

NBER WORKING PAPER SERIES

FINANCIAL INTEGRATION AND LIQUIDITY CRISES

Fabio Castiglionesi  
Fabio Feriozzi  
Guido Lorenzoni

Working Paper 23359  
<http://www.nber.org/papers/w23359>

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
April 2017

We thank Fernando Broner, Emmanuel Farhi, Andreas Pfingsten, Chen Zhou and seminar participants at University of Maryland, NY Fed, Philadelphia Fed, Bundesbank (Frankfurt), Dutch Central Bank, Instituto de Empresa (Madrid), Université de Montreal, Cass Business School (London), Warwick Business School, Lancaster Business School, European University Institute (Florence), IHS (Vienna), LUISS (Rome), Tor Vergata (Rome), Singapore Management University, National University of Singapore, University of New South Wales, EFA (Frankfurt), SED (Istanbul), ESEM (Barcelona), 10th World Congress Econometric Society (Shanghai), 11th Euro-Area Business Cycle Network Workshop (Amsterdam) and 3rd RBA Workshop (Sydney) for useful comments. Fabio Feriozzi gratefully acknowledges the financial support of the Spanish Ministry of Economy and Competitiveness under grant no. ECO2014-59491-P. The usual disclaimer applies. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

NBER working papers are circulated for discussion and comment purposes. They have not been peer-reviewed or been subject to the review by the NBER Board of Directors that accompanies official NBER publications.

© 2017 by Fabio Castiglionesi, Fabio Feriozzi, and Guido Lorenzoni. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Financial Integration and Liquidity Crises  
Fabio Castiglionesi, Fabio Feriozzi, and Guido Lorenzoni  
NBER Working Paper No. 23359  
April 2017  
JEL No. F36,G15,G21

**ABSTRACT**

The paper analyzes the effects of financial integration on the stability of the banking system. Financial integration allows banks in different regions to smooth local liquidity shocks by borrowing and lending on a world interbank market. We show under which conditions financial integration induces banks to reduce their liquidity holdings and to shift their portfolios towards more profitable but less liquid investments. Integration helps reallocate liquidity when different banks are hit by uncorrelated shocks. However, when a correlated (systemic) shock hits, the total liquid resources in the banking system are lower than in autarky. Therefore, financial integration leads to more stable interbank interest rates in normal times, but to larger interest rate spikes in crises. These results hold in a setup where financial integration is welfare improving from an ex ante point of view. We also look at the model's implications for financial regulation and show that, in a second-best world, financial integration can increase the welfare benefits of liquidity requirements.

Fabio Castiglionesi  
Tilburg University  
Finance Department  
fabio.castiglionesi@uvt.nl

Fabio Feriozzi  
IE Business School  
IE University  
Madrid  
Fabio.Feriozzi@ie.edu

Guido Lorenzoni  
Department of Economics  
Northwestern University  
2211 Campus Dr  
Evanston, IL 60208  
and NBER  
guido.lorenzoni@northwestern.edu

# 1 Introduction

Financial integration allows banks located in different countries to smooth local liquidity shocks by borrowing and lending on the world interbank market. Everything else equal, this should have a stabilizing effect on financial markets, thanks to the additional sources of short-term funds banks can rely on to cover a liquidity shortage. That is, financial integration should help dampen the effects of local liquidity shocks. However, an easier access to liquid resources on the interbank market changes the ex ante incentives of banks when they make their lending and portfolio decisions. In particular, banks that can readily obtain short-term funds on the market may choose to hold lower reserves of safe, liquid assets and choose less liquid and/or more risky investment strategies. Once this endogenous response is taken into account, the equilibrium effects of financial integration on financial stability are less clear. If a correlated shock hits all banks—a “systemic” event—the lower holdings of liquid reserves in the banking system can lead to a larger increase in interbank rates. In this paper, we explore these effects showing that financial integration can lead to lower holdings of liquid assets, to more severe crises and, in some cases, to increased volatility in interbank markets. Moreover, we show that all these effects can take place in an environment where financial integration is welfare improving from an ex ante point of view.

Prior to the onset of the 2007/2008 financial crisis, Larry Summers remarked that “changes in the structure of financial markets have enhanced their ability to handle risk in normal times” but that “some of the same innovations that contribute to risk spreading in normal times can become sources of instability following shocks to the system.”<sup>1</sup> Our model focuses on a specific form of structural change in financial markets—financial integration in interbank markets—and formalizes the contrasting effects of this structural change in normal times and in crises.

A crucial step of our analysis is to understand the effects of integration on the banks’ investment decisions ex ante and, in particular, on the response of equilibrium liquidity holdings. The direction of this response is non obvious because two forces are at work. On the one hand, more integrated banks have more opportunities to borrow if they are hit by a liquidity shock. This lowers their incentives to hold reserves of liquid assets. On the other hand, more integrated banks also have more opportunities to lend their excess liquidity when they do not need it. This increases their incentives to hold liquid reserves. In our model we capture these two forces and show that under reasonable parameter restrictions the first force dominates and financial integration leads to lower holdings of liquid assets.

We consider an economy with two ex ante identical regions. In each region banks offer state contingent deposit contracts to consumers, they invest the consumers’ savings and allocate funds to them when they are hit by liquidity shocks à la Diamond and Dybvig (1983). Banks can invest in two assets: a liquid short-term asset and an illiquid long-term asset. When the two regions are

---

<sup>1</sup>*Financial Times*, December 26, 2006.

hit by different liquidity shocks there are gains from trade from sharing liquid resources through the world interbank market. However, when the two regions are both hit by a high liquidity shock, there is an economy-wide liquidity shortage and the presence of the world interbank market is of little help. We consider different configurations of regional liquidity shocks allowing for various degrees of correlation between regional shocks. When the correlation between regional shocks is higher (i.e., the probability of the world-wide shocks is higher) there is more aggregate uncertainty and the gains from integration are lower.

We analyze the optimal investment decision of banks in autarky and under financial integration and show that, under some conditions, banks invest a smaller fraction of their portfolio in liquid assets under integration. Moreover, we show that this effect is stronger when there is less aggregate uncertainty. In this case, it is more likely that banks are hit by different shocks, there is better scope for coinsurance, and the ex ante incentive to hold liquid assets is lower.

We then look at the implications for the equilibrium distribution of interest rates in the interbank market, comparing the equilibrium under integration and in autarky. When the two regions are hit by different shocks financial integration tends to reduce interest rates in the region hit by the high liquidity shock. This is the stabilizing effect of financial integration. However, financial integration makes things worse in the state of the world where both regions are hit by a high liquidity shock. In this case, since banks are holding overall lower liquid reserves, the economy-wide liquidity shortage is more severe and there is a spike in interest rates. Therefore, financial integration tends to make the distribution of interest rate more skewed, with low and stable interest rates in normal times, in which regional shocks offset each other, and occasional spikes when an economy-wide shock hits. If the probability of an economy-wide shock is small enough the overall effect of integration is to reduce interest rate volatility. However, if there is a sufficient amount of aggregate uncertainty interest rate volatility can increase as a consequence of financial integration.

We also look at the model implications for the equilibrium distribution of consumption, showing the real implications of financial integration are similar to the implications for the interest rate: the distribution of consumption tends to become more skewed after integration and can display higher volatility.

We conduct our exercise in the context of a model with minimal frictions, where banks allocate liquidity efficiently by offering fully state contingent deposit contracts. In this setup equilibria are Pareto efficient and the increased market volatility that can follow financial integration is not necessarily a symptom of inefficiency. In fact, in our model financial integration is always welfare improving. Although this result clearly follows from the absence of frictions in the model, it points to a more general observation: the negative effects of integration on volatility should not be taken as unequivocal evidence that integration is undesirable ex ante.

To analyze the implications of our mechanism for regulation, we also consider a variant of our model where banks borrow and lend from each other on an ex post spot market instead of writing

state-contingent credit lines *ex ante*. In this case, banks typically hold an inefficient amount of liquid resources as they essentially free ride on each other's liquidity holdings. In this context, we show that financial integration can make the free-riding problem worse and that the welfare benefits of regulation can be larger under financial integration than under autarky.

## 1.1 Related literature

There is a large literature on the role of interbank markets as a channel for sharing liquidity risk among banks. In particular, our paper is related to Bhattacharya and Gale (1987), Allen and Gale (2000), and Freixas, Parigi and Rochet (2000), who analyze the functioning of the interbank market in models where banks act as liquidity providers à la Diamond and Dybvig (1983).

Bhattacharya and Gale (1987) consider banks that are subject to idiosyncratic and privately observed liquidity shocks. Banks coinsure each other against such shocks through an *ex post* borrowing and lending market. They show that banks free-ride on market liquidity and end up holding an inefficiently low amount of liquid resources in their portfolios. Differently from Bhattacharya and Gale (1987), we allow for a systematic component in liquidity shocks and assume that banks can (partially) coinsure each other through an *ex ante* interbank market. As a consequence, banks' liquidity holdings are always efficient in our model, and we show that a reduction in the systematic component of the liquidity uncertainty (e.g., through financial integration) can reduce the efficient level of liquidity, and induce higher volatility in the economy.

Allen and Gale (2000) and Freixas, Parigi and Rochet (2000) are concerned with the fact that interbank linkages can act as a source of contagion, generating chains of bank liquidations. In this paper instead we focus on how different degrees of interbank market integration affect *ex ante* investment decisions. For this reason, we simplify the analysis and rule out bank runs and liquidations by allowing for fully state contingent deposit contracts.

Other papers have used a setup where the scope for coinsurance is limited. Allen and Gale (1998) show that in the presence of aggregate uncertainty on long-term returns, bank runs can be a way of implementing optimal risk sharing when banks are restricted to offer non-contingent deposit contracts. Differently from them, we focus on aggregate liquidity uncertainty and study what happens to interbank rates and consumption volatility when the degree of uncertainty is reduced. Allen and Carletti (2006) show how financial innovation, in the form of new credit risk transfer instruments, can improve risk sharing in some circumstances but can lead to contagion and create welfare losses in others. Castiglionesi et al. (2014) extend the setup used in this paper by introducing bank capital as an additional tool for handling the liquidity risk. Consistently with the model predictions, they document in a large sample of US banks that bank capital is negatively associated with interbank activity. We instead focus on the effect of banks' liquidity choices on both consumption and interest rate volatility.

A paper which also emphasizes the potentially destabilizing effects of integration is Freixas and Holthausen (2005), who point out that integration may magnify the asymmetry of information,

as banks start trading with a pool of foreign banks on which they have less precise information. Here we abstract from informational frictions in interbank markets, either in the form of asymmetric information (as, e.g., in Rochet and Tirole, 1996) or moral hazard (as, e.g., in Brusco and Castiglionesi, 2007). Our paper is also related to Holmstrom and Tirole (1998), who emphasize the different role of aggregate and idiosyncratic uncertainty in the optimal allocation of liquidity.

The literature has emphasized a number of potential inefficiencies generating excessive illiquidity during systemic crises. In Wagner (2008) interbank lending may break down due to moral hazard. Allen, Carletti and Gale (2009) show that inefficiencies in the interbank market arise because interest rates fluctuate too much in response to shocks, precluding efficient risk sharing. Acharya, Shin and Yorulmazer (2011) analyze the optimal (private) choice of banks liquidity holding when banks do not use interbank markets to coinsure each other but sell their assets to raise liquidity. If assets are sold at fire-sale prices in a crisis, banks hold inefficiently low liquidity during booms and excessively high liquidity during crises. In Acharya, Gromb and Yorulmazer (2012) inefficiencies in interbank lending arise due to monopoly power. Castiglionesi and Wagner (2013) show how inefficient liquidity provision among banks is due to the presence of non exclusive contracts. This paper emphasizes the fact that the instability associated to integration can also be the product of an efficient response of banks' investment decisions. In Section 6 we consider one potential source of inefficiency—the presence of spot markets—and we analyze the optimal regulatory response to integration. Our approach to optimal regulation builds on Lorenzoni (2008), Allen and Gale (2004) and Farhi, Golosov and Tsivinsky (2009). Bengui (2014) studies the implications for international policy coordination of cross-border liquidity externalities similar to those explored in this paper.

Caballero and Krishnamurthy (2009) have emphasized that the higher demand for liquid stores of values by emerging economies is an endemic source of financial instability. This demand pressure stretches the ability of financial institutions in developed countries to transform illiquid assets into liquid liabilities, pushing them to hold larger holdings of risky assets. Here we emphasize a different but complementary channel by emphasizing the endogenous illiquidity generated by the increased access to international interbank markets.

The remainder of the paper is organized as follows. In the rest of the introduction, we lay out the empirical motivation for our theoretical work. Section 2 presents the model. In Section 3 we characterize the equilibrium in autarky and under financial integration. Section 4 contains our main results on the effects of integration on liquid asset holdings. Section 5 analyzes the consequences of integration on the depth of systemic crises, both in terms of interbank market interest rates and in terms of consumption. Section 6 analyzes liquidity coinsurance with spot markets and its implications for regulation under financial integration. Section 7 concludes. All the proofs are in the Appendix.

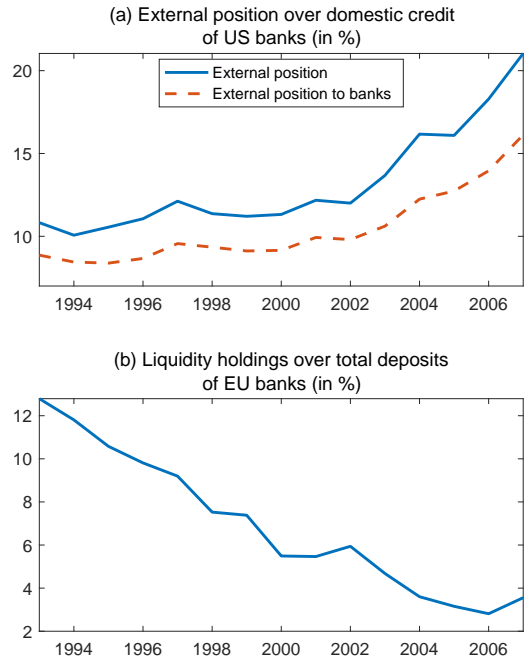


Figure 1: Integration and liquidity in the US.

## 1.2 Some motivating facts

To motivate our analysis, we begin by briefly documenting the increase in financial market integration and the contemporaneous reduction in the holding of liquid assets in the banking system, in the run-up to the global financial crisis of 2007-2008.

The years preceding the crisis witnessed a dramatic increase in the international integration of the banking system. Panel (a) in Figure 1 documents the increase in the cross-border activities of US banks between 1993 and 2007. To take into account the overall growth of the banking activities, we look at the ratio of the external positions of US banks towards all foreign financial institutions to total domestic credit.<sup>2</sup>

This ratio goes from 11% to 21% between 1993 and 2007. If we restrict attention to the external position of the US banks towards foreign banks, the same ratio goes from 8% to 16%. In panel (b) of Figure 1, we look at the holdings of liquid assets by US banks over the same time period. In particular, we look at the ratio of liquid assets to total deposits.<sup>3</sup> This ratio

<sup>2</sup>The external position of US and Euro area banks is measured as banks cross-border total assets, which are retrieved from the BIS locational data. Domestic credit, liquidity holdings (reserves and claims on central government) and total deposits (demand deposits, time and savings deposits, money market instruments and central government deposits) are from the International Financial Statistics of the IMF.

