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ABSTRACT

During the recent U.S. financial crisis, the large decline in aggregate output and labor was accompanied by both a tightening of financial conditions and a large increase in the dispersion of growth rates across firms. The tightened financial conditions manifested themselves as increases in firms' credit spreads and decreases in both equity payouts and debt purchases. These features motivate us to build a model in which increased volatility of firm level productivity shocks generates a downturn and worsened credit conditions. The key idea in the model is that hiring inputs is risky because financial frictions limit firms' ability to insure against shocks. Hence, an increase in idiosyncratic volatility induces firms to reduce their inputs to reduce such risk. We find that our model can generate most of the decline in output and labor in the Great Recession of 2007–2009 and the observed tightening of financial conditions.

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During the recent U.S. financial crisis, the economy experienced a severe contraction in economic activity and a tightening of financial conditions. At the micro level, the crisis was accompanied by large increases in the cross-section dispersion of firm growth rates (Bloom et al. 2014). At the macro level, it was accompanied by a large decline in labor and output. During the crisis financial, conditions tightened in that firms' credit spreads increased, and both equity payouts and debt purchases decreased. Finally, aggregate total factor productivity fell only slightly. Motivated by these observations, we build a quantitative general equilibrium model with heterogeneous firms and financial frictions in which increases in volatility at the firm level lead to increases in the cross-section dispersion of firm growth rates, a worsening of financial conditions, and decreases in aggregate labor and output with small movements in measured total factor productivity.

The key idea in the model is that hiring inputs to produce output is a risky endeavor. Firms must hire inputs to produce and take on the financial obligations to pay for them before they receive the revenues from their sales. In this context, any idiosyncratic shock that occurs between the time of production and the receipt of revenues makes hiring inputs risky. When financial markets are incomplete in that firms have only debt contracts to insure against such shocks, they must bear this risk. This risk has real consequences if, when firms cannot meet their financial obligations, they must experience a costly default. In the model, an increase in uncertainty arising from an increase in the volatility of idiosyncratic productivity shocks makes the revenues from any given amount of labor more volatile and the probability of a default more likely. In equilibrium, an increase in volatility leads firms to pull back on their hiring of inputs.

We quantify our model and ask, can an increase in the volatility of firm-level idiosyncratic productivity shocks that generates the observed increase in the cross-section dispersion in the recent recession lead to a sizable contraction in aggregate economic activity and tighter financial conditions? We find that the answer is yes. Our model can generate most of the decline in output and employment seen in the Great Recession of 2007–2009. During this event, our model can also generate increases in firm credit spreads, as well as reductions in debt purchases and equity payouts comparable to those observed in the data. More generally, we find that the model generates labor fluctuations that are large relative to those in output,

similar to the relationship in the data. The ability to generate such a pattern has been a major goal of the business cycle literature. Underlying these aggregate macro predictions, our model contains a rich set of micro predictions. We compare the model to firm-level data and show that it generates data consistent with the distributions and covariates of firm spreads, leverage, debt purchases, and equity payouts.

Our model has a continuum of heterogeneous firms that produce differentiated products. The productivity of these firms is subject to idiosyncratic shocks. The volatility of these shocks is stochastically time varying, and these *volatility* shocks are the only aggregate shocks in the economy.

The model has three key ingredients. First, firms hire their inputs—here, labor—and produce before they know their idiosyncratic shocks. That hiring labor is a risky investment is a hallmark of quantitative search and matching models but is missing from most simple macroeconomic models that have, essentially, static labor choices. Here we capture that feature in a simple way: firms commit to hiring labor before they experience idiosyncratic shocks. Second, financial markets are incomplete in that firms have access only to state-uncontingent debt and can default on it. Firms face interest rate schedules for borrowing that reflect their default probabilities and are increasing in their borrowing and labor choices and depend on all shocks. Third, motivated by the work of Jensen (1986), we introduce an incentive problem in that managers can divert free cash flow to projects that benefit them at the expense of firms. This incentive problem creates an *agency friction* that makes it optimal for firms to limit the free cash flow. This limitation makes the firm less able to self-insure against shocks.

Given these ingredients, when firms choose their inputs, they face a trade-off between the expected return on hiring workers and the risk of default. As firms increase their labor, they increase the expected return conditional on not defaulting, but they increase the probability of default. For a given variance of idiosyncratic productivity shocks, they choose labor to balance off the increase in expected return against the costs from increasing default probabilities. The increase in the probability of default has two costs: it increases the probability of liquidation and increases the interest rate firms pay on their borrowing. These effects constitute an extra cost of increasing labor and thus distort the firm's optimal labor choice.

When the variance of the idiosyncratic productivity shocks increases at a given level of labor and borrowing, the probability of default increases and the interest rate schedule tightens, both of which increase the distortions for labor and borrowing. Firms become more cautious in the face of such an increase in variance and respond by decreasing labor and borrowing. In equilibrium, default probabilities and credit spreads increase. At the aggregate level, these firm-level responses imply that when volatility increases, aggregate output and employment both fall, debt purchases are reduced, and credit spreads increase.

The result that firms decrease employment when the variance of idiosyncratic productivity shocks increases depends critically on our assumptions of incomplete financial markets and the agency friction. If firms had access to complete financial markets, an increase in volatility would lead to no change in their employment; firms would simply restructure the pattern of payments across states so that they would never default. With incomplete markets and default risk, firms have a precautionary motive to self-insure by maintaining a buffer stock of unused credit. If incentives to self-insure are sufficiently strong, firms build such a large stock that they can greatly dampen fluctuations in labor. We introduce an agency friction that limits the incentives to build up such a large buffer stock.

We are motivated to introduce agency frictions by a large literature in finance that argues that there are substantial agency costs of maintaining a large buffer stock of unused credit and that these agency costs help explain why firms typically have large amounts of debt. In particular, Jensen (1986) argues that, in practice, if firms retain a large buffer, managers use these funds in ways that benefit their private interests rather than shareholder interests. Since shareholders understand these incentives, they give the managers incentives to pay out funds immediately rather than retain them. We model this Jensen effect by assuming that managers can divert the buffer stock to projects that benefit them at the expense of the firm. In the presence of such a friction, the firm finds it optimal to limit the size of the buffer stock and maintain high debt levels.

We show that a quantitative version of the model can successfully reproduce the main real and financial variables at both the macro and micro level. We choose the parameters of the idiosyncratic firm productivity shock process, including those governing the aggregate volatility shocks, so that the model produces the observed time variation in the cross-sectional

dispersion of the growth rate of sales.

At the macro level, the model produces 86% percent of the observed fluctuations in output and fluctuations in labor that, as in the data, are about 30% more volatile than those in output. The model generates many features of observed financial variables including countercylical spreads and procyclical debt purchases and equity payouts. At the micro level, the model reproduces the main patterns of the cross-sectional distributions of financial variables including spreads, leverage, debt purchases, and equity payouts. It is also consistent with the observed positive correlation of spreads with leverage.

We view our model as providing a new mechanism that links increases in firm-level volatility to downturns. To keep the model simple, we have abstracted from additional forces that would lead it to generate a slow recovery, as has been observed following the Great Recession. In so doing, we follow the spirit of much of the work on the Great Depression, including Cole and Ohanian (2004), that divides the analysis of the downturn and recovery into mechanisms that generate the sharp downturn and mechanisms that generate a slow recovery.

Related Literature. Our work is motivated by the evidence of Bloom et al. (2014) that uses detailed Census micro data to document that the dispersion of plant-level shocks to total factor productivity is strongly countercyclical, rising steeply in recessions. In terms of causality, Bloom et al. (2014) ask whether volatility drives the cycle or whether recessions drive increases in volatility. In particular, they search for but find no evidence that the increase in the variance of these shocks is driven by level, or first-moment, shocks and conclude that the causation seems to run from volatility to the cycle and not the reverse.

Our work is related to models that link time-varying volatility to downturns. Bloom (2009) and Bloom et al. (2014) consider models with frictionless financial markets. As such, their models do not attempt to deliver on our prime motivation: linking the drop in economic activity during the Great Recession to a tightening of financial conditions. Instead, their work focuses on adjustment costs, which implies that firms halt their investment and hiring when hit by a high volatility shock, but such firms have zero credit spreads. We purposefully abstract from such adjustment costs to emphasize the financial frictions that lay at the heart of our mechanism, and in contrast to Bloom et al. (2014), show that the

empirical implications of the model are consistent with micro and macro financial data.

Schaal (2014) endogenizes adjustment costs to labor by incorporating time-varying volatility shocks into a search model of the labor market. He shows that while an increase in idiosyncratic volatility leads to an increase in unemployment, it actually leads to an increase in output. A main success of his model is that a given drop in aggregate TFP generates a larger increase in unemployment than Shimer's (2005) search model. In contrast to our work, however, this framework cannot account for any of the downturn in output during the Great Recession from the observed increase in volatility.

As in our work, Christiano, Motto, and Rostagno (2009) and Gilchrist, Sim, and Zakrajsek (2010) explore the business cycle implications of volatility shocks in environments with financial frictions. While both of these studies are complementary to ours, we focus on different issues. Christiano, Motto, and Rostagno (2009) show that, in a dynamic stochastic general equilibrium model with nominal rigidities and financial frictions, volatility shocks to the quality of capital account for a significant portion of the fluctuations in output. In contrast to our work, they focus solely on aggregate implications and abstract from any features of the micro data, such as the observed high persistence in firm-level productivity shocks and the distribution of real and financial outcomes at the firm level, such as sales growth and spreads. Gilchrist, Sim, and Zakrajsek (2010) have a frictionless labor market and instead focus on the dynamics of investment. Differently from us, they abstract from any feature that can generate the large observed labor wedge in the Great Recession documented by Brinca, Chari, Kehoe, and McGrattan (2016).

Our work is also related to studies on heterogeneous firms and financial frictions. For example, Cooley and Quadrini (2001) develop a model of heterogeneous firms with incomplete financial markets and default risk and explore its implications for the dynamics of firm investment growth and exit. In other work, Cooley, Marimon, and Quadrini (2004) find in a general equilibrium setting that limited enforceability of financial contracts amplifies the effects of technology shocks on output.

Finally, several researchers, including Buera and Shin (2010), Buera, Kaboski, and Shin (2011), and Midrigan and Xu (2014), have used heterogeneous firm models without aggregate shocks to help account for the relation between financial frictions and the level of

development.

A recent literature, linking financial frictions and business cycles, has developed quantitative business cycle models in which the exogenous shock is directly to the credit constraint. (See, for example, the work of Guerrieri and Lorenzoni (2011), Perri and Quadrini (2011), Jermann and Quadrini (2012), and Midrigan and Philippon (2016).) As do we, this literature aims at generating business cycle fluctuations without large fluctuations in aggregate productivity. One difference is that in our model, the tightening of the credit constraint is endogenously linked to our volatility shocks, measured from firm-level data, while in this literature the shock to the credit constraint is exogeneous and chosen based only on aggregate data. Khan and Thomas (2013) have exogenous shocks directly to the collateral constraint in a model with heterogeneous firms subject to investment adjustment costs. They find that a shock that tightens the collateral constraint can generate a long-lasting recession. Their model differs from ours in that they abstract from any labor market frictions and, in it, credit spreads are zero. Our work is complementary to this literature.

As shown in Eggertsson and Krugman (2012), financial shocks can lead to reductions in aggregate demand when coupled with sticky prices. In a quantitative model, Midrigan and Philippon (2016) find that shocks that directly tighten credit constraints in an economy with sticky prices can account for under half of the employment decline during the Great Recession because these shocks cause the economy to hit the zero lower bound. We think of our work as complementary to theirs. Adding to our model sticky prices and a zero lower bound constraint would simply amplify our effects.

1. Our Mechanism in a Simple Example

Before we turn to our full model, we construct a simple example to illustrate our mechanism in its starkest and most intuitive form. Specifically, we show how, in the presence of financial frictions, fluctuations in the volatility of idiosyncratic shocks give rise to distortions that generate fluctuations in labor. To do so, we compare the optimal labor choice of firms in two environments: one in which they can fully insure against shocks and one in which they cannot insure at all.

Consider a model with a continuum of firms that solve one-period problems. Firms

begin with some debt obligations b, produce using the technology $y = \ell^\alpha$, and maximize equity payouts, which must be nonnegative. They choose the amount of labor input ℓ to hire before the idiosyncratic shock z for this product is realized. These shocks are drawn from a continuous distribution $\pi_z(z)$ with volatility σ_z. The demand for a given firm's product is given by

$$(1) \quad y = \left(\frac{z}{p}\right)^\eta Y,$$

where Y is aggregate output. As we discuss later, the shock z can be interpreted as a productivity shock. At the end of the period, after shock z is realized, a firm chooses the price p for its product and sells it. If a firm has sufficient revenues from these sales, it pays equity holders its revenues net of its wage bill $w\ell$ and debt obligations. This firm also receives a continuation value V, here simply modeled as a positive constant. If the firm cannot pay its wage bill and debt, it defaults, equity payouts are zero, and the firm also receives a continuation value of zero.

Consider, first, what happens when financial markets are complete. Imagine that a firm chooses the state-contingent pattern of repayments $b(z)$ to meet its total debt obligations b and, hence, faces the constraint

$$(2) \quad \int_0^\infty b(z)\pi_z(z)dz = b.$$

The firm chooses labor and state-contingent debt to solve the following problem:

$$\max_{\ell, b(z)} \int_0^\infty [p(z,\ell)\ell^\alpha - w\ell - b(z)]\pi_z(z)dz + V$$

subject to (2) and the nonnegative equity payout condition

$$(3) \quad p(z,\ell)\ell^\alpha - w\ell - b(z) \geq 0,$$

where $p(z,\ell) = zY^{1/\eta}\ell^{-\alpha/\eta}$ is the price the firm sets to sell all of its output and is derived from (1) and $y = \ell^\alpha$. Assume that the debt b is small enough so that it can be paid for

by the profits of the firm. Hence, with complete financial markets, the firm can guarantee positive cash flows in every state in period 1 by using state-contingent debt $b(z)$, and the equity payout constraint is not binding.

