ ), plus the contractual payment  $W^{jb}$ . Eq. (3.3) requires that this gain from deception not exceed the payoff from honestly revealing the good state  $W^{jg}$ . It is not necessary to introduce a symmetric constraint to dissuade the entrepreneur from dishonestly claiming times are good; it is easy to show this constraint would never bind.

The formal contracting problem is to choose  $K_2^j$ ,  $y^{jk}$  and  $W^{jk}$  for k = g, b, to maximize (3.1) subject to (3.2), (3.3), the capacity constraint (2.4b), the non-negativity constraint on final wealth (2.9), and the feasibility requirement that  $K_2^j$ ,  $q^{jk}$ ,  $y^{jk} \ge 0$ . The solution follows.

The entrepreneur will always operate at full capacity in the good state; it would only be wasteful to install more capital than needed in good times. Eq. (2.4b) thus holds with equality:

$$K_2^{j} = y^{jq}$$
 (3.4)

Input choice for the good state is given by

$$\pi^{g}[\psi^{g} - c'(y^{jg})] - r_{a}/\theta_{a} = 0 \qquad (3.5)$$

The first term in eq. (3.5) is the expected marginal benefit from increasing  $y^{jq}$ : the probability of the good state times the gain in output from employing an additional unit of capital in that state. The second term is the marginal cost, given that increasing capacity one unit requires investing  $1/\theta_2$  units of the numeraire good. Let  $y^{q*}$  be the value of  $y^{jq}$  which satisfies eq. (3.5). Note that it is unaffected by the incentive constraint (3.3);  $y^{q*}$  is thus the unconstrained optimal (first-best) choice of  $y^{jq}$ .

Input choice for the bad state depends on whether the incentive constraint (3.3) is binding. Let  $y^{b^0}$  be the value of  $y^{j^b}$  arising when the incentive constraint is relaxed (i.e., the first-best value of  $y^{j^b}$ ). Then from (2.4a),  $y^{b^0}$  satisfies

$$1 - c'(y^{b^*}) = 0 \tag{3.6}$$

where eq. (3.6) incorporates the restriction that  $\psi^{b}$  equals unity (see eq. (2.2).) In the unconstrained optimum,  $y^{jb}$  adjusts until the change in output at the margin equals zero. The cost of financing capital investment is irrelevant to the decision because there is excess capacity in the bad state. Condition (2.5a) guarantees that  $y^{b^{\bullet}} < y^{g^{\bullet}} = K_{2}^{j}$ . Nonetheless, operating at  $y^{b^{\bullet}}$  might not be feasible. The problem is that the bad-state input choice affects the gain from falsely announcing bad times. Unreported income  $\Delta y^{jb}$  is increasing in  $y^{jb}$ . To credibly claim times are bad, the entrepreneur may have to set  $y^{jb}$  below  $y^{b^{\bullet}}$ .

When the incentive constraint binds, the optimal contract fixes the entrepreneur's bad-state payoff at its lowest feasible value, zero;

<sup>&</sup>lt;sup>5</sup>Since  $y^{g^*}$  does not depend on economic conditions in period one, neither  $y^{jg}$  nor  $i_1^j$  depend on period one outcomes (i.e.,  $y^{gg} = y^{bg}$  and  $i_1^g = i_i^b$ ) In contrast,  $y^{jb}$  is state-dependent when the incentive constraint is binding, as will be seen shortly.

$$W^{jb} = 0$$
 (3.7)

Reducing  $W^{jb}$  is desirable because it lowers the entrepreneur's incentive to misreport the good state as the bad.

A condition uniquely determining  $y^{jb}$  follows from using eqs. (3.2) and (3.7) to eliminate  $W^{jg}$  and  $W^{jb}$  from the incentive constraint (3.3):

$$[R_{2}(y^{g^{*}}, y^{j^{b}}) + r_{2}W^{j}]/\pi^{g} - \Delta y^{j^{b}} = 0 , \text{ if } y^{j^{b}} > 0$$
(3.8)  
$$y^{j^{b}} = 0 , \text{ otherwise}$$

where  $y^{g^*}$  is given by eq. (3.5); and where the function  $R_t(a,b)$  is the entrepreneur's expected net gain from operating her project in period t, with a and b being the input levels for the good and bad states respectively. It is given by

$$R_{t}(a,b) = \pi^{q}[(1+\Delta)a - c(a)] + \pi^{b}[b - c(b)] - (r_{t}/\theta_{t})a \qquad (3.9)$$

Eq. (3.8) (when it holds) restricts  $y^{jb}$  below  $y^{b^{\bullet}}$  to ensure that the entrepreneur's payoff from honestly revealing the good state is at least the same as her gain from falsely announcing the bad state.<sup>6</sup>

Whether the incentive constraint is binding depends on the entrepreneur's beginning-of-period account balance. A rise in  $W^{j}$  enlarges the amount that the entrepreneur can commit to lenders should the bad state occur, which consequently raises the payoff that she can obtain in the good state.  $W^{jg}$  thus increases while  $W^{jb}$  remains fixed at zero. This relaxes the incentive