<sup>3</sup>Following Freedman and Click [17] we look at the “liquidity ratio” given by liquid assets over total deposits. The numerator is given by the sum of reserves and claims on central government from the IMF International

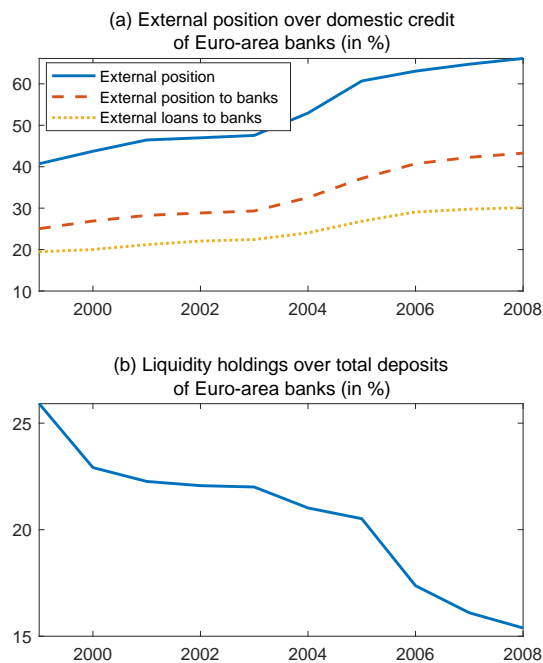


Figure 2: Integration and liquidity in the Euro area.

decreased from 13% in 1993 to 3.5% in 2007, and its correlation with the external position of US banks towards all foreign financial institutions is -0.77 during this period (with a p-value well below 1%). The US banking system clearly displayed a combination of increased integration and increased illiquidity.

A similar pattern arises if we look at banks in the Euro area. In panel (a) of Figure 2 we plot the ratio of the total external position of Euro-area banks to domestic credit and in panel (b) we plot their liquidity ratio.<sup>4</sup> The external position goes from 40% of domestic credit to 66% between 1999 and 2008. Restricting attention to the external position towards Euro-area borrowers (i.e., banks located in a Euro-area country different than the originating bank), we observe an increase from 25% until 43% in the same period. Finally, if we restrict attention to external loans to foreign banks, we see an increase from 19% to 30%. At the same time, also in the Euro area we see a reduction in liquidity holdings, with a liquidity ratio going from 26% in 1999 to 15% in 2008. The correlation between banks liquidity ratio and their total external position in the Euro area during the period 1999-2008 is -0.95 and highly significant.

Figure 3 shows similar results for Germany alone and allows us to go back to 1993. Notice that the level of financial integration and the liquidity ratio of the German banking system were

---

Financial Statistics. The denominator is the sum of demand deposits, time and savings deposits, money market instruments and central government deposits, also from the IMF International Financial Statistics.

<sup>4</sup>The construction of the ratios and the data sources are the same as for Figure 1.



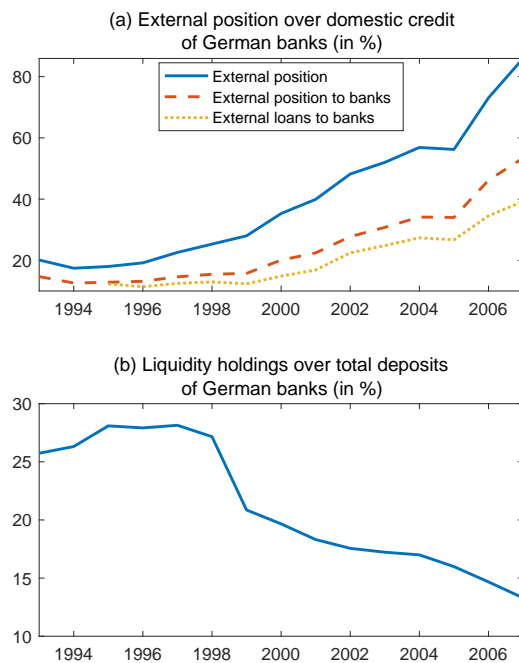


Figure 3: Integration and liquidity in Germany.

relatively stable until 1998. The increase in financial integration and the concomitant decrease in the liquidity ratio only start in 1999. The correlation of the two series between 1993 and 2007 is however -0.92 and highly significant.

It is well known that these trends in financial integration have been accompanied by ambiguous changes in the stability of interbank markets. Liquidity premia in interbank markets can be measured in terms of spreads (e.g., the spread between the LIBOR rate and the Overnight Indexed Swap rate, or the spread between the LIBOR and the secured interbank government REPOs). These spreads have been unusually low and stable in the period preceding the financial crisis. However, since the onset of the crisis in the summer of 2007 these spreads have become extremely volatile, reflecting a protracted illiquidity problem in international interbank markets.

Summing up, the time series evidence suggests that there has been a considerable increase in world financial integration in the years before the crisis and that this increase has been associated to a decline in liquidity ratios, to low and stable interbank rates before the crisis and to very volatile rates in the crisis. These are the basic observations that motivate our model. In particular, we uncover a possible theoretical reason behind the observed negative relationship between financial integration and banks' liquidity holding, and we analyze the consequences if this negative relationship on the depth of systemic crises (in term of spikes in interest rates and drop of consumption).

It is also useful to mention some cross sectional evidence showing that banks in emerging

economies, which are typically less integrated with the rest of the world, tend to hold larger liquid reserves. Freedman and Click [17] show that banks in developing countries keep a very large fraction of their deposits in liquid assets. The average liquidity ratio for developing countries is 45% against an average ratio of 19% for developed countries.<sup>5</sup> This evidence is at least consistent with the view that financial integration may affect banks' liquidity ratios. Another type of cross sectional evidence which is interesting for our exercise is in Ranciere, Tornell and Westerman [22], who show that countries affected by large financial crises display both higher variance and larger negative skewness in credit growth than countries characterized by a more stable financial system. Since the former group of countries are also more open to international capital flows, this evidence provides some support to our view that important implications of financial integration are to be observed in both the variance *and* the skewness of real aggregates.

## 2 The model

In this section, we present a simple model of risk sharing among banks located in different regions. The model is a two-region version of Diamond and Dybvig (1983) and is similar to Allen and Gale (2000), but we allow for fully contingent deposit contracts.

Consider an economy with three dates,  $t = 0, 1, 2$ , and a single consumption good that serves as the numeraire. There are two assets, both in perfectly elastic supply. The first, called the “liquid asset”, yields one unit of consumption at date  $t + 1$  for each unit of consumption invested at date  $t$ , for  $t = 0, 1$ . The second, called the “illiquid asset”, yields  $R > 1$  units of consumption at date 2 for each unit of consumption invested at date 0.

There are two ex ante identical regions,  $A$  and  $B$ . Each region contains a continuum of ex ante identical consumers with an endowment of one unit of consumption good at date 0. In period 1, agents are hit by a preference shock  $\theta \in \{0, 1\}$  which determines whether they like to consume in period 1 or in period 2. Their preferences are represented by the expected utility function

$$E[\theta u(c_1) + (1 - \theta)u(c_2)],$$

where  $u(\cdot)$  is continuously differentiable, increasing and strictly concave and satisfies the Inada condition  $\lim_{c \rightarrow 0} u'(c) = \infty$ . We will call *early* and *late* consumers, respectively, the consumers hit by the shocks  $\theta = 1$  and  $\theta = 0$ .

The uncertainty about preference shocks is resolved in period 1 as follows. First, an aggregate shock  $s$  is realized that determines the regional liquidity shocks  $\omega^A(s)$  and  $\omega^B(s)$ , which are the fractions of early consumers, respectively, in regions  $A$  and  $B$ . Then, preference shocks are randomly assigned to the consumers in each region. The preference shocks are privately observed by the consumers, while the aggregate state  $s$  is publicly observed.

---

<sup>5</sup>Clearly, there can be other reasons behind this difference, besides international integration. Acharya, Shin and Yorulmazer (2011) emphasize that banks in poor legal and regulatory environments may find difficult to raise liquidity against future profits (or external finance) and thus hoard greater amount of liquidity.

For most of the paper we assume a binary, symmetric distribution of regional liquidity shocks, with two possible realizations,  $\omega_H$  and  $\omega_L$ , where  $\omega_H > \omega_L$ . To allow for various degrees of correlation between regions, we assume that with probability  $p$  the two regions receive different shocks and with probability  $1 - p$  they receive the same shock. Namely, there are four possible states of the world  $s \in S = \{HH, LH, HL, LL\}$ , where the first letter denotes the shock in region  $A$  and the second the shock in region  $B$ . The probabilities of each state, denoted by  $\pi(s)$ , are in Table 1.

Table 1: Regional liquidity shocks

State $s$	$\omega^A(s)$	$\omega^B(s)$	$\pi(s)$
$HH$	$\omega_H$	$\omega_H$	$(1 - p) / 2$
$LH$	$\omega_L$	$\omega_H$	$p / 2$
$HL$	$\omega_H$	$\omega_L$	$p / 2$
$LL$	$\omega_L$	$\omega_L$	$(1 - p) / 2$

A higher value of the parameter  $p$  implies a lower correlation between regional shocks and more scope for interregional risk sharing. When  $p = 1/2$  the shocks are independent. It will be convenient to denote the average value of possible liquidity shocks as

$$\omega_M \equiv (\omega_H + \omega_L) / 2.$$

While most of the analysis focuses on this simple shock structure, some of our results hold more generally for any discrete and symmetric joint distribution of regional shocks. The robustness of other results will be explored numerically under richer shock structures in Online Appendix C.

In each region  $i = A, B$ , there is a competitive banking sector. Banks offer fully state contingent deposit contracts: at time 0, each consumer transfers his initial endowment to a regional bank, which invests  $y^i$  in the liquid asset and  $1 - y^i$  in the illiquid asset; at time 1, after the aggregate shocks  $s$  is publicly observed, the consumer reveals his preference shock to the regional bank and receives the consumption vector  $(c_1^i(s), 0)$  if he is an early consumer and the consumption vector  $(0, c_2^i(s))$  if he is a late consumer. Therefore, a deposit contract in region  $i$  is fully described by the array

$$\{y^i, (c_1^i(s))_{s \in S}, (c_2^i(s))_{s \in S}\}.$$

To simplify the notation, in what follows we will drop the regional index  $i$  whenever no confusion is possible. We will also often refer to a representative bank in region  $i$  simply as bank  $i$ .

### 3 Equilibrium

In this section we characterize the equilibrium of the model under two alternative assumptions on financial openness. First, we look at autarky, in which banks located in different regions are

not allowed to trade. Next, we look at a regime of financial integration, in which banks can trade state contingent claims.

### 3.1 Autarky

Consider first autarky. Take a representative bank in region  $A$ . Given competition in the banking sector, the representative bank chooses a contract that maximizes the expected utility of the consumers in the region. Proceeding backwards, we will first analyze the problem of allocating optimally a certain level of liquidity  $y$  after a realization of the regional shock  $\omega^A(s)$ , and then look at the ex ante problem of choosing  $y$  optimally.

Let us first define the value function  $V(y, \omega)$  that gives the expected utility of the consumers serviced by a bank, when there are  $\omega$  early consumers and  $y$  units of the liquid asset available:<sup>6</sup>

$$V(y, \omega) \equiv \max_{c_1, c_2} \quad \omega u(c_1) + (1 - \omega) u(c_2) \quad (1)$$

$$s.t. \quad \omega c_1 \leq y,$$

$$(1 - \omega) c_2 \leq R(1 - y) + (y - \omega c_1).$$

The first constraint is a liquidity constraint which requires total payments to early consumers to be covered by the returns of the liquid asset. If this constraint is slack, the residual funds  $y - \omega c_1 > 0$  are reinvested in the liquid asset in period 1. When the liquidity constraint is slack, we say that there is *positive rollover*. The second constraint requires total payments to late consumers to be covered by the returns of the illiquid asset plus the returns of the liquid investment made in period 1.<sup>7</sup>

The following lemma summarizes some useful properties of  $V$ .

**Lemma 1** *The value function  $V(y, \omega)$  is continuous, differentiable and strictly concave in  $y$  and  $\partial V(y, \omega) / \partial y$  is non-decreasing in  $\omega$ .*

Turning to the choice of  $y$  ex ante, it is given by the solution to the problem:

$$\max_y \sum_{s \in S} \pi(s) V(y, \omega^A(s)). \quad (2)$$

The next proposition characterizes the autarky allocation in region  $A$ . Not surprisingly, given the lack of insurance opportunities across regions, the optimal allocation only depends on the regional liquidity shock in region  $A$ .

---

<sup>6</sup>It is convenient to define the value function using generic notation for the control variables  $c_1, c_2$  and for the shock  $\omega$ . The regional indexes  $A$  and  $B$  and the dependence on the state  $s$  will be reintroduced later to describe equilibrium allocations.

<sup>7</sup>Although consumers' preference shocks are private information, it is easy to show that the consumers' incentive compatibility constraints are satisfied at the optimum.

**Proposition 1** *The optimal allocation under autarky in region A satisfies*

$$c_1^A(HH) = c_1^A(HL) < c_1^A(LH) = c_1^A(LL) \leq c_2^A(LH) = c_2^A(LL) < c_2^A(HH) = c_2^A(HL).$$

*It is never optimal to have positive rollover in states HH and HL. If positive rollover occurs in states LH and LL then  $c_1^A(LH) = c_1^A(LL) = c_2^A(LH) = c_2^A(LL)$ .*

The fact that it is never optimal to have positive rollover when the regional shock is high is intuitive. If there is positive rollover with the high liquidity shock, then there must also be positive rollover with the low liquidity shock. But then some of the funds invested in the short asset at date 0 will be rolled over with certainty, yielding a return of 1 in period 2, while it would be more profitable to invest them in the illiquid asset which yields  $R > 1$ . On the other hand, if  $\omega_L$  is sufficiently low, it may be optimal not to exhaust all the liquid resources to pay early consumers. In this case, the optimal allocation of funds between periods 1 and 2 requires that the marginal utility of early and late consumers be equalized, which in turn requires consumption to be constant over time.

Proposition 1 shows that in autarky there is uncertainty about the level of consumption at time  $t$  and, in particular,

$$u'(c_1^A(HL)) > u'(c_1^A(LH)) \quad \text{and} \quad u'(c_2^A(HL)) < u'(c_2^A(LH)).$$

Given the symmetry of the problem,  $c_1^A(LH) = c_1^B(HL)$  and  $c_2^A(LH) = c_1^B(HL)$ . These inequalities imply that it would be welfare improving to reallocate resources from the region hit by the low liquidity shock to the region hit by the high liquidity shock in period 1, and reallocate resources in the opposite direction in period 2. Financial integration allows this mutually beneficial trade to take place.

### 3.2 Integration

We now turn to the case of financial integration, in which banks located in different regions can insure against regional liquidity shocks by trading contingent credit lines. In particular, under financial integration we have a banking system where:

1. each regional bank offers deposit contracts to the consumers in its own region;
2. regional banks offer each other *contingent credit lines* of the following form: if the two regions are hit by different shocks, the bank hit by the high liquidity shock can receive resources from the bank hit by the low liquidity shock in period 1 and repay in period 2.

Notice that this mechanism does not eliminate aggregate uncertainty: it is possible to co-insure in states  $LH$  and  $HL$ , but in states  $HH$  and  $LL$  co-insurance is not possible.

A contingent credit line for bank  $A$  is denoted by the pair  $(m_1^A(s), m_2^A(s))$ : the bank receives  $m_1^A(s)$  in period 1 and repays  $m_2^A(s)$  in period 2.<sup>8</sup> So, in the presence of contingent credit lines, the liquidity constraint at date 1 and the budget constraint at date 2 for bank  $A$  take the following form:

$$\omega^A(s)c_1^A(s) \leq y + m_1^A(s),$$

and

$$(1 - \omega^A(s))c_2^A(s) \leq Rx + (y + m_1^A(s) - \omega^A(s)c_1^A(s)) - m_2^A(s)$$

where  $x$  denotes the investment in the illiquid asset at  $t = 0$ . Letting  $Q_1(s)$  and  $Q_2(s)$  denote the prices of Arrow securities in state  $s$ , the budget constraint at date 0 for bank  $A$  takes the form

$$x + y + \sum_{s \in S} (Q_1(s)m_1^A(s) - Q_2(s)m_2^A(s)) \leq 1.$$

Notice that the interest rate implicit in the credit line is given by the ratio  $Q_1(s)/Q_2(s)$ .