With complete markets, the firm's optimal labor choice ℓ^* is such that the expected marginal product of labor is a constant markup over the wage

$$(4) \quad Ep(z, \ell^*)\alpha \left(\ell^*\right)^{\alpha-1} = \frac{\eta}{\eta - 1}w.$$

This first-order condition shows that with complete financial markets, fluctuations in the volatility of the idiosyncratic shock z that do not affect its mean will have no impact on a firm's labor choice, since $p(z, \ell)$ is linear in z.

Now consider what happens when financial markets are not complete. The existing debt is state uncontingent, so firms have no way to insure against idiosyncratic shocks. Here, firms with large employment have to default and exit when they experience low productivity shocks, since cash flow is insufficient to cover the wage bill plus debt repayments. Effectively, the firm chooses its labor input ℓ as well as a cutoff productivity $\hat{z}$ below which it defaults, where for any ℓ, $\hat{z}$ is the lowest z such that $p(z, \ell)\ell^\alpha \geq w\ell + b$, where $p(z, \ell)$ is described above. Thus, the firm solves the following problem:

$$\max_{\ell, \hat{z}} \int_{\hat{z}}^{\infty} [p(z, \ell)z\ell^\alpha - w\ell - b]\pi_z(z)dz + \int_{\hat{z}}^{\infty} V\pi_z(z)dz$$

subject to $p(\hat{z}, \ell)\ell^\alpha - w\ell - b = 0$. This last condition defines the cutoff productivity $\hat{z}$ below which the firm defaults, because for any $z < \hat{z}$, the firm would have negative equity payouts. The cutoff $\hat{z}$ is increasing in labor because as labor ℓ is increased, the wage bill $w\ell$ increases by larger amounts than the revenues $p(z, \ell)\ell^\alpha$. The larger the level of labor ℓ, the larger the probability of default for the firm.

In this environment, the optimal choice of labor does not simply maximize period 1 profits as it does with complete financial markets. Here, the firm balances the marginal increase in profits from an increase in ℓ with the increased costs arising from a higher probability

of default that such an increase entails. The choice of ℓ^* satisfies

$$(5) \quad E(p(z,\ell^*)|z \geq \hat{z})\alpha\ell^{*\alpha-1} = \frac{\eta}{\eta-1}\left[w + V\frac{\pi_z(\hat{z})}{1-\Pi_z(\hat{z})}\frac{d\hat{z}}{d\ell^*}\right],$$

where $p(\hat{z},\ell^*)\ell^{*\alpha} - w\ell^* - b = 0$ and $\Pi_z(z)$ is the distribution function associated with the density $\pi_z(z)$.

When financial markets are incomplete and firms face default risk, the choice of ℓ equates the effective marginal product of labor in the states in which the firm is operative to the marginal costs arising from increasing labor, which includes the wage and the loss in future value. This loss in future value arising from default risk and encoded in the second term of the right-hand side of condition (5) distorts the firm's first-order condition and creates a wedge between the marginal product of labor and the wage.

Now, in contrast to what happens in complete financial markets, here fluctuations in the volatility of idiosyncratic shocks do affect the first-order condition of labor. Increases in volatility typically increase the hazard rate $\pi_z(\hat{z})/[1-\Pi_z(\hat{z})]$, which in turn leads to a larger distortion and a smaller labor input for any given wage w and aggregate output Y. More precisely, in the appendix, we assume that z is lognormally distributed with $E(z) = 1$ and $var(\log z) = \sigma_z^2$. We show that if the value of continuation V is sufficiently large, then a mean-preserving spread, namely an increase in σ_z, leads to a decrease in labor ℓ. The intuition for this result is that an increase in volatility increases the risk of default; hence, firms have incentives to lower this risk by reducing their labor input.

Note that the first-order condition (5) shares some features with that for the choice of capital in standard costly state verification models, as in Bernanke, Gertler, and Gilchrist (1999). In particular, the lost resources from default in our example play a similar role as the lost resources from monitoring in the costly state verification framework.

2. Model

We now turn to our general model, namely, a dynamic open economy model that incorporates financial frictions and variations in the volatility of idiosyncratic shocks at the firm level. The economy has continuums of final goods, intermediate goods firms, and households. The final goods firms are competitive and have a technology that converts intermediate goods

into a final good. This technology is subject to idiosyncratic shocks that affect the productivity of intermediate goods used to produce final goods. The volatility of these shocks is stochastically time varying, and these *volatility* shocks are the only aggregate shocks in the economy.

The intermediate goods firms are monopolistically competitive and use labor to produce differentiated products. They can only borrow state-uncontingent debt and are allowed to default on both their debt and payments to workers. If they default, they exit the market with zero value. New firms enter to replace defaulting firms that exit. Households have preferences over consumption and leisure, provide labor services to intermediate goods firms, and own all firms.

At the end of any given period, firms decide how many workers to hire for the next period and how much to borrow, while households decide how much labor to supply to the market for the next period. In the beginning of the next period, aggregate and idiosyncratic shocks are realized. Intermediate goods firms set their prices, produce, sell their products to final goods firms, choose whether to pay their existing debts and their wage bill, and distribute dividends. The final goods firms buy the intermediate goods and produce. Potential new firms decide whether to enter the market. Households consume and receive payments on their assets.

A. Final and Intermediate Goods Firms

The final good is traded on world markets and has a price of one. The final good Y_t is produced from a fixed variety of nontraded intermediate goods $i \in [0, 1]$ via the technology

$$(6) \quad Y_t \leq \left(\int z_t(i) y_t(i)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}},$$

where the elasticity of demand η is greater than one. Final goods firms choose the intermediate goods $\{y_t(i)\}$ to solve

$$(7) \quad \max_{\{y_t(i)\}} Y_t - \int p_t(i) y_t(i) di$$

subject to (6), where $p_t(i)$ is the price of good i relative to the price of the final good. This problem yields that the demand $y_t(i)$ for good i is

$$(8) \quad y_t(i) = \left(\frac{z_t(i)}{p_t(i)}\right)^{\eta} Y_t.$$

Intermediate goods firms produce differentiated goods that are subject to idiosyncratic productivity shocks z_t that follow a Markov process with transition function $\pi_z(z_t|z_{t-1}, \sigma_{t-1})$, where σ_{t-1} is an aggregate shock to the standard deviation of the idiosyncratic productivity shocks. The aggregate shock σ_t follows a Markov process with transition function $\pi_\sigma(\sigma_t|\sigma_{t-1})$. Firms are also subject to i.i.d. idiosyncratic revenue shocks κ_t that have a distribution $\Phi(\kappa)$. These firms are monopolistically competitive and produce output y_t using technology $y_t = \ell_t^\alpha \ell_{mt}^\theta$, where ℓ_t is the input of workers and ℓ_{mt} is the input of a single manager where $0 < \alpha < 1$. Since each active firm uses one manager, we simply impose $\ell_{mt} = 1$ from now on.

In the current setup, since the final goods production function has no value added but rather simply combines the intermediate goods, we can alternatively reinterpret our setup as follows. The aggregator of final goods is $Y_t = \left(\int \tilde{y}_t(i)^{\frac{\eta-1}{\eta}} di\right)^{\frac{\eta}{\eta-1}}$ and each final good i produces $\tilde{y}_t(i) = z_t^{(\eta-1)/\eta} \ell_t^\alpha \ell_{mt}^\theta$ units of good i with associated demand function $\tilde{y}_t(i) = p_t(i)^{-\eta} Y_t$. Here, when measured in logs, the TFP of a firm is proportional to z_t. This alternative interpretation is useful to keep in mind when using the data to help set the parameters for z_t.

After all shocks are realized, each firm decides on the price $p_t(i)$ of its product and decides whether to repay or default on its wage bill and debt. Since firms face demand curves with an elasticity larger than 1, they always choose prices to sell all of their output, and, hence, we can set $p_t(i) = z_t(i) \left(Y_t/\ell_t^\alpha(i)\right)^{1/\eta}$ and eliminate prices as a choice variable from now on. Firms that default exit, whereas firms that continue choose new debt b_{t+1} and labor input ℓ_{t+1} at the end of period t, paying the associated wage bill only after they produce. This debt contract pays off b_{t+1} at $t+1$ as long as a firm chooses not to default at $t+1$ and gives the firm $q_t b_{t+1}$ at t where, as we show later, the bond price q_t is a function that reflects the compensation for the loss in case of default.

Firms pay their equity holders their revenues net of production costs and net payments on debt. Equity payouts d_t are restricted to be nonnegative and satisfy the *nonnegative equity*

payout condition

$$(9) \quad d_t = p_t \ell_t^{\alpha} - w_t \ell_t - w_{mt} - \kappa_t - b_t + q_t b_{t+1} \geq 0,$$

where w_t is the wage of workers and w_{mt} is the wage of managers. Firms use variations in equity payouts to help buffer shocks. It will turn out that this motive leads equity payouts to be procyclical, as they are in the data. We follow Jermann and Quadrini (2012) in interpreting reductions in equity payouts as issuing equity.

It is convenient for the recursive formulation to define the cash-on-hand x_t as

$$(10) \quad x_t = p_t \ell_t^{\alpha} - w_t \ell_t - w_{mt} - \kappa_t - b_t.$$

The *idiosyncratic state* of a firm, (z_t, x_t), records the current idiosyncratic shock z_t and its cash-on-hand x_t, whereas the *aggregate state* $S_t = (\sigma_t, \Upsilon_t)$ records the current aggregate shock σ_t and the distribution Υ_t over idiosyncratic states. It is permissible to index a firm by its idiosyncratic state (z_t, x_t) rather than its index i because all intermediate goods firms with the same idiosyncratic state take the same decisions.

We provide a brief overview of the firm's problem before we formally describe it. The firm's value is the discounted value of its stream of equity payouts. In each period the firm chooses current equity payouts, the default decision, borrowing, and next period's labor. The firm has a budget constraint, a nonnegativity condition on equity payouts, and an agency friction constraint derived from the manager's incentive problem. Firms default only when their value is less than or equal to zero. Since equity payouts are nonnegative, the firm's value is always nonnegative. Since the firm will never default if it can pay positive equity payouts in the current period, it follows that the firm will default only if there is no feasible choice for it that leads to nonnegative equity payouts, that is, it defaults only if its budget set is empty. Using this logic, we can set up the firm's problem.

Financial Frictions

We turn now to discussing the bond price and the default decision that determines it and then turn to the agency friction. The bond price $q_t = q(S_t, z_t, \ell_{t+1}, b_{t+1})$ reflects the

compensation for the loss in case of default and depends on the current aggregate state S_t, the firm's current idiosyncratic shock z_t, and two decisions of the firm—its labor input ℓ_{t+1} and its borrowing level b_{t+1}. To derive when firms default, let $M(S_t, z_t)$ be the *maximal borrowing*, namely the largest amount a firm can borrow, given the price of debt schedule q, that is,

$$(11) \quad M(S_t, z_t) = \max_{\{\ell_{t+1}, b_{t+1}\}} q(S_t, z_t, \ell_{t+1}, b_{t+1}) b_{t+1},$$

and let $\bar{\ell}(S_t, z_t)$ and $\bar{b}(S_t, z_t)$ be the labor and debt plan associated with this maximal borrowing. Let $\kappa^*_{t+1} = \kappa^*(S_t, S_{t+1}, z_{t+1}, \ell_{t+1}, b_{t+1})$ be the highest level of the revenue shock such that if at this level a firm borrows this maximal amount, it can just satisfy the nonnegative equity payout condition. From (9), this *cutoff level* of revenue shock satisfies

$$(12) \quad \kappa^*_{t+1} \equiv \kappa^*(S_t, S_{t+1}, z_{t+1}, \ell_{t+1}, b_{t+1}) = p_{t+1}\ell^{\alpha}_{t+1} - w_{t+1}\ell_{t+1} - w_{mt+1} - b_{t+1} + M(S_{t+1}, z_{t+1}),$$

where $p_{t+1} = z_{t+1}\left(Y_{t+1}/\ell^{\alpha}_{t+1}\right)^{1/\eta}$. Wages for workers and managers, $w_{t+1} = w(S_t)$ and $w_{mt+1}(S_t)$, depend on the aggregate state S_t because they are determined at the end of period t. Aggregate output $Y_{t+1} = Y(S_t)$ also depends on the aggregate state S_t because it is based on choices made at the end of period t and the distribution of idiosyncratic productivity shocks at $t+1$, which is known at the end of period t.

The default decision thus has a cutoff form: repay in period $t+1$ if the revenue shock $\kappa \leq \kappa^*_{t+1}$, which occurs with probability $\Phi(\kappa^*_{t+1})$, and default otherwise. Hence, the bond price schedule that ensures that lenders break even is defined by

$$(13) \quad q(S_t, z_t, \ell_{t+1}, b_{t+1}) = \beta \sum_{\sigma_{t+1}, z_{t+1}} \pi_\sigma(\sigma_{t+1}|\sigma_t)\pi_z(z_{t+1}|z_t, \sigma_t)\Phi(\kappa^*_{t+1}),$$

where β is the discount factor of risk-neutral international intermediaries and the aggregate state S_t evolves according to its transition function.

All firms, even those that default, choose prices and produce. Defaulting firms with enough revenues to cover their wage bill, namely those with $p_t\ell^{\alpha}_t - w_t\ell_t - w_{mt} - \kappa_t \geq 0$, pay this wage bill in full, and those with insufficient revenues to cover current wages pay out of

all of their revenues to labor. Defaulting firms can pay their wage bill in full if

$$(14)\quad \kappa_t \leq \bar{\kappa}(S_t, z_{t+1}, \ell_{t+1}, b_{t+1}) = p_{t+1}(S_t, z_{t+1}, \ell_{t+1})\ell_{t+1}^{\alpha} - w(S_t)\ell_{t+1} - w_{mt+1}.$$

Consider next the agency friction. This friction captures the tensions between shareholders and managers discussed by Jensen (1986). The idea is that if the plans of the firm do not exhaust most of the credit available to the firm, then managers are tempted to access this unused credit and use the resulting funds to benefit their private interests relative to the shareholder interests. When shareholders choose their borrowing, they understand the incentives of managers to divert unused credit. This agency friction will end up implying a constraint on the maximum amount of unused credit or equivalently, a constraint on the minimum amount of borrowing.