 $<sup>^{6}</sup>_{\text{The effect of a rise in }y^{jb}}$  on the entrepreneur's net gain from cheating  $(\Delta y^{jb} + W^{jb} - W^{jg})$  is  $\Delta - \pi^{b} [1-c'(y^{jb})]/\pi^{g} > 0$  (given eq. (2.5b)). Thus,  $y^{jb}$  must decline to satisfy the incentive constraint.

constraint, permitting input use during bad times, y<sup>lb</sup>, to rise. Differentiating (3.8) yields

$$\frac{\partial \mathbf{y}^{jb}}{\partial \mathbf{W}^{j}} = \mathbf{r}_{2} \Delta \mathbf{\pi}^{q} [1 - \delta(1 - c'(\mathbf{y}^{jb})] > 0 \qquad (3.10)$$

for  $y^{jb} \in [0, y^{b^*})$ , where  $\delta = \pi^b / \Delta \pi^g < 1$  (by eq. (2.5b)).

If  $W^{j}$  is below a threshold value, the entrepreneur cannot obtain funds to operate her project because she cannot offer lenders a competitive return. This value, W, is obtained by setting  $y^{jb} = 0$  in eq. (3.8); and is given by

$$\underline{W} = -R_2(y^{q^{\bullet}}, 0)/r_2$$
(3.11)

The number - <u>W</u> is interpretable as the entrepreneur's period one "financial capacity," since it reflects the maximum in-debt she can be at the end of period one in order to function in period two. According to eq. (3.11), financial capacity equals the present value of "collateralizable" expected profits,  $R_2(y^{g^{\bullet}}, 0)$ , the expected net project yield when the incentive problem is severest.<sup>7</sup>

Conversely, when  $W^{j}$  is greater than or equal to an upper limit  $\overline{W}$ , the entrepreneur's wealth is sufficient to guarantee that the incentive constraint is not binding at the first best allocation. In this region,  $y^{jb}$  is set at  $y^{b^{\bullet}}$  and is unaffected by changes in  $W^{j}$ . The limit  $\overline{W}$  is found by setting  $y^{jb} = y^{b^{\bullet}}$ ; and is given by <sup>8</sup>

 $<sup>^{7}</sup>$ R ( $y^{q}^{\bullet}$ ,0) is "collaterizable" because it is the secure portion of expected future profits, the amount that can be guaranteed no matter how bad the incentive problem gets. This is because a contract offered when the incentive problem is severest sets  $y^{jq}$  at  $y^{q}^{\bullet}$  (as always) and  $y^{b}$  at zero. Fixing  $y^{b}$  at zero guarantees the contract is always incentive-compatible.

<sup>&</sup>lt;sup>8</sup>Note that  $\overline{W} > \underline{W}$  since  $\pi^{g} \Delta y^{b^{\bullet}} - \pi^{b} (y^{b^{\bullet}} - c(y^{b^{\bullet}})) > 0$ . While  $\overline{W}$  always exceeds  $\underline{W}$ , it may be less than or equal to zero if the gain from cheating is not large. Otherwise, it is positive.

$$\overline{W} = [\pi^{9} \Delta y^{b^{\bullet}} - R_{2}(y^{9^{\bullet}}, y^{b^{\bullet}})]/r_{2}$$
(3.12)

A value function expressing the entrepreneur's expected discounted period two payoff under the optimal program as a function of  $W^{J}$ , for  $W^{J} \geq \underline{W}$ , is defined by

$$V(W^{J}) = [R_{2}(y^{g^{\bullet}}, y^{J^{b}}) + r_{2}W^{J}]/r_{2}$$
(3.13)

Let  $V_{H}^{J}$  and  $V_{HH}^{J}$  be the first and second partial derivatives of  $V(\cdot)$  with respect to  $W^{J}$ :

$$V_{ij}^{J} = \{1 - \delta[1 - c'(y^{Jb})]\}^{-1} \ge 1$$
(3.14)

$$V_{HH}^{J} = -\delta \mathbf{c}''(\mathbf{y}^{Jb}) [V_{H}^{J}]^{2} \cdot \frac{\partial \mathbf{y}^{Jb}}{\partial \mathbf{W}^{J}} \le 0 \qquad (3.15)$$

Figure 1 illustrates the behavior of  $V(W^{J})$ . It equals zero at  $W^{J} = W$ , and is strictly increasing. The slope equals  $V_{W}^{J}$ , the shadow value of wealth. The function is strictly concave over the interval  $[W, \overline{W})$ , with a slope exceeding unity.  $V_{W}^{J}$  exceeds unity in this range because additional  $W^{J}$ increases the entrepreneur's expected project return by allowing  $y^{Jb}$  to rise. Further,  $V_{WW}^{J}$  is negative because of the concavity built in the production relation (2.4a).<sup>9</sup> When  $W^{J} \geq \overline{W}$ ,  $V(W^{J})$  is linear and its slope is unity. Because  $y^{Jb}$  is fixed at  $y^{b^{\bullet}}$ , more  $W^{J}$  simply adds to the entrepreneur's net worth without affecting her expected project return.