Under financial integration, the representative bank in region  $A$  chooses the levels of investment  $x$  and  $y$ , the state contingent consumption allocations  $c_1^A(s)$  and  $c_2^A(s)$ , and the state contingent credit lines  $m_1^A(s)$  and  $m_2^A(s)$ , to maximize the expected utility of the consumers subject to the constraints above. The representative bank in region  $B$  solves an analogous problem. Market clearing in financial markets requires the credit line sold by bank  $A$  in state  $s$  to be equal to the credit line purchased by bank  $B$  in the same state, that is,

$$m_t^A(s) + m_t^B(s) = 0 \quad \text{for all } s \in S \text{ and } t = 1, 2.$$

Given the symmetry of the environment, from now on we focus on characterizing the symmetric equilibrium of the integrated economy (which can be shown to exist and be unique). The next lemma shows that the two-region economy under financial integration behaves as a one-region economy in which the regional shocks  $\omega^A(s)$  and  $\omega^B(s)$  are replaced by the average shock.

**Lemma 2** *Under financial integration, the economy behaves as a one-region economy exposed to the liquidity shock*

$$\Omega(s) \equiv (\omega^A(s) + \omega^B(s))/2.$$

Therefore, in states  $HL$  and  $LH$  consumption is equalized across regions and across states:

$$c_t^A(HL) = c_t^B(HL) = c_t^A(LH) = c_t^B(LH) \quad \text{for } t = 1, 2.$$

In states  $HH$  and  $LL$  coinsurance is not possible and  $\Omega(s)$  is equal to either  $\omega_H$  or  $\omega_L$ , so, conditional on  $y$ , the allocation is the same as in autarky. On the other hand, in states  $HL$  and  $LH$ ,  $\Omega(s) = \omega_M$  and the two regions equalize their consumption by drawing on their contingent

---

<sup>8</sup>By letting  $m_1^A(s)$  and  $m_2^A(s)$  to be negative, we also cover the case in which bank  $A$  is a net supplier of credit lines in state  $s$ .

credit lines. Given the symmetry of the environment, the credit lines bought and sold by bank  $A$  at date 0 have the same value, so the budget constraint at  $t = 0$  boils down to  $x + y = 1$ . Moreover, both regions choose the same values for  $x$  and  $y$ .

Notice that the contingent credit lines are uniquely determined if the equilibrium features no rollover in states  $HL$  and  $LH$ . In this case, the credit line purchased in equilibrium by bank  $A$  in state  $HL$  is:

$$(m_1^A(HL), m_2^A(HL)) = ((\omega_H - \omega_M) c_1(HL), (\omega_M - \omega_H) c_2(HL)).$$

An identical credit line is *sold* in equilibrium by bank  $A$  in state  $LH$ . If instead positive rollover occurs in states  $HL$  and  $LH$ , the contingent credit lines are indeterminate, because banks are indifferent between using a contingent credit line that transfers wealth one for one between periods 1 and 2 and investing in the liquid asset. This indeterminacy, however, does not affect the consumption allocation or the interest rates, which are the objects of interest in the rest of the paper.

Lemma 2 implies that optimal liquidity under integration can be found solving the following problem, which is similar to problem (2) in autarky:

$$\max_y \sum_{s \in S} \pi(s) V(y, \Omega(s)). \quad (3)$$

Also, similarly to Proposition 1, we obtain the following characterization of the equilibrium allocation.

**Proposition 2** *The equilibrium allocation under financial integration gives the same consumption levels  $c_t^A(s) = c_t^B(s) = c_t(s)$  to consumers in both regions and satisfies:*

$$c_1(HH) < c_1(HL) \leq c_1(LL) \leq c_2(LL) \leq c_2(HL) < c_2(HH).$$

*Positive rollover can occur: (i) in states  $LL$ ,  $HL$  and  $LH$ ; (ii) only in state  $LL$ ; or (iii) never.*

As in the autarky case rollover never occurs in the less liquid state of the world (here state  $HH$ ). However, rollover can occur in the state where both regions are hit by the low liquidity shock and also in the intermediate states where only one region is hit by the high liquidity shock.

## 4 Integration and illiquidity

In the rest of the paper, we analyze the effects of financial integration by comparing the two regimes introduced in the last section: autarky and integration. In this section, we analyze how financial integration affects liquid asset holdings  $y$ . In the next section we analyze how it affects the severity of crises.

The basic idea of this section is the following: under financial integration there is more scope for coinsurance and banks are less concerned about holding a buffer of liquid assets, because they

expect to be able to borrow from banks located in the other region in states of the world in which the regional shocks are uncorrelated. While this argument is intuitive, the result is non-obvious because two forces are at work. On the one hand, integration means that banks can borrow on the interbank market when they are hit by a high (uncorrelated) liquidity shock. On the other hand, integration also means that banks can lend their excess liquidity on the interbank market when they are hit by a low (uncorrelated) liquidity shock. The first effect lowers the ex ante value of liquidity in period 1, reducing the banks' incentives to hold liquid assets. But the second effect goes in the opposite direction, increasing the incentive to hold liquid assets. In the rest of this section, we derive conditions under which the first effect dominates and integration leads to lower liquid holdings.

The first order condition of the autarky problem (2) is<sup>9</sup>

$$\frac{1}{2} \frac{\partial V(y, \omega_H)}{\partial y} + \frac{1}{2} \frac{\partial V(y, \omega_L)}{\partial y} = 0, \quad (4)$$

while the first order condition of the problem under integration (3) is

$$p \frac{\partial V(y, \omega_M)}{\partial y} + (1-p) \left( \frac{1}{2} \frac{\partial V(y, \omega_H)}{\partial y} + \frac{1}{2} \frac{\partial V(y, \omega_L)}{\partial y} \right) = 0. \quad (5)$$

Comparing the expressions on the left-hand sides of (4) and (5) shows that the difference between the marginal value of liquidity under integration and in autarky is:

$$\frac{p}{2} \left( \frac{\partial V(y, \omega_M)}{\partial y} - \frac{\partial V(y, \omega_H)}{\partial y} \right) + \frac{p}{2} \left( \frac{\partial V(y, \omega_M)}{\partial y} - \frac{\partial V(y, \omega_L)}{\partial y} \right). \quad (6)$$

The two expressions in brackets are the formal counterparts of the two opposing forces discussed above. With probability  $p/2$  a region is hit by the high liquidity shock and borrows from the other region hit by the low liquidity shock. In that state of the world, financial integration reduces the marginal value of liquidity for the borrowing region by

$$\frac{\partial V(y, \omega_M)}{\partial y} - \frac{\partial V(y, \omega_H)}{\partial y} \leq 0, \quad (7)$$

where the inequality follows from Lemma 1 and  $\omega_M < \omega_H$ . At the same time, also with probability  $p/2$ , the same region is hit by the low liquidity shock and lends to the region hit by the high shock. In that state of the world, the marginal gain from being able to share its liquidity with the high-shock region is

$$\frac{\partial V(y, \omega_M)}{\partial y} - \frac{\partial V(y, \omega_L)}{\partial y} \geq 0, \quad (8)$$

where the inequality also follows from Lemma 1 and  $\omega_M > \omega_L$ . Liquidity is less valuable at the margin under financial integration if the difference in (7) is larger in absolute value than the difference in (8). We will now provide conditions for this to be true.

---

<sup>9</sup>The Inada conditions for  $u(\cdot)$  ensure that we are always at an interior optimum.



The next proposition shows that a sufficient condition for investment in the liquid asset to be lower under financial integration is a sufficiently low value of the rate of return  $R$ . As stated in the proposition, the result generalizes beyond our binary shock environment to any discrete, symmetric joint distribution of shocks.

**Proposition 3** *If the return  $R$  of the long asset is smaller than some cutoff  $\hat{R} > 1$ , then the equilibrium investment in the liquid asset  $y$  is lower under financial integration than in autarky. The result holds for any discrete, symmetric joint distribution of the regional liquidity shocks.*

To gain intuition for this result, we make two observations. First, remember that banks face a tradeoff between liquidity and profitability in choosing  $y$ , and  $R$  reflects the opportunity cost of investing in the liquid asset. So, when  $R$  is sufficiently small, banks choose liquidity holdings large enough that the liquidity constraint is binding only in the high shock states—states  $HL$  and  $HH$  in autarky, and state  $HH$  under integration. Remember also from Propositions 1 and 2 that no matter how small  $R$  is, as long as it is greater than 1, the liquidity constraint is always binding in those states.

Second, notice that in states with positive rollover it is optimal to equalize the consumption of early and late consumers, setting both equal to  $y + R(1 - y)$ . This happens because with positive rollover, the liquid asset has a gross return of 1, so that the optimality condition for consumption takes the form  $u'(c_1) = u'(c_2)$ . This implies that in states with positive rollover the ex ante marginal value of liquidity is

$$\frac{\partial V(y, \omega)}{\partial y} = u'(c_1) - Ru'(c_2) = (1 - R)u'(y + R(1 - y)).$$

The crucial property of this expression is that it is independent of the fraction of early consumers  $\omega$ .

Combining these two observations, when  $R$  is close enough to 1, the marginal gain of being able to share liquidity in state  $LH$  for bank  $A$  is zero, as both under autarky and under integration liquidity is abundant and we have positive rollover, so  $\partial V(y, \omega_L)/\partial y = \partial V(y, \omega_M)/\partial y$ . Therefore, expression (8) is zero and our second effect is muted. The proof of Proposition 3 completes the argument by showing that (7) is strictly negative, so the marginal value of liquidity is overall negative and there is lower liquidity under integration than in autarky. In short, an integrated bank in region  $A$  perceives the interbank market as a source of cheaper liquidity in state  $HL$ , but not as a profitable opportunity to invest excess liquidity in state  $LH$ . So it lowers its liquid investment ex ante.

As the discussion above shows, the condition in Proposition 3 is a relatively stringent sufficient condition, since it essentially ensures that the second effect is zero. Let  $y^{Aut}$  and  $y^{Int}$  denote the equilibrium investment in the liquid asset under autarky and, respectively, under financial integration. A weaker sufficient condition for  $y^{Int} < y^{Aut}$  is given by the following proposition, which can be proved for any joint distribution of regional liquidity shocks.

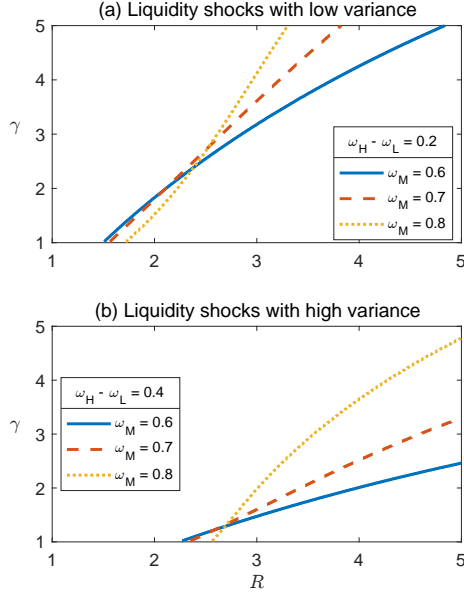


Figure 4: Regions of parameters in which  $y^{Int} < y^{Aut}$ .

**Proposition 4** *Consider a version of the model where the regional liquidity shocks follow a general symmetric joint distribution. If the marginal value of liquidity  $\partial V(y^{Aut}, \omega) / \partial y$  is convex in  $\omega$  on the support of  $\omega$ , then the equilibrium investment in the liquid asset is lower under financial integration than in autarky.*

Unfortunately, it is not easy to derive general conditions on fundamentals which ensure the convexity of  $\partial V(y, \omega) / \partial y$  in  $\omega$ . Therefore, we now turn to numerical examples to show the parameter configurations under which our result holds.

We assume a CRRA utility function with relative risk aversion equal to  $\gamma$ . In panel (a) of Figure 4, we fix the difference between the two possible realizations of the liquidity shock to be  $\omega_H - \omega_L = 0.2$ , and we explore what happens for three values of the average shock  $\omega_M = 0.6, 0.7, 0.8$ . The areas to the left of the lines displayed in the figure identify pairs  $(R, \gamma)$  for which  $y^{Int} < y^{Aut}$ . Notice that this inequality holds for any  $p \in (0, 1]$ , since the sign of expression (6) is independent of  $p$ .<sup>10</sup> The figure shows that if the coefficient of risk aversion is greater than 1 the condition is satisfied for reasonably low values  $R$ —namely for  $R \leq 1.5$ . For higher values of  $R$  the condition  $y^{Int} < y^{Aut}$  is satisfied only if agents are sufficiently risk averse. Panel (b) in Figure 4 is analogous to panel (a), except that we use a higher variance for the liquidity shocks, setting  $\omega_H - \omega_L = 0.4$ . An increase in the variance of  $\omega$  tends to enlarge the region where  $y^{Int} < y^{Aut}$  holds.

<sup>10</sup>Our calculations also show that  $y^{Int} < y^{Aut}$  holds for all  $R$ , when either one of the following conditions is satisfied:  $\gamma < 1$  or  $\omega_M < 0.5$ .

We conclude this section with comparative statics with respect to  $p$ . We use the notation  $y^{Int}(p)$  to denote the dependence of optimal investment under integration on the parameter  $p$ .

**Proposition 5** *If  $y^{Int}(p_0) < y^{Aut}$  for some  $p_0 \in (0, 1]$  then the equilibrium investment in the liquid asset under integration  $y^{Int}(p)$  is decreasing in  $p$  and is smaller than the autarky level  $y^{Aut}$  for all  $p \in (0, 1]$ .*

As  $p$  increases, the probability of a correlated shock decreases and the scope for coinsurance increases. Therefore, the marginal value of liquidity falls and banks hold smaller liquidity buffers. Notice also that as  $p \rightarrow 0$ , the possibility of coinsurance disappears and the integrated economy converges to the autarky case. Combining Propositions 3 and 5, shows that if  $R$  is below some cutoff  $\hat{R}$  then investment in the liquid asset under integration is everywhere decreasing in  $p$ .

## 5 The depth of systemic crises

In this section, we analyze the implications of financial integration for the depth of systemic crises. Here we simply identify a systemic crisis with a realization of the shock  $HH$ . We first consider the effects of this shock on prices, looking at interest rates on deposit contracts and on the interbank market. Then we look at the effect of the shock on quantities, focusing on the response of consumption.

### 5.1 Interest rates

To measure the price of liquidity, we focus on the interest rate between  $t = 1$  and  $t = 2$  implicit in the interbank credit lines.<sup>11</sup> This interest rate is denoted by  $r(s)$  and defined by

$$1 + r(s) = \frac{Q_1(s)}{Q_2(s)} = \frac{u'(c_1(s))}{u'(c_2(s))}. \quad (9)$$

Notice that in the autarky case, it is straightforward to allow banks in a given region to trade state contingent credit lines among themselves. Of course, in autarky these banks are all hit by the same shock, so trading in credit lines will be zero. But the relative price  $r$  will be well defined and given by (9) as in the case of integration. The interest rate  $r$  is also the price at which banks are willing to trade on an ex post spot market for one-period loans at  $t = 1$ , if such a market is open in addition to the market for ex ante credit lines.<sup>12</sup>

An alternative is to look at the interest rate implicit in the terms of the deposit contracts. A deposit contract offers the option to withdraw  $c_1(s)$  in  $t = 1$  or  $c_2(s)$  in  $t = 2$ . So the implicit

<sup>11</sup>This is an ex post notion of the price of liquidity. Ex ante, i.e. at  $t = 0$ , the opportunity cost of investing in the liquid asset is equal to the forgone return on the illiquid asset,  $R$ . Therefore, the ex ante price of liquidity is determined by the technology and is equal to  $R$  independently of the degree of financial integration.

<sup>12</sup>In Section 6 below we study the case in which only the spot market can be used to trade liquidity and credit lines are not available.

interest rate on deposits  $r_d(s)$  is defined by

$$1 + r_d(s) = \frac{c_2(s)}{c_1(s)}. \quad (10)$$

With CRRA utility the interest rate on deposits is a monotone transformation of the interest rate on the credit lines. In the special case of log utility, the two interest rates coincide. From now on, we mostly focus on the interest rate on credit lines.