A large number of potential managers either can work in intermediate goods firms with w_{mt} or can use a backyard technology to produce $\bar{w}_m$ units of final goods each period. Given the large number of potential managers, competition implies that managers earn $w_{mt} = \bar{w}_m$. If a firm leaves too much unused credit, the manager of that firm will make the following deviation. After the firm borrows $q_t b_{t+1}$, the manager diverts the unused credit, the maximal credit $M(S_t, z_t)$ minus the actual credit used by the firm $q_t b_{t+1}$. The manager uses these resources to hire workers $\ell_{s,t+1}$ for a side project chosen to just exhaust the unused credit at the current wages; that is, $\ell_{s,t+1}$ solves

$$(15)\quad M(S_t, z_t) - q_t b_{t+1} = w_{t+1}\ell_{s,t+1},$$

where $q_t = q(S_t, z_t, \ell_{t+1}, b_{t+1})$. This side project produces a good with a current period payoff for the manager of

$$(16)\quad \lambda \ell_{s,t+1}^{\alpha} E_t p_{s,t+1}$$

where the fraction λ determines the profitability of this side project, $\ell_{s,t+1} = (M(S_t, z_t) - q_t b_{t+1})/w_{t+1}$, and $p_{s,t+1} = z_{t+1}(Y_{t+1}/\ell_{s,t+1}^{\alpha})^{1/\eta}$. In this diversion, the deviating manager changes the plan of the firm from ℓ_{t+1},b_{t+1} to the plan $\bar{\ell}(S_t, z_t)$, $\bar{b}(S_t, z_t)$, which allows maximal

borrowing, $M(S_t, z_t)$, and the creditors adjust the price of debt in (13) to be consistent with the deviating plan so that they break even. After diversion, the firm fires the manager. After being fired, the manager regains the ability to either work in the market or use the backyard technology with probability θ. A manager will not divert unused credit if the diversion payoff is sufficiently small relative to the wage in that

$$(17) \quad E_t \lambda p_{s,t+1} \ell_{s,t+1}^{\alpha} + \theta\beta \frac{\bar{w}_m}{1-\beta} \leq \frac{\bar{w}_m}{1-\beta}.$$

The left side of this constraint captures both the current period payoff of diverting funds plus the present value of payoffs after being fired, while the right side is the present value of wages if the manager never diverts funds. To prevent diversion, firms leave small enough unused credit such that the value of the side project the manager can undertake is sufficiently small, which means choosing borrowing to be sufficiently high so that the *agency friction* constraint

$$(18) \quad q(S_t, z_t, \ell_{t+1}, b_{t+1}) b_{t+1} \geq M(S_t, z_t) - F_m(S_t, z_t)$$

holds. Here $F_m(S_t, z_t)$ is the maximum amount of unused credit, or free cash flow, that prevents diversion and is obtained by substituting (15) into (17) using $p_{t+1} = z_{t+1} \left(Y_{t+1}/\ell_{t+1}^{\alpha} \right)^{1/\eta}$ and rearranging to get

$$(19) \quad F_m(S_t, z_t) = \left[\frac{1}{E_t \lambda z_{t+1} Y_{t+1}^{1/\eta}} \frac{(1-\theta\beta)\bar{w}_m}{1-\beta} \right]^{\frac{\eta}{\alpha(\eta-1)}} w_{t+1},$$

where $w_{t+1} = w(S_t)$ and $Y_{t+1} = Y(S_t)$. This maximum cash flow depends on the side project technology, the manager's wage, the probability of a fired manager regaining a job, and the wage rate of workers.

The agency friction constraint plays an important role in our model. In the model, the combination of uncontingent debt and the nonnegative equity payout condition restricts the ability of the firm to choose the size of employment to maximize expected profits. That restriction gives firms an incentive to build up a large buffer stock of unused credit, which would allow the firm to self-insure against idiosyncratic shocks. This constraint makes building up

such a buffer stock unattractive.

Most dynamic models of financial frictions face a similar issue. The financial frictions, by themselves, make internal finance through retained earnings more attractive than external finance. Absent some other force, firms build up their savings and circumvent these frictions. In the literature, the forces used include finite lifetimes (Bernanke, Gertler, and Gilchrist (1999), Gertler and Kiyotaki (2011)), impatient entrepreneurs (Kiyotaki and Moore (1997)), and the tax benefits of debt (Jermann and Quadrini (2012)). For a survey of these forces and the role they play, see Quadrini (2011).

The Firm's Recursive Problem

Consider now the problem of an incumbent firm. Let $V(S_t, z_t, x_t)$ denote the value of the firm after shocks are realized in period t. The value of such a firm is

$$V(S_t, z_t, x_t) = 0$$

for any state (S_t, z_t, x_t) such that the budget set is empty in that even if it borrows the maximal amount, it cannot make nonnegative equity payouts, that is, $d_t = x_t + M(S_t, z_t) < 0$. For all other states (S_t, z_t, x_t), the budget set is nonempty, and firms continue their operations and choose labor ℓ_{t+1}, new borrowing b_{t+1}, and equity payouts d_t to solve

$$V(S_t, z_t, x_t) = \max_{\{\ell_{t+1}, b_{t+1}, d_t\}} d_t + \sum_{\sigma_{t+1}, z_{t+1}} Q(\sigma_{t+1}|\sigma_t)\pi_z(z_{t+1}|z_t, \sigma_t) \int^{\kappa^*_{t+1}} V(S_{t+1}, z_{t+1}, x_{t+1}) d\Phi(\kappa)$$

subject to the nonnegative equity payout condition

$$(20) \quad d_t = x_t + q_t b_{t+1} \geq 0$$

and the agency friction constraint (18) where q_t and κ^*_{t+1} are given in (12) and (13). The law of motion for aggregate states $S_{t+1} = (\Upsilon_{t+1}, \sigma_{t+1})$ has

$$(21) \quad \Upsilon_{t+1} = H(\sigma_{t+1}, S_t),$$

where σ_{t+1} follows the Markov chain $\pi_\sigma(\sigma_{t+1}|\sigma_t)$ and the cash-on-hand tomorrow,

$$x_{t+1} = p_{t+1}\ell_{t+1}^{\alpha} - w_{t+1}\ell_{t+1} - \bar{w}_m - b_{t+1} - \kappa_{t+1}. \tag{22}$$

In (22), the price $p_{t+1} = z_{t+1}\left(Y_{t+1}/\ell_{t+1}^{\alpha}\right)^{1/\eta}$ is defined by the demand function, wages for workers and managers $w_{t+1} = w(S_t)$, and output $Y_{t+1} = Y(S_t)$. As we show below, $Q(\sigma_{t+1}|\sigma_t)$ equals $\beta\pi_\sigma(\sigma_{t+1}|\sigma_t)$. This problem gives the decision rules for labor $\ell_{t+1} = \ell(S_t, z_t, x_t)$, borrowing $b_{t+1} = b(S_t, z_t, x_t)$, and equity payouts $d_t = d(S_t, z_t, x_t)$.

Now consider firm entry. The model has a continuum of potential entering firms every period, each of which draws an entry cost ω from a lognormal distribution with mean $\bar{\omega}$ and standard deviation σ_ω with cdf $\Psi(\omega)$ and density $\psi(\omega)$. To enter, firms have to pay an entry cost ω in period t and decide on the labor input ℓ_{t+1}^{e} for the following period. An entering firm must borrow to pay the entry cost and current equity payouts.

Firms that enter in period t draw their idiosyncratic productivity in period $t+1$ according to the density $\pi_z(z_{t+1}|z_e, \sigma_t)$ where z_e simply indexes the density from which entrants draw their productivity shocks. Such firms can borrow up to a maximum of $M(S_t, z_e)$. We assume that from the measure of potential entrants with entry costs smaller than the maximal borrowing, namely those with $\omega \leq M(S_t, z_e)$, a subset is chosen randomly so that the measure of entering firms equals the measure of exiting firms. All such firms have an incentive to enter. An entering firm solves the same problem as an incumbent firm with $x = -\omega$ and $z = z_e$.

Characterizing Firms' Decisions

The following lemma characterizes some properties of a firm's decision rules.

Lemma. For $x < -M(S_t, z_t)$, the firm defaults. For $x \geq -M(S_t, z_t)$, there exists a cutoff level of cash-on-hand, $\hat{x}(S_t, z_t)$, such that for $x < \hat{x}$, the nonnegative equity payout constraint is binding and the value of borrowing $q'b'$ increases one-for-one as cash-on-hand decreases, whereas for $x \geq \hat{x}$, the nonnegative equity payout constraint is slack and the bond price, labor, and borrowing do not vary with cash-on-hand while equity payouts increase one-for-one with cash-on-hand.

Proof. From the definition of $M(S_t, z_t)$, for any level of $x < -M(S_t, z_t)$, the budget

set is empty and the firm necessarily defaults. For $x \geq -M(S_t, x_t)$, we construct the cutoff level of cash-on-hand by solving a relaxed version of the firm's problem for the optimal levels of new borrowing and labor in which we drop the nonnegative equity payout constraint for the current period only, namely $\tilde{V}(S_t, z_t, x_t) =$

$$x_t + \max_{\{\ell_{t+1}, b_{t+1}\}} q_t b_{t+1} + \beta \sum_{\sigma_{t+1}, z_{t+1}} \pi_\sigma(\sigma_{t+1}|\sigma_t)\pi_z(z_{t+1}|z_t, \sigma_t) \int^{\kappa^*_{t+1}} V(S_{t+1}, z_{t+1}, x_{t+1}) d\Phi(\kappa)$$

subject to the agency friction constraint (18) where the cash-on-hand tomorrow x_{t+1} is given in (22) and the aggregate state evolves according to the state evolution equation. Note that cash-on-hand x_t enters simply as an additive constant in the objective function and not in any constraint. Hence, the *relaxed solution* does not vary with x_t and has the form $\hat{\ell}(S_t, z_t)$ and $\hat{b}(S_t, z_t)$ so that the associated bond price $\hat{q} = q(S_t, z_t, \hat{\ell}, \hat{b})$ and the value of borrowing, denoted $\hat{q}\hat{b}$, also do not vary with x_t. The *cutoff level* of cash-on-hand is defined by $-\hat{x}(S_t, z_t) \equiv \hat{q}\hat{b}$. For a level of cash-on-hand below this cutoff level, the nonnegative equity constraint binds, the firm chooses its borrowing level so that equity payouts are zero. For cash-on-hand above this cutoff level, the optimal level of borrowing does not vary with x and is given by the solution to the relaxed problem $\hat{q}\hat{b}$. Because the associated equity payouts satisfy $d = x + \hat{q}\hat{b}$, they increase one-for-one with x. Clearly, the multiplier on the nonnegative equity payout constraint $\gamma(S_t, z_t, x_t) = 0$ if $x \geq \hat{x}$ and $\gamma(S_t, z_t, x_t) > 0$ if $x < \hat{x}$. *Q.E.D.*

Consider next the firm's first-order conditions for labor ℓ' and new borrowing b'. The first-order condition with respect to labor is a generalization of the corresponding first-order condition (5) from our simple example, namely

$$\text{(23)} \quad \alpha (\ell')^{\alpha-1} \int_{s' \in R} \pi(s'|z, \sigma) p' ds' = \frac{\eta}{\eta - 1} \left[w' + \frac{E_{z',\sigma'} V'^* \phi(\kappa^*) \left(\frac{-\partial \kappa^*}{\partial \ell}\right) + \frac{(1+\gamma+\mu)}{\beta} \left(-\frac{\partial q}{\partial \ell'}\right) b'}{E_{z',\sigma'} \int_0^{\kappa^*} (1 + \gamma') \, d\Phi(\kappa')} \right],$$

where $s' = \{z', \sigma', \kappa'\}$, $p' = z' \left(Y(S)/\ell'^\alpha\right)^{1/\eta}$ and the probability density $\pi(s'|z, \sigma)$ is given by

$$\pi(s'|z, \sigma) = \frac{\pi_\sigma(\sigma'|\sigma)\pi_z(z'|z, \sigma)\phi(\kappa') (1 + \gamma')}{E_{z',\sigma'} \int_0^{\kappa^*} (1 + \gamma') \, d\Phi(\kappa')},$$

the repayment set is $R = \{(z', \sigma', \kappa') : \kappa \leq \kappa^*(S, S', z', \ell', b')\}$, and the multipliers γ and

μ are associated with the nonnegative equity payout condition (9) and the agency friction constraint (18).

The optimal labor choice equates the weighted expected marginal benefit of labor to expected marginal cost times a markup. This expected benefit, given by the left side of (23), is calculated using the "distorted" probability density $\pi(s'|z,\sigma)$. This benefit weights the marginal product in future states taking into account two forces. First, it puts weight only on states in which the firm repays the debt tomorrow because whenever the firm defaults, its shareholders receive zero. Second, it puts more weight on states in which the nonnegative equity payout condition tomorrow is binding in that $\gamma' > 0$. Here $1+\gamma'$ is the shadow price of cash-on-hand and reflects the marginal value of internal funds to a firm.

The expected marginal cost of labor, given by the right side of (23), equals the marked-up value of the wage and a wedge. The first term in this wedge is the loss in value from default incurred from hiring an additional unit of labor and is similar to the wedge in the simple example. This term is proportional to $V'^{*}\phi(\kappa^{*})\left(\frac{-\partial\kappa^{*}}{\partial\ell}\right)$ where $V'^{*}\phi(\kappa^{*})$ is the firm's future value evaluated at the default cutoff weighted by the probability of the cutoff and $\frac{-\partial\kappa^{*}}{\partial\ell}$ captures how the cutoff changes with labor. Since the cutoff decreases with labor, at least for low values of z, this first term is generally positive and acts like a tax on labor.

The second term in the wedge, which was not present in the simple example, comes from the decrease in the bond price from hiring an extra unit of labor. The wedge is scaled by the expected value of the shadow price in nondefault states.

In the simple example we abstracted from new borrowing. Here we do not. The first-order condition with respect to new borrowing is

$$(24)\quad (1+\gamma+\mu)\left[q+b'\frac{\partial q}{\partial b'}\right]=\beta E_{z',\sigma'}\int_{0}^{\kappa^{*}}(1+\gamma')\,d\Phi(\kappa')+\beta E_{z',\sigma'}V'^{*}\phi(\kappa^{*}).$$

The optimal level of new borrowing equates the effective marginal benefit of new borrowing to the expected marginal cost. Borrowing one more unit gives a direct increase in current resources of q and leads to a fall in the price of existing debt, giving a total change in current resources of $q+b'\partial q/\partial b'$. On the margin, these resources help relax both the nonnegative equity payout condition and the agency friction constraint and, hence, are valued at the sum

of the multipliers on these constraints. This marginal borrowing relaxes the agency constraint because by issuing more debt, there is less unused credit that the manager can use for its side project.