<sup>&</sup>lt;sup>9</sup>In Bernanke and Gertler (1987, 1989),  $V(\cdot)$  is convex over a region where  $V(\cdot) > 0$ . This increasing marginal return to wealth arises because project sizes are fixed. It introduces risk-loving behavior to marginal (less efficient) entrepreneurs, making them willing to enter fair lotteries. In this paper, project size is variable continuously and the production technology is concave; as a result,  $V(\cdot)$  is strictly concave in the positive orthant, so that risk-loving behavior does not arise.

Finally, what are the effects of changes in the interest rate  $r_2$  and the investment technology parameter  $\theta_2^{\,2}$  When informational asymmetries are absent, shifts in  $r_2$  and  $\theta_2$  alter the level of capacity investment and hence the quantity of capital employed in the good state  $y^{9^{\circ}}$  (see eq. (3.5)). Once informational problems are present, however, changes in  $r_2$  and  $\theta_2$  may also affect capacity utilization in the bad state,  $y^{1b}$ , in a way which magnifies the overall effect on expected output.<sup>10</sup> A rise in  $r_2$  lowers capacity investment and therefore lowers  $y^{9^{\circ}}$ . The drop in  $y^{9^{\circ}}$  forces  $y^{1b}$  down to dissuade the entrepreneur from claiming hard times (presuming the entrepreneur is a net debtor; i.e.,  $y^{9^{\circ}}/\theta_2 - W^1 > 0$ ). Conversely, a rise in  $\theta_2$  stimulates capacity investment and  $y^{9^{\circ}}$ , thereby increasing  $y^{1b}$  as well.

3b. Solution to the long-term contracting problem. We now move back to period zero and solve a contracting problem for period one which determines the entrepreneur's end-of-period-one state-contingent account balance  $W^{j}$  (for j = g, b). Once this is done, it is simple to characterize the optimal long-term contract.

First, express the entrepreneur's expected discounted final period payoff in terms of the value function  $V(W^{j})$ :

$$E\{\tilde{W}_{2}/r_{1}r_{2}\} = \sum_{j} \pi^{j} V(W^{j})/r_{1}$$
(3.16)

Next, note that lenders must receive a competitive return, which requires

$$\sum_{j} \pi^{j} [\psi^{j} x^{j} - c(x^{j}) - W^{j}] = r_{1} [x^{9} / \theta_{1} - W_{0}]$$
(3.17)

where eqs. (2.1) and (2.3a) are used to eliminate i and  $q^{j}$ .<sup>11</sup> Eq. (3.17)

<sup>11</sup>It is straightforward to show that if the entrepreneur satisfies the one

<sup>&</sup>lt;sup>10</sup>Farmer (1985) derives the result that the interest rate affects capacity utilization in this kind of environment.

embeds the result that the quantity of capital  $K_1$  installed in period zero for use in period one will equal the quantity of input  $x^9$  employed in the good productivity state, in analogy to the optimum for period two (see the previous section). Thus,  $x^9/\theta_1 - W_0$  is the amount borrowed at time zero.

As before, restrict attention to contracts where the entrepreneur has no incentive to lie. This requires

$$V(W^{g}) \ge \Delta x^{b} + V(W^{b})$$
(3.18)

where the payoffs are measured in terms of period one wealth. Similar to the previous case,  $\Delta x^{b}$  is the unreported income the entrepreneur earns from lying about the good state.<sup>12</sup> It does not enter the value function on the right side of eq. (3.18) additively with  $W^{b}$ . This is because the entrepreneur cannot use  $\Delta x^{b}$  to improve the terms of her period two contract in the same way she can use  $W^{b}$ , else she would reveal her dishonesty.

Another restriction is that the entrepreneur must be able to honor any liability she incurs at the end of period one.<sup>13</sup> (She incurs a liability if  $W^{j}$  is negative). This requires

period budget constraints (3.2) and (3.17), then she automatically satisfies her lifetime budget constraint (2.7).

<sup>&</sup>lt;sup>12</sup>The entrepreneur may secretly store unreported earnings. Her gain measured in period one wealth is thus  $r_2\Delta x^b/r_2 = \Delta x^b$ .