Notice that the equilibrium characterizations in Proposition 1 (in autarky) and in Proposition 2 (integration) imply that  $r$  is always greater than or equal to 0. It is strictly positive if the liquidity constraint is binding and is zero otherwise.<sup>13</sup> Since in autarky the interest rates can be different in the two regions, we focus on the equilibrium behavior of the interest rate in region  $A$ . By symmetry, identical results hold for region  $B$ , inverting the role of states  $HL$  and  $LH$ .

The following proposition holds for both interest rates  $r$  and  $r_d$ .

**Proposition 6** *All else equal, if  $R$  is below some cutoff  $\hat{R}$ , the equilibrium interest rate in region  $A$  is higher under integration than in autarky in state  $HH$ , lower in state  $HL$ , equal in states  $LH$  and  $LL$ .*

In state  $HL$  the banks in region  $A$  reap the benefits of integration as they are allowed to borrow at a lower interbank rate than in autarky. This is the stabilizing effect of financial integration. However, when the correlated shock  $HH$  hits, the interest rate increases more steeply than in autarky due to a systemic shortage of liquidity, i.e., a lower  $y$ . This shortage is simply a result of the optimal ex ante investment in liquid assets characterized in Proposition 3.

Proposition 6 can be extended to any discrete, symmetric distribution of liquidity shocks along the lines of Proposition 3. In that extension, the highest possible realization of the liquidity shock  $\omega_N$ <sup>14</sup> plays the role of the high shock  $\omega_H$  in the binary case and the proposition holds if we make the following substitutions: replace state  $HH$  with the  $s$  in which  $\omega^A(s) = \omega^B(s) = \omega_N$ ; replace state  $HL$  with all the  $s$  in which  $\omega^A(s) = \omega_N$  and  $\omega^B(s) \neq \omega_N$ ; replace states  $LH$  and  $LL$  with all remaining states.

What does this result implies for the volatility and skewness of the interest rate distribution? The answer depends on the value of  $p$ . The comparative static result in Proposition 5 implies that for larger values of  $p$  the spike in the price of liquidity in state  $HH$  will be worse, as banks will hold less liquidity ex ante. At the same time, when  $p \rightarrow 1$ , the probability of a systemic event goes to zero, so the spike happens with smaller probability. The combination of these effects suggests that the volatility of the interest rate may increase or decrease with  $p$ , while the distribution will tend to be more positively skewed when  $p$  is larger. We now explore these effects formally.

First, let us look at a numerical example. The utility function is CRRA with relative risk aversion  $\gamma = 1$ , the rate of return on the illiquid asset is  $R = 1.15$ , and the liquidity shocks

<sup>13</sup>The same properties hold for  $r_d$ .

<sup>14</sup>See the notation in the proof of Proposition 3.

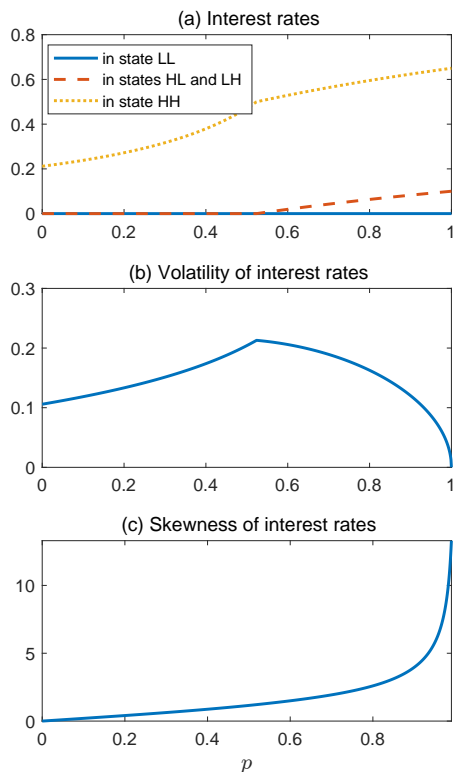


Figure 5: Interest rates.

are  $\omega_L = 0.4$  and  $\omega_H = 0.6$ . Panel (a) in Figure 5 plots the interest rate in different states, for different values of the parameter  $p$ . Remember that in states  $HL$  and  $LH$  interest rates are equalized. Notice also that the case of autarky coincides with the case  $p = 0$ . Panel (b) plots the standard deviation of  $r$  and panel (c) plots the skewness of  $r$ , measured by its third standardized moment:<sup>15</sup>

$$sk(r) \equiv E \left[ (r - E[r])^3 \right] / (Var[r])^{3/2}.$$

The example in Figure 5 shows that it is possible for financial integration to make interest rates both more volatile and more skewed. Clearly, if  $p = 1$  interest rate volatility disappears with integration as banks can perfectly coinsure their local shocks. However, when there is a sufficient amount of residual aggregate uncertainty, i.e., when  $p$  is sufficiently smaller than 1, financial integration increases interest rate volatility. Moreover, for all levels of  $p < 1$  financial integration makes the interest rate distribution more skewed.

We now provide some sufficient conditions for having an increase in volatility and skewness under integration. First, we show that in some circumstances the increase in banks' illiquidity

<sup>15</sup>Alternative skewness indexes (e.g., Pearson's skewness coefficients) will have the same sign as  $sk(r)$  although, clearly, different magnitudes.

identified in the previous section dominates the stabilizing effects of integration, and volatility is higher under integration than in autarky.

**Proposition 7** *Suppose rollover is optimal in all states except  $HH$  for all  $p$  in some interval  $(0, p_0]$ , then there is a  $p < p_0$  such that financial integration increases interest rate variance:*

$$\text{Var}(r^{Int}) > \text{Var}(r^{Aut}).$$

When rollover is optimal in all states except  $HH$  the interest rate has a binary distribution. Let the function  $f(p)$  denote the interest rate in state  $HH$  as a function of  $p$ . Then the interest rate is  $f(p)$  with probability  $(1 - p)/2$  and 0 with probability  $1 - (1 - p)/2$ , so the variance is:

$$\text{Var}(r^{Int}) = \frac{1}{4} (1 - p^2) (f(p))^2. \quad (11)$$

Moreover, as  $p \rightarrow 0$  the interest rate distribution converges to its autarky value. Therefore, to prove Proposition 7 it is enough to prove that the expression on the right-hand side of (11) is strictly increasing in  $p$  at  $p = 0$ . Since the changes in  $(1 - p^2)$  are of second order importance at  $p = 0$ , this is equivalent to proving that the crisis interest rate  $f(p)$  is strictly increasing in  $p$ . This can be shown by an argument similar to the one behind Proposition 5: as  $p$  increases, banks' liquidity holdings are reduced and so the crisis interest rate is higher. The details of the proof are in the Appendix.

Notice that the hypothesis of Proposition 7—positive rollover in all states except  $HH$ —holds when  $R$  is sufficiently close to 1 (see the proof of Proposition 3 for a formal argument). The example in Figure 5 satisfies this hypothesis, and, indeed, displays increasing volatility for low levels of  $p$ . Notice also that the relation between  $p$  and interest rate volatility cannot be everywhere increasing, because the variance is positive as  $p \rightarrow 0$  and goes to 0 as  $p \rightarrow 1$ . Therefore, under the assumptions of Proposition 7 there is a non-monotone relation between  $p$  and the variance of  $r$ : increasing for low values of  $p$  and eventually decreasing.

Let us now look at the model implications for skewness. In autarky, the interest rate follows a symmetric binary distribution, so in this case the skewness  $sk(r^{Aut})$  is zero. If we use the shorthand notation  $M$  to refer to states  $HL$  and  $LH$ , under integration the interest rate takes the three values  $r(HH)$ ,  $r(M)$ , and  $r(LL)$  with probabilities, respectively,  $(1 - p)/2$ ,  $p$ , and  $(1 - p)/2$ . The following lemma allows us to characterize the sign of the skewness of this distribution.

**Lemma 3** *The skewness of the interest rate distribution is positive if and only if*

$$r(HH) - r(M) > r(M) - r(LL).$$

Notice that if rollover is optimal in states  $LL$  and  $M$ , then  $r(M) - r(LL) = 0$  and  $r(HH) - r(M) > 0$  so the lemma immediately implies that the distribution is positively skewed. However, the result is more general, as shown in the following proposition.

**Proposition 8** *The interest rate distribution is positively skewed under financial integration and zero in autarky.*

Can we generalize Propositions 6, 7 and 8 beyond the simple binary shock structure studied here? As argued above, it is possible to consider a general discrete distribution of shocks and, by choosing  $R$  small enough, ensure that in equilibrium there is rollover in all states except the one with the highest realization of the liquidity shock in both regions. This line of argument essentially reduces the general case to the binary case. So a more interesting question is whether we can extend our argument to cases in which the liquidity constraint is binding for different realizations of the liquidity shock and not only for the highest one. In general, it is not easy to obtain analytical characterization results for a model with these features and the reason is that two opposing forces are at work. Think of the move from autarky to integration in two steps. First, there are the effects of integration *for a given level* of liquid investment  $y$ . Keeping  $y$  fixed, integration improves the opportunities for risk sharing, thus leading to less frequent crises. Second, we have the effects of the endogenous adjustment of  $y$ . If the conditions in Proposition 4 are satisfied, investment in liquid assets is reduced. This implies that when both regions have a large fraction of early consumers, liquidity is overall scarcer and so interest rates are higher. To see these forces at work in our example, Figure 6 shows the effects of integration on interest rate volatility for given  $y$  and the effect of changing  $y$ . As we can see, the first effect goes in the direction of reducing volatility, while the second effect goes in the opposite direction and dominates for low values of  $p$ . In Online Appendix B, we present conditions that ensure that the first effect reduces volatility for a general shock structure. Proposition 4, on the other hand, shows conditions that ensure that the second effect reduces  $y$  and thus increases volatility. Whether the second effect is strong enough to dominate the first depends on the model parameters. In Online Appendix C we explore numerically the strength of the two effects in a model with a richer, continuous shock structure, in which the liquidity constraint is binding for a whole interval of shocks. In particular, we study a joint distribution with marginals distributed as beta random variables and show that under a range of parameters our results for volatility hold when  $R$  is not too large and regional shocks are not too dissimilar from one another (i.e., their negative correlation is not too high).

While skewness necessarily increases with integration when regional liquidity shocks follow a symmetric binary distribution, the magnitude of the response clearly depends on the strength of the illiquidity effect studied in Section 4, which tends to magnify the spike in interest rates in a crisis. With continuous liquidity shocks the distribution of interest rates is typically skewed also in autarky and the skewness can either increase or decrease with integration because of its contrasting effects. Online Appendix C shows by means of examples that higher skewness of interest rates tend to occur under integration in the same cases when volatility increases, that is, when  $R$  is not too large and regional shocks are not too dissimilar from one another.

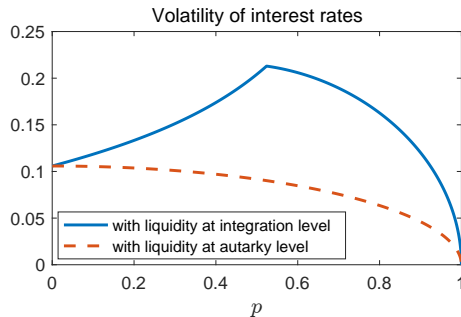


Figure 6: Decomposing the volatility of interest rates.

## 5.2 Consumption and welfare

To assess the real consequences of integration it is useful to look at the effects of a systemic crisis on consumption in  $t = 1$ . We show that the implications on the real side are similar to those obtained in terms of interest rates: financial integration can make the distribution of consumption more volatile and more skewed.

Let us begin with a numerical example. Figure 7 characterizes the distribution of consumption in  $t = 1$  in the same example used for Figure 5. The volatility of consumption is non-monotone in  $p$  and is higher than in autarky for intermediate values of  $p$ . The distribution of consumption is symmetric in autarky and negatively skewed in integration, with more negative skewness for larger values of  $p$ .

From an analytical point of view, it is possible to obtain the analog of Proposition 7 for consumption and show that when rollover is optimal in all states except  $HH$ , consumption volatility is always larger under integration if  $p$  is not too large. This helps us understand the increasing portion of the relation in panel (b) of Figure 7.

Similarly to interest rates, also consumption has a symmetric binary distribution in autarky, with zero skewness. To show that the consumption distribution becomes negatively skewed under integration we need some restrictions on parameters, as shown in the following proposition.

**Proposition 9** *Consumption is negatively skewed under integration iff*

$$y^{Int} > \frac{\omega_M \omega_H R}{\omega_M \omega_H R + 2\omega_H - \omega_M(1 + \omega_H)}. \quad (12)$$

In the Online Appendix B, we use some numerical examples to investigate the set of parameters under which condition (12) is satisfied.

Turning to welfare, our model, due to complete markets, leads to first-best efficiency. So the increase in consumption volatility and skewness is not a symptom of inefficiency. Moreover, a higher level of  $p$ , by increasing the possibility for coinsurance is beneficial in terms of ex ante welfare, as shown in the following proposition.



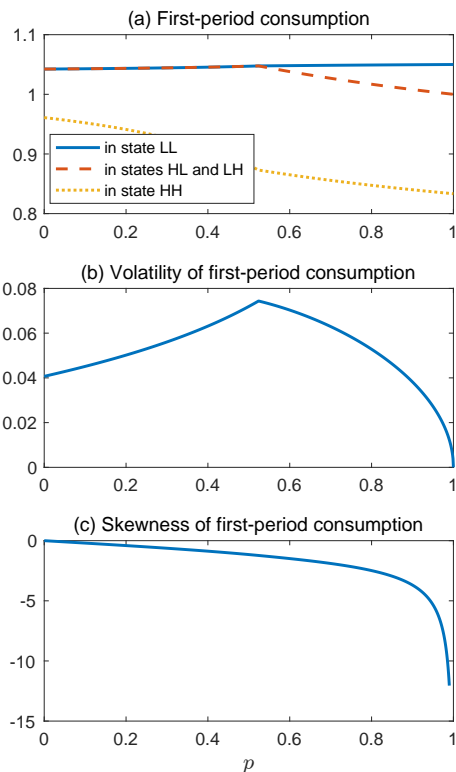


Figure 7: First-period consumption.

**Proposition 10** *The ex ante expected utility of consumers is higher under integration than in autarky for all  $p \in (0, 1]$ , and is strictly increasing in  $p$ .*

In the next section, we consider the case of incomplete markets, in which fully state contingent credit lines are not available, to investigate one potential source of inefficiency.

## 6 Spot markets and regulation

In this section, we ask what happens when the allocation of liquidity in period 1 is determined by a spot interbank market rather than by state-contingent credit lines. That is, we assume that banks can only borrow and lend from each other ex post, after the liquidity shocks have been realized.

One motivation for this extension is realism, as a large fraction of interbank lending takes the form of spot transaction. A second motivation is that this extension moves away from a first-best world and allows us to discuss the potential role of regulation. With spot interbank markets, the equilibrium is not in general constrained efficient, as shown in Lorenzoni (2001), Allen and Gale (2004), and Farhi, Golosov and Tsivinsky (2009), and there is a role for welfare-improving

prudential regulation in the form of liquidity requirements. In our setup, we show that financial integration can increase the benefits of liquidity requirements.

## 6.1 Setup

The technology and preferences are as in the baseline model and so is the shock structure. However, we now consider a different structure for the interbank market, and assume that banks cannot write state contingent credit lines. They can only borrow and lend from each other ex post on a competitive spot market where they trade consumption goods in  $t = 1$  for consumption goods in  $t = 2$  at the interest rate  $r$ .<sup>16</sup> Now, the two regimes of autarky and integration correspond, respectively, to the case in which there are two separate interbank markets in period 1, and to the case in which there is a single integrated market.