The marginal cost of borrowing, given by the right side of this condition, consists of two terms. The first term reflects the cost of repaying but is relevant only in repayment states and is weighted by the shadow price of cash-on-hand in those states, namely $1 + \gamma'$. The second term is the loss in value from default.

It is useful to contrast these first-order conditions to those that would arise in a version of the model with complete markets and no agency frictions. In that version, the first-order condition for labor would be

$$(25) \quad \alpha \left(\ell'\right)^{\alpha-1} \int_{z'} \pi_z(z'|z,\sigma) p' dz' = \frac{\eta}{\eta - 1} w',$$

where $p' = z' \left(Y(S)/\ell'^{\alpha}\right)^{1/\eta}$, which is the same as in our simple example (5). As in that example, labor is chosen statically by the firm so as to equate the current marginal product of labor to a markup over the current wage. The first-order condition for state-contingent borrowing is, of course, trivial in the complete market model without agency frictions because the firm no longer has to consider how borrowing affects the future probability of default or the manager's incentive constraint.

B. Households

There are a large number of identical households. Each household is a family with a continuum of members. At the beginning of period t, each household elastically supplies a measure L_t of labor to the labor market as workers and has a mass of managers $\mu > 1$ that earn an effective wage $\bar{w}_m$ either by working as managers or by working in home production. On the firm side, each firm decides how much labor it wishes to hire at the going wage. The labor L_t that the household supplies gets distributed across firms according to their relative demands. In particular, households cannot pick which firm they work for; rather they supply labor to the market. After the aggregate shock σ_t and the idiosyncratic shocks are realized, the households choose their consumption C_t and state-contingent asset holdings $A_{t+1} = \{A_{t+1}(\sigma_{t+1})\}$, get paid their wages W_t, and receive aggregate equity payouts D_t from

their ownership of the intermediate goods firms.

The state of the household is their vector of assets A_t and the beginning-of-period state S_{t-1}. The recursive problem for households is the following:

$$(26)\quad V^H\left(A_t, S_{t-1}\right) = \max_{L_t} \left\{ \sum_{\sigma_t} \pi_\sigma(\sigma_t|\sigma_{t-1}) \max_{C_t(\sigma_t), \{A_{t+1}(\sigma_{t+1})\}} \left[U(C_t, L_t) + \beta V^H(A_{t+1}, S_t)\right] \right\}$$

subject to their budget constraint for each σ_t,

$$C_t(\sigma_t) + \sum_{\sigma_{t+1}} Q(\sigma_{t+1}|\sigma_t) A_{t+1}(\sigma_{t+1}, \sigma_t) = W_t(S_{t-1})L_t + \mu w_m + A_t(\sigma_t) + D_t(\sigma_t, S_{t-1}),$$

where A_t is the vector $\{A_t(\sigma_t)\}$, and the aggregate law of motion for S_{t-1} is given in (21). In our open economy the state-contingent prices $Q(\sigma_{t+1}|\sigma_t)$ are equal to $\beta\pi_\sigma(\sigma_{t+1}|\sigma_t)$. In the budget constraint $W_t(S_{t-1})L_t$ is the total wage payments to the measure of labor L_t where $W_t(S_{t-1})$ is referred to as the *effective wage*. This wage is determined before the shock σ_t is realized and is a function of the aggregate state S_{t-1}. Aggregate equity payouts $D_t(\sigma_t)$ are determined after the shock σ_t is realized and hence are functions of the aggregate state S_{t-1} and the shock σ_t. The first-order condition for labor is

$$(27)\quad -\frac{\sum_{\sigma_t} \pi_\sigma(\sigma_t|\sigma_{t-1}) U_L(C_t(\sigma_t), L_t)}{\sum_{\sigma_t} \pi_\sigma(\sigma_t|\sigma_{t-1}) U_C(C_t(\sigma_t), L_t)} = W_t(S_{t-1}).$$

Using the envelope condition and $Q(\sigma_{t+1}|\sigma_t) = \beta\pi_\sigma(\sigma_{t+1}|\sigma_t)$, the first-order condition for consumption implies

$$(28)\quad U_C(C_t(\sigma_t), L_t) = U_C(C_{t+1}(\sigma_{t+1}), L_{t+1}).$$

The aggregate equity payout that households receive each period is the sum of all the equity payouts from intermediate goods firms so that

$$(29)\quad D_t(\sigma_t, S_{t-1}) = \int d(x, z, \sigma_t, H(\sigma_t, S_{t-1})) dH(\sigma_t, S_{t-1}).$$

The household's problem (26) gives the decision rule for labor, $L_t\left(S_{t-1}\right)$, and the decision

rules for consumption and bond holdings, $C_t(\sigma_t, S_{t-1})$ and $A_{t+1}(\sigma_{t+1}|\sigma_t, S_{t-1})$.

The aggregate wage payments that households receive from all firms is $W_t L_t$, whereas w_t is the face value of wages that an individual firm offers but may not pay. A given firm pays the full face value of wages w_t if $\kappa < \kappa_t^*$ or $\kappa < \bar{\kappa}_t$. We denote the corresponding repayment set as

$$\Omega_R(S_{t-1}, S_t, z_t, x_{t-1}, z_{t-1}) = \{\kappa : \kappa \leq \kappa_t^* \text{ or } \kappa \leq \bar{\kappa}_t\},$$

where κ_t^* and $\bar{\kappa}_t$ are given in (12) and (14) where we evaluate $\ell_t = \ell(S_{t-1}, z_{t-1}, x_{t-1})$ and $b_t = b(S_{t-1}, z_{t-1}, x_{t-1}))$. Let Ω_D be the *default set* in which the firm pays less than the full face value. The aggregate wage payments at t that a household receives from firms, $W_t(S_{t-1})L_t(S_{t-1}) =$

$$\int \pi_\sigma(\sigma_t|\sigma_{t-1})\pi_z(z_t|z_{t-1}, \sigma_{t-1}) \left[\int_{\kappa\in\Omega_R} w_{t-1}\ell_t \Upsilon_{t-1} d\Phi(\kappa) + \int_{\kappa\in\Omega_D} \max\{p_t \ell_t^\alpha - \bar{w}_m - \kappa, 0\}\Upsilon_{t-1} d\Phi(\kappa)\right]$$

where the first integral is taken over $\{x_{t-1}, z_{t-1}, \sigma_t, z_t\}$. Here the household understands that when it supplies a measure of workers to the market in which each firm is promising a face value of wage w_t, once default has been taken into account, the effective wage will only be $W_t < w_t$.[1]

C. Equilibrium

Here we specify the equilibrium conditions in our model for aggregates in $t+1$. Market clearing in the labor market requires that the amount of labor demanded by firms equals the amount of labor supplied by households,

$$\int \ell_{t+1}(S_t, z_t, x_t) d\Upsilon_t(z_t, x_t) = L_{t+1}(S_t)\,. \tag{30}$$

[1] In our quantitative model, firms almost always pay their wage bill, so that W_t is within .01% of w_t. In the appendix, we also allow for the firm to default on the wages of managers, but in our quantitative model with bounded supports on shocks, this never arises. The firm never defaults on managers' wage payments so that it pays the constant amount $\bar{w}_m$.

Output satisfies

$$(31)\quad Y(S_t) \leq \left[\int_{z_{t+1},x_t,z_t} \pi_z(z_{t+1}|z_t,\sigma_t) z_{t+1} y_{t+1}(S_t,z_t,x_t)^{\frac{\eta-1}{\eta}} d\Upsilon_t(z_t,x_t)\right]^{\frac{\eta}{\eta-1}},$$

where $y_{t+1} = \ell_{t+1}^{\alpha}$. The measure of exiting firms at $t+1$ when the aggregate shocks is σ_{t+1} is

$$E_{t+1}(\sigma_{t+1}, S_t) = \int_{z_{t+1},x_t,z_t} \int_{\kappa \geq \kappa_{t+1}^*} \pi_z(z_{t+1}|z_t,\sigma_t) d\Phi(\kappa) d\Upsilon_t(z_t,x_t).$$

The transition function for the measure of firms is $\Upsilon_{t+1} = H(\sigma_{t+1}, S_t)$, which consists of incumbent firms that do not default at time $t+1$ and new entrant firms, and is implicitly defined by $H(x_{t+1}, z_{t+1}; \sigma_{t+1}, S_t) =$

$$\int \Lambda(x_{t+1}, z_{t+1}, x_t, z_t|\sigma_t, S_t) d\Upsilon_t(z_t,x_t) + E_{t+1}(\sigma_{t+1}, S_t) \frac{\psi(-x_{t+1}) I_{\{z_{t+1}=z_e,\ -x_{t+1} \leq M(\sigma_{t+1}, H(\sigma_{t+1},S_t), z_e)\}}}{\int_{\omega \leq M(\sigma_{t+1}, H(\sigma_{t+1},S_t), z_e)} d\Psi(\omega)}$$

The first term comes from the incumbents. To understand this term, note that when the aggregate state is S_t and the current aggregate shock is σ_{t+1}, the probability that an incumbent firm with some state (z_t, x_t) transits to state (z_{t+1}, x_{t+1}) is given by Λ. Here $\Lambda(x_{t+1}, z_{t+1}, x_t, z_t|\sigma_{t+1}, S_t) = \pi_z(z_{t+1}|z_t,\sigma_t)\phi(\kappa_{t+1})$ if, at that state (z_t, x_t), the decision rules $\ell_{t+1} = \ell(S_t, z_t, x_t)$, $b_{t+1} = b(S_t, z_t, x_t)$ together with the given κ_{t+1} produce the particular level of cash-on-hand x_{t+1}, so that κ_{t+1} satisfies

$$\kappa_{t+1} = z_{t+1} Y(S_t)^{1/\eta} \ell_{t+1}^{\alpha(\frac{\eta-1}{\eta})} - w(S_t)\ell_{t+1} - w_m - b_{t+1} - x_{t+1}$$

and $\kappa_{t+1} \leq \kappa^*(S_t, \sigma_{t+1}, H(S_t, \sigma_{t+1}), z_{t+1}, \ell_{t+1}, b_{t+1})$ so the firm does not default. If not, then $\Lambda(x_{t+1}, z_{t+1}, x_t, z_t|\sigma_t, S_t) = 0$.

The second term comes from the new entrants. To understand this term, note that the probability that a new entrant's state (z_{t+1}, x_{t+1}) is equal to the density of the entry cost $\psi(-x_{t+1})$ conditional on $z_{t+1} = z_e$ and the entry cost $\omega = -x_{t+1}$ being less than the borrowing limit so that $-x_{t+1} \leq M(\sigma_{t+1}, H(\sigma_{t+1}, S_t), z_e)$. The term in the denominator scales the total measure of new entrants so that it equals the total measure of exiting firms.

Given the initial distribution Υ_0 and an initial aggregate shock σ_0, an *equilibrium*

consists of policy and value functions of intermediate goods firms $\{d(S_t, z_t, x_t),\ b(S_t, z_t, x_t),$ $\ell(S_t, z_t, x_t), \gamma(S_t, z_t, x_t), V(S_t, z_t, x_t)\}$; of households $C(\sigma_t, S_{t-1})$, $L(S_{t-1})$, and $A(\sigma_{t+1}|\sigma_t, S_{t-1})$; the wage rate $w(S_{t-1})$ and discount bond price $Q(\sigma_{t+1}, \sigma_t)$; bond price schedules $q(S_t, z_t, \ell_{t+1}, b_{t+1})$; and the evolution of aggregate states Υ_t governed by the transition function $H(\sigma_t, S_{t-1})$, such that for all t (i) the policy and value functions of intermediate goods firms satisfy their optimization problem, (ii) household decisions are optimal, (iii) the bond price schedule satisfies the break-even condition, (iv) the labor market clears, and (v) the evolution of the measure of firms is consistent with the policy functions of firms, households, and shocks.

3. Quantitative Analysis

We begin with a description of the data we use, discuss our parameterization, and describe how we choose parameters using a moment-matching exercise. Since our model is highly nonlinear and has occasionally binding constraints, we explain our algorithm in some detail.

We then explore the workings of our model starting at the firm level. We begin with an analysis of interest rate spreads and decision rules and how these shift with aggregate volatility. We study the impulse responses for a firm's labor in response to an increase in aggregate volatility. We illustrate the importance of the financial structure by contrasting the response of a firm in our baseline model to one of a firm with frictionless financial markets. We then compare firm-level statistics in the model and the data.

We then turn to the model's predictions for aggregate variables. We begin with business cycle moments and then show that the model can account for many of the patterns of aggregates during the Great Recession.

A. Data

We use a combination of quarterly aggregate data from the national income and product accounts (NIPA), Bureau of Labor Statistics (BLS), the Federal Reserve's flow of funds accounts, Moody's, and firm-level data from Compustat since 1985. From NIPA we use GDP and from BLS we use hours. From the flow of funds we use information on equity and debt for the nonfinancial corporate sector to construct our aggregate measures for equity payouts and debt purchases. From Compustat we construct five firm-level series: sales growth, leverage,

equity payouts, debt purchases, and spreads.

Consider first the firm-level series from Compustat. As in Bloom (2009), we restrict the sample for firms to those with at least 100 quarters of observations since 1970. We define sales *growth* for each firm as $(s_{it} - s_{it-3})/0.5(s_{it} + s_{it-3})$ where s_{it} is the nominal sales for firm i at time t deflated by the consumer price index. Here we follow Davis and Haltiwanger (1992) in defining growth as being relative to the average level in order to have a measure that is less sensitive to extreme values of sales. We follow Bloom (2009) in computing growth rates across four quarters to help eliminate the strong seasonality evident in the data. Using the panel data on firm growth rates, we construct the time series of the *interquartile range* (IQR) of sales growth across firms for each quarter. We define *leverage* as debt, defined as the sum of short-term and long-term debt, divided by average sales, which is the average of sales over the past eight quarters expressed in annual terms. We define *equity payouts* as the average across the previous four quarters of the ratio of the sum of dividends and net equity repurchases to average sales. We define *debt purchases* as the average across the previous four quarters of the ratio of the change in total firm debt to average sales. To construct the spread for a given firm, we use Compustat to obtain the credit rating for each firm in each quarter and then proxy the firm's spread using Moody's spread for that credit rating in the given period.