<sup>&</sup>lt;sup>13</sup>Here it is assumed that contracts are enforceable so that the entrepreneur must honor any liability she can feasibly absorb. This contrasts with sovereign lending, where contracts are unenforceable and borrowers are thus able to renegotiate debts (Bulow and Rogoff (forthcoming)). The Bulow-Rogoff scenario may be relevant as well to domestic lending situations where the punishments that courts can inflict on delinquent borrowers are sufficiently limited. Constraining the ability to enforce contracts here would strengthen the basic points made since it would further increase the importance of borrower net worth.

$$V(W^{D}) \ge 0 \tag{3.19}$$

(Eqs. (3.18) and (3.19) ensure that  $V(W^q) \ge 0$ .) If this condition is not satisfied, the entrepreneur cannot obtain funding to operate her project in period two, and thus cannot pay off her debt. Because it essentially requires that  $W^b \ge \underline{W}$ , eq. (3.19) may be termed the "financial capacity constraint" (see the previous section, especially eq. (3.11)).<sup>14</sup>

The multi-period contracting problem thus collapses to choosing  $W^{j}$  and  $x^{j}$ , for j = g, b, to maximize (3.16) subject to (3.17) - (3.19), plus the requirement that  $x^{j}$  be non-negative. There are two key differences from the one period problem presented in section 2. First, the entrepreneur's net financial position can be negative at the end of any production period except the last. This implies that the contract may offer lenders contingent claims on the expected future project rents as a device to improve the entrepreneur's current incentives. Second, even though the entrepreneur is risk neutral over period two consumption, she is effectively risk averse at time zero over period one wealth, for a certain range. The entrepreneur's objective is strictly concave in  $W^{j}$  over a certain interval due to the role of period one wealth in reducing agency costs for period two. She may thus prefer to smooth the realizations of  $W^{j}$ , holding everything else constant.

The optimal arrangement works as follows: Like before, capital input in the good state is set at its first best value; here denoted  $x^{9^*}$ , and given by

-18-

<sup>&</sup>lt;sup>14</sup>The condition that <u>final</u> wealth be non-negative - here eq. (2.9) - is commonly known as a "limited liability constraint" (see Sappington (1983)). The financial capacity constraint, in comparison, is a restriction on <u>interim</u> wealth; it requires that the entrepreneur's interim wealth be sufficient to guarantee that she can feasibly satisfy any given constraint on her final wealth.

$$\pi^{g}(\psi^{g} - c'(x^{g^{\bullet}})) - r_{1}/\theta_{1} = 0$$
 (3.20)

Further, when the incentive constraint is relaxed, capacity usage in the bad state is fixed at its first best optimum,  $x^{b^{\bullet}}$ , given by

$$1 - c'(x^{b^{\bullet}}) = 0 \tag{3.21}$$

If the incentive constraint binds, capacity utilization in the bad state is distorted, also as before. In this case, the following three conditions jointly govern the values of  $x^b$  and  $W^b$ :

$$V(W^{g}(W^{b}, x^{b})) - V(W^{b}) - \Delta x^{b} = 0, \qquad (ic) \qquad (3.22)$$

and either,

$$V(W^b) = 0,$$
 (fc) (3.23)

or,

$$V_{W}^{g} \pi^{b} (1 - c'(x^{b})) / \pi^{g} - \pi^{b} \Delta (1 - V_{W}^{g} V_{W}^{b}) = 0 \qquad (ws) \qquad (3.24)$$

The implicit function for the good state payoff,  $W^{9}(W^{b}, x^{b})$ , is obtained from manipulating eq. (3.17); and is given by

$$W^{g}(W^{b}, x^{b}) = [R_{1}(x^{g^{\bullet}}, x^{b}) + r_{1}^{W}_{0} - \pi^{b}W^{b}]/\pi^{g}$$
(3.25)

where  $R_i(\cdot, \cdot)$  is the expected gain from operating a project in period one, defined by eq. (3.9).  $W^{g}(W^{b}, x^{b})$  is decreasing in  $W^{b}$  and increasing in  $x^{b}$ .

Eq. (3.22) is the incentive constraint modified by using eq. (3.25) to eliminate W<sup>9</sup>. It is portrayed as the (ic) curve in Figures 2 and 3. The

curve is downward sloping in the region of the equilibrium.<sup>15</sup> This is because, in this region, both a rise in  $x^b$  and a rise in  $W^b$  increase the entrepreneur's gain from falsely claiming bad times.

Eqs. (3.23) and (3.24) cannot hold simultaneously. The former applies when the financial capacity constraint, eq. (3.19), is binding. It is portrayed as the (fc) curve in Figures 2 and 3. We know from before that the value of  $W^b$  which satisfies this restriction equals minus the present value of expected period two profits contingent on  $y^b$  equaling zero,  $-R_2(y^{q^*}, 0)/r_2$  (=  $\underline{W}$ ). Since this minimum depends only on anticipated period two gains and not  $x^b$ , the (fc) curve is horizontal.

The financial capacity constraint need not bind, however. The entrepreneur may prefer to set  $W^b$  above <u>W</u> since she is risk averse over a certain range of period one wealth realizations. The benefit of raising  $W^b$  is to narrow the spread between it and  $W^9$ . The cost is the decline in  $x^b$ 

The (ic) curve's slope has the same sign as the term  $\Omega \equiv V_{\mu}^{g} \delta(1-c'(x^{b})-1)$ , where  $\delta = \pi^{b}/\pi^{g} \Delta < 1$  (see eqs. (2.5b) and (3.10)).  $\Omega$  may be positive at  $x^{b} = 0$ if  $V^{g}$  is sufficiently greater than unity. However,  $\Omega$  is always negative at any equilibrium point below  $x^{b^{e}}$ .