Consider a bank in region  $A$ , facing the interest rate  $r(s)$  on the spot interbank market. The bank's problem is now:

$$\begin{aligned} \max_{y, c_1^A(\cdot), c_2^A(\cdot)} \quad & \sum_{s \in S} \pi(s) [\omega^A(s) u(c_1^A(s)) + (1 - \omega^A(s)) u(c_2^A(s))] \\ \text{s.t.} \quad & (1 + r(s)) \omega^A(s) c_1^A(s) + (1 - \omega^A(s)) c_2^A(s) \leq (1 + r(s)) y + R(1 - y). \end{aligned} \quad (13)$$

The single bank does not perceive a liquidity constraint, because it can freely borrow on the competitive interbank market. However, the liquidity constraint is present at the aggregate level and operates through the market clearing condition. In particular, in autarky, the market clearing condition is

$$\omega^A(s) c_1^A(s) \leq y, \quad (14)$$

which must hold as an equality if  $r(s) > 0$ . Under integration, the market clearing condition is instead

$$\omega^A(s) c_1^A(s) + \omega^B(s) c_1^B(s) \leq 2y,$$

which also must hold as an equality if  $r(s) > 0$ .

In both the cases of autarky and integration, the first order condition for  $y$  for an individual bank in region  $A$  solving (13) can be written as:

$$\sum_{s \in S} \pi(s) u'(c_1^A(s)) (1 + r(s) - R) = 0, \quad (15)$$

given that the Lagrange multiplier on the budget constraint is  $\lambda(s) = \pi(s) u'(c_1^A(s))$ . However, the equilibrium interest rates are different in the two cases, leading to different values of  $y$ .

A symmetric equilibrium in the economy with spot interbank markets is given by interest rates and quantities such that banks in each region optimize and markets clear. Notice that because of symmetry, equilibrium liquidity holdings end up being the same in the two regions.

---

<sup>16</sup>Notice that if this spot market is open and banks cannot monitor each other trades, then the market for contingent credit lines is useless (see Farhi, Golosov and Tsivinsky, 2009).

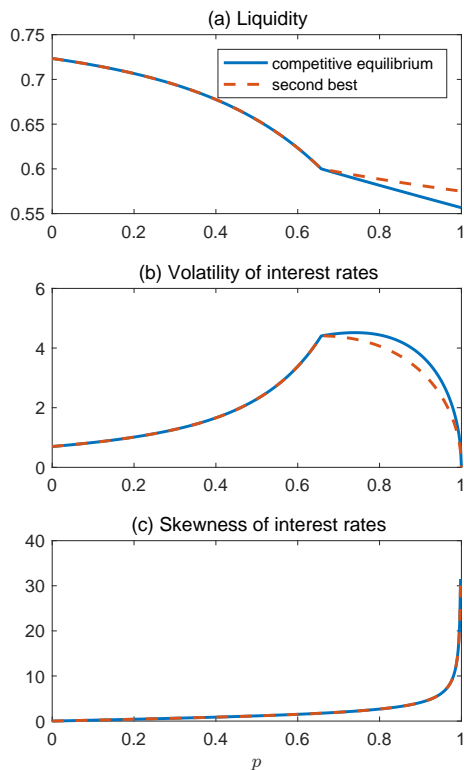


Figure 8: Spot markets, competitive equilibrium, and second best.

## 6.2 Integration and regulation

From a positive point of view the predictions of the model under spot markets are broadly similar to those under state contingent credit lines. In particular, Figure 8 shows liquidity holdings, volatility and skewness of the interest rate for different values of  $p$ . Recall that a value of  $p = 0$  coincides with the autarky case. The numerical values used for this example are:

$$\gamma = 2, \quad R = 1.2, \quad \omega_H = 0.8, \quad \omega_L = 0.2.$$

The figure shows that in the spot market economy a similar mechanism is at work as in the case of complete markets: financial integration leads to a portfolio shift for banks away from the liquid asset, as banks anticipate the ability to borrow when uncorrelated shocks hit. This can lead to higher volatility of interest rates in equilibrium.

Let us turn to constrained efficiency, focusing first on the case of integration. Consider a planner who can dictate the liquidity holdings in the two regions  $y^A$  and  $y^B$  at date 0. The allocation of liquidity in  $t = 1$  still has to go through the spot market, possibly because of informational limitations on the planner's side.<sup>17</sup> So the only way in which the planner can affect

<sup>17</sup>Farhi, Golosov and Tsivinsky (2009) justify the inability of the planner to intervene at date 1 in terms of its

the final allocation is through the choice of  $y^A$  and  $y^B$ . With a slight abuse of notation, let's use  $r(s, y^A, y^B)$  to denote the equilibrium interest rate in state  $s$  that arises conditional on bank liquidity holdings in regions  $A$  and  $B$ .

Given symmetry, it is natural to look at a planner that gives equal weights to the consumers in the two regions and maximizes

$$W = \frac{1}{2} \sum_i \sum_s \pi(s) [\omega^i(s) u(c_1^i(s)) + (1 - \omega^i(s)) u(c_2^i(s))],$$

subject to budget constraints as in problem (13), except with  $r(s, y^A, y^B)$  replacing  $r(s)$ . The planner's first order condition for  $y^A$  is

$$\begin{aligned} \sum_s \pi(s) u'(c_1^A(s)) (1 + r(s, y^A, y^B) - R) \\ + \sum_i \sum_s \pi(s) u'(c_1^i(s)) (y^i - \omega^i(s) c_1^i(s)) \frac{\partial r(s, y^A, y^B)}{\partial y^A} = 0. \end{aligned} \quad (16)$$

The first term on the left-hand side is equal to the left-hand side of the private optimality condition (15). The second term is a pecuniary externality which determines a difference between the social and the private marginal value of liquidity and causes inefficiency in the laissez-faire equilibrium. When more liquid assets are available at the aggregate level interest rates are lower. This induces a reallocation of resources from lending banks, with  $y^i - \omega^i(s) c_1^i(s) > 0$ , to borrowing banks, with  $y^i - \omega^i(s) c_1^i(s) < 0$ . If the marginal utility of the second type of banks is larger, this reallocation yields a positive gain in social welfare, which is not internalized by individual banks ex ante. Basically, we have a situation in which banks free ride on each others' liquid holdings, because in the spot market they do not fully capture the gain from providing liquidity.

The following proposition shows conditions under which borrowing banks have indeed higher marginal utility than lending banks and the previous argument applies. It also covers the knife-edge case of log utility, in which the spot market replicates the first-best allocation and the equilibrium is constrained efficient.

**Proposition 11** *With CRRA preferences, if  $\gamma = 1$  the equilibrium under integration is constrained efficient. If  $\gamma > 1$  and  $r(M) > 0$  the equilibrium under integration is constrained inefficient and banks hold too little liquidity from a social welfare point of view.*

Similar derivations can be done in the case of autarky, except that the pecuniary externality now only includes the term with  $i = A$  as the interest rate in region  $B$  is unaffected by liquidity holdings in region  $A$ . In the case of autarky the pecuniary externality has no bite because two cases are possible: either the interest rate is 0 and the expression  $\partial r / \partial y^A$  is zero, or the interest rate is positive and the expression  $y^A - \omega^A(s) c_1^A$  is zero by market clearing. We conclude that in

---

limited ability to monitor hidden trades.

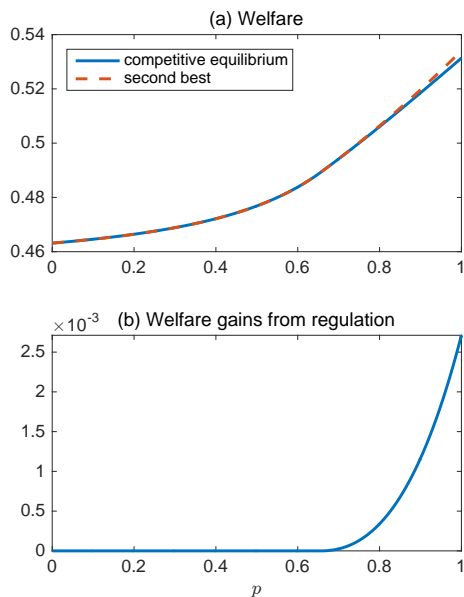


Figure 9: Welfare gains of liquidity regulation.

this model financial integration makes regulation more desirable, as it increases the scope for ex post trading and thus increases the scope for pecuniary externalities.

Figure 8 illustrates these results by showing the planner second best level of liquidity (dashed red line in panel (a)), and the implied levels of volatility and skewness (dashed red lines in panels (b) and (c)). Notice that the presence of inefficiency depends on whether or not there is rollover in state  $M$ , that is, in the states in which co-insurance is possible. When  $r(M) = 0$  the two regions have the same consumption levels in those states and the pecuniary externality is muted. This is visible in the figure since whether or not  $r(M) = 0$  depends on the value of  $p$ . When  $p$  is low enough liquidity is abundant and there is positive rollover in state  $M$ . For low values of  $p$  the solid blue line and dashed red line coincide. When  $p$  is higher there is more room for co-insurance but there is also room for free-riding, as now liquidity is scarce in state  $M$ . In this case, the equilibrium is inefficient and the dashed red line is lower than the solid blue line. Notice however that even though crises are less severe in the regulated economy, the skewness statistic does not capture this as interest rates in the second best are lower both in states  $M$  and  $HH$ , so skewness is not affected by the reduction in  $y$ .

Figure 9 shows the effects of integration and of financial regulation on welfare. The top panel plots  $\frac{1}{1-\gamma} \ln(-W)$  in the unregulated competitive equilibrium and at the second best.<sup>18</sup> The bottom panel plots the difference between the lines in the top panel, and so it captures the welfare gains from regulation in terms of equivalent consumption. For low values of  $p$  the equilibrium is constrained efficient and there are no welfare gains. Welfare gains are positive

<sup>18</sup>This transformation gives us welfare in equivalent consumption terms.

for higher values of  $p$  and they are increasing in  $p$ : the scope for coinsurance increases, but so does the scope for free-riding. Notice that in our example the welfare gains are small. We have explored other parametrizations and noticed that higher values of  $\gamma$  and larger volatility in  $\omega$  can deliver larger gains from regulation (as they increase the difference in marginal utilities and the level of trading). Notice also that gains from regulation are small when compared to the welfare gains from integration that can be gleaned from the top panel, comparing welfare at high values of  $p$  to welfare at  $p = 0$  (which corresponds to autarky). So the main lesson from this exercise is that financial integration can make regulation more desirable, by increasing the distance between laissez faire and second best, even though integration is per se (even absent regulation) welfare improving.

We have explored the second-best policy in our baseline model in which all banks have the same shock in each region. This makes the result that integration increases the need for regulation particularly stark, as autarky is constrained efficient. In a previous version of the paper, we explored variants of the model in which trading among banks in the same region can take place in financial autarky. This was done by introducing a non-degenerate distribution of liquidity shocks within each region. In that case, the welfare gains from regulation can be positive both in autarky and under integration. Integration allows banks to better smooth liquidity shocks in normal times. This implies that marginal utilities are less affected by the bank specific shock  $\omega$ , reducing the benefits from regulation. On the other hand, integration leads to larger trading across regions, increasing the effects of the pecuniary externality. In the examples we explored, on net, the benefits from regulation are always larger under integration, as in the case studied here. But, unlike in the example presented here, the benefits of regulation can be hump-shaped in  $p$  instead of always increasing.

## 7 Concluding remarks

In this paper we explore the consequences of financial integration for banks and the real economy. Consistently with aggregate trends in the US and the Euro area in the pre-crisis period, we show a model where the banks' liquidity can decline with more integration. This happens because integration improves the opportunities to coinsure liquidity risk, and therefore reduces the incentives to hold costly liquid reserves. As a result, financial integration has opposing effects on the stability of the banking system: interbank interest rates are less sensitive to local liquidity shocks, but react more to systemic events. These effects can result in higher volatility of interest rates and consumption levels, yet integration is desirable because it improves welfare. We also show that in a second-best world the welfare gains of liquidity regulation can be larger in an integrated economy.

For the sake of simplicity, the model analyzed in the paper abstracts from a number of issues. In particular, we have ignored the possibility that banks operate in countries that use different

currencies. Depending on circumstances, exchange rate fluctuations can either hinder or facilitate the ability of banks to use the world interbank market to insure local liquidity shocks. Exchange rate risk per se makes holdings of liquidity in a different currency a worse buffer against liquidity shocks, by making the domestic value of the buffer subject to currency risk. On the other hand, if exchange rate movements are correlated with domestic liquidity conditions, reserves of foreign liquidity can provide better state contingency than domestic liquidity. Think of a bank in an emerging economy facing liquidity withdrawals at a time in which the domestic currency is depreciating. For such a bank, dollar reserves may be especially useful. We leave to future work the investigation of liquidity insurance in a multi-currency world.<sup>19</sup>

Second, in the paper we have restricted domestic consumers to be serviced by domestic banks. It would be interesting to investigate models in which consumers can also get liquidity services by directly accessing large international banks. One can conjecture that this channel would act as a substitute for financial integration through the interbank market, but there are interesting open issues regarding the speed at which foreign retail investors and financial institutions flow in and out of domestic financial markets, and how this affects liquidity. This is also an open issue for future work.

Third, through most of the paper we have focused on a fully efficient setup, where the liquidity holdings of the banks are both privately and socially efficient. We did so to emphasize that there are fundamental forces behind systemic illiquidity, which are not necessarily a symptom of inefficiency. At the same time, we know that various market failures can exacerbate illiquidity problems ex post, and dampen the incentives for precautionary behavior ex ante. We have introduced one such market failure in Section 6 and used it to discuss the merits of prudential regulation in a financially integrated world. However, more work remains to be done to understand and evaluate other potential inefficiencies and the appropriate instruments to address them.

---

<sup>19</sup>See Bocola and Lorenzoni (2017).

## References

- [1] Acharya, V., D. Gromb, and T. Yorulmazer (2012), “Imperfect Competition in the Inter-Bank Market for Liquidity as a Rationale for Central Banking,” *American Economic Journal: Macroeconomics*, **4** (2): 184-217.
- [2] Acharya, V., H. S. Shin, and T. Yorulmazer (2011), “Crisis Resolution and Bank Liquidity,” *Review of Financial Studies*, **24**(6): 2166-2205.
- [3] Allen, F., and E. Carletti (2006), “Credit Risk Transfer and Contagion,” *Journal of Monetary Economics*, **53** (1): 89-111.
- [4] Allen, F., E. Carletti, and D. Gale (2009), “Interbank Market Liquidity and Central Bank Intervention,” *Journal of Monetary Economics*, **56** (5): 639-652.
- [5] Allen, F., and D. Gale (1998), “Optimal Financial Crises,” *Journal of Finance*, **53** (4), 1245-1284.
- [6] Allen, F., and D. Gale (2000), “Financial Contagion,” *Journal of Political Economy*, **108** (1): 1-33.
- [7] Allen, F., and D. Gale (2004), “Financial Intermediaries and Markets,” *Econometrica*, **72** (4): 1023-1061.
- [8] Bengui, J. (2014), “Macroprudential Policy Coordination ”Working Paper, University of Montreal.
- [9] Bhattacharya, S., and D. Gale (1987), “Preference Shocks, Liquidity and Central Bank Policy,” in W. Barnett and K. Singleton, eds., *New Approaches to Monetary Economics*, Cambridge University Press, Cambridge.
- [10] Bocola, L. and G. Lorenzoni (2017), “Confidence Crises, Exchange Rates, and the Role of Reserves” mimeo, Northwestern University.
- [11] Brusco, S., and F. Castiglionesi (2007), “Liquidity Coinsurance, Moral Hazard, and Financial Contagion,” *Journal of Finance*, **62** (5): 2275-2302.
- [12] Caballero, R. J., and A. Krishnamurthy (2009), “Global Imbalances and Financial Fragility,” *American Economic Review*, **99** (2): 584-588.
- [13] Castiglionesi, F., F. Feriozzi, G. Lóránth, and L. Pelizzon (2014), “Liquidity Coinsurance and Bank Capital,” *Journal of Money Credit and Banking*, **46** (2-3), 409-443.
- [14] Castiglionesi, F., and W. Wagner (2013), “On the Efficiency of Bilateral Interbank Insurance,” *Journal of Financial Intermediation*, **22** (2): 177-200.