Consider next the aggregate measures for equity payouts and debt purchases from the flow of funds. We use data from the nonfinancial corporate sector, and, in contrast to the firm-level definitions, we define equity payouts and debt purchases relative to GDP rather than sales. We use the NIPA data for GDP and employment. For more details, see the appendix.

B. Parameterization and Quantification

Here we discuss how we parameterize preferences and technologies and choose the parameters of the model.

Parameterization

We assume the utility function, has the additively separable form

$$(32) \quad U(C,L) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{L^{1+\nu}}{1+v}.$$

Consider next the parameterization of the Markov processes over idiosyncratic shocks and aggregate shocks to volatility. We want the parameterization to allow for an increase in the volatility of the idiosyncratic productivity shock z while keeping fixed the mean level of this shock. We assume a discrete process for idiosyncratic productivity shocks that approximates the autoregressive process,

$$(33) \quad \log z_t = \mu_t + \rho_z \log z_{t-1} + \sigma_{t-1}\varepsilon_t,$$

where the innovations $\varepsilon_t \sim N(0,1)$ are independent across firms. We choose $\mu_t = -\sigma_{t-1}^2/2$ so as to keep the mean level of z across firms unchanged as σ_{t-1} varies. We assume that the volatility shock σ_t takes on two values, a high value, σ_H, and a low value, σ_L, with transition probabilities determined by the probabilities of remaining in the high and low volatility states, p_{HH} and p_{LL}.

Next, the revenue shock κ_t is assumed to be normal with mean $\bar{\kappa}$ and standard deviation σ_κ. Notice that in the definition of equity payouts, (9), the manager's wage and the revenue shock enter symmetrically, so that only the sum, $\bar{w}_m + \kappa_t$, matters for decisions. From the definition of free cash flow in (19), we see that in the agency friction constraint, only the ratio $\tilde{\lambda} \equiv \lambda\left(1 - \beta(1-\theta)\right)/\bar{w}_m$ matters. Hence, we only parameterize σ_κ, $\bar{w}_m + \bar{\kappa}$, and $\tilde{\lambda}$ and refer to $\tilde{\lambda}$ as the *agency friction*.

We divide the parameters into two groups. We using existing studies to assign some parameters and use a moment-matching exercise to assign others.

Assigned Parameters

The assigned parameters are $\Theta_A = \{\beta, v, \sigma, \alpha, \eta, \rho_z\}$. Many of these parameters are fairly standard, and we choose them to reflect commonly used values. The model is quarterly. In terms of preferences, we set the discount factor $\beta = .99$ so that the annual interest rate is

4%, and we set $\nu = 0.5$, which implies a labor elasticity of 2. This elasticity is in the range of elasticities used in macroeconomic work, as reported by Rogerson and Wallenius (2009). We also redo our experiments with $\nu = 1$, which implies a labor elasticity of 1, and find similar results. We set $\sigma = 2$, a common estimate in the business cycle literature. Although given the risk-sharing condition (28) and the separable utility (21), this parameter matters little for fluctuations.

Consider the parameters governing production. For the intermediate goods production function, we set the parameter α equal to the labor share of 0.70. We interpret there to be two other fixed factors, managerial input and capital, which receive a share of 0.30. For the final goods production function, we choose the elasticity of substitution parameter $\eta = 5.75$ so as to generate a markup of 20%, which is in the range estimated by Basu and Fernald (1997). We choose the serial correlation of the firm-level productivity shock $\rho_z = .91$. This value is consistent with the estimates of Foster, Haltiwanger, and Syverson (2008) for measures of their traditional TFP index, which measures the dollar value of output deflated by a four-digit industry-level deflator.

Parameters from Moment Matching

The parameters set in the moment-matching exercise are

$$\Theta_M = \left\{ \sigma_H, \sigma_L, p_{HH}, p_{LL}, \bar{\kappa} + \bar{w}_m, \sigma_\kappa, \tilde{\lambda}, z_e, \bar{\omega}, \sigma_\omega \right\}.$$

We target ten moments. The first four are the mean, standard deviation, autocorrelation, and skewness of the IQR of sales growth. The next three are the median firm spread and its standard deviation and the median firm leverage. To calculate these medians, we first calculate for each period the median spread and leverage in the cross section and then report the medians of the constructed time series. Likewise, the standard deviation of the median spread is the standard deviation of the cross-sectional medians. The final three are the mean productivity and mean employment of entrants relative to incumbents, as reported by Lee and Mukoyama (2012), and an average leverage of entrants equal to that of incumbents.

Our model is highly nonlinear, and all parameters affect all the moments. Nevertheless, some parameters are more important for certain statistics. The mean IQR is largely driven by

the mean volatility shock σ_t. The IQR standard deviation is determined largely by the distance between σ_L and σ_H, and the IQR autocorrelation is determined by the levels of the transition probabilities p_{LL} and p_{HH} of these shocks. The IQR skewness is controlled by the difference in these transition probabilities. In our calibration, p_{LL} is sufficiently larger than p_{HH} so that, on average, high volatility shocks are realized relatively infrequently. This leads to skewness because the resulting IQR reflects the disproportionate probability that is put on the low volatility shocks. The median spread and its standard deviation are affected by the standard deviation of the revenue shocks and the agency friction. The median leverage is largely determined by the mean revenue shock and the agency friction. The relative productivity, employment, and leverage of entrants are determined by z_e, $\bar{\omega}$, and σ_ω.

The parameters we use are reported in Table 1. In Table 2, we report the target moments in the data and the model. Overall, the model produces similar statistics for the IQR, spreads, and leverage.

C. Algorithm

Here we provide an overview of the algorithm we use to solve the model and relegate the detailed description to the appendix.

To solve its problem, each firm needs to forecast next period's wage $w(S)$ and output $Y(S)$, and it needs a transition law for the aggregate state. In practice, it is infeasible to include the entire distribution Υ in the state. Instead, we follow a version of Krusell and Smith (1998) to approximate the forecasting rules for the firm. We do so by approximating the distribution of firms Υ with lags of aggregate shocks, $(\sigma_{-1}, \sigma_{-2}, \sigma_{-3}, k)$ where k records how many periods the aggregate shocks have been unchanged. Here $k = 1, \ldots, \bar{k}$ and $\bar{k}$ is the upper bound on this number of periods. In a slight abuse of notation, we use $S = (\sigma, \sigma_{-1}, \sigma_{-2}, \sigma_{-3}, k)$ in the rest of this description of the algorithm to denote our approximation to the aggregate state. The law of motion of our approximation to the aggregate state is given by $H(\sigma', S) = (\sigma', \sigma, \sigma_{-1}, \sigma_{-2}, k')$ with $k' = k + 1$ if $\sigma' = \sigma = \sigma_{-1} = \sigma_{-2}$ and 0 otherwise.[2]

[2] To help motivate this approach to approximating the state, note that for any initial distribution Υ_0, after sufficiently many periods the distribution over Υ_t becomes independent of this initial distribution and instead only depends on the history of aggregate shocks $(\sigma_0, ..., \sigma_t)$. We think of our approximation is simply a truncation of that history.

We start with an initial guess of two arrays for the aggregate wages, $w^0(S)$, and output, $Y^0(S)$, referred to as *aggregate rules*. We then solve the model with two loops: an inner and an outer loop.

In the inner loop, taking as given the current set of aggregate rules, we first solve for the bond price schedule by iterating on the borrowing limit $M(S, z)$ in (11), the default cutoff $\kappa^*(S, S', z', \ell', b')$ in (12), and the bond price $q(S, z, \ell', b')$ in (13) until convergence. Given the resulting bond price schedule, we then iteratively solve each firm's optimization problem using a combination of policy function and value function iteration until convergence. In the iterations we also iterate on a set of arrays of grids $\{X(S, z)\}$ where the set of points $X(S, z) = \{x_1, \ldots, x_N\}$ varies with (S, z). We begin with an initial guess on the array of grids $\{X^0(S, z)\}$, the multiplier function on the nonnegative equity payout condition $\{\gamma^0(S, z, x)\}$ and the value function $\{V^0(S, z, x)\}$ where the multiplier function and value function are defined for all values of x in a range $[-M(S, z), \infty]$. For each iteration n, given the array of grids, the multipliers, and the value function from the previous iteration, we solve for the updated array of grids $\{X^{n+1}(S, z)\}$, multiplier function $\{\gamma^{n+1}(S, z, x)\}$, and value function $\{V^{n+1}(S, z, x)\}$ in two steps. In these steps we use the result that for all cash-on-hand levels x greater than some cutoff level $\hat{x}(S, z)$, the nonnegative equity payout condition is not binding and the decision rules for labor and debt do not vary with cash-on-hand x. We refer to the associated values of labor and debt as the *nonbinding levels of labor and debt* and denote them by $\hat{\ell}(S, z)$ and $\hat{b}(S, z)$.

In particular, given the multipliers $\{\gamma^n(S, z, x)\}$ and the value function $\{V^n(S, z, x)\}$ in the first step, we solve for these nonbinding levels. To do so, we solve a relaxed problem in which we drop both the nonnegative equity payout constraint and the agency friction constraint and then check whether the constructed tentative solutions satisfy the agency friction condition. If so, then we set the nonbinding levels of labor and debt equal to the tentative solutions. If not we impose that the agency friction constraint binds and define these non-binding levels to be the resulting solution. We then define the cutoff level

$$\hat{x}(S, z) = -q(S, z, \hat{\ell}(S, z), \hat{b}(S, z))\hat{b}(S, z)$$

and construct the new grid by setting $x_1 = -M(S,z)$ and $x_N = \hat{x}(S,z)$. In the second step, we solve for the decisions and multipliers at intermediate points using the firm first-order conditions and the nonnegative equity payout condition. Finally, we update the value function. We iterate on these steps until the grids, the multipliers, and the value functions converge.

In the outer loop, taking as given the converged decisions from the inner loop, we start with a distribution of firms $\Upsilon_0(z,x)$ and simulate the economy for T periods. In each period t, we record firms' labor choices $\{\ell_{t+1}(z,x)\}$, borrowing $\{b_{t+1}(z,x)\}$, and default decisions $\{\iota_t(z,x)\}$ as well as wages and aggregate output. We then project the simulated values for wages and output on a set of dummy variables corresponding to the state S. We use the fitted values as the new aggregate rules $w(S) = w^{k+1}(S)$ and $Y(S) = Y^{k+1}(S)$.

Given the new guesses for the aggregate rules, we then go back to the inner loop and first iterate on the bond price schedule to convergence and then, using the new bond price schedule, iterate on the grids, multipliers, and value functions to convergence. Then, given these converged values, we simulate the economy and construct new guesses for aggregate rules. We then repeat the procedure until the arrays of aggregate output and wages converge.

D. Firm-Level Decisions and Responses

We begin by studying firm spread schedules, decision rules, and responses to an aggregate shock.

Spread Schedules

The bond price schedule that a given firm faces, $q(S_t, z_t, \ell', b')$, depends on the aggregate state S_t, the firm's idiosyncratic shock z_t, and the firm's choice of labor and borrowing. The bond price schedule maps into a *spread schedule* that firms face on their borrowing given by

$$spr(S_t, z_t, \ell', b') = \frac{1}{q(S_t, z_t, \ell', b')} - \frac{1}{\beta}.$$

In Figure 1 we display the spread schedules. To graph this schedule, we need to choose a particular (S_t, z_t) pair. In both panels, we choose the idiosyncratic shock z to be in

the median level in the distribution. For the lines in panel A, the aggregate state, denoted S^L, has aggregate shock σ_L and a distribution of firm-level states Υ that emerges after a long sequence of low volatility shocks. The lines in panel A show how the resulting spread schedule varies with borrowing for two different levels of labor, that is, the lines are the function $spr(S^L, z, \ell', \cdot)$ evaluated at two levels of ℓ'.

As this figure shows, if a firm chooses higher levels of borrowing, it faces higher spreads. The reason is simply that for a given level of labor, the higher the level of debt, the greater the tendency for firms to default. These lines also show that if a firm chooses a higher level of labor, it faces higher spreads. The logic behind this feature is more subtle: a higher level of labor is associated with higher spreads because firms default more in the low z states, and, on the margin, a higher level of labor tends to decrease profits in such z states. Hence, hiring more labor increases the default probability and, hence, drives up the interest rate paid by firms.

The lines in panel B show how the spread schedule shifts with aggregate volatility: the low volatility state S^L described earlier and the corresponding high volatility state S^H that has aggregate shock σ_H and a distribution of firm-level states Υ that emerges after a long sequence of high volatility shocks. When the economy shifts from the low volatility state to the high volatility state, spreads increase for any level of borrowing because this shift increases the probability that a firm will default. The firm is more likely to default both because the probability of low idiosyncratic productivity shocks has increased and also because the ability to borrow in the future has become more restricted.

Decision Rules

Consider next the firm's decision rules. In Figure 2 we again consider a firm at the median level of idiosyncratic shock z and at the low volatility aggregate state S^L. We graph, as a function of cash-on-hand, the firm's choices of new labor ℓ', the value of new borrowing qb', the equilibrium spread at its optimal choices, equity payouts d, and the multiplier on the nonnegative equity payout condition γ.

These decision rules and multipliers have the features highlighted in the lemma. For cash-on-hand $x < -M(S, z)$, the firm defaults whereas for cash-on-hand above this level,

it repays. As panel A shows, the multiplier $\gamma(S^L, z, x)$ on the nonnegative equity payout constraint (20) hits zero at a cutoff level $\hat{x} = \hat{x}(S, z)$. Above this cutoff, labor, the value of borrowing, and the equilibrium spread do not vary with cash-on-hand and the value of equity increases one-for-one with cash-on-hand. Below this cutoff, the nonnegative equity payout constraint binds so that the value of borrowing, $qb' = -x$, increases one-for-one with decreases in x.