First suppose the intersection of the (ic) and (fc) curves defines the equilibrium, as in Figure 2. The (ic) curve intersects the horizontal axis at  $W^b = R_1(x^{q^0}, 0) + r_{10} \ge \underline{W}$ . Because the (ic) curve must cut the (fc) curve from above, its slope must be negative at the intersection. To see that the equilibrium is unique, note that any other intersection must lie to the right of the first one since the (ic) curve is monotone. However, a second intersection to the right of the first is impossible. It is easy to verify that  $\Omega$  would have to be negative at this point, which is not feasible since the slope of the (ic) curve has to be positive here.

Now suppose the intersection of the (ic) and (ws) curves defines the equilibrium, as in Figure 3. At this point,  $V_{W}^{9}\delta(1-c'(x^{b})-1 = -\pi^{9}V_{W}^{9}/V_{W}^{b} < 0$  (from eq. (3.24)). Thus  $\Omega < 0$ ; and therefore the slope of the (ic) curve is negative at the equilibrium. It is easy to verify that the equilibrium is unique, using the same kind of reasoning as in the previous case.

<sup>&</sup>lt;sup>15</sup>At any equilibrium point where the (ic) curve is binding, the slope of this curve is negative. Further, the equilibrium is unique.

required by the incentive condition. Eq. (3.24) reflects this tradeoff; and in this case, it replaces eq. (3.23) as a restriction on  $x^b$  and  $W^b$ . The condition sets the expected net benefit from increasing  $x^b$  equal to zero, holding constant  $W^b$ . The first term is the marginal gain, the entrepreneur's benefit from  $W^q$  rising in response to the increase in expected output. The second is the marginal cost, her loss in expected utility from having the gap between  $W^q$  and  $W^b$  widened to satisfy the incentive constraint.

Figures 2 and 3 portray eq.(3.24) as the (ws) curve (for wealthsmoothing). The curve slopes upward. A rise in W<sup>b</sup> lowers the marginal cost of increasing x<sup>b</sup> by reducing the difference between the shadow values of wealth in the good and bad states. Thus, x<sup>b</sup> must also rise to keep eq. (3.24) satisfied. The restriction is never satisfied for values of x<sup>b</sup> below a minimum  $\underline{x}^{b} \in (0, x^{b^{\bullet}})$ . Below  $\underline{x}^{b}$ , the marginal benefit from raising x<sup>b</sup> always exceeds the marginal cost. This occurs because  $V_{W}^{g}/V_{W}^{b}$  has a lower bound above zero (and below unity). Finally, if  $x^{b} < x^{b^{\bullet}}$ , then  $W^{b} < \overline{W}$ , the minimum value of wealth needed to relax the incentive constraint (see eq. (3.12)). Eq. (3.24) requires that  $V_{W}^{g} < V_{W}^{b}$  when  $x^{b} < x^{b^{\bullet}}$ . This is possible only if  $W^{b} < \overline{W}$ .

3c. Equilibrium. Assume parameters are chosen to guarantee inventory accumulation is always positive.<sup>16</sup> The returns on storage  $r_1$  and  $r_2$  thus become the period one and two competitive equilibrium interest rates. Correspondingly, the state-contingent quantities defined in the multi-period contract are equilibrium values; and together with eqs. (2.10a) and (2.10b), they define equilibrium per capita output for each period.<sup>17</sup>

<sup>&</sup>lt;sup>16</sup>There will be inventory accumulation in period one if  $W_0 > x^{q^{\bullet}}/\theta_1$ ; and in period two, if  $r_1(W_0 - x^{q^{\bullet}}/\theta_1) + \pi^{q}[x^{q^{\bullet}}-c(x^{q^{\bullet}})] + \pi^{b}[x^{b^{\bullet}}-c(x^{b^{\bullet}})] > y^{q^{\bullet}}/\theta_2$ .

<sup>&</sup>lt;sup>17</sup>The equilibrium is Pareto-optimal. The decentralized equilibrium can be reproduced as a solution to a social planning problem with suitably chosen weights assigned to individual utilities. See Townsend (1988).

Since the first best optimum is straightforward, this section concentrates on the case where the incentive constraint is binding. An immediate result is that period one capacity utilization in the bad state,  $x^b$ , must lie below its unconstrained optimum to satisfy the incentive condition, in analogy to the short-term contracting problem studied earlier. It follows that period one output per capita must be lower than its first best value as well (see eq. (2.10a)).

An outcome of the multi-period setting is that output may exhibit positive serial correlation, due to the link between entrepreneurial account balances and real economic activity. Different kinds of serial correlation are possible, depending on the initiating disturbance. An entrepreneur's expected second period output depends positively on her first period productivity shock  $(\tilde{\psi}_1)$  since her good state payoff, W<sup>9</sup>, exceeds her bad state payoff, W<sup>b</sup> (see eq. (3.19)). Since this shock is independent across projects, however, the serial correlation it induces at the individual level vanishes in the aggregate. In contrast, economy-wide shocks - shifts in the common technology parameters (the  $\theta_t$ s), in the returns on storage (the  $r_t$ s), or in initial wealth W<sub>o</sub> - have persistent aggregate effects as well. Economy-wide disturbances in period one affect average entrepreneurial account balances at the end that period; and in this way, they influence per capita output in period two.