- [15] Diamond, D., and P. Dybvig (1983), “Bunk Runs, Deposit Insurance and Liquidity,” *Journal of Political Economy*, **91** (3): 401-419.
- [16] Farhi, E., M. Golosov, and A. Tsyvinski (2009), “A Theory of Liquidity and Regulation of Financial Intermediation,” *Review of Economic Studies*, **76** (3): 972-992.
- [17] Freedman, P., and R. Click (2006), “Banks that Don’t Lend? Unlocking Credit to Spur Growth in Developing Countries,” *Development Policy Review*, **24** (3): 279-302.
- [18] Freixas, X., and C. Holthausen (2005), “Market Interbank Integration under Asymmetric Information,” *Review Financial Studies*, **18** (2): 459-490.
- [19] Freixas, X., B. Parigi, and J. C. Rochet (2000), “Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank,” *Journal of Money, Credit and Banking*, **32**: 611-638.
- [20] Holmstrom, B., and J. Tirole (1998), “Private and Public Supply of Liquidity,” *Journal of Political Economy*, **106** (1): 1-40.
- [21] Lorenzoni, G. (2008), “Inefficient Credit Booms,” *Review of Economic Studies*, **75** (3): 809-833.
- [22] Ranciere, R., A. Tornell, and F. Westermann (2008), “Systemic Crises and Growth,” *Quarterly Journal of Economics*, **128** (1): 359-406.
- [23] Rochet, J. C., and J. Tirole (1996), “Interbank Lending and Systemic Risk,” *Journal of Money, Credit and Banking*, **28** (4): 733-762.
- [24] Wagner, W. (2008), “The Homogenization of the Financial System and Liquidity Crisis,” *Journal of Financial Intermediation*, **17** (3): 330-356.

## Appendix

Throughout this appendix, we use  $C_1(y, \omega)$  and  $C_2(y, \omega)$  to denote the optimal policies associated to the value function (1) defined in the text. The following lemma is an extended version of Lemma 1, so the proof also proves that lemma.

**Lemma 4** *The value function  $V(y, \omega)$  is strictly concave, continuous and differentiable in  $y$ , with*

$$\partial V(y, \omega) / \partial y = u'(C_1(y, \omega)) - Ru'(C_2(y, \omega)). \quad (17)$$

*The policies  $C_1(y, \omega)$  and  $C_2(y, \omega)$  are given by*

$$\begin{aligned} C_1(y, \omega) &= \min \left\{ \frac{y}{\omega}, y + R(1 - y) \right\}, \\ C_2(y, \omega) &= \max \left\{ R \frac{1 - y}{1 - \omega}, y + R(1 - y) \right\}. \end{aligned}$$

*The partial derivative  $\partial V(y, \omega) / \partial y$  is non-increasing in  $\omega$ .*

**Proof.** Continuity and weak concavity are easily established. Differentiability follows using concavity and a standard perturbation argument to find a differentiable function which bounds  $V(y, \omega)$  from below. From the envelope theorem

$$\partial V(y, \omega) / \partial y = \lambda + (1 - R)\mu,$$

where  $\lambda$  and  $\mu$  are the Lagrange multipliers on the two constraints. The problem first order conditions are

$$\begin{aligned} u'(C_1(y, \omega)) &= \lambda + \mu, \\ u'(C_2(y, \omega)) &= \mu, \end{aligned}$$

which substituted in the previous expression give (17). Considering separately the cases  $\lambda > 0$  (no rollover) and  $\lambda = 0$  (rollover), it is then possible to derive the optimal policies. Strict concavity can be proven directly, substituting the expressions for  $C_1(y, \omega)$  and  $C_2(y, \omega)$  in (17) and showing that  $\partial V / \partial y$  is strictly decreasing in  $y$ . This last step uses the strict concavity of  $u(\cdot)$  and  $R > 1$ . Substituting  $C_1(y, \omega)$  and  $C_2(y, \omega)$  in (17) also shows that  $\partial V(y, \omega) / \partial y$  is non-decreasing in  $\omega$ .

■

**Lemma 5**  $C_1(y, \omega) \leq C_2(y, \omega)$  for all  $y \geq 0$  and  $\omega \in (0, 1)$ . In particular we distinguish two cases:

(i) *If  $y > \omega R / (1 - \omega + \omega R)$  there is rollover and the following conditions hold*

$$\frac{y}{\omega} > C_1(y, \omega) = C_2(y, \omega) = y + R(1 - y) > R \frac{1 - y}{1 - \omega},$$

(ii) If  $y \leq \omega R / (1 - \omega + \omega R)$  there is no rollover and the following conditions hold

$$C_1(y, \omega) = \frac{y}{\omega} \leq y + R(1 - y) \leq R \frac{1 - y}{1 - \omega} = C_2(y, \omega),$$

where the inequalities are strict if  $y < \omega R / (1 - \omega + \omega R)$  and hold as equalities if  $y = \omega R / (1 - \omega + \omega R)$ .

**Proof.** The proof follows from inspection of  $C_1(y, \omega)$  and  $C_2(y, \omega)$  in Lemma 4. ■

An immediate consequence of Lemma 5 is the following corollary.

**Corollary 1** *If rollover is optimal in problem (1) for some pair  $(y, \omega)$  then it is also optimal for any pair  $(y, \omega')$  with  $\omega' < \omega$ .*

**Proof of Proposition 1.** Rewrite problem (2) as

$$\max_y \frac{1}{2} V(y, \omega_L) + \frac{1}{2} V(y, \omega_H). \quad (18)$$

The first order condition of this problem and Lemma 4 imply that  $y^{Aut}$  is characterized by

$$\sum_{\omega=\omega_L, \omega_H} \frac{1}{2} (u'(C_1(y^{Aut}, \omega)) - Ru'(C_2(y^{Aut}, \omega))) = 0. \quad (19)$$

The Inada condition for  $u(\cdot)$  ensures that we have an interior solution  $y^{Aut} \in (0, 1)$ . Notice that if positive rollover is optimal when the regional shock is  $\omega_H$ , it is also optimal with  $\omega_L$  by Corollary 1. But then Lemma 5 implies that the left-hand side of (19) is equal to

$$(1 - R) u'(y^{Aut} + R(1 - y^{Aut})) < 0,$$

leading to a contradiction. Therefore, no positive rollover occurs in autarky when the regional shock is  $\omega_H$ . We can then distinguish two cases, which correspond to the two cases in Lemma 5. We use  $L^*$  to denote the states  $LL$  and  $LH$ , since allocations are identical in those states. Similarly we use  $H^*$ . Also, we drop the regional index  $i = A$ , with the understanding that we refer to the allocation in region  $A$ .

(i) Solution with roll over in state  $L$ . In this case

$$c_1(L^*) = c_2(L^*) = y^{Aut} + R(1 - y^{Aut}).$$

This condition together with (19) (and  $R > 1$ ) implies

$$u'(c_1(H^*)) - Ru'(c_2(H^*)) > 0,$$

which in turns implies  $c_1(H^*) < c_2(H^*)$  and, by Lemma 5, we have  $c_1(H^*) < y^{Aut} + R(1 - y^{Aut}) < c_2(H^*)$ .

(ii) Solution without roll over in state  $L$ . Then  $c_1(L^*) = y^{Aut}/\omega_L > y^{Aut}/\omega_H = c_1(H^*)$  and  $c_2(H^*) = R(1 - y^{Aut})/(1 - \omega_H) > R(1 - y^{Aut})/(1 - \omega_L) = c_2(L^*)$ . So we have

$$c_1(H^*) < c_1(L^*) \leq c_2(L^*) < c_2(H^*).$$

■

**Proof of Lemma 2.** The argument is standard, so we only provide a sketch of the proof. To construct a symmetric equilibrium, we characterize the problem of a social planner who assigns equal weights to the two regions and show that it can be implemented by a symmetric equilibrium with state contingent credit lines. The planner problem is:

$$\begin{aligned} \max_{y, (c_t^i(s))_{i=A,B;t=1,2}} \quad & \sum_{s \in S} \pi(s) \sum_{i=A,B} \omega^i(s) u(c_1^i(s)) + (1 - \omega^i(s)) u(c_2^i(s)) \\ \text{s.t.} \quad & \sum_{i=A,B} \omega^i(s) c_1^i(s) \leq 2y \quad \forall s, \\ & \sum_{i=A,B} (1 - \omega^i(s)) c_2^i(s) \leq 2R(1 - y) + 2y - \sum_{i=A,B} \omega^i(s) c_1^i(s) \quad \forall s. \end{aligned}$$

From the optimality conditions for consumption we get

$$u'(c_t^A(s)) = u'(c_t^B(s)), \quad \forall s, \forall t,$$

which imply, by strict concavity of  $u$ , that  $c_t^A(s) = c_t^B(s)$ . Denoting the equalized consumption levels with  $c_t(s)$  we can then rewrite the problem as

$$\begin{aligned} \max_{y, (c_t(s))_{t=1,2}} \quad & \sum_{s \in S} \pi(s) [\Omega(s) u(c_1(s)) + (1 - \Omega(s)) u(c_2(s))] \\ \text{s.t.} \quad & \Omega(s) c_1(s) \leq y \quad \forall s, \\ & (1 - \Omega(s)) c_2(s) \leq R(1 - y) + y - \Omega(s) c_1(s) \quad \forall s. \end{aligned}$$

The equilibrium prices can be derived from the Lagrange multipliers of the planner problem. Contingent credit lines that support this allocation are given by:

$$m_1(s) = (\omega(s) - \Omega(s)) c_1(s),$$

and

$$m_2(s) = -(\omega(s) - \Omega(s)) c_2(s).$$

Symmetry implies that the cost of the credit lines bought and sold by a given region are equal, so the budget constraint of each regional bank at date 0 is satisfied. ■

**Proof of Proposition 2.** Consider the program under integration (3). Optimal liquidity under integration  $y^{Int}$  solves

$$\max_y pV(y, \omega_M) + (1 - p) \left[ \frac{1}{2} V(y, \omega_H) + \frac{1}{2} V(y, \omega_L) \right]. \quad (20)$$

Using Lemma 4 the first order condition for this problem can be written as

$$p [u' (C_1 (y^{Int}, \omega_M)) - Ru' (C_2 (y^{Int}, \omega_M))] + \sum_{\omega=\omega_L, \omega_H} \frac{1-p}{2} [u' (C_1 (y^{Int}, \omega)) - Ru' (C_2 (y^{Int}, \omega))] = 0. \quad (21)$$

The Inada condition for  $u(\cdot)$  ensures that we have an interior solution. Notice that, by an argument similar to that used in Proposition 1, rollover cannot be optimal in state  $HH$ . Notice also that, if rollover is optimal in state  $HL$  and  $LH$  then it is also optimal in  $LL$ . Hence, we distinguish three cases. To save notation, we use the label  $M$  to denote the states  $HL$  and  $LH$ , since they have the same allocation.

(i) Rollover in states  $LL$ ,  $HL$  and  $LH$ . Notice that, since

$$c_1(M) = c_1(LL) = c_2(LL) = c_2(M) = y^{Int} + R(1 - y^{Int}),$$

$c_1(HH) = c_2(HH)$  is incompatible with (21), and we must have  $c_1(HH) < c_2(HH)$ . Lemma 5 then yields the desired inequalities.

(ii) Rollover only in state  $LL$ . From Lemma 5 we have

$$c_1(M) = \frac{y^{Int}}{\omega_M} \leq y^{Int} + R(1 - y^{Int}) \leq R \frac{1 - y^{Int}}{1 - \omega_M} = c_2(M)$$

and, since  $\omega_H > \omega_M$ , the desired inequalities follow.

(iii) No rollover. Applying Lemma 5 and  $\omega_H > \omega_M > \omega_L$  yields the desired inequalities.

■

**Proof of Proposition 3.** The proof is in two steps.

*Step 1.* First, we show that there is a cutoff  $\hat{R} > 1$  such that if  $R < \hat{R}$ , then rollover occurs in all states except  $HH$ . Consider the optimality condition (21). Substituting for  $C_t(y, \omega)$  from Lemma 5, some algebra then shows that as  $R \rightarrow 1$  (from above) we have  $y^{Int} \rightarrow \omega_H$  (from below) and  $C_t(y^{Int}, \omega) \rightarrow 1$  for all  $t$  and all  $\omega$ . That is, as the return on the long asset approaches 1, the optimal solution is to hold the minimal amount of liquidity that ensures that all consumers consume 1 in all states of the world. At the limit, the liquidity constraints  $\omega c_1 < y$  are slack for  $\omega \in \{\omega_L, \omega_M\}$  since  $\omega_M \cdot 1 < \omega_H = y^{Int}$  and  $\omega_L \cdot 1 < \omega_H = y^{Int}$ . A continuity argument then shows that the same constraints must be slack for  $\omega \in \{\omega_L, \omega_M\}$ , for all  $R$  below some cutoff  $\hat{R}$ .

*Step 2.* Consider an equilibrium under integration where rollover is positive in all states except  $HH$ . In this case the optimal liquidity  $y^{Int}$  satisfies the first order condition

$$\frac{1-p}{2} (u' (C_1 (y^{Int}, \omega_H)) - Ru' (C_2 (y^{Int}, \omega_H))) + \frac{1+p}{2} (1-R) u' (y^{Int} + R(1 - y^{Int})) = 0.$$

Now,  $R > 1$  implies  $u'(C_1(y^{Int}, \omega_H)) > Ru'(C_2(y^{Int}, \omega_H))$ . Therefore, we obtain

$$\frac{1}{2}(u'(C_1(y^{Int}, \omega_H)) - Ru'(C_2(y^{Int}, \omega_H))) + \frac{1}{2}(1 - R)u'(y^{Int} + R(1 - y^{Int})) > 0.$$

Since  $C_1(y^{Int}, \omega_L) = C_2(y^{Int}, \omega_L) = y^{Int} + R(1 - y^{Int})$ , this condition can be rewritten, using Lemma 4, as

$$\frac{1}{2} \frac{\partial V(y^{Int}, \omega_H)}{\partial y} + \frac{1}{2} \frac{\partial V(y^{Int}, \omega_L)}{\partial y} > 0. \quad (22)$$

Now consider the autarky problem written in the form (18). Since the problem is concave condition (22) implies that the optimal solution  $y^{Aut}$  must be to the right of  $y^{Int}$ .