Consider the rest of the patterns in these rules. For the multiplier, note from panel E that, below this cutoff, the multiplier increases as x decreases because the nonnegative equity payout condition restricts plans for labor and borrowing more as x falls further.

For labor, note from panel A that below this cutoff level, labor at first decreases and then starts increasing. The decreasing part is straightforward: as cash-on-hand decreases today, the firm has to borrow more to increase qb' and the current multiplier γ increases. This increase in the multiplier increases the shadow price of labor and thus the wedge in (23). The firm responds by decreasing its labor to decrease the spread and reduce the wedge. The increasing part is more subtle. As cash-on-hand decreases sufficiently, the new borrowing necessary to meet the nonnegative equity payout condition increases so much that the default rate and the spread increase rapidly. Hence, conditional on repaying, the idiosyncratic shock z is higher, and so the relevant marginal product of labor, given by the left side of (23), increases. The firm responds by increasing its level of labor accordingly.

For the spread, note from panel C that, below the cutoff $\hat{x}$, the spread increases as x decreases. Briefly, the quantitative impact of the increased borrowing on spreads outweighs the effect from changing labor, so the spread increases.

Decision Rules and Aggregate Volatility

Consider next how an increase in volatility shifts the decision rules. Figure 3 shows how an increase in volatility shifts the decision rules for labor, the value of borrowing, and equity payouts as well as the equilibrium spread at these optimal choices. As we did earlier, in each panel for the idiosyncratic shock, we consider a value of z at the median level of the distribution. We see that an increase in volatility shifts down the decision rules for labor, value of borrowing, and equity payouts and increases the equilibrium spread. The intuition

is that the increase in volatility makes the firm more cautious in hiring labor because of the increase in the wedges in the first-order conditions described earlier. This caution also extends to the value of borrowing and equity payouts. The increase in volatility induces firms to reduce their level of borrowing and equity payouts. Nevertheless, the equilibrium spread increases because, as described above, the spread schedule is more restricted when volatility is high.

The change in decisions for labor, value of borrowing, and equity payouts can be thought of as coming from two parts. The first is the partial equilibrium effect, namely for a given level of wages and aggregate output, firms tend to decrease their labor, borrowing, and equity payouts for precautionary reasons. The second is the general equilibrium effect, namely as volatility increases, wages fall and aggregate output falls. The lower wages induce firms to hire more labor, whereas the lower aggregate output induces firms to hire less labor. Quantitatively, the wage effect dominates, so the general equilibrium effect tends to dampen the drop in labor relative to the partial equilibrium effect.

Impulse Responses for a Firm's Labor

We want to contrast the firm's response for labor with and without frictions. We focus on the impulse response for labor for a firm with the median z and κ as volatility switches from low to high. Along this impulse response, we keep the level of both z and κ at their median levels. The responses of this firm are driven by three factors that are exogenous to its choices: the change in the probability of future levels of z, which are drawn from a more dispersed distribution than under low volatility; the change in the wage and aggregate demand; and the resulting change in the schedule for borrowing that it faces.

Specifically, we suppose that the aggregate state in period 0 is S^L, and in period 1 the economy switches to the high volatility state and stays there throughout the experiment, eventually ending in S^H.

A Firm in the Baseline Model Consider first a firm in the baseline model. In Figure 4 we see that on impact the firm decreases its labor by 4%, and, after four periods, labor drops a total of about 8% and stays persistently at a depressed level. The firm becomes cautious in its hiring decisions for two reasons that are driven by the increased volatility. First, the firm

now fears receiving a very low idiosyncratic shock z at which, at its original level of labor, it will have to default. Second, spread schedules tighten, and firms understand that if they do indeed receive a very low idiosyncratic productivity shock, they will be unable to borrow as easily as they could when volatility was low. This shift in the spread schedule thus reinforces the tendency of firms to be cautious in hiring.

In general equilibrium, since this increase in volatility leads overall employment to fall, it also leads to a fall in wages and a fall in aggregate demand, in the sense that the Y_t term in (8) falls, so that the demand schedule facing each firm shifts inwards.

A Firm with Frictionless Financial Markets To isolate the firm-level effects from the general equilibrium effects, we suppose that a single firm operates without frictions in the midst of an economy in which all other firms face the frictions in the baseline model. This lack of frictions is modeled by allowing this firm to borrow using complete markets and assuming there is no agency friction so that labor satisfies (25). The upshot of these assumptions is that this frictionless firm faces the same aggregate wages and demand schedule as do the firms in our baseline model.

In Figure 4 we compare the impulse response of labor for this frictionless firm to the corresponding impulse response for a firm in our baseline model. We normalize the values of labor to be equal before the shock. (Absent this normalization, the level of labor for the frictionless firm is about 30% higher than that of the firm in the baseline model.) As this figure makes clear, such a firm actually *increases* its labor when volatility increases. There are three effects: the fall in wages increases this firm's incentives to hire workers, the inward shift in the demand for its product reduces its incentives, and the lack of frictions implies that the firm can insure against all the increase in idiosyncratic risk from the more dispersed distribution of productivity shocks. On net, the wage effect dominates and the firm hires more workers.

Note that in our model with frictions, the firm also faces a net positive effect from the general equilibrium forces, dominated by the fall in wages, but the frictions that make the firm cautious outweigh this effect. Of course, if in our baseline model we make wages sticky, then we would have dampened this general equilibrium effect, and the resulting drop in labor

would have been much larger.

E. Firm Moments

Before we present the model's aggregate implications, we show that the model can produce the broad patterns in firm-level statistics. Our earlier moment-matching exercise ensured that the model was consistent with some basic features of firms' financial conditions, including median spread, median leverage, and the dispersion of sales growth. Here we take a closer look at firm-level statistics in the model and the data.

Table 3 presents some moments of the cross-sectional distribution of firms. Consider first the spreads. In each period, we compute the spread at the first, second, and third quartiles in the distribution of spreads and then consider the time series median of spreads at each quartile. In Table 3 we see that in the model, the spreads are a bit higher and more dispersed than they are in the data. For example, the median of the spread at the 50th percentile is 2.8 in the model and 1.3 in the data, whereas the median spread at the 75th percentile is 6.3 in the model and 2.1 in the data.

Consider next the distribution of the growth of sales. In the model, we abstracted from any force that leads to trend growth in a firm's sales, such as an upward drift in the size of z. Because of this abstraction, by construction the median of the 50th percentile of firms' growth in the model is zero. To make the statistics in the data comparable to those in the model, we subtract the median of the 50th percentile of firms' growth. We see that the model does a good job of replicating this distribution.

In terms of leverage, the median of the 50th percentile in the model is similar to that in the data (29 in the model and 26 in the data), but the model's distribution is more compressed than in the data. The distribution of debt purchases in the model is also roughly similar to that in the data.

Finally, consider the equity payouts. In the model, we have firms with decreasing returns in the variable input, namely labor, which can be thought of as arising from a fixed factor, such as land or a fixed capital stock. Thus, the equity payouts to agents outside the firm should be thought of as the sum of payments to the fixed factor and the payments to the owners of the firm. In the data, of course, equity payments are reported net of the payments

to land and capital. To make the numbers in the model comparable to those in the data, in both we subtract out the median of the median equity-payouts-to-sales ratio. We see that the model gives a wider dispersion in the equity-payouts-to-sales ratio than in the data.

Consider next the correlations of firm-level variables with leverage displayed in Table 4. For a given variable, such as spreads, we compute the correlation of each firm's spread with its leverage over time and report the median of these correlations across firms. The correlation between spreads and leverage is positive in both the model and the data. This correlation arises because firms that have low cash-on-hand tend to have high spreads and high leverage. Firms tend to have high leverage when they have low cash-on-hand because higher debt decreases cash-on-hand one-for-one, as seen from the definition (10). As we described earlier using the decision rules, firms tend to have high spreads when they have low cash-on-hand. The correlation between growth and leverage is also positive in the model and the data. In the model, firms with high growth are those that receive relatively high productivity shocks. The increase in productivity allows firms to borrow more at the same rate. This effect induces firms to take on more debt and, thus, higher leverage.

Next, the correlation between debt purchases and leverage is also positive in both the model and the data. In the model, firms with low cash-on-hand have higher leverage, as explained, and tend to borrow more. Finally, equity payouts are nearly uncorrelated with leverage in both the model and the data. In the model, there are two opposing forces. One force is that firms with low cash-on-hand and high leverage tend to have low equity payouts. The opposing force is that firms with high current productivity tend to have high leverage and high equity payouts. These two forces tend to cancel out each other and lead to small correlations between equity payouts and leverage.

F. Business Cycle Moments

So far we have focused on firm-level moments. We are also interested in the moments of aggregate variables in our model over the business cycle. In Table 5 we report for both the data and the model the standard deviations of output, labor, aggregate debt purchases relative to output, and aggregate equity payouts relative to output, as well as the correlations of these variables with output. We also report the correlations of median spread and IQR

with output. The output and labor series are logged and HP-filtered quarterly data from 1985:1 to 2013:1.

Table 5 shows that volatility shocks at the micro level lead output in the model to fluctuate nearly as much as in the data. Since we are abstracting from all other shocks that contribute to fluctuations in output, we think of this result as showing that micro-level volatility shocks can potentially account for a sizable fraction of the volatility in aggregate output.

More interesting is the behavior of labor. Recall that one of the main problems of business cycle models with only productivity shocks is that they generate a much lower volatility of labor relative to output than is observed in the data. Here, instead, there is no such problem: the relative volatility of labor to output is very similar in the model and the data (1.31 in the model versus 1.26 in the data). Moreover, as the lower part of the table shows, in both the model and the data, labor is highly correlated with output. This result represents the primary success of the model for business cycle moments.

Consider now the statistics for the IQR. The standard deviation of IQR is close in the model and the data by our calibration. The interesting result here is that the data show that, over the last 30 years, the correlation between IQR and output is negative. The correlation is -.27 in the data and -.45 in the model. In comparing these numbers, it is useful to remember that if we add in other aggregate shocks that we have abstracted from, such as aggregate productivity shocks, this correlation will be weakened in the model and hence become closer to the data.

Turning to financial variables, we see that the volatility of the median spread, one of our calibration moments, is close in the model and the data. More interesting is that the model produces a key feature of the data: firm spreads are countercyclical. Specifically, in the model and the data, the median spread is negatively correlated with output: -.33 in the model and -.31 in the data. The volatility of the ratio of debt purchases to output in the model is 2.51 and close to the corresponding value in the data of 2.83. This debt purchases ratio is positively correlated with output in the model (.21) although less so than in the data (.75). Equity payouts are somewhat less volatile in the model than in the data: the volatility of the ratio of equity payouts to output is 1.76 in the model and 2.74 in the data. This equity

payout ratio is also positively correlated with output in the model (.18), although somewhat less so than in the data (.45).

Although we abstract from aggregate productivity shocks, our model generates modest movements in measured TFP because, with our financial frictions, labor is not efficiently allocated across firms. One way to illustrate the fluctuations in measured TFP is to define a measure of aggregate TFP as an outsider would, namely, $Y_t/L_t^{\alpha}K_t^{1-\alpha}$ with $K_t = \bar{K}$. We find that the correlation of TFP and output is positive in the model and in the data (.40 and 0.56 respectively), but the fluctuations in measured TFP in the model are about a third as volatile as those in the data.

G. The Great Recession of 2007–2009

Here we ask how much of the movement in aggregates in the recession of 2007–2009 can be accounted for by our model. We show that our model can account for much of this movement.

In this experiment, we choose a sequence of volatility shocks so that the IQR of sales growth in the model reproduces the corresponding IQR path in the data. We think of this procedure as using the data and the model to back out the realized sequence of volatility shocks. We plot the aggregate series after detrending them with a linear trend and normalizing them by first observation.

Baseline Model

In Figure 5A we show the IQR of sales growth for both the data and the model. As the figure shows, the IQR increased substantially during the recession, from 0.16 in 2007:3 to almost 0.34 in 2009:2. Note that the IQR reached its highest level since 1985 at the height of the Great Recession.

The model generates substantial declines in aggregate output and labor over this period. In Figure 5B, we see that over the period 2007:3 to 2009:2, the model generates a decline in output of 9.5%, whereas in the data, output declines 9.2%. Figure 5C shows that the dynamics of labor are similar to those of output: the model produces about a 9.7% decline in labor, whereas in the data, labor declines about 8.7%. Thus, the model can account for essentially all of the contraction in output and labor that occurred in the Great Recession.

The Great Recession had sizable changes in financial variables. Consider first the median spread across firms. Figure 5D shows that in the data, the median spread increases about 575 basis points by 2008:4, whereas in the model, it increases about 480 basis points by 2009:2. Note that in the model, the peak of the spread occurs two quarters later than it does in the data. The reason is that in the data, the IQR is highest at the end of the recession, and in the model the spread is largest when the IQR is highest.

Figure 5E shows the pattern of aggregate debt purchases over output. By the end of the recession, debt purchases had fallen by 7.5% in the data and 9.5% in the model. This pattern of debt purchases implies that the outstanding level of firm debt slowly falls over the recession. Figure 5F plots equity payouts over output. In the data, equity payouts over output fall about 3% by the end of the recession, whereas in the model they fall more, by about 8%.

Here we have focused on the Great Recession of 2007–2009. We have not tried to account for the slow recovery following the end of the recession in 2009. As it stands, our model cannot account for the slow recovery. The reason is twofold. First, in the data, our measure of volatility, the IQR of sales growth, falls relatively quickly post-2009. Second, our model has a tight link between volatility and output so that when volatility falls, output recovers. One reason for this tight connection is that agents know exactly when the volatility shifts. A more elaborate stochastic structure on information in which agents receive only noisy signals of the underlying aggregate shocks, such as in Kozlowski, Veldkamp, and Venkateswaran (2016), would allow the model to break this tight connection. Another reason is that we have abstracted from other mechanisms, such as adjustment costs in debt or in labor, search frictions, and so on, that stretch out the impact of shocks on aggregates. Finally, we have abstracted from other shocks, including policy uncertainty shocks, that Baker, Bloom, and Davis (2012) show actually increase further after the end of the Great Recession. While it is conceptually straightforward to extend the model to have a more elaborate information structure, various adjustment costs, search frictions, and more elaborate shocks, it is computationally infeasible for us to do so.