A financial mechanism also makes beliefs about values of the period two economy-wide parameters matter to period one behavior. The exact link, though, depends on whether the financial capacity constraint is binding along with the incentive constraint. Each case is discussed below; and the effects of shifts in period one parameters are detailed as well. It is also demonstrated that behavior resembling reliquification is possible when the financial capacity constraint is relaxed.

-22-

<u>Case 1:  $V(W^b) = 0$ </u>. The financial capacity constraint is binding. Eqs. (3.22) and (3.23) thus jointly determine  $x^b$  and  $W^b$ . Figure 2 portrays the equilibrium as the intersection of the (ic) and (fc) curves.  $x^b$  is below  $x^{b^a}$ and  $W^b$  is fixed at its minimum feasible level <u>W</u>. A rise in either initial wealth  $W_0$  or the period one technology parameter  $\theta_1$  shifts the (ic) curve rightward, increasing  $x^b$  and leaving  $W^b$  unchanged.<sup>18</sup> Both adjustments relax the incentive constraint by permitting the spread between  $W^a$  and  $W^b$  to widen; this allows  $x^b$  to increase. Per capita output in period one thus rises. Further, the disturbances are "positively" transmitted into period two. The average end-of-period account balance rises since  $W^a$  is greater for every entrepreneur. Expected per capita output for period two goes up as a result. By analogous reasoning, a rise in the interest rate  $r_1$  shifts the (ic) curve inward, ultimately lowering both per capita output in period one and expected per capita output for period two.

Anticipated future economic conditions also affect current behavior. An expected rise in the future technology parameter  $\theta_2$  is expansionary. The resulting increase in expected period two profits raises the value of accumulating wealth in the good state,  $V(W^9)$ , which relaxes the incentive constraint. (Recall that  $V(W^b)$  is fixed at zero.) In addition to this "incentive" effect, there is also a "financial capacity" effect that is

(estimates of)  $\theta_{j}$  outweigh the benefits.

-23-

 $<sup>^{18}</sup>$  Think of shifts in any economy-wide parameters as occurring <u>prior</u> to contracting. As a matter of theory, in this setting, the optimal contract will have (the risk-neutral) lenders perfectly insure borrowers' net worth against <u>post</u>-contracting fluctuations in aggregate variables. In practice, however, this perfect insurance of borrowers against aggregate shocks appears to rarely arise. In the real world, lenders are typically risk averse, possibly making them unwilling to perfectly insure borrowers against aggregate risks. Also, in the context of the example here, one could imagine that measurement error and delay in the reporting of aggregate quantities could make it difficult for individuals to unravel the precise values of parameters such as  $\theta_2$ ; this could make the overall costs of introducing contingencies on

reinforcing. The anticipated rise in future unencumbered profits raises the maximum liability that the entrepreneur can absorb in the event of a current bad outcome. This lowers  $W^b$ , allowing  $W^g$  to rise, further reducing the entrepreneur's incentive to cheat. The (fc) curve moves downward, dominating a simultaneous downward shift by the (ic) curve, so that  $x^b$  increases on net. Conversely, a rise in  $r_2$  reduces both the shadow value of good state wealth and financial capacity, which in turn lowers  $x^b$ . In both cases, the effect of the disturbance persists into the second period, due to the impact on the average entrepreneurial account balance.

<u>Case 2:  $V(W^b) > 0$ </u>. Improvements in the economic situation relax the financial capacity constraint. These improvements are mirrored in rightward shifts of the (ic) curve and/or downward shifts of the (fc) curve, either of which increases  $x^b$ . After a point, the intersection of the (ic) and (ws) curves defines the equilibrium; the (fc) curve becomes irrelevant. As figure 3 portrays,  $x^b$  is further below  $x^{b^*}$  than it would be if the financial capacity constraint was still binding (holding everything else constant). Correspondingly,  $W^b$  lies above W. Entrepreneurs now accumulate wealth in the bad state at the cost of production being lower than otherwise.<sup>19</sup> This behavior resembles reliquification; and it arises here because the dynamic agency problem introduces a penchant for wealth-smoothing by making an entrepreneur's expected earnings a concave function of her account balance.