*General distribution.* With a general discrete distribution with  $N$  possible realizations (in increasing order)  $\{\omega_1, \omega_2, \dots, \omega_N\}$ , the same arguments for Step 1 apply, since we can choose  $\hat{R}$  such that the liquidity constraint is slack in all states except in the state  $s = NN$  in which both regions have the highest shock  $\omega_N$ . Moving to Step 2, the first order condition will be

$$\begin{aligned} \pi(NN)(u'(C_1(y^{Int}, \omega_N)) - Ru'(C_2(y^{Int}, \omega_N))) \\ + (1 - \pi(NN))(1 - R)u'(y^{Int} + R(1 - y^{Int})) = 0, \end{aligned}$$

which implies

$$\begin{aligned} \pi(N)(u'(C_1(y^{Int}, \omega_N)) - Ru'(C_2(y^{Int}, \omega_N))) \\ + (1 - \pi(N))(1 - R)u'(y^{Int} + R(1 - y^{Int})) > 0, \end{aligned}$$

where  $\pi(N)$  denotes the marginal probability of  $\omega^A = \omega_N$  and satisfies  $\pi(N) > \pi(NN)$  for any joint distribution such that  $\pi(Nn) > 0$  for some  $n < N$ . The last condition corresponds to

$$\sum_{s \in S} \pi(s) \frac{\partial V(y^{Int}, \omega^A(s))}{\partial y} > 0,$$

and the usual concavity argument completes the proof. ■

**Proof of Proposition 4.** The convexity assumption implies, for all  $s$ ,

$$\frac{\partial V(y^{Aut}, \Omega(s))}{\partial y} < \frac{1}{2} \frac{\partial V(y^{Aut}, \omega^A(s))}{\partial y} + \frac{1}{2} \frac{\partial V(y^{Aut}, \omega^B(s))}{\partial y}.$$

Taking expectations on both sides, using symmetry and the optimality of  $y^{Aut}$  under autarky, implies  $E[\partial V(y^{Aut}, \Omega(s)) / \partial y] < 0 = E[\partial V(y^{Int}, \Omega(s)) / \partial y]$ . Concavity of the optimization problem implies  $y^{Aut} > y^{Int}$ . ■

**Proof of Proposition 5.** Suppose  $y_0^{Int} \equiv y^{Int}(p_0)$  is optimal under integration for  $p = p_0$ . If  $y_0^{Int} < y^{Aut}$  the optimality condition in autarky and the strict concavity of  $V$  imply that

$$\frac{1}{2} \frac{\partial V(y_0^{Int}, \omega_H)}{\partial y} + \frac{1}{2} \frac{\partial V(y_0^{Int}, \omega_L)}{\partial y} > 0.$$

This inequality, combined with (5), optimality under integration, implies that

$$\frac{\partial V(y_0^{Int}, \omega_M)}{\partial y} < 0.$$

Now consider an integrated economy with any  $p > p_0$ . The previous inequalities together with (5) imply that

$$\begin{aligned} p \frac{\partial V(y_0^{Int}, \omega_M)}{\partial y} + (1-p) \left( \frac{1}{2} \frac{\partial V(y_0^{Int}, \omega_H)}{\partial y} + \frac{1}{2} \frac{\partial V(y_0^{Int}, \omega_L)}{\partial y} \right) &< \\ p_0 \frac{\partial V(y_0^{Int}, \omega_M)}{\partial y} + (1-p_0) \left( \frac{1}{2} \frac{\partial V(y_0^{Int}, \omega_H)}{\partial y} + \frac{1}{2} \frac{\partial V(y_0^{Int}, \omega_L)}{\partial y} \right) &= 0. \end{aligned}$$

The concavity of  $V$  then implies that  $y^{Int}(p) < y^{Int}(p_0)$ , which immediately implies that  $y^{Int}(p) < y^{Aut}$ . It remains to show that  $y^{Int}(p) < y^{Aut}$  and that  $y^{Int}(p)$  is decreasing in  $p$  for  $p < p_0$ . Now suppose, by contradiction that for some  $0 < p' < p_0$  we have  $y^{Int}(p') \geq y^{Aut}$ . Then an argument symmetric to the one above shows that  $y^{Int}(p)$  must be non-decreasing in  $p$  for all  $p > p'$ . This implies  $y^{Int}(p_0) \geq y^{Int}(p') > y^{Aut}$ , a contradiction. It follows that  $y^{Int}(p) < y^{Aut}$  for all  $p \in (0, 1]$ . The argument used above starting at  $p_0$  then applies to all points in  $(0, 1]$ , so  $y^{Int}(p)$  is everywhere decreasing in  $p$ . ■

**Proof of Proposition 6.** As in the proof of Proposition 3, the condition  $R < \hat{R}$  implies that, under integration, there is positive rollover and the interest rate is 0 in all states except  $HH$ . In autarky, there is positive rollover in states  $HL$  and  $HH$ . This immediately implies that  $r^{Int}(HL) = 0 < r^{Aut}(HL)$ . To complete the proof it remains to show that  $r^{Int}(HH) > r^{Aut}(HH)$ . This follows immediately from the fact that  $y^{Int} < y^{Aut}$ , from Proposition 3, and that with no rollover the interest rates are given by

$$1 + r^{Int}(HH) = \left( \frac{R(1 - y^{Int}) / (1 - \omega_H)}{y^{Int} / \omega_H} \right)^\gamma > \left( \frac{R(1 - y^{Aut}) / (1 - \omega_H)}{y^{Aut} / \omega_H} \right)^\gamma = 1 + r^{Aut}(HH).$$

■

**Proof of Proposition 7.** To complete the argument in the text we need to prove that

$$\frac{d}{dp} (1 - p^2) (f(p))^2 = -2p (f(p))^2 + 2(1 - p^2) f(p) f'(p) > 0,$$

at  $p = 0$ . That  $f(0) > 0$  follows from the fact that the liquidity constraint is binding in state  $HH$ . Therefore, we need to show  $f'(0) > 0$ . Given that the liquidity constraint is binding in state  $HH$  we have

$$r(HH) = \left( \frac{R(1 - y) / (1 - \omega_H)}{y / \omega_H} \right)^\gamma, \quad (23)$$

which is decreasing in  $y$ . Therefore, to prove that this expression is increasing in  $p$ , we need to prove that  $y$  is decreasing in  $p$ . Since the liquidity constraint is only binding in  $HH$ , differentiating

the first order condition (21) yields

$$\frac{dy}{dp} = \frac{u' \left( \frac{y}{\omega_H} \right) - Ru' \left( R \frac{1-y}{1-\omega_H} \right) - (1-R) u' (y + R(1-y))}{\frac{1-p}{\omega_H} u'' \left( \frac{y}{\omega_H} \right) + \frac{1-p}{1-\omega_H} R^2 u'' \left( R \frac{1-y}{1-\omega_H} \right) + (1-R)^2 (1+p) u'' (y + R(1-y))}.$$

The expression at the denominator is negative. The expression at the numerator at  $p = 0$  is

$$u' \left( \frac{y}{\omega_H} \right) - Ru' \left( R \frac{1-y}{1-\omega_H} \right) - (1-R) u' (y + R(1-y)) = 2(R-1) u' (y + R(1-y)) > 0,$$

where the equality follows from the first-order condition (21) at  $p = 0$ . ■

**Proof of Lemma 3.** The third moment around the mean is

$$\frac{p(1-p)(r(LL) + r(HH) - 2r(M))}{8} \times [(2p-1)(r(LL) + r(HH) - 2r(M))^2 + 3(r(HH) - r(LL))^2].$$

We want to show that this quantity has the same sign as  $r(LL) + r(HH) - 2r(M)$ . To this end, we need to show that the following expression is positive:

$$(2p-1)(r(LL) + r(HH) - 2r(M))^2 + 3(r(HH) - r(LL))^2.$$

This can be proved observing that this expression is increasing in  $p$  and is positive at  $p = 0$ . To prove the last claim notice that  $r(LL) \leq r(M)$  implies  $r(HH) - 2r(LL) \geq r(HH) - 2r(M)$  which implies  $r(HH) - r(LL) \geq r(HH) + r(LL) - 2r(M)$  that, together with  $r(LL) < r(HH)$ , implies

$$3(r(HH) - r(LL))^2 - (r(LL) + r(HH) - 2r(M))^2 > 0.$$

■

**Proof of Proposition 8.** From Proposition 2 three cases are possible. In the first case, rollover occurs in states  $LL$ ,  $LH$  and  $HL$  and  $r(HH) + r(LL) - 2r(M) = r(HH) - 1 > 0$ . In the second case, rollover only occurs in state  $LL$  and we have  $r(LL) = 1$  and

$$r(HH) = \frac{R(1-y^{Int})}{y^{Int}} \frac{\omega_H}{1-\omega_H}, \quad r(M) = \frac{R(1-y^{Int})}{y^{Int}} \frac{\omega_M}{1-\omega_M}.$$

Then

$$r(HH) + r(LL) - 2r(M) \geq \frac{R(1-y^{Int})}{y^{Int}} \left( \frac{\omega_H}{1-\omega_H} + \frac{\omega_L}{1-\omega_L} - 2 \frac{\omega_M}{1-\omega_M} \right) > 0,$$

where the inequality follows from the convexity of the function  $\omega/(1-\omega)$  and the fact that  $\omega_M = (1/2)\omega_H + (1/2)\omega_L$ . A similar argument applies in the third case, in which rollover never occurs. ■



**Proof of Proposition 9.** First, notice that Lemma 3 applies to any random variable taking three values  $x_1 < x_2 < x_3$  with probabilities  $(1-p)/2, p, (1-p)/2$ . So the distribution of consumption is negatively skewed iff

$$c(HH) + c(LL) - 2c(M) < 0. \quad (24)$$

From Lemma 5, we have positive rollover in states  $LL$ ,  $HL$  and  $LH$  if

$$y^{Int} > \frac{\omega_M R}{1 + R(1 - \omega_M)}.$$

Since in this case  $c(HH) < c(LL) = c(M)$  and condition (24) is immediately satisfied. If instead

$$\frac{\omega_L R}{1 + R(1 - \omega_L)} < y^{Int} \leq \frac{\omega_M R}{1 + R(1 - \omega_M)}$$

positive rollover is optimal in state  $LL$  but not in states  $HL$  and  $LH$  and  $HH$ , so

$$c_1(LL) = (1 - y^{Int})R + y^{Int}, \quad c_1(M) = y^{Int}/\omega_M, \quad c_1(HH) = y^{Int}/\omega_H.$$

Substituting in (24) yields

$$(1 - y^{Int})R + y^{Int} + y^{Int}/\omega_H - 2y^{Int}/\omega_M < 0,$$

and rearranging gives  $y^{Int} > \hat{y}$  (with  $\hat{y}$  defined in the statement of the proposition). Finally, if

$$y^{Int} < \frac{\omega_L R}{1 + R(1 - \omega_L)}$$

then rollover is never optimal and we have

$$c_1(LL) = y^{Int}/\omega_L, \quad c_1(M) = y^{Int}/\omega_M, \quad c_1(HH) = y^{Int}/\omega_H.$$

In this case consumption is always positively skewed, because the convexity of the function  $1/\omega$  implies

$$1/\omega_L + 1/\omega_H > 2/\omega_M.$$

Combining the three cases discussed above and noticing that

$$\hat{y} \in \left( \frac{\omega_L R}{1 + R(1 - \omega_L)}, \frac{\omega_M R}{1 + R(1 - \omega_M)} \right),$$

shows that  $y^{Int} > \hat{y}$  is a necessary and sufficient condition for negative skewness of first-period consumption. ■

**Proof of Proposition 10.** Take any  $y \in (0, 1)$  and let  $(c_1^H, c_2^H)$  and  $(c_1^L, c_2^L)$  be optimal consumption allocations for the problem in (1) with, respectively,  $\omega = \omega_H$  and  $\omega = \omega_L$ . Inspecting

the constraints shows that the following are feasible consumption allocations for the same problem with  $\omega = \omega_M$ :

$$\begin{aligned}\hat{c}_1 &= \frac{\omega_H}{\omega_H + \omega_L} c_1^H + \frac{\omega_L}{\omega_H + \omega_L} c_1^L, \\ \hat{c}_2 &= \frac{1 - \omega_H}{2 - \omega_H - \omega_L} c_2^H + \frac{1 - \omega_L}{2 - \omega_H - \omega_L} c_2^L.\end{aligned}$$

The strict concavity of  $u(\cdot)$  implies that

$$\begin{aligned}\omega_M u(\hat{c}_1) + (1 - \omega_M) u(\hat{c}_2) &\geq \\ &\geq \omega_M \left[ \frac{\omega_H}{\omega_H + \omega_L} u(c_1^H) + \frac{\omega_L}{\omega_H + \omega_L} u(c_1^L) \right] + \\ (1 - \omega_M) &\left[ \frac{1 - \omega_H}{2 - \omega_H - \omega_L} u(c_2^H) + \frac{1 - \omega_L}{2 - \omega_H - \omega_L} u(c_2^L) \right] \\ &= \frac{1}{2} V(y, \omega_H) + \frac{1}{2} V(y, \omega_L),\end{aligned}$$

where the inequality holds strictly if  $c_1^H \neq c_1^L$ . Since  $(\hat{c}_1, \hat{c}_2)$  is feasible, we further have

$$V(y, \omega_M) \geq \omega_M u(\hat{c}_1) + (1 - \omega_M) u(\hat{c}_2).$$

We conclude that

$$V(y, \omega_M) \geq \frac{1}{2} V(y, \omega_H) + \frac{1}{2} V(y, \omega_L),$$

with strict inequality if the optimal consumption levels for the problems associated to  $V(y, \omega_H)$  and  $V(y, \omega_L)$  satisfy  $c_1^H \neq c_1^L$ .

For any  $p \in [0, 1]$  let  $W(p)$  denote the expected utility of consumers:

$$W(p) = \max_y pV(y, \omega_M) + (1 - p) \left( \frac{1}{2} V(y, \omega_H) + \frac{1}{2} V(y, \omega_L) \right).$$

The envelope theorem implies

$$W'(p) = V(y^{Int}(p), \omega_M) - \frac{1}{2} V(y^{Int}(p), \omega_H) - \frac{1}{2} V(y^{Int}(p), \omega_L). \quad (25)$$

Moreover, Proposition 2 (and Proposition 1 for the case  $p = 0$ ) show that the solutions to the problems associated to  $V(y^{Int}(p), \omega_H)$  and  $V(y^{Int}(p), \omega_L)$  yield  $c_1^H < c_1^L$ . We conclude that the expression on the right-hand side of (25) is strictly positive, so  $W'(p) > 0$  for all  $p \in [0, 1]$ . Since  $W(0)$  corresponds to the expected utility in autarky, this proves both statements in the proposition. ■

### Proof of Proposition 11.

Standard derivations show that with spot markets and CRRA preferences consumption in period 1, for a given preference shock  $\omega$  and interest rate  $r > 0$  is

$$c_1 = \frac{(1 + r)y + R(1 - y)}{\omega(1 + r) + (1 - \omega)(1 + r)^{\frac{1}{\gamma}}}.$$

Differentiating, gives

$$\frac{dc_1}{d\omega} = c_1 \frac{(1+r)^{\frac{1}{\gamma}} - (1+r)}{\omega(1+r) + (1-\omega)(1+r)^{\frac{1}{\gamma}}},$$

which is negative if  $\gamma > 1$ , since  $r > 0$ . Moreover, it is easy to show that  $d(\omega c_1)/d\omega > 0$  independently of  $\gamma$ , as

$$\frac{dc_1}{d\omega} \frac{\omega}{c_1} > -1.$$

Therefore, when  $\gamma > 1$  the region with the higher level of  $\omega$  must be the borrowing region as it has the higher level of  $\omega c_1$  and by market clearing only one region can be a net lender. Moreover, the region with the higher level of  $\omega$  has the lower  $c_1$  and thus the higher marginal utility  $u'(c_1)$ . Therefore, the pecuniary externality in (16) is negative, as  $\partial r/\partial y < 0$ . ■

# Online Appendix for Castiglionesi, Feriozzi, and Lorenzoni (2017)

## A. Integration and volatility with fixed $y$

With CRRA preferences, for a given level of liquidity  $y$  the interest rate as a function of the liquidity shock  $\omega$  is

$$r = \frac{u'(c_1)}{u'(c_2)} - 1 = h(\omega)$$

where the function  $h$  is defined as follows

$$h(\omega) \equiv \max\left\{\left(\frac{1-\omega}{\omega}\right)^{-\gamma} \left(\frac{y}{R(1-y)}\right)^{-\gamma} - 1, 0\right\}$$

**Lemma 6** *If  $\gamma \geq 1$  the function  $h$  is convex in  $\omega$*

**Proof.** Notice that

$$\left(\frac{1-\omega}{\omega}\right)^{-\gamma} = \left(\frac{1}{1-\omega} - 1\right)^{\gamma}$$

is convex and so is the constant function. The maximum of two convex functions is convex. ■

What happens to the volatility of  $r$  if two regions integrate but keep the level of  $y$  unchanged? With integration every pair of realizations  $\omega^A$  and  $\omega^B$  in the two regions is replaced by

$$\Omega = \frac{1}{2}\omega^A + \frac{1}{2}\omega^B.$$

So, assuming a symmetric joint distribution of the shocks, going from integration to autarky is the same as having a mean preserving spread of the distribution of  $\omega$  and the question we want to address in general is what happens to the moments of  $h(\omega)$ ? The following proposition provides a sufficient condition on the curvature of  $h$  that ensures that the volatility of interest rates decreases with integration.