Lower Labor Elasticity

So far we have assumed a Frisch labor supply elasticity of $1/\nu = 2$. Here we redo our Great Recession experiment with a lower labor elasticity of $1/\nu = 1$. When we change the labor supply elasticity, we do not adjust the other parameters in our moment-matching exercise, so this experiment should be thought of as a simple comparative statics exercise. In Figure 6 we see that the financial variables are affected little by this change. The main effects are that both output and labor fall less than they did in our baseline model. For example, in the baseline model, by the second quarter of 2009, output has fallen by 9.5% and labor has fallen by 9.7%, whereas with the lower elasticity, the corresponding falls are 7.4% and 6.7%, respectively.

In the appendix, we report all the statistics corresponding to those in Tables 2–5 for this lower elasticity. The basic pattern is that with a lower labor elasticity, the financial variables change little, whereas output and labor become less volatile. For example, moving from the benchmark to the lower labor elasticity results in a drop in the volatility of output from .97 to .75 and a drop in the relative volatility of labor to output from 1.31 to 1.22.

Frictionless Financial Markets

To help isolate the quantitative role of frictions in our baseline model, it is useful to contrast the implications for output and labor for our model with frictionless financial markets, namely complete markets and no agency frictions. In contrast to our earlier study of a firm's impulse response, here we are considering the full general equilibrium effects with endogenous wages and aggregate demand.

As Figures 7A and 7B show, with frictionless financial markets, output and labor both increase sharply when volatility rises: output increases about 8% and labor about 6%. The channel by which this increase takes place is referred to as the *Oi-Hartman-Abel* effect, based on the work of Oi (1961), Hartman (1972), and Abel (1983). The mechanism is that when the distribution of z spreads out and z is serially correlated, firms with high z tend to hire relatively more of the factor inputs. To understand why, consider a mean-preserving spread in the distribution of idiosyncratic productivity shocks. With a more spread out distribution, a firm in the upper fraction of the distribution now has a higher level than it did under a

less spread out distribution. With serially correlated productivity shocks, a firm knows that if its productivity shock is high today, then its mean productivity shock tomorrow will also be high. All else equal, this prediction leads the firm to increase its labor.

In our baseline model, these same Oi-Hartman-Abel forces are present but to a much weaker degree because firms are unable to insure against the risk of a low realization of z. With financial frictions, if a firm in the upper fraction of the distribution sharply increases the amount of labor it hires, then, in the unfortunate circumstance that the realized level of z in the next period is actually very low, it will default. This inability to insure against the low realization of z shocks makes such a firm cautious and undoes the Oi-Hartman-Abel effect.

4. Conclusion

Many observers believe that the depth of the Great Recession was due to the interaction of shocks with financial frictions. We have formalized this idea in a model with heterogeneous firms that face default risk and time-varying volatility shocks. We find that fluctuations in the volatility of idiosyncratic productivity shocks lead to quantitatively sizable contractions in economic activity as well as tightening in financial conditions. In the model, as in the recent recession, we observe a large increase in the cross-sectional dispersion of firm growth rates and a large decline in aggregate labor and output, accompanied by a tightening of financial conditions that manifest themselves with increases in firm credit spreads and declines in debt purchases and equity payouts. Hence, we think of our model as a promising parable for the Great Recession of 2007–2009.

A critical feature of our analysis is the use of micro firm-level data for both disciplining the parameterization of the model and checking many empirical predictions of the model mechanisms. We use firm-level data on time-varying volatility, credit spreads, and leverage to parameterize the volatility shocks, including the increase in volatility during the Great Recession, and the magnitude of the financial frictions in our model. We then show that the resulting model predictions for the distributions of firm growth rates, credit spreads, debt, and equity, as well as firm covariates among these variables, resemble the patterns of the micro firm-level data. Hence, the macro predictions of the model occur in a framework that

is consistent with micro observations. We view this attempt to connect the macro and micro predictions to be a strength of the paper and a useful addition to the growing literature using heterogeneous firm models for business cycles, which, unlike this paper, has, with few exceptions, not confronted the micro-level predictions of their models.

We think the quantitative framework developed in this paper, business cycles with firm-level default risk, can be used for studying other applications. One area of interest is financial regulation. The framework can be useful in studying the real implications of financial regulation that change firms' borrowing incentives. Another application is the monetary policy transmission to the real economy through changes in firms' financial conditions. Finally, as explored in Arellano, Bai, and Bocola (2016), the framework is useful for studying the connection between sovereign debt crises and firm default risk.

References

Abel, Andrew B. 1983. "Optimal Investment Under Uncertainty." *American Economic Review* 73(1): 228–233.

Arellano, Cristina, Yan Bai, and Luigi Bocola. 2016. "Private Leverage and Sovereign Default." Mimeo Federal Reserve Bank of Minneapolis.

Baker, Scott, Nicholas Bloom, and Steven J. Davis. 2012. "Measuring Economic Policy Uncertainty." Stanford University Working Paper.

Basu, Susanto, and Fernald, John G. 1997. "Returns to Scale in U.S. Production: Estimates and Implications." *Journal of Political Economy* 105(2): 249–283.

Bernanke, Ben S., Mark L. Gertler, and Simon Gilchrist. 1999. "The Financial Accelerator in a Quantitative Business Cycle Framework." In *Handbook of Macroeconomics*, ed. J. B. Taylor and M. Woodford, vol. 1, chap. 21: 1341–1393. Amsterdam: New Holland.

Bloom, Nicholas. 2009. "The Impact of Uncertainty Shocks." *Econometrica* 77(3): 623–686.

Bloom, Nicholas, Max Floetotto, Itay Saporta-Eksten, Nir Jaimovich, and Stephen Terry. 2014. "Really Uncertain Business Cycles." Stanford University Working Paper.

Brinca, Pedro, V.V. Chari, Patrick J. Kehoe, and Ellen R. McGrattan. 2016. "Accounting for Business Cycles." Federal Reserve Bank of Minneapolis Staff Report 531.

Buera, Francisco J. and Yongseok Shin. 2010. "Financial Frictions and the Persistence of History: A Quantitative Exploration." NBER Working Paper 16400.

Buera, Francisco J., Joseph P. Kaboski, and Yongseok Shin. 2011. "Finance and Development: A Tale of Two Sectors." *American Economic Review* 101(5): 1964–2002.

Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan. 2007. "Business Cycle Accounting." *Econometrica* 75(3): 781–836.

Christiano, Lawrence, Roberto Motto, and Massimo Rostagno. 2009. "Financial Factors in Business Cycles." Northwestern University Working Paper.

Cole, Harold L. and Lee E. Ohanian 2004. "New Deal Policies and the Persistence of the Great Depression: A General Equilibrium Analysis." *Journal of Political Economy* 112 (4): 779-816.

Cooley, Thomas F., and Vincenzo Quadrini. 2001. "Financial Markets and Firm

Dynamics." *American Economic Review* 91(5): 1286–1310.

Cooley, Thomas, Ramon Marimon, and Vincenzo Quadrini. 2004. "Aggregate Consequences of Limited Contract Enforceability." *Journal of Political Economy* 112(4): 817–847.

Davis, Steven, and John Haltiwanger. 1992. "Gross Job Creation, Gross Job Destruction, and Employment Reallocation." *Quarterly Journal of Economics* 107(3):819-863.

Eggertsson, Gauti B., and Paul Krugman. 2012. "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach." *Quarterly Journal of Economics* 127(3): 1469–1513.

Foster, Lucia, John Haltiwanger, and Chad Syverson. 2008. "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability? *American Economic Review* 98(1): 394–425.

Gertler, Mark and Nobuhiro Kiyotaki. 2011. "Financial Intermediation and Credit Policy in Business Cycle Analysis." In *Handbook of Monetary Economics*, ed. B. M. Friedman and M. Woodford, vol. 3A, chap. 11: 547–599. Amsterdam: North-Holland.

Gilchrist, Simon, Jae Sim, and Egon Zakrajsek. 2010. "Uncertainty, Financial Frictions, and Investment Dynamics." Working paper, Boston University.

Guerrieri, Veronica and Guido Lorenzoni. 2011. "Credit Crises, Precautionary Savings and the Liquidity Trap." NBER Working Paper 17583.

Hall, Robert E. 2005. "Employment Fluctuations with Equilibrium Wage Stickiness" American Economic Review 95 (1): 50–65.

Hartman, Richard. 1972. "The Effects of Price and Cost uncertainty on Investment." *Journal of Economic Theory* 5(2): 258–266.

Jensen, Michael C. 1986. "The Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers." *American Economic Review* 76(2): 323–330.

Jermann, Urban, and Vincenzo Quadrini. 2012. "Macroeconomic Effects of Financial Shocks." *American Economic Review* 102(1): 238–271.

Khan, Aubhik, and Julia K. Thomas. 2013. "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity." *Journal of Political Economy* 121(6): 1055–1107.

Kiyotaki, Nobuhiro, and John Moore. 1997. "Credit Cycles." *Journal of Political*

Economy 105(2): 211–248.

Kozlowski, Julian, Laura Veldkamp, and Venky Venkateswaran. 2016. "The Tail that Wags the Economy: Belief-Driven Business Cycles and Persistent Stagnation." Mimeo NYU.

Krusell, Per, and Anthony A. Smith, Jr. 1998. "Income and Wealth Heterogeneity in the Macroeconomy." *Journal of Political Economy* 106(5): 867-896.

Lee, Yoonsoo, and Toshihiko Mukoyama. 2012. "Entry, Exit, and Plant-level Dynamics over the Business Cycle." University of Virginia Working Paper.

Midrigan, Virgiliu, and Thomas Philippon. 2016. "Household leverage and the recession." Mimeo NYU.

Midrigan, Virgiliu, and Daniel Y. Xu. 2014. "Finance and Misallocation: Evidence from Plant-Level Data." *American Economic Review,* 104(2): 422–458.

Oi, Walter Y. 1961. "The Desirability of Price Instability under Perfect Competition." *Econometrica* 29(1): 58–64.

Perri, Fabrizio, and Vincenzo Quadrini. 2011. "International Recessions." NBER Working Paper 17201.

Rogerson, Richard and Johanna Wallenius. 2009. "Micro and Macro Elasticities in a Life Cycle Model with Taxes." *Journal of Economic Theory* 144(6): 2277–2292.

Quadrini, Vincenzo. 2011. "Financial Frictions in Macroeconomic Fluctuations." *Economic Quarterly* 97(3): 209–254. Federal Reserve Bank of Richmond.

Schaal, Edouard. 2014. "Uncertainty and Unemployment." *Econometrica.* Mimeo NYU.

Shimer, Robert. 2005. The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review* 95 (1): 25-49.

Table 1: Parameter Values

Parameters from Moment Matching		
Volatility levels	$\sigma_H = 0.12, \sigma_L = 0.09$	IQR mean and std
Volatility transition	$p_{HH} = 0.84, p_{LL} = 0.94$	IQR auto. and skewness
Revenue shock process	$\bar{\kappa} + \bar{w}_m = 0.005$, $\sigma_\kappa = 0.036$	Spread median and std
Agency friction	$\tilde{\lambda} = 0.079$	Leverage median
Entry	$z_e/\bar{z} = 0.64, \bar{\omega} = 1, \sigma_\omega = 2$	Productivity, labor, and leverage for new entrants
Assigned Parameters		
Persistent z	$\rho_z = 0.9$	Haltiwanger et al. (08)
Labor elasticity	$\nu = 0.5$	Rogerson and Wallenius (09)
Labor share	$\alpha = 0.7$	National accounts
Markup	$\eta/(\eta - 1) = 1.21$	Basu and Fernald (97)
Discount rate	$\beta = 0.99$	Annual interest rate 4%
Curvature	$\sigma = 2$	Standard business cycle models

Table 2: Moment-Matching Exercise

Moments		
	Data	Model
Mean IQR (%)	21	18
Std deviation IQR (%)	3.5	3.6
Autocorrelation IQR (%)	82	87
Skewness IQR	1.2	0.9
Median spread (%)	1.3	2.8
Std. median spread (%)	1.1	0.9
Median leverage (%)	26	29
Relative entrant:		
Productivity (%)	75	67
Labor (%)	60	50

Table 3: Firm Distributions

	Percentile		
	25	50	75
Data (%)			
Spread	1	1.3	2.1
Growth	-9	0	11
Leverage	9	26	62
Debt purchases	-10	0	21
Equity payouts	-4	0	12
Model (%)			
Spread	1.1	2.8	6.3
Growth	-7	0	9
Leverage	25	29	33
Debt purchases	-14	0	16
Equity payouts	-19	0	23

In the data and the model, for each variable and quarter, we calculate the 25th, 50th and 75th percentiles across firms. Then we report the median of each time series. Growth and dividends are reported relative to the median 50th percentile. Data are from Compustat. See variable definitions in the data appendix.

Table 4: Firm Correlations

Median Corr. with Leverage (%)		
	Data	Model
Spread	10	20
Growth	9	28
Debt purchases	45	59
Equity payouts	-5	13

For the data and the model, we compute for each firm the correlation between its spread, growth, debt change, and dividends with its leverage across time. We report the median correlation across firms. Data are from Compustat. See variable definitions in the data appendix.

Table 5: Business Cycles

	Data	Model
St. Deviations (%)		
Output	1.13	0.97
Labor (rel output)	1.26	1.31
IQR	3.50	3.62
Spread	1.10	0.91
Debt purchases/output	2.51	2.83
Equity payouts/output	1.76	2.74
Corr. with Output (%)		
Labor	81	94
IQR	-27	-45
Spread	-55	-33
Debt purchases/output	75	21
Equity payouts/output	45	18

Data for output and labor are from NIPA, data for IQR (interquantile range of sales growth) and spread are from Compustat and Moody's, and data for debt change and dividends are from the flow of funds. Output and labor are logged and filtered with a Hodrick-Prescott filter with a smoothing parameter equal to 1600. See details of the variable definitions in the data appendix.