As in case 1, increases in  $W_0$  and  $\theta_1$  relax the incentive constraint, moving the (ic) curve rightward. The (ws) curve moves leftward. A rise in either parameter increases  $W^9$ . Holding  $x^b$  constant,  $W^b$  must rise to satisfy the entrepreneur's desire for wealth-smoothing implicit in eq. (3.24). The combined effect of the movement in the two curves is that  $W^b$  increases. The

-24-

<sup>&</sup>lt;sup>19</sup>Calomiris and Hubbard (1987) and Leach (1988) obtain related results in environments with single-period contracting.

overall increase in per capita account balances implies that per capita output in period two will go up as well. Interestingly, when  $W^b$  is initially very low, the reliquification effect (the inward movement of the (ws) curve) may be sufficiently strong to make  $x^b$  fall. However, as  $W^b$  nears  $\overline{W}$ , the marginal gain from reliquification declines, so that  $x^b$  responds positively to rises in  $W_0$ and  $\theta_1$  (i.e., the movement of the (ic) curve dominates). Conversely, an increase in  $r_1$  moves the (ic) curve inward and the (ws) curve outward, in the end reducing both current and future output per capita. Finally, the effects of changes in the period two parameters  $\theta_2$  and  $r_2$  are indeterminate in this case. Unlike the earlier case, the net impact on incentives is indeterminate; this is because  $V(W^b)$  is no longer fixed.

With enough improvement in the economic situation, the incentive constraint will not bind. For example, if  $W_0$  or  $\theta_1$  increases sufficiently or  $r_1$  declines sufficiently, then  $x^b$  converges to its unconstrained optimum  $x^{b^e}$ . The (ic) curve moves far enough to the right so that it no longer intersects the (ws) curve below  $x^{b^e}$ .

# 4. Extension to the N Period Case

First imagine adding a period at the beginning, so that time starts in period "minus one." There now exist three production periods: zero, one and two. The algorithm for solving the long-term contracting problem here follows closely the one for the problem with two production periods, presented in section 3. Let  $v^{0}(W_{0})$  be a value function which expresses the entrepreneur's expected final consumption under the optimal contract as a function of  $W_{0}$ , her wealth entering period zero. The period zero value function  $v^{0}(\cdot)$  has the same general properties as the period one value function  $V(\cdot)$ , derived in section 3a. There is a minimum,  $\underline{W}_{0}$ , below which the entrepreneur is too uncreditworthy to obtain funding to operate her project; i.e.,  $v^{0}(\underline{W}_{0}) = 0$ . Over the interval  $[\underline{W}_{0}, \overline{W}_{0})$ , the function is strictly concave with a slope exceeding unity because increments of  $W_{0}$  in this range reduce period one agency costs.<sup>20</sup> Finally,  $V^{0}(\cdot)$  is linear with a slope of unity when  $W_{0}$  is greater than or equal to  $\overline{W}_{0}$ , the threshold value of period zero wealth at which agency costs vanish.

The three period contracting problem collapses to finding the state-contingent values for period zero wealth,  $W_0^9$  and  $W_0^b$ , to maximize the entrepreneur's expected final wealth. This programming problem is qualitatively similar to the one presented earlier for the two period case since  $V^0(\cdot)$  has the general form of  $V(\cdot)$ . The same can thus be said about the solution. These conclusions extend easily to the N period case. By using backward induction, it is straightforward to show that the value function at any (admissible) period minus t,  $V^{-t}(\cdot)$ , has the same general form as the period one value function  $V(\cdot)$ .

Quantitative differences may arise, of course. Extending the number of production periods increases the entrepreneur's financial capacity; that is, the minimum initial account balance she needs to operate in period minus t,  $\underline{W}_{-t}$ , declines as t rises.<sup>21</sup> The minimum for the case of three production periods,  $\underline{W}_{n}$ , is given by

$$\frac{W_0}{r_0} = - \left[ \frac{R_1(x^{g^*}, 0)}{r_1} + \frac{R_2(y^{g^*}, 0)}{r_1} \right]$$
(4.1)

As before, financial capacity (in this example,  $-W_{0}$ ,) equals the present

 $<sup>{}^{20}</sup>V_{W}^{0} = V_{W}^{9}/[1-V_{H}^{9}\delta(1-c'(x^{b})] > 1$ , where  $V_{W}^{9}$  is given by eq. (3.14). Further,  $V_{WW}^{0}$  is negative in this region, reflecting a diminishing effect of additional wealth on agency costs.

<sup>&</sup>lt;sup>21</sup>The optimal contract thus may call for partial "debt-forgiveness" in the event of a string of bad project outcomes; this arises simply because the maximum liability the entrepreneur can bear declines as the project nears the terminal period.

value of collateralizable future project rents, defined in section 3a. Lengthening the horizon increases this present value, as a comparison of eqs. (3.11) and (4.1) indicates.

Less clear is the temporal behavior of  $\overline{W}_{t}$ , the wealth level at period minus t required to ensure functioning at the unconstrained optimum in each subsequent production period. This value for the three period case,  $\overline{W}_{0}$ , is given by

$$\overline{W}_{0} = [\pi^{9} \Delta x^{b^{\bullet}} - R_{1}(x^{9^{\bullet}}, x^{b^{\bullet}})]/r_{1} + [\pi^{9} \Delta y^{b^{\bullet}} - R_{2}(y^{9^{\bullet}}, y^{b^{\bullet}})]/r_{1}r_{2}$$
(4.2)

It follows from eqs. (3.12) and (4.2) that  $\overline{W}_0$  equals the present value of the sum of the minimum levels of wealth that would alleviate the incentive problem each period under single period contracting.<sup>22</sup> If these minimum levels are positive each period then  $\overline{W}_{-t}$  rises as the horizon increases; more initial wealth is needed to perfectly ensure that the entrepreneur's account balance can remain sufficiently in surplus in the event of a sustained string of bad project outcomes. Conversely, if they are negative then  $\overline{W}_{-t}$  declines. In this case, lengthening the horizon reduces the entrepreneur's incentive to deviate from the first best. This is because the expected gain from honestly operating at the unconstrained optimum each period always exceeds the expected gain in unreported income, obtained from falsely pretending times are bad (see eq. (4.2)).