**Proposition 12** *Suppose  $\gamma \geq 1$  and the distribution of  $\omega$  in autarky satisfies*

$$(h(\omega) - E[h(\Omega)])h''(\omega) + h'(\omega)^2 > 0$$

*for all  $\omega$  in the support of the autarky distribution. Then if the regions integrate and the choice of  $y$  is kept fixed, interest rate volatility decreases.*

**Proof.** Define a parametric family of distributions for  $\omega$  with CDF

$$F(\omega, \alpha) = \alpha F_{Aut}(\omega) + (1 - \alpha) F_{Int}(\omega),$$

for  $\alpha \in [0, 1]$ , where  $F_{Aut}$  is the distribution under autarky and  $F_{Int}$  is the distribution under integration. Let  $[a, b]$  denote the support of the distribution of  $\omega$ . Define the first and second moment of the interest rate distribution as functions of  $\alpha$

$$M_1(\alpha) = \int_a^b h(\omega) f(\omega, \alpha) d\omega,$$

$$M_2(\alpha) = \int_a^b (h(\omega))^2 f(\omega, \alpha) d\omega.$$

Integrating by parts and using the fact that all distributions  $F(\omega, \alpha)$  have the same mean, gives the following two results

$$\frac{d(M_1(\alpha))^2}{d\alpha} = 2M_1(\alpha) \int_a^b h''(\omega) \left( \int_a^\omega \frac{\partial F(s, \alpha)}{\partial \alpha} ds \right) d\omega$$

and

$$M_2'(\alpha) = \int_a^b \left[ 2h(\omega)h''(\omega) + 2(h'(\omega))^2 \right] \left( \int_a^\omega \frac{\partial F(s, \alpha)}{\partial \alpha} ds \right) d\omega.$$

Since the variance of the interest rate is  $M_2 - M_1^2$ , the variance is increasing in  $\alpha$  if

$$\int_a^b \left[ [h(\omega) - M_1(\alpha)]h''(\omega) + (h'(\omega))^2 \right] \left( \int_a^\omega \frac{\partial F(s, \alpha)}{\partial \alpha} ds \right) d\omega > 0. \quad (26)$$

Since  $F_{Aut}$  is a mean preserving spread of  $F_{Int}$ , we have

$$\int_a^\omega \frac{\partial F(s, \alpha)}{\partial \alpha} ds = \int_a^\omega (F_{Aut}(s) - F_{Int}(s)) ds > 0 \text{ for all } \omega.$$

Moreover  $M_1(1) > M_1(\alpha)$ , because  $h$  is a convex function when  $\gamma \geq 1$ . So the condition in statement of the proposition is sufficient to ensure that  $[h(\omega) - M_1(\alpha)]h''(\omega) + (h'(\omega))^2 > 0$  for all  $\omega$  in the support of the autarky distribution. This completes the argument for inequality (26) and completes the proof. ■

## B. More on consumption skewness

To illustrate numerically when condition (12) is satisfied we assume again a CRRA utility with relative risk aversion  $\gamma$ . In all our examples, the condition holds when  $\gamma$  is above some cutoff, irrespective of the value of  $p$ . Table 1 shows this threshold for  $\gamma$  in some numerical examples. It is clear that for  $R \leq 3$  a value of  $\gamma > 3$  is sufficient to obtain negative skewness. Table 2 shows the standard deviation and skewness of consumption in various examples where  $R = 1.4$ ,  $\omega_L = 0.35$ , and  $\omega_H = 0.65$ .

## C. Alternative shock distributions

Consider a version of the model of financial integration presented in Section 2 where regional liquidity shocks are identically distributed, continuous random variables with expected value  $\mu$ . Let  $\sigma^2$  denote their common variance and  $\rho = cov(\omega^A, \omega^B)/\sigma^2$  their linear correlation. In autarky, the optimal level of liquidity in region  $i = A, B$  maximizes  $E[V(y, \omega^i)]$ , where the expectation is taken with respect to the regional shock  $\omega^i$ . Let  $\Omega = (\omega^A + \omega^B)/2$  denote the average liquidity shock in the two-region economy. The argument behind Lemma 2 can be extended to show that,

Table 1: Cutoffs for  $\gamma$ 

$R$	$\omega_L = 0.35$	$\omega_L = 0.1$
	$\omega_H = 0.65$	$\omega_H = 0.9$
1.1	0.25	0.15
1.2	0.48	0.28
1.4	0.88	0.52
1.8	1.56	0.92
2.2	2.07	1.24
2.6	2.52	1.50
3.0	2.89	1.72

Table 2: Standard deviation and skewness of first-period consumption

$p$	$\gamma = 4$		$\gamma = 0.75$		$\gamma = 0.5$	
	Std Dev	Skew	Std Dev	Skew	Std Dev	Skew
0	0.036	0	0.204	0	0.275	0
0.1	0.040	-0.201	0.200	-0.032	0.262	0.091
0.2	0.045	-0.408	0.194	-0.040	0.248	0.192
0.3	0.050	-0.629	0.186	-0.028	0.233	0.306
0.4	0.058	-0.873	0.175	0.002	0.217	0.438
0.5	0.066	-0.155	0.162	0.053	0.199	0.596
0.6	0.077	-1.500	0.147	0.128	0.178	0.794
0.7	0.089	-1.960	0.129	0.237	0.155	1.062
0.8	0.080	-2.653	0.107	0.411	0.127	1.477
0.9	0.058	-4.023	0.076	0.763	0.090	2.334
1	0	0	0	0	0	0

under integration, the optimal level of liquidity in each region maximizes  $E[V(y, \Omega)]$ , where the expectation is taken with respect to  $\Omega$ .

Using the law of iterated expectations, the difference between the marginal value of liquidity under integration and in autarky in region  $i$  can be written as

$$E[\partial V(y, \Omega)/\partial y - \partial V(y, \omega^i)/\partial y \mid \omega^i > \Omega] \Pr(\omega^i > \Omega) \\ + E[\partial V(y, \Omega)/\partial y - \partial V(y, \omega^i)/\partial y \mid \omega^i < \Omega] \Pr(\omega^i < \Omega).$$

Given the monotonicity properties of  $\partial V(y, \omega)/\partial y$ , the first term is non negative while the second is non positive. This expression captures in a more general setup the opposing effects of integration on banks' optimal liquidity holding that are discussed in Section 4 for the case of binary shocks.

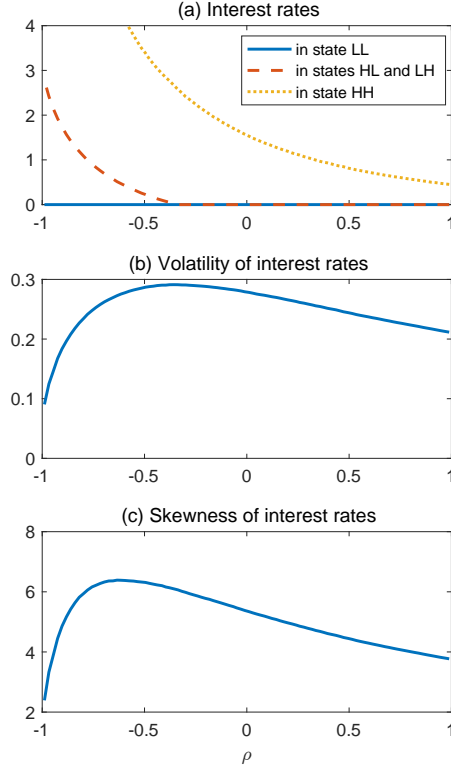


Figure 1: Interest rates.

Notice that  $\Omega$  has the same expected value as the regional shocks, but its variance is  $(1 + \rho)\sigma^2/2$ . This variance can be taken as a measure of aggregate liquidity uncertainty and declines as  $\rho$  approaches  $-1$ . In the extreme situation where  $\rho = -1$ , the average shock  $\Omega$  is equal to  $\mu$  with probability 1, in which case consumers can be completely insured against liquidity shocks. If instead  $\rho = 1$ , the regional shocks are identical (i.e.,  $\omega^A = \omega^B$ ) with probability 1, and integration offers no coinsurance possibility.<sup>1</sup> In this case,  $\Omega$  is distributed as the regional shocks, which means that banks's behavior in case of financial integration is the same as in autarky.

We now turn to numerical examples in which the continuous joint distribution of regional liquidity shocks is parametrized as follows. Let  $(U^A, U^B)$  be two random variables distributed according to a bivariate Gaussian copula. Define

$$(X^A, X^B) = (H^{-1}(U^A, \alpha, \beta), H^{-1}(U^B, \alpha, \beta)),$$

where  $H^{-1}(\cdot, \alpha, \beta)$  is the inverse of the cumulative distribution function of a beta random variable with shape parameters  $\alpha$  and  $\beta$ . It follows that both  $X^A$  and  $X^B$  are identically-distributed, beta

<sup>1</sup>To see this, notice that a linear correlation of 1 means that there exists two real numbers  $a$  and  $b$  such that  $b > 0$  and  $\omega^A = a + b\omega^B$ . However,  $\omega^A$  and  $\omega^B$  are identically distributed with, in particular, identical mean and variance. Hence,  $\mu = a + b\mu$  and  $\sigma^2 = b^2\sigma^2$ , which imply that  $a = 0$  and  $b = 1$ .

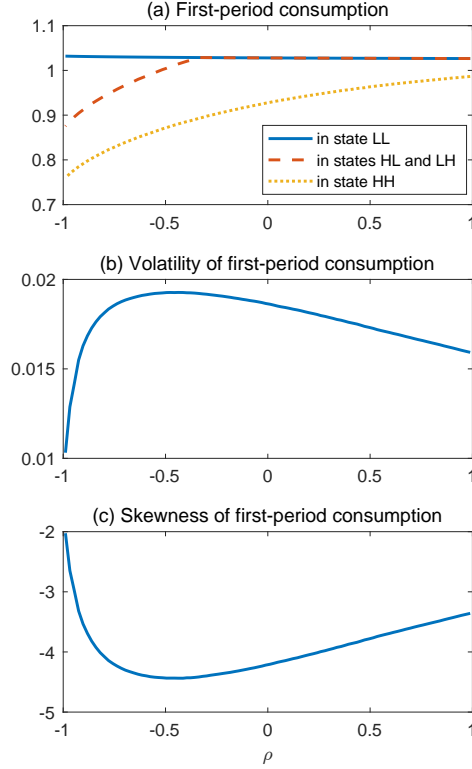


Figure 2: First-period consumption.

random variables. Let  $\rho = \text{corr}(X^A, X^B)$  denote their linear correlation. The liquidity shock in region  $i$  is then defined as  $\omega^i = \omega_L + (\omega_H - \omega_L)X^i$ , with  $0 \leq \omega_L < \omega_H \leq 1$ . In this way shocks are restricted to the range  $[\omega_L, \omega_H]$  and maintain a linear correlation of  $\rho$ . The numerical example assumes  $\alpha = \beta = 1.1$ ,  $\omega_L = 0.2$  and  $\omega_H = 0.5$ . This implies that, in each region, the liquidity shock has a symmetric distribution with an inverted-U shape on the support  $[0.2, 0.5]$ , a mean of 0.35 and a standard deviation of 0.084. The utility function is a CRRA with relative risk aversion of 5 and, finally, the return on the illiquid asset is  $R = 1.05$ .<sup>2</sup>

In order to facilitate the comparison of this example with that presented in the case of binary shocks, in Figure 1 we call “low” a liquidity shock that is equal to the mean ( $\omega = 0.35$ ), and we call “high” a shock that is 1.5 standard deviations above the mean ( $\omega = 0.474$ ). Panel (a)

<sup>2</sup>The numerical example is obtained as follows. First, we generate a large number of random draws  $(U^A, U^B)$  from a bivariate Gaussian copula distribution with assigned linear correlation. We then apply the transformation  $(X^A, X^B) = (F^{-1}(U^A, \alpha, \beta), F^{-1}(U^B, \alpha, \beta))$  and compute  $\rho = \text{corr}(X^A, X^B)$ . The simulated liquidity shocks  $(\omega^A, \omega^B)$  and their average  $\Omega$  are then obtained as described in the text. Based on the simulated realizations of  $\Omega$  we can finally estimate its probability distribution, which is then used to solve numerically for the level of liquidity that maximizes  $E[V(y, \Omega)]$ . Other quantities, such as interest rates and consumption levels, are obtained starting from the calculated optimal level of liquidity.



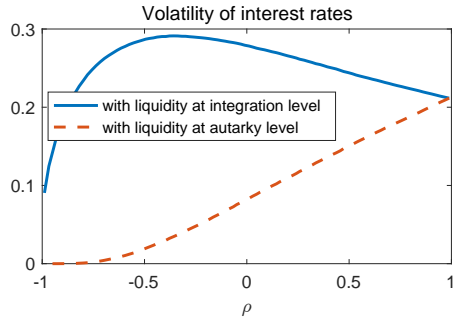


Figure 3: Decomposing the volatility of interest rates.

of Figure 1 plots the interest rate  $r$  in three alternative cases: i) both regions are hit by low liquidity shocks ( $LL$ ); ii) the liquidity shock is high in one region and low in the other ( $LH, HL$ ); and iii) both regional shocks are high ( $HH$ ). It is useful to recall that the scope for integration increases as  $\rho$  moves away from 1 and towards  $-1$ , at which point aggregate liquidity uncertainty disappears. When  $\rho = 1$  the two regions have identical liquidity shocks and the (local) interest rate in autarky is the one for case  $HH$  or  $LL$ , depending on whether the local shock is high or low. In Panel (a) of Figure 1 we can recognize the contrasting effects of financial integration. Consider for example what happens upon integration when  $\rho = 0$ . If a region has high liquidity needs, the interest rate under financial integration is lower than in autarky if in the other region liquidity needs are low (stabilizing effect). However, if also the other region has high liquidity needs, the interest rate is larger under integration than in autarky (destabilizing effect).<sup>3</sup> Panels (b) and (c) respectively plot the standard deviation and the skewness of interest rates for different values of  $\rho$ . The picture that emerges is very similar to that in Figure 5: due to its contrasting effects, financial integration can make interest rates and consumption levels more volatile and more skewed also in the case of continuous shocks. This happens for a relatively small value of  $R$  and a large value of  $\gamma$  in the example in Figure 1. However, provided that  $\rho$  is close enough to 1 and other parameters are held constant, it is sufficient that  $R$  does not exceed 1.3 or  $\gamma$  is above 0.8 for integration to increase the volatility of interest rates. On the other hand, provided again that  $\rho$  is close enough to one and given other parameters, the skewness of interest rates increases with integration for all value of  $R$  below 4.5 and for essentially any value of  $\gamma > 0$ . Figure 2 makes a similar point for consumption, whose standard deviation and (negative) skewness also increase under integration for values of  $\rho$  that are close enough to 1.

Finally, Figure 3 decomposes the effects of integration on interest rate volatility in two steps. First, the dashed red line shows what would happen if liquidity did not change upon integration. Interest rate volatility would necessarily decline in this case because, with fixed liquidity, only the stabilizing effect of integration would be at work. Second, the solid blue line shows what

<sup>3</sup>These effects could also be highlighted with alternative choices of the high and low shocks, as long as the high shock is large enough to ensure that the liquidity constraint is binding when it hits both regions.

happens when banks adjust their liquidity holdings. In this case, the endogenous reduction in banks' liquidity introduces a destabilizing effect of integration, which results in larger interest rate spikes in case of economy-wide liquidity shocks, and can ultimately increase the volatility of interest rate.