Figure 1: Bond Spread Schedule

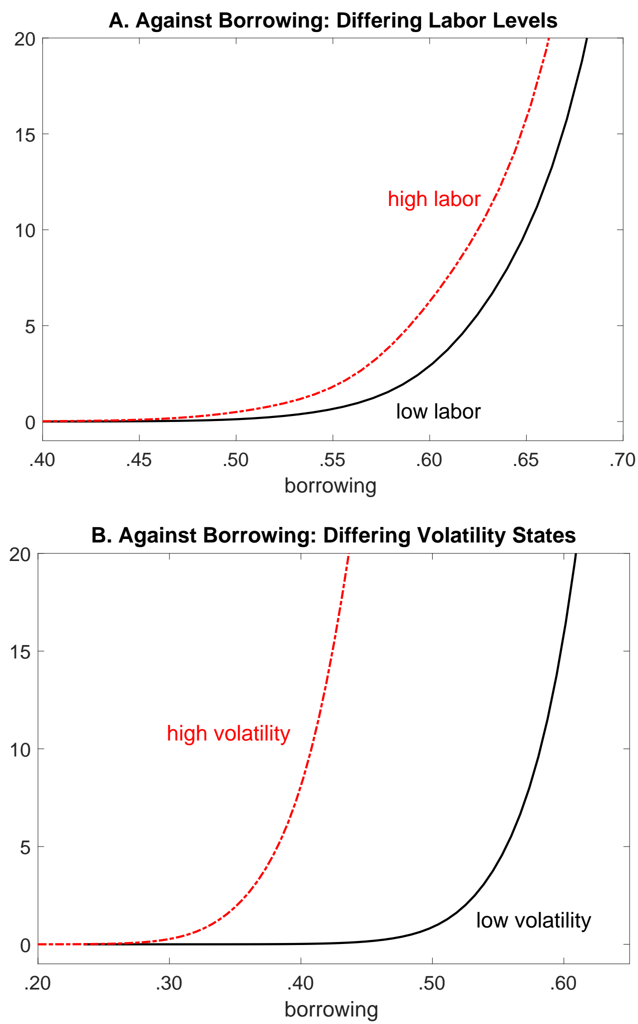

Figure 2: Firm Decision Rules

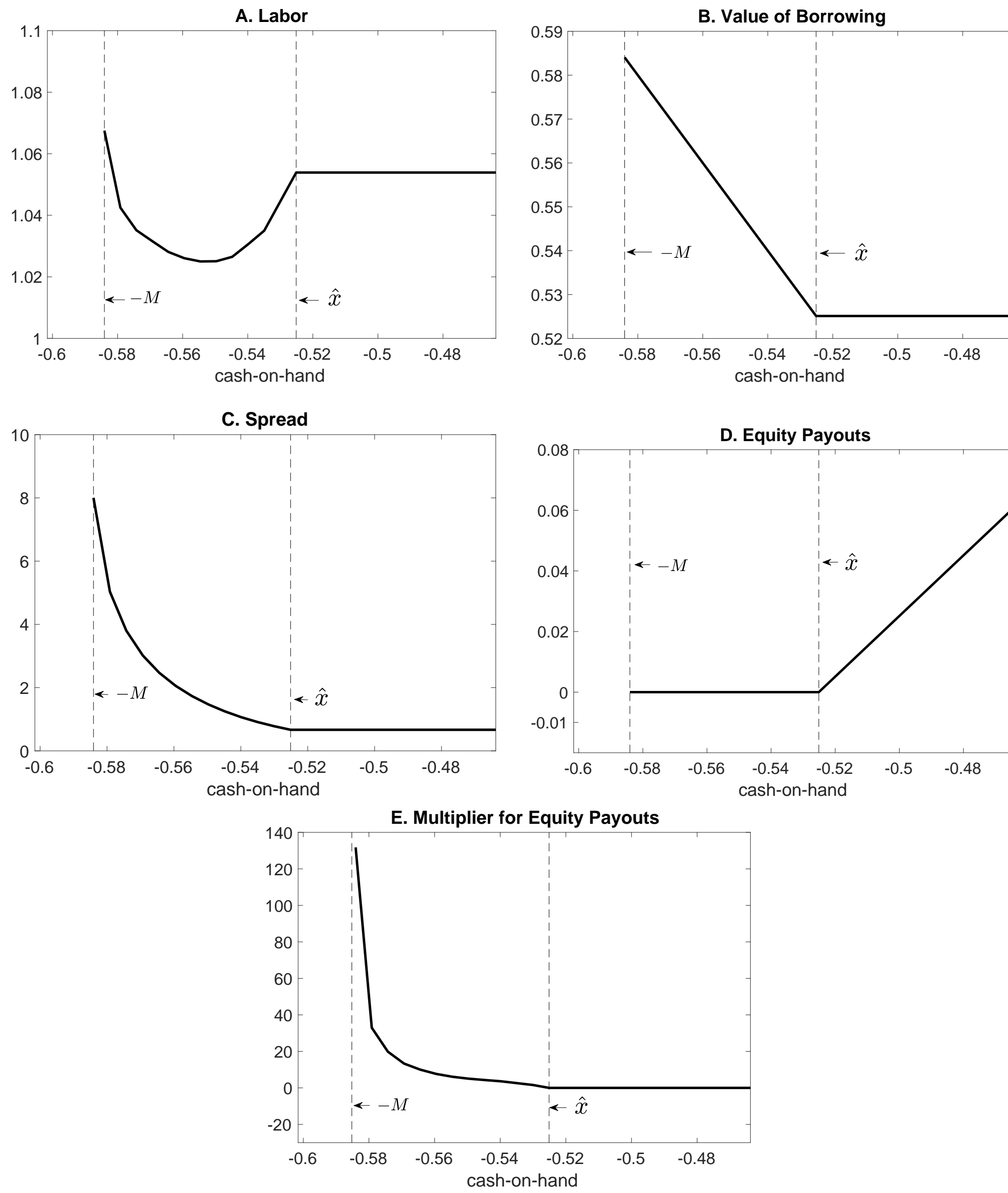

Figure 3: Firm Decision Rules

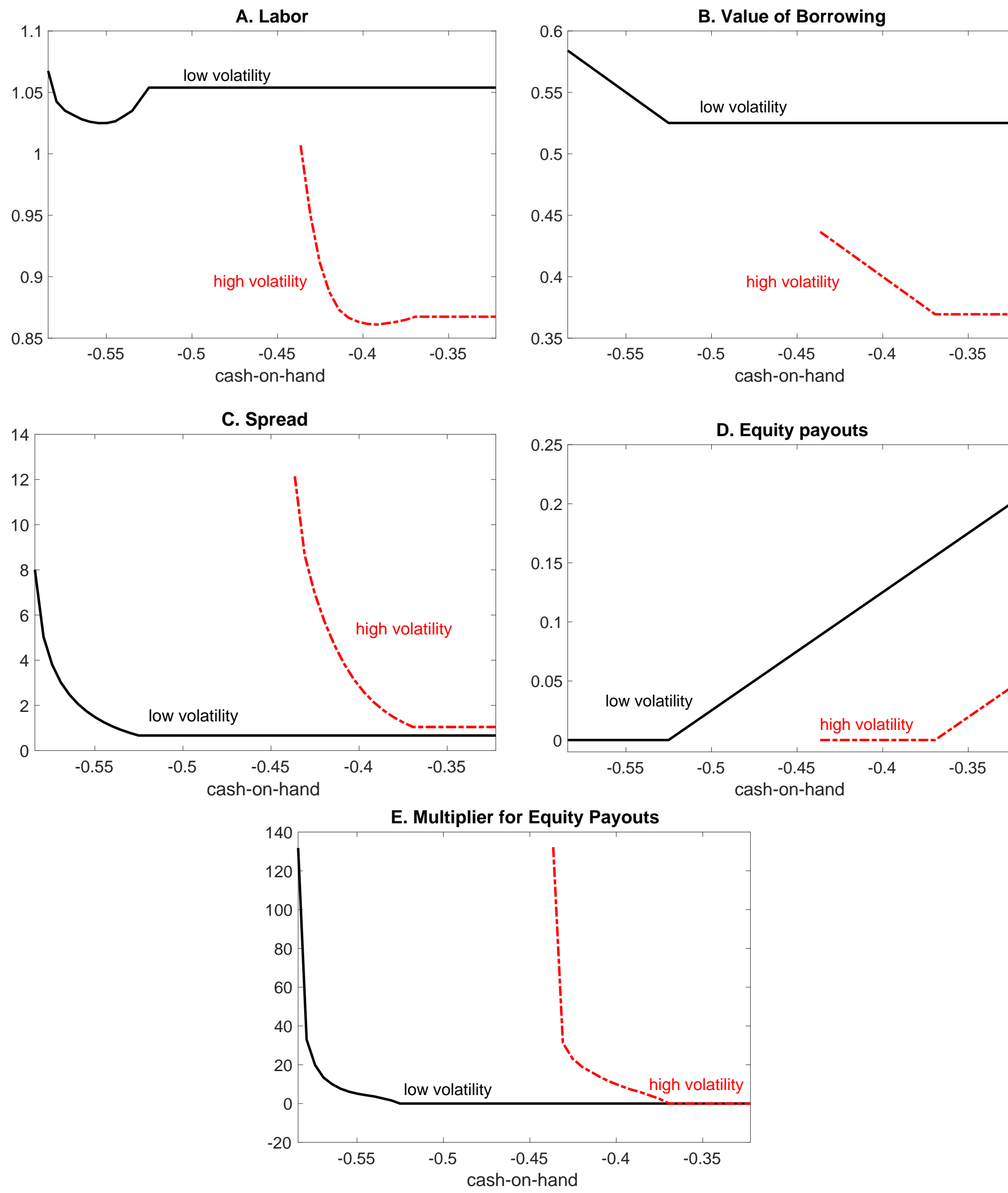

Figure 4: Firm Labor Impulse Responses

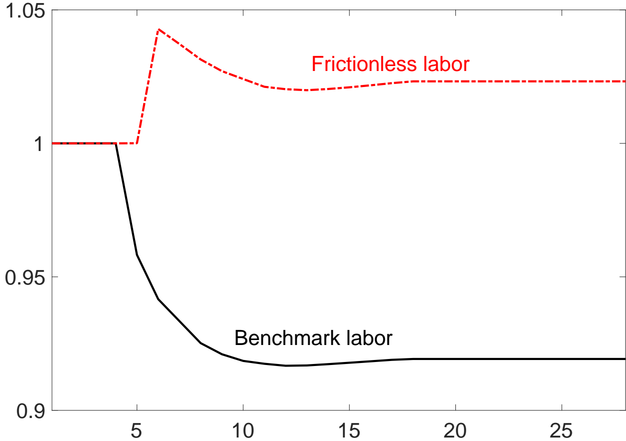

Figure 5: Great Recession Event

Figure 6: Great Recession Event, Lower Labor Elasticity

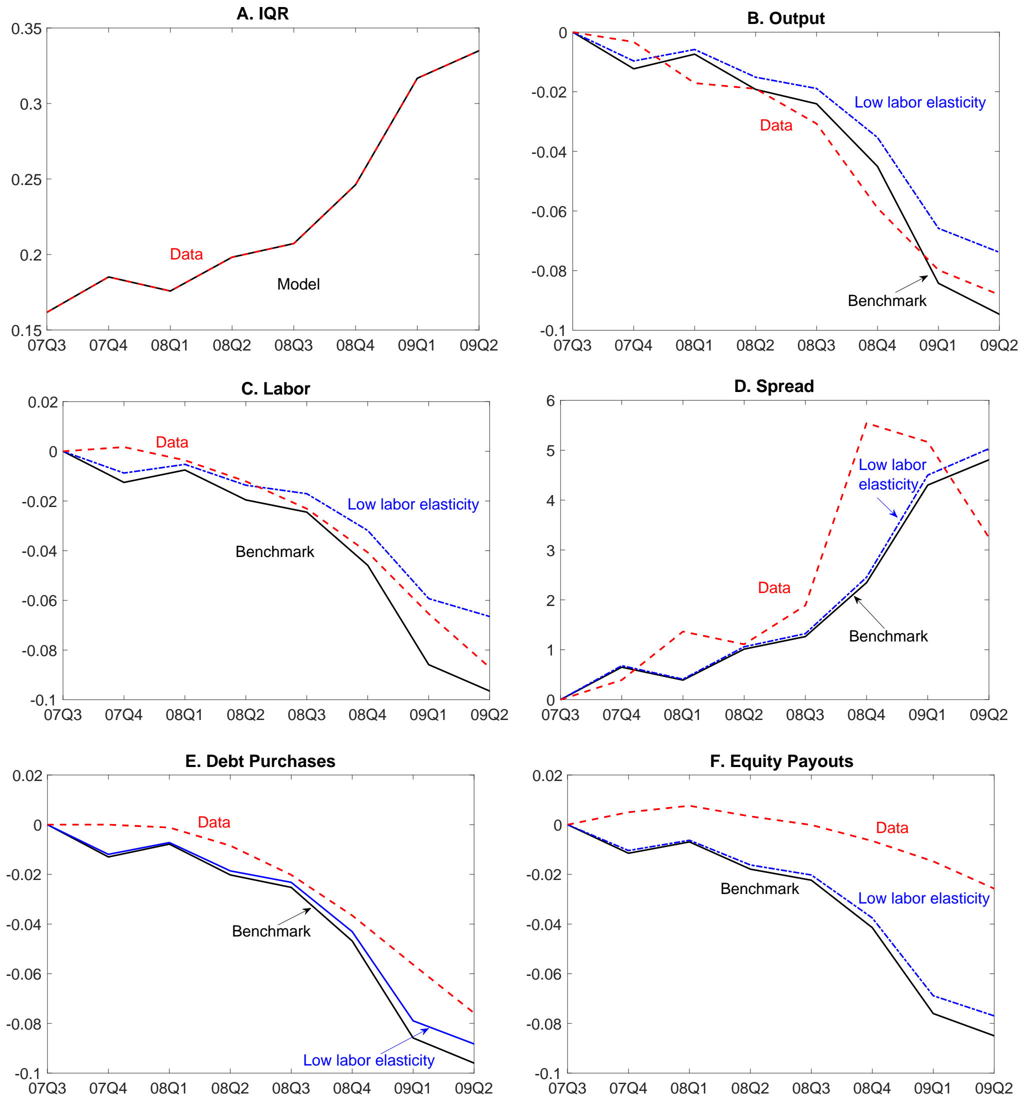

Figure 7: Great Recession Event, Frictionless Financial Markets