# 6. Concluding Remarks

This paper characterizes a multi-period production economy in which borrowers and lenders enter optimal long-term financial contracts. A key

<sup>&</sup>lt;sup>22</sup>It is easy to show that  $\overline{W}_0 > \underline{W}_0$  using the same basic means of proof that  $\overline{W} > W$  (see footnote 8).

feature of the equilibrium is that aggregate production and borrowers' capacity to absorb debt - their "financial capacity" - are jointly determined endogenous variables, in the spirit of Gurley and Shaw (1955). Expectations of future economic conditions govern financial capacity, which in turn influences current capacity utilization. Further, disturbances in the present may persist into the future by influencing borrowers' net asset positions. Finally, borrowers may substitute future for current production by preserving their assets in hard times, behavior akin to reliquification.

The exact determination of financial capacity and of how it may feed back into the real equilibrium is of course more complex than this paper portrays. A major omission is role of financial intermediaries. Certainly, secular movements in financial capacity are also tied to the development of intermediation.<sup>23</sup> And breakdowns in intermediation are likely a key aspect of depressions. Nonetheless, the factor emphasized here, collateralizable future profits, may be relevant as well to explaining the kind of short run variation in financial capacity needed to make the general story apply to ordinary business fluctuations. Finally, while the analysis makes no attempt to model growth, it does suggest that the evolution of productivity (profitability) in an economy and of financial capacity may be intimately-connected processes.

-28-

<sup>&</sup>lt;sup>23</sup>See Bencivenga and Smith (1988) and Greenwood and Jovanovic (1988) for recent treatments of the role of financial intermediation in growth.





Figure 2

#### References

- Bencivenga, Valerie and Bruce Smith, "Financial Intermediation and Endogenous Growth," University of Western Ontario, mimeo, February 1988.
- Bernanke, Ben, "Non-Monetary Effects in the Propagation of the Great Depression," <u>American Economic Review</u> 73, June 1983, 257-76.
- Bernanke, Ben and Mark Gertler, "Agency Costs, Net Worth, and Business Fluctuations," <u>American Economic Review</u>, forthcoming 1989.

, "Financial Fragility and Economic Performance," NBER Working Paper no. 2318, July 1987.

- Bulow, Jeremy and Kenneth Rogoff, "A Constant Recontracting Model of Sovereign Debt," Journal of Political Economy, forthcoming.
- Calomiris, Charles and Glenn Hubbard, "Firm Heterogeneity, Internal Finance, and Credit Rationing," NBER Working Paper no.2497, December 1987.
- Eckstein, Otto and Allen Sinai, "The Mechanisms of the Business Cycle in the Postwar Era," in Robert Gordon, <u>The American Business Cycle in the</u> <u>Postwar Era</u>, University of Chicago Press for NBER, 1986.
- Fazzari, Stephen, Glenn Hubbard and Bruce Peterson, "Financing Constraints and Corporate Investment," <u>Brookings Papers on Economic Activity</u>, 1988 a:1, 141-195.
- Farmer, Roger, "Implicit Contracts with Asymmetric Information and Bankruptcy: The Effect of Interest Rates on Layoffs," <u>Review of Economic Studies</u> 52, 1985, 427-442.
- Gertler, Mark, "Financial Structure and Aggregate Economic Activity: An Overview," <u>Journal of Money, Credit and Banking</u> 20 Part II, August 1988, 559-588.
- Green, Edward, "Lending and the Smoothing of Uninsurable Income," University of Pittsburgh, mimeo, 1987.
- Greenwood, Jeremy and Boyan Jovanovic, "Financial Development, Growth, and the Distribution of Income," mimeo, University of Western Ontario, 1988.
- Grossman, Sanford and Oliver Hart, "Implicit Contracts under Asymmetric Information," <u>Quarterly Journal of Economics</u> 71, 1983, 123-157.
- Gurley, John and Edward Shaw, "Financial Aspects of Economic Development," <u>American Economic Review</u> 45, September 1955, 515-538.
- Leach, John, "Underemployment with Liquidity-Constrained Multi-period Firms," Journal of Economic Theory 44, February 1988, 81-98.
- Sappington, David "Limited Liability Contracts Between Principal and Agent," Journal of Economic Theory 29, February 1983, 1-21.
- Townsend, Robert, "Information-Constrained Insurance: The Revelation Principle Extended," <u>Journal of Monetary Economics</u> 21, March-May 1988, 411-